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SELECTED METHODS OF REJECTING ARGUMENTS FOR DETERMINISM IN TENSE LOGIC SYSTEMS

Abstract. In this article we will consider the application of tense logics enriched with additional modal operators or additional logical values to construct logical systems in which the thesis of logical determinism cannot be expressed.

Keywords: determinism, tense logic, modal operators.

Branched time tense logic systems are often proposed as a solution to the problem of logical determinism and to reject the argument for determinism. Opponents of such a solution have raised the issue of reducing the tree-like structure to a single branch, the branch along which reality is realized.

The solution to the problem of reducing a tree-like structure to a linear structure – which occurs when choosing the actual future from the tree of possible futures – seems to be the modalization of tensed operators. Robert McArthur argues that one should not choose one branch from the branching temporal structure and call it the actual future. McArthur calls any branch an "accessible future".

1. Modal-temporal operators: $F^{\Diamond}, F^{\Box}, G^{\Diamond}, G^{\Box}$

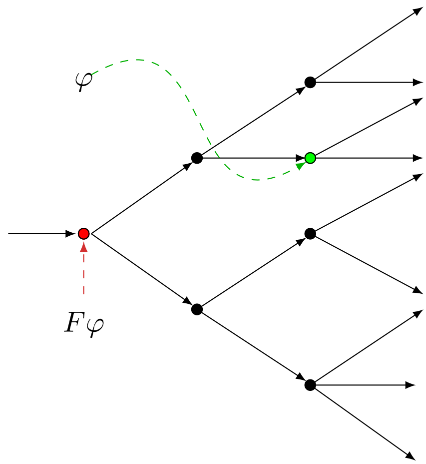
The concept of "actual future" does not appear in temporal logic systems with modal-tense operators. These operators were introduced by McArthur to consider the following question:

How can one distinguish between the actual future in 'tomorrow will be...' and 'tomorrow should be...' and 'tomorrow is possible...'?

In a model where time has a linear structure, the three statements are semantically equivalent. In a model with branching time, there is a hidden modalization of the F operator. Therefore, we must answer the following question:

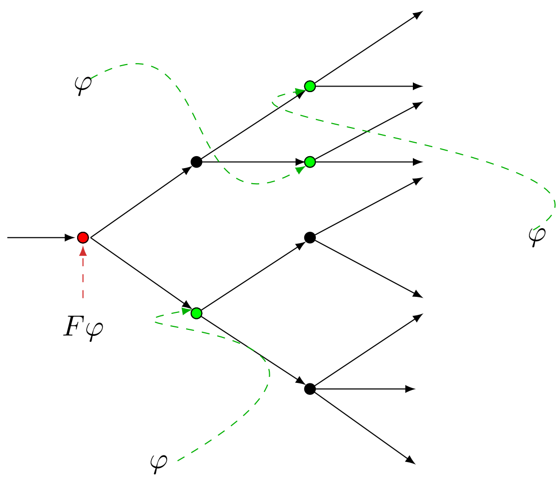
Is it sufficient for the sentence $F\varphi$ to be true at time t in a branching time model for:

1) *φ is true at least one moment later than t belonging to any branch of the branching temporal structure*



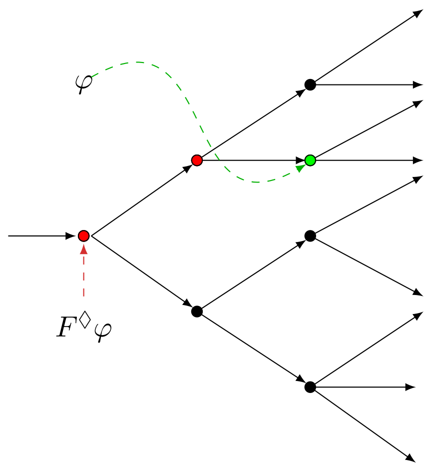
whether

2) *φ is true at least at one moment later than t on every branch of the branching temporal structure?*



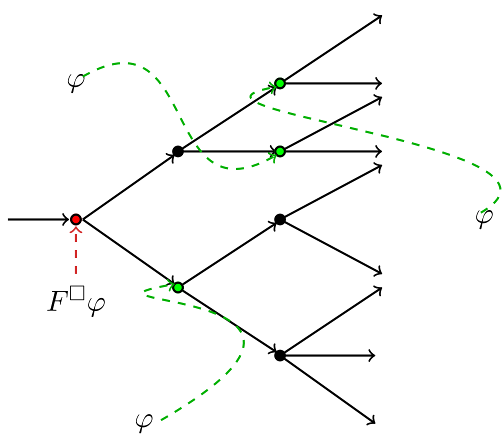
To distinguish between 1) and 2), McArthur introduces modalized counterparts of the F operator, denoted as $F^{\Diamond}$ and $F^{\Box}$. The notation $F^{\Diamond}\varphi$ encodes the expression "possible that φ will be true", while $F^{\Box}\varphi$ encodes the expression "necessary that φ will be true".

A fragment of an example time structure in which $F^{\Diamond}\varphi$ is satisfied at time t is presented in the following diagram:



A necessary condition for the truth of the expression $F^{\Diamond}\varphi$ at time t is the existence of at least one branch at a later time t' than t where φ is true. Therefore, the $F^{\Diamond}$ operator satisfies part 1) of the posed question.

Part 2) of the posed question is satisfied by the $F^{\Box}$ operator. The fragment of the time structure in which the sentence $F^{\Box}\varphi$ is satisfied at time t is shown in the figure:



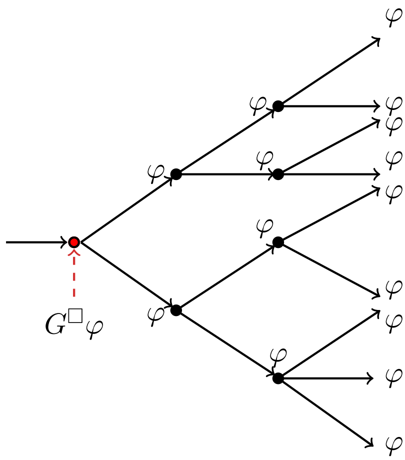
The sentence $F^{\Box}\varphi$ is true at moment t if and only if for every branch that contains t, there exists a moment t' later than t such that φ is true at t'.

The operator $G^{\Box}$ is defined using the operator $F^{\Diamond}$. The following definition is assumed:

Definition 1

$G^{\Box}\varphi \equiv \neg F^{\Diamond}\neg\varphi$.

The sentence $\neg F^{\Diamond}\neg\varphi$ is understood as "it is not true that in the future it is possible for φ to be false". This sentence, in the framework of tense logic built on classical logic, can also be interpreted as "in the future, it is always necessary that φ is true". A fragment of an example temporal structure in which $\neg F^{\Diamond}\neg\varphi$ (i.e., $G^{\Box}\varphi$) is true at moment t is presented in the following diagram:



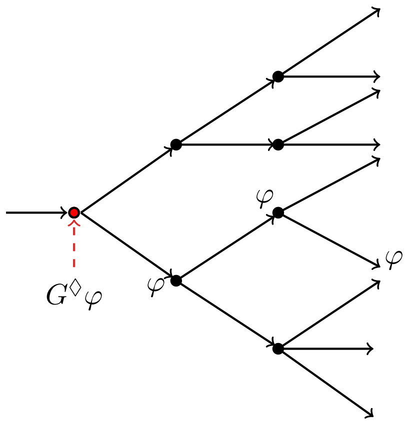
The $G^{\Diamond}$ operator is defined using the $F^{\Box}$ operator by the following definition:

Definition 2

$G^{\Diamond}\varphi \equiv \neg F^{\Box}\neg\varphi$.

$G^{\Diamond}\varphi$ is true at time t if there is a branch in the branching time structure that contains t and in which φ is true at any later time in that branch.

A fragment of an example temporal structure in which $G^{\Diamond}\varphi$ is satisfied at time t is shown in the diagram:



Later in the text, two logics are considered with the presented operators: minimal logic and modal-temporal logic, whose semantics is based on branching time.

1.1. $K_t^{\Box}$ – minimal logic of modal-time operators

The axiomatization of minimal tense logic, called $K_t^{\Box}$, with the operators $G^{\Diamond}$, $G^{\Box}$, $F^{\Box}$, $F^{\Diamond}$, and the operators H and P understood in the standard way, was given by W. A. Smirnov[1]. $K_t^{\Box}$ is the counterpart of the minimal tense logic K_t. This logic is built on classical logic, and its specific axioms are:

1. $G^{\Box}(\varphi \to \psi) \to (G^{\Box}\varphi \to G^{\Box}\psi)$,
2. $H(\varphi \to \psi) \to (H\varphi \to H\psi)$,
3. $\varphi \to G^{\Box}P\varphi$,
4. $\varphi \to HF^{\Diamond}\varphi$,
5. $G^{\Box}(\varphi \to \psi) \to (F^{\Box}\varphi \to F^{\Box}\psi)$.

Rules:

R1: Modus Ponens, R2: $\dfrac{\varphi}{G^{\Box}\varphi}$, R3: $\dfrac{\varphi}{H\varphi}$ R4: $\dfrac{\varphi}{F^{\Box}\varphi}$.

In $K_t^{\Box}$, no conditions are imposed on the temporal structure.

However, for rejecting arguments for determinism, it is important to construct such a logic in which modal temporal operators occur and semantics is based on branching time.

1.2. Logic $K_b^{\Box}$ by J. Burgess

J. Burgess introduced the axiomatization of tense logic with modal-tense operators such that the time structure was left-linear and branching in the future. This logic is denoted by $K_b^\Box$ and was presented in his work *Decidability for branching time* (Studia Logica, Vol. 39, 1980, pp. 203-218).

Semantics of $K_b^\Box$

Definition 3

Let $\mathcal{T} = \langle T, R\rangle$ be a tree. A valuation is a function $V : X \rightarrow 2^T$. Valuation V can be extended to any sentence of the language of the $K_b^\Box$ system. Let B_t be the set of branches containing t. The truth of a sentence φ at moment t $(\in T)$ is defined as follows:

a) $\mathfrak{M}, t \models \varphi$	$\equiv$	$t \in V(\varphi)$, if φ is a propositional letter,
b) $\mathfrak{M}, t \models \neg\varphi$	$\equiv$	it is not true that $\mathfrak{M}, t \models \varphi$,
c) $\mathfrak{M}, t \models \varphi \rightarrow \psi$	$\equiv$	it is not true that $\mathfrak{M}, t \models \varphi$ or $\mathfrak{M}, t \models \psi$,
e) $\mathfrak{M}, t \models H\varphi$	$\equiv$	for any $t' \in T$ if $t'Rt$, then $\mathfrak{M}, t' \models \varphi$,
f) $\mathfrak{M}, t \models G^\Box\varphi$	$\equiv$	for any $t' \in T$ if tRt', then $\mathfrak{M}, t' \models \varphi$,
g) $\mathfrak{M}, t \models F^\Box\varphi$	$\equiv$	for any b $(\in B_t)$ there exists t' $(\in b)$ such that tRt' and $\mathfrak{M}, t' \models \varphi$.

Axioms

1. $G^\Box(\varphi \rightarrow \psi) \rightarrow (G^\Box\varphi \rightarrow G^\Box\psi)$,
2. $H(\varphi \rightarrow \psi) \rightarrow (H\varphi \rightarrow H\psi)$,
3. $G^\Box(\varphi \rightarrow \psi) \rightarrow (F^\Box\varphi \rightarrow F^\Box\psi)$,
4. $H\,\varphi \rightarrow \neg H\neg\varphi$,
5. $G^\Box\varphi \rightarrow F^\Box\varphi$,
6. $G^\Box\varphi \rightarrow \neg F^\Box\neg\varphi$,
7. $H\,\varphi \rightarrow H\,H\varphi$,
8. $G^\Box\varphi \rightarrow G^\Box G^\Box\varphi$,
9. $F^\Box F^\Box\varphi \rightarrow F^\Box\varphi$,
10. $\varphi \rightarrow G^\Box\neg H\neg\varphi$,
11. $\varphi \rightarrow H\neg G^\Box\neg\varphi$,
12. $H\varphi \rightarrow [\varphi \rightarrow (G^\Box\varphi \rightarrow G^\Box H\varphi)]$,
13. $H\varphi \rightarrow [\varphi \rightarrow (\neg F^\Box\neg\varphi \rightarrow \neg F^\Box\neg H\varphi)]$,
14. $F^\Box G^\Box\varphi \rightarrow G^\Box F^\Box\varphi$

Rules

R1: Modus Ponens, R2: $\dfrac{\varphi}{G^\Box\varphi}$ R3: $\dfrac{\varphi}{H\varphi}$, and a non-standard rule:

R4[2]: If φ is not a subformula of ψ, then:

$$\frac{\vdash_{K_b^\square} H\neg\varphi \to [\neg\varphi \to (G^\square\varphi \to \psi)]}{\vdash_{K_b^\square} \psi}$$

In $K_b^\square$, there are definitional relationships between the operators $G^\Diamond$ and $F^\square$ as mentioned earlier.

Given the definitional relationship between the operators $G^\Diamond$ and $F^\square$ from Axiom 11, the thesis of the $K_b^\square$ system is the formula:

$$\varphi \to HF^\Diamond\varphi.$$

This expression is not, however, a formulation of the thesis of determinism. Under the given interpretation of modal temporal operators, it is read as follows:

If it is true that φ, then it is true that it has always been the case in the past that φ will be possible in the future.

This statement only asserts that if φ is true, then it has always been the case in the past that φ will be possible in the future. The expression $\varphi \to HF^\Diamond\varphi$ is therefore not a formulation of the determinism thesis in the language of $K_b^\square$. It only encodes the fact that if φ is true, then the possibility of φ occurring in the future was determined in the past.

However, the expression $\varphi \to HF^\square\varphi$ is not a theorem of the $K_b^\square$ system, which represents the logical formulation of the determinism thesis. It reads as follows:

If it is true that φ, then it is true that it has always been the case in the past that φ is necessary in the future.[3]

1.3. The Logic of Modal and Temporal Operators

Ewa Orłowska proposes the construction of a tense logic system for non-deterministic time[4]. In this proposal, the concept of non-deterministic time is based on the concept of a non-deterministic world. A non-deterministic world is a set of equally possible states. For a given world representing the present, past worlds are its subsets. Future worlds, on the other hand, are sets that contain the world representing the present. It is assumed, therefore, that worlds undergo changes, and the way of changing is through expansion.

In this approach, indeterminism occurs at two levels. The first is the acceptance of the existence of non-deterministic worlds containing accidental events. The second is the recognition of the future as reachable in a non-deterministic way. Consequently, for any world m, there are many past worlds from which m can be reached, and there are many future worlds with respect to m that are reachable from m.

The primary operators assumed are:

H – it was always the case that,

G – it is necessary that it will always be the case that[5],

$\Box$ – it is necessary that at some point it will be the case that.

The alphabet of the considered system is the alphabet of classical propositional calculus enriched with the operators $G, H, \Box$. A set of sentences is defined in the usual way.

Semantics

A model is a structure $\mathfrak{M} = (S, \mathcal{S}, V)$, where S is a non-empty set (whose elements are called states, moments or events), $\mathcal{S} \subseteq 2^S$ is a family of possible worlds, and $V : \mathfrak{X} \to 2^{2^S}$ is a valuation function such that for any p ($\in Var$), $V(p) \subseteq \mathcal{S}$ (intuitively, $V(p)$ contains only those worlds in which p is a true sentence).

The concept of sentence satisfaction in a model is defined as follows:

Definition 4

For a model $\mathfrak{M} = (S, \mathcal{S}, V)$, world m and sentences φ, ψ:

a) $\mathfrak{M}, m \models \varphi \quad \equiv \quad m \in V(\varphi)$, where $\varphi \in \mathfrak{X}$,

b) $\mathfrak{M}, m \models \neg\varphi \quad \equiv \quad$ it is not true that $\mathfrak{M}, m \models \varphi$,

c) $\mathfrak{M}, m \models \varphi \to \psi \quad \equiv \quad$ if $\mathfrak{M}, m \models \varphi$, then $\mathfrak{M}, m \models \psi$,

d) $\mathfrak{M}, m \models H\varphi \quad \equiv \quad \forall m' \in \mathcal{S}$, if $m' \subseteq m$, then $\mathfrak{M}, m' \models \varphi$,

e) $\mathfrak{M}, m \models G\varphi \quad \equiv \quad \forall m' \in \mathcal{S}$, if $m \subseteq m'$, then $\mathfrak{M}, m' \models \varphi$,

f) $\mathfrak{M}, m \models \Box\varphi \quad \equiv \quad \forall m' \in \mathcal{S}$, if $m \cap m' \neq \emptyset$, then $\mathfrak{M}, m' \models \varphi$.

Axioms

A1. All sentences of the language of the considered system that have the form of a tautology of classical propositional calculus,

A2. $G(\varphi \to \psi) \to (G\varphi \to G\psi)$,

A3. $H(\varphi \to \psi) \to (H\varphi \to H\psi)$,

A4. $\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$,

A5. $\varphi \to GP\varphi$,

A6. $\varphi \to HF\varphi$,

A7. $H\varphi \to \varphi$,

A8. $G\varphi \to \varphi$,

A9. $\Box\varphi \to \varphi$,

A10. $G\varphi \to GG\varphi$,

A11. $H\varphi \to HH\varphi$,

A12. $\Box\varphi \to H\Box G\varphi$,

A13. $\varphi \to \Box\Diamond\varphi$.

Rules

R1: Modus Ponens R2: $\dfrac{\varphi}{G\varphi}$, R3: $\dfrac{\varphi}{H\varphi}$, R4: $\dfrac{\varphi}{\Box\varphi}$.

Other propositional conjunctions and tense operators are defined as follows:

Definition 5

a) $\varphi \vee \psi \quad \equiv \quad \neg\varphi \rightarrow \psi$,
b) $\varphi \wedge \psi \quad \equiv \quad \neg(\neg\varphi \rightarrow \psi)$,
c) $P\varphi \quad \equiv \quad \neg H\neg\varphi$,
d) $F\varphi \quad \equiv \quad \neg G\neg\varphi$,
e) $\Diamond\varphi \quad \equiv \quad \neg\Box\neg\varphi$.

Despite its rich language, which includes both tense operators and modal operators, this logic is not entirely satisfactory when it comes to rejecting arguments in favor of logical determinism. Indeed, the statement of logical determinism can be expressed within this logic as a theorem. To demonstrate this, the following theorem will first be proven:

Theorem 1

$\vdash \neg F\neg\varphi \rightarrow G\varphi$

Proof.

1) $\neg\neg\varphi \rightarrow \varphi$ – A1,
2) $G(\neg\neg\varphi \rightarrow \varphi)$ – 1,R2,
3) $G\neg\neg\varphi \rightarrow G\varphi$ – 2,A2,R1,
4) $\neg G\neg\neg\varphi \rightarrow F\neg\varphi$ – def. 5,
5) $\neg F\neg\varphi \rightarrow \neg\neg G\neg\neg\varphi$ – 4,A1,R1,
6) $(\neg F\neg\varphi \rightarrow \neg\neg G\neg\neg\varphi) \rightarrow [(\neg\neg G\neg\neg\varphi \rightarrow G\neg\neg\varphi) \rightarrow (\neg F\neg\varphi \rightarrow G\neg\neg\varphi)]$ – A1,
7) $(\neg\neg G\neg\neg\varphi \rightarrow G\neg\neg\varphi) \rightarrow (\neg F\neg\varphi \rightarrow G\neg\neg\varphi)$ – 5,6,R1,
8) $\neg F\neg\varphi \rightarrow G\neg\neg\varphi$ – A1,7,R1,
9) $(\neg F\neg\varphi \rightarrow G\neg\neg\varphi) \rightarrow [(G\neg\neg\varphi \rightarrow G\varphi) \rightarrow (\neg F\neg\varphi \rightarrow G\varphi)]$ – A1,
10) $(G\neg\neg\varphi \rightarrow G\varphi) \rightarrow (\neg F\neg\varphi \rightarrow G\varphi)$ – 8,9,R1,
11) $\neg F\neg\varphi \rightarrow G\varphi$ – 3,10,R1.

$\Box$

In the following theorem, it is shown that the expression $F\neg\varphi \vee F\varphi$, which can be considered as the statement of logical determinism, is a theorem of the discussed logic.

Theorem 2 (Surowik 2001)
$\vdash F\neg\varphi \vee F\varphi$

Proof.

1) $\varphi \to HF\varphi$ – A6,
2) $HF\varphi \to F\varphi$ – A7,
3) $(\varphi \to HF\varphi) \to [(HF\varphi \to F\varphi) \to (\varphi \to F\varphi)]$ – A1,
4) $[(HF\varphi \to F\varphi) \to (\varphi \to F\varphi)]$ – 1,3,R1,
5) $\varphi \to F\varphi$ – 2,4,R1,
6) $G(\varphi \to F\varphi)$ – 5,R2,
7) $G(\varphi \to F\varphi) \to (G\varphi \to GF\varphi)$ – A2,
8) $G\varphi \to GF\varphi$ – 6,7,R1,
9) $GF\varphi \to F\varphi$ – A8,
10) $(G\varphi \to GF\varphi) \to [(GF\varphi \to F\varphi) \to (G\varphi \to F\varphi)]$ – A1,
11) $(GF\varphi \to F\varphi) \to (G\varphi \to F\varphi)$ – 8,10,R1,
12) $G\varphi \to F\varphi$ – 9,11,R1,
13) $(\neg F\neg\varphi \to G\varphi) \to [(G\varphi \to F\varphi) \to (\neg F\neg\varphi \to F\varphi)]$ – A1,
14) $(G\varphi \to F\varphi) \to (\neg F\neg\varphi \to F\varphi)$ – Theorem 1,13,R1,
15) $\neg F\neg\varphi \to F\varphi$ – 12,14,R1,
16) $F\neg\varphi \vee F\varphi$ – 15,A1,R1.

□

2. Three-valued tense logics

The literature also discusses the problem of constructing tense logic systems based on three-valued logic. This is related to the possibility of rejecting arguments for logical determinism by giving up the principle of bivalence. An example of such a system was proposed by A. N. Prior[6].

2.1. A. N. Prior's three-valued tense logic

This is a logic in which both propositional letters and statements of the form $F\varphi$ and $P\varphi$ can be assigned a truth value different from classical values.

Let $\mathfrak{X}$ be the set of propositional letters. Let T be the set of moments of time. A valuation is a function $V : \mathfrak{X} \times T \to \{0, \frac{1}{2}, 1\}$.

The function V can be extended to the set of statements in the following way:

Definition 6

For any φ,

a) $V^*(\varphi, t) = 1 \quad \equiv \quad V(\varphi, t) = 1$, if $\varphi \in \mathfrak{X}$,
b) $V^*(\varphi, t) = 0 \quad \equiv \quad V(\varphi, t) = 0$, if $\varphi \in \mathfrak{X}$,
c) $V^*(\varphi, t) = \frac{1}{2} \quad \equiv \quad V(\varphi, t) = \frac{1}{2}$, if $\varphi \in \mathfrak{X}$,
d) $V^*(F\varphi, t) = 1 \quad \equiv \quad \exists t_1\, [tRt_1$, and $V^*(\varphi, t_1) = 1]$,
e) $V^*(F\varphi, t) = 0 \quad \equiv \quad \forall t_1\, [$ if tRt_1, then $V^*(\varphi, t_1) = 0]$,
f) $V^*(F\varphi, t) = \frac{1}{2} \quad \equiv \quad$ it is not true that $V^*(F\varphi, t) = 1$ and it is not true that $V^*(F\varphi, t) = 0$.

For the operator P, the function V^* is defined analogously. ‘1’ and ‘0’ denote classical truth values of true and false, respectively, while ‘$\frac{1}{2}$’ denotes a neutral value. The truth values of expressions built using ‘not’ and ‘implies’ are determined according to the matrices of Łukasiewicz three-valued logic. The expressions:

$$F\varphi \vee F\neg\varphi$$

and

$$P\varphi \vee P\neg\varphi$$

assuming the lack of both beginning and end of time, the statements of the predeterminism and postdeterminism theses are not laws of this logic[7]. As a non-deterministic system, this system is not fully satisfactory because the laws of the system are: $\varphi \to PF\varphi$ (assuming the lack of beginning of time) and $\varphi \to FP\varphi$ (assuming the lack of end of time), which respectively express the principle of causality and the principle of effects[8]. The laws of this logic also include the following sentences:

$$\varphi \to HF\varphi$$

and

$$\varphi \to GP\varphi$$

the expressions corresponding to the theses of predeterminism and post-determinism respectively. Tense operators do not lose their deterministic character even though more than two truth values are adopted.

2.1.1. Logic K_{t_3} by N. Rescher and A. Urquhart

Another proposal for a three-valued tense logic is the logic K_{t_3} constructed by N. Rescher and A. Urquhart[9]. K_{t_3} is also not entirely satisfactory as an indeterministic logic for at least two reasons. Firstly, the axioms of this system are

1) $\neg G \neg H \varphi \rightarrow \varphi$

and

2) $\neg H \neg G \varphi \rightarrow \varphi$.

From 1) we obtain[10]

3) $\varphi \rightarrow G \neg H \neg \varphi$.

From 2) we obtain

4) $\varphi \rightarrow H \neg G \neg \varphi$.

Expressions 3) and 4) are respectively statements of the predeterminism thesis and the postdeterminism thesis in the language of K_{t_3}.

Secondly, similar to A. N. Prior's three-valued tense logic, the laws of K_{t_3} are:

$$F\varphi \vee F\neg\varphi$$

and

$$P\varphi \vee P\neg\varphi$$

which in the language of K_{t_3} also represent respectively the predeterminism thesis and the postdeterminism thesis.

In the discussed systems of three-valued tense logic, if it was possible to reject the argument in favor of determinism based on the principle of excluded middle, it was impossible to reject the argument based on the principle of causality, as the formulation of this principle was a law of the discussed logics. Another three-valued tense logic, free from these drawbacks, is discussed next.

2.1.2. Logic $Ł_m^3$ by K. Trzęsicki

Another proposal for a three-valued tense logic is the logic constructed by Kazimierz Trzęsicki[11]. This logic is denoted by $Ł_m^3$. The construction of this logic was motivated by intuitions expressed by J. Łukasiewicz[12]. The definitions of the tense operators given for $Ł_m{}^3$ are such that in the case of accepting only two truth values, these operators are equivalent to Prior's operators P and F, but do not have a deterministic character.

Sentences:

$$F\varphi \vee F\neg\varphi$$

and

$$P\varphi \vee P\neg\varphi$$

expressing the thesis of predeterminism and the thesis of postdeterminism are not laws of the logic $Ł^3_m$[13]. The expression stating the principle of causality, $\varphi \to PF\varphi$, is also not a law of $Ł^3_m$[14].

Among the considered three-valued tense logics, $Ł^3_m$ seems to be the most adequate for the logical description of an indeterminate world. Sentences expressing the thesis of determinism are not laws of this logic, even when assuming that time has no beginning, end, it is transitive and linear.

When considering the tense logic systems, the question arises whether it is possible to construct an indeterministic tense logic in which:
- there are only two truth values,
- no conditions are imposed on the structure of time,
- no other specific operators appear in the language of this logic besides tense operators.

The answer to this question seems to be the construction of an intuitionistic tense logic.

NOTES

[1] See W. A. Smirnov, *Logiczeskije sistiemy z modalnymi vriemiennymi operatorami*, Modalnyje i vriemiennyje logiki, Moscow 1979, pp. 89-98.

[2] The occurrence of the R4 rule in the $K_b^{\Box}$ system was indicated as a flaw of this logic. The problem was raised of how to axiomatize the logic of modal-temporal operators with semantics based on branching time, so that there are no non-standard rules in the system. (See A. S. Karpienko, *Fatalizm i słuczajność buduszczego*, Moscow 1990, p. 175.)

[3] One can point to a branching time structure in which the expression $p \to HF^{\Box}p$ is not satisfied. Such a structure is a tree $\mathcal{T}$ containing a branch b such that for any $t \in b$, the statement $\neg p$ is true and there exists a moment $t_1 \notin b$ in $\mathcal{T}$ where the statement p is true. At moment t_1, $p \to HF^{\Box}p$ is not satisfied.

[4] See E. Orłowska, *Tense logic for nondeterministic time*, Bulletin of The Section of Logic, Vol. 11, Nr 3/4, 1982, pp. 127–133.

[5] It is not clear why the author of the system introduces necessity only in the interpretation of the G operator, while the definitions of the satisfaction of the statements $G\varphi$ and $H\varphi$ are symmetric with respect to time.

[6] See A. N. Prior, *Time and Modality*, Clarendon Press, Oxford 1957.

[7] Cf. K. Trzęsicki, *Logika operatorów czasów gramatycznych...*, Białystok 1986, p. 296.

[8] **The principle of effects** is symmetrical to the principle of causality and states: "If φ is a fact at a time t, then at some time t_1 later than t there will be a fact ψ which is an effect of φ." Cf. K. Trzęsicki, *Zasada przyczynowosci i skutków (związki logiczne)*, Studia Filozoficzne, 1988, p. 190.

[9] See N. Rescher, A. Urquhart, *Temporal Logic*, Wien New York 1971, pp. 219–224.

[10] By replacing φ with $\neg\varphi$ in 1) and applying transposition.

[11] See K. Trzęsicki, *Logika operatorów czasów gramatycznych...*, 1986, pp. 298–328.

[12] See J. Łukasiewicz, *O determinizmie,...*

[13] See K. Trzęsicki, *Logika operatorów czasów gramatycznych...*, p. 306.

[14] Ibid, p. 307.

R E F E R E N C E S

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