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TENSE LOGICS AND DETERMINISM

Abstract. In this article, we will discuss the concept of logical determinism and arguments in favor of determinism based on the law of excluded middle and the principle of causality. We will also discuss minimal tense logic and tense logic of branched time, constructed to reject the thesis of logical determinism.

Keywords: determinism, tense logic, causality.

One of the main logical-philosophical problems is to determine the mutual relationships between concepts of time, truth, necessity and possibility, determinism and indeterminism. To accurately characterize the concepts of determinism and indeterminism, the temporal aspect of the world must be taken into account. Logical tools used to investigate and formally describe these relationships include **tense logic**[1], in which they appear as logical expressions whose meaning is determined by reference to time.

There are many tense logic systems in the literature of the subject. In this text only those logics that are relevant to the presented argumentation and provide a broader context for the considerations being made are discussed. These logics, according to their creators, were supposed to belong to the class of **indeterministic logics**, i.e., logics whose laws are not statements that express the thesis of determinism. Through the language of such logics, it is possible to describe an indeterminate world[2].

The discussed tense logics are representative of research aimed at creating a logic whose language does not exclude the possibility of describing contingent events. However, tense logics of branching time, modal-time operators, and many-valued tense logics are not fully satisfactory in relation to the problem of describing an indeterminate world. To enable the description of an indeterminate world using the mentioned tense logics, at least one of the following conditions must be met:

– acceptance of a specific semantic time structure (branching structure),
– introduction of additional logical values (many-valued tense logics),
– enrichment of the logic language not only with tense operators but also with modal or modal-time operators.

Not all of these conditions, however, are sufficient[3] to construct an indeterministic tense logic. In the case of most of the discussed logics, their laws are statements in the language of these logics that express the thesis of determinism.

1. Logical determinism

The position that the principles of logic are sufficient to derive the thesis of determinism is called **logical determinism**[4]. According to logical determinists, the principle of bivalence or the law of excluded middle provides a sufficient basis for constructing an argument for determinism without the need to appeal to extra-logical principles. Discussions of arguments for determinism[5] date back to ancient times. The problem was discussed by Aristotle. His clear formulation is found – in particular – in Chapter IX of the *Hermeneutics*. Aristotle assumes that statements about what was or what is are true or false. In his opinion, the assumption that all statements describing future events are true or false provides a basis for constructing an argument for determinism. Aristotle writes:

> As regards particular statements and statements about the future, the case is different. For if every affirmation and negation is either true or false, every attribute must belong or not belong to every subject. If, then, one man says truly that a thing is and another says truly that it is not, there must be a third man for whom the first speaks truly, and for whom the second speaks falsely. For if that were not so, everything would be the same. If, for instance, it is true to say that a thing is white or not white, one of the two must be true and the other false; and if it is neither white nor not white, one cannot say truly that it is white or that it is not white, but only in some qualified sense, e.g. 'It is white' or 'It is not white' relatively to sight or to touch or to some other standard. Similarly, then, it is impossible that there should be a difference of opinion about what has happened or will happen in the future, unless one thing is not true and another true. But when a man says that what is not is, or that what is, is not, we know at once that he does not speak truly, and this is the case whether he says it ignorantly or intentionally. And this being impossible of particular statements, it is impossible of statements about the future also; for these are of the same character with particular statements, since they also are concerned with things that are not necessary. Thus the result is that nothing is or happens by chance, but everything for a reason and of necessity.[6]

If all statements about future events were true or false, then the events to which these statements refer would be determined. If all future events are determined, there are no accidental future events, and everything that happens, happens necessarily. Therefore, the proposition that statements about future events are true or false implies that in addition to the past and present, the future is logically determined.

Currently, the problem of logical determinism has been studied by Jan Łukasiewicz among others. In his article "On Determinism," he presents the following understanding of determinism:

"If an object **A** has a property **b** at a given time **t**, then in any earlier time than **t**, it is true that **A** has the property **b** at time **t**." This is the so-called principle of determinism in its semantic formulation. As Zbigniew Jordan writes, "It does not refer to events, but to statements asserting events and speaks of certain special properties of true statements. According to this principle, the predicate 'true' is an absolute predicate... If a statement is true, it is true regardless of the person, place, and time in which it is asserted to be true."

According to Kazimierz Trzęsicki, we distinguish between the thesis of predeterminism and the thesis of postdeterminism.

The thesis of predeterminism (PRE-DET) states that:

if it is a fact that φ, then it has always been a fact that φ will be the case.

The thesis of postdeterminism (POST-DET) states that:

if it is a fact that φ, then it will always be a fact that φ was the case.

We say that the thesis of determinism occurs only when the thesis of predeterminism and the thesis of postdeterminism are present. In the literature, the term "thesis of determinism" usually corresponds to our formulation of the thesis of predeterminism[7]. In further considerations, unless stated otherwise, the term "thesis of determinism" will be understood as it is commonly done in the literature.

There are two arguments for determinism. One of them stems from the logical principle of bivalence or the semantic principle of excluded middle, and the other stems from the principle of causality.

The principle of bivalence states that every statement about the future is either true or false, and if this is the case, then future events must be determined. The principle of causality suggests that every event is caused by a previous event, and if all events are caused, then the future is determined by the present and the past.

The principle of bivalence:

Every sentence is either true or false.

Let φ be a sentence. Assuming the notations:
$t(\varphi)$ – φ is true,
$f(\varphi)$ – φ is false,
the principle of bivalence can be written as follows:

$$t(\varphi) \stackrel{\bullet}{\vee} f(\varphi).$$[8]

Semantic principle of excluded middle:

Of two sentences, one of which is the negation of the other, one must be true.

This principle, using the previously adopted notations, can be expressed as:

$$t(\varphi) \stackrel{\bullet}{\vee} t(\neg\varphi).$$

When Tarski's schema is in place, which states that the expression "it is true that φ" is equivalent to φ, the semantic principle of excluded middle can be written as:

$$\varphi \vee \neg\, \varphi.$$

If the thesis is the expression $\varphi \vee \neg\varphi$, and Tarski's schema holds, as well as the following definition of falsehood:

$$f(\varphi) \text{ is equivalent to } t(\neg\varphi),$$

then

$$t(\varphi) \stackrel{\bullet}{\vee} f(\varphi)$$

also holds.

In systems where Tarski's schema and the given definition of falsehood hold, the law of bivalence and the law of excluded middle are therefore equivalent.

Assuming the **principle of necessity**[9], formulated as:

If φ is true, then φ is necessary

it is possible to perform a formal reconstruction of the argument for determinism from both the principle of bivalence and the semantic principle of excluded middle.

Let $\Box(\varphi)$ mean "φ is necessary." The principle of necessity can then be written as:

$$t(\varphi) \to \Box(\varphi)$$

Let φ be a sentence. The reconstruction of the argument for determinism from the principle of bivalence is as follows:

1) $t(\varphi) \to \Box(\varphi)$ – the principle of necessity,
2) $f(\varphi) \to t(\neg\varphi)$ – the definition of falsity,
3) $t(\neg\varphi) \to \Box(\neg\varphi)$ – the principle of necessity,
4) $f(\varphi) \to \Box(\neg\varphi)$ – based on 2, 3, and the inference rule $\frac{\varphi\to\psi,\psi\to\gamma}{\varphi\to\gamma}$,
5) $t(\varphi)\ \dot{\vee}\ f(\varphi)$ – principle of bivalence,
6) $\Box(\varphi)\ \dot{\vee}\ \Box(\neg\varphi)$ – based on 1, 4, 5, and the inference rule $\frac{\varphi\to\gamma,\psi\to\delta,\varphi\dot{\vee}\psi}{\gamma\dot{\vee}\delta.}$

If one accepts the universal character of the principle of bivalence, then statements about future events are either true or false. Moreover, if the principle of necessity is accepted, then one can say that any future event is necessary to happen, or it is necessary that the event will not happen in the future.

The reconstruction of the argument from the semantic principle of excluded middle for determinism is as follows:

1) $t(\varphi) \to \Box(\varphi)$ – the principle of necessity,
2) $t(\neg\varphi) \to \Box(\neg\varphi)$ – the principle of necessity,
3) $t(\varphi)\ \dot{\vee}\ t(\neg\varphi)$ – the semantic principle of excluded middle,
4) $\Box(\varphi)\ \dot{\vee}\ \Box(\neg\varphi)$ – based on 1,2,3 and $\frac{\varphi\to\gamma,\psi\to\delta,\varphi\dot{\vee}\psi}{\gamma\dot{\vee}\delta}$.

It has been shown that if we accept the principle of necessity and one of two principles: the principle of bivalence or the semantic principle of the excluded middle, then for any expression φ (in particular for any expression describing future events), it is necessary that φ is true, or it is necessary that it is not true that φ. This would mean that all future events occur necessarily, that is, they are determined. Therefore, there would be no accidental future events, and the thesis of determinism would hold.

If everything that will be is already true today and has always been true, then the future is established just like the past, and it differs from the past only in that it has not yet been realized.

Jan Łukasiewicz was among the researchers who investigated the status of future accidental events. He believed that expressions about future accidental events are neither true nor false. Therefore, he assigns such expressions an additional logical value denoted by "$\frac{1}{2}$". This led Łukasiewicz to construct many-valued logics, whose language, in th author's opinion, is suitable for describing the indeterminate world. However, Tadeusz Kotarbiński already believed that the significance of many-valued logic for solving the problem of determinism is controversial.[10]

1.1. The Principle of Causality and Determinism

The starting point of the argument for determinism is the principle of causality, which can be expressed as follows:

The Principle of Causality

Every event Z that occurs at time t has a cause in some event Z_1 that occurred at a time t_1 earlier than t. At every later time after t_1 and before t, an event that is the effect of Z_1 and the cause of Z occurs.[11]

The weaker formulation of the principle of causality is as follows:

If Z is an event at time t, then at some earlier time t_1, an event Z_1 occurred that is the cause of Z.[12]

The principle of causality, both in its stronger and weaker senses, does not exclude the existence of a cause of an event φ that is simultaneous with that event.

Therefore, if there is a cause of some event, then due to the state of affairs it co-constitutes, the event that is the effect of that cause is inevitable[13]. However, the reverse is not true. The current inevitability of an event does not necessarily mean the current existence of its cause. For example, it is inevitable that a certain person will die, although there may not yet be a direct cause of their death[14].

In a set of events, some events are causally related to other events. It is assumed that this relation is transitive, meaning that if event Z_1 is the cause of event Z_2 (event Z_2 is the effect of event Z_1), and event Z_2 is the cause of event Z_3 (event Z_3 is the effect of event Z_2), then event Z_1 is the cause of event Z_3 (event Z_3 is the effect of event Z_1). Thus, events Z_1, Z_2, and Z_3 form a causal chain. In a set of events ordered by a causal relation, each event has its cause in other events that precede it in the causal chain. Therefore, causal chains are infinite[15]. The infinity of causal chains is not, however, a sufficient condition for stating that every event has eternal causes, i.e. is determined. Causal chains can be infinite and limited in time. The necessary condition for the infinity and limitedness in time of causal chains is the density of the causal relation[16]. The principle of causality postulates that for every event, there is its earlier cause. Each event in the causal chain is situated at a different moment in time than the moments in which the other events in the causal chain are situated.

If we assume that the measure of change (i.e., the transition from cause to effect) is time, then the necessary condition for the existence of infinite and time-limited causal chains is temporal density. Therefore, assuming temporal density, the thesis of determinism does not follow from the principle of causality. Jan Łukasiewicz demonstrated this[17]. It is usually assumed that physical time is continuous, and therefore dense. Thus, the thesis of determinism does not follow from the principle of causality under the usually accepted properties of time. To demonstrate that the thesis of determinism follows from the principle of causality, it is necessary to assume the discreteness of time. Causal chains then extend indefinitely into the past[18], which is interpreted as the existence of eternal causes for every event. Every event that has eternal causes is a determined event.

Indeterminists usually assume that the past is determined, while the future is not. Therefore, we should consider the difference between the past and the future, and thus highlight the difference between the "earlier-later" and "later-earlier" relations, such as determining the direction of time.

One of the arguments for highlighting the difference between the past and the future is the McTaggart paradox of the unreality of time. This paradox is related to two fundamental ways of understanding time.

On the one hand, time is understood as a dynamic process in which events are arranged according to the distinction between the past, present, and future. What is happening now will become past, and what will happen will become present. Past events become more distant, and future events less distant from the present. This is a dynamic concept of time, called the A-theory.

On the other hand, the same events whose temporal characteristics change in relation to the past, present, and future are arranged in an order that is determined by the relation of "before-after." This is a static concept of time called the B-theory. In this approach, the temporal characteristics of all events exist as given facts, for example, the event "Socrates' death" is always earlier than the event "The death of Casimir the Great."

It seems that the A-theory should be derivable from the B-theory, and the B-theory from the A-theory. However, this is not the case, and it is the essence of McTaggart's paradox. It turns out that the static conception of time can be derived from the dynamic one, but not vice versa. It is argued that the dynamic conception of time is contradictory, and time turns out to be unreal and illusory. Supporters of the dynamic conception of time argue that there are ontological differences between past and future events. Past events have already happened, while future events are only possible. This difference can be explained, for example, by the logical asymmetry between past and future. All statements about the past are either true or false, while some statements about the future are neither true nor false. Therefore, it can be said that every past event is determined, but it cannot be said that any future event is determined.

Determination should be considered in a temporal context. Therefore, a better approximation of the concept of determination is obtained by introducing temporal expressions into the language, which would indicate the time distance between the moment of utterance and the moment when the fact mentioned in the utterance will take place. This is realized in various systems of temporal logic.

2. Tense logics

Logic in which the meanings of logical constants are determined by reference to time and expressed as temporal expressions is called **tense logic**. A. N. Prior was a precursor of tense logic[19]. One of Prior's fundamental concepts was the temporal interpretation of modal operators. In "classical" tense logics, tense operators F, G, P, and H are used and understood as follows:

$F\varphi$ – it will be the case that φ

$P\varphi$ – it was the case that φ

$G\varphi$ – it will always be the case that φ

$H\varphi$ – it has always been the case that φ

Typically, in tense logics built on classical propositional logic, the operators F and G as well as P and H are mutually definable[20].

2.1. K_t – minimal tense logic

The basic deductive system of tense logic is K_t. It is a system introduced by E. J. Lemmon[21]. K_t is a tense logic system built on classical logic. It is a minimal system, which formally means that any other tense logic system built on classical logic is richer than K_t.

The language of K_t is the language of classical propositional calculus enriched with tense operators G and H. The set of sentences in the language of K_t is defined in a standard way.

In semantic considerations of K_t, time is assumed to have a point-like structure[22]. Let T be a non-empty set (elements of this set are called "time moments" or simply "times"). Let R be a binary relation defined on T. Time $\mathcal{T}$ is understood as a pair $\langle T, R\rangle$. A valuation is a function

$V : \mathfrak{X} \to 2^T$[23]. A model, denoted by "$\mathfrak{M}$," is an ordered triple $\langle T, R, V\rangle$, i.e., $\mathfrak{M} = \langle T, R, V\rangle$. For the purposes of further considerations, the notation is adopted:

"$\mathfrak{M}, t \models \varphi$" means that the sentence φ is true in the model $\mathfrak{M}$ at the moment t[24].

The function V can be uniquely extended to the set of sentences in the language of K_t. The truth of a sentence φ in a model $\mathfrak{M}$ at time moment t is defined as follows:

Definition 1

For any φ, model $\mathfrak{M}$ and moment of time t holds:

a) $\mathfrak{M}, t \models \varphi \quad \equiv \quad t \in V(\varphi)$, if $\varphi \in \mathfrak{X}$,

b) $\mathfrak{M}, t \models \neg\varphi \quad \equiv \quad$ not $\mathfrak{M}, t \models \varphi$,

c) $\mathfrak{M}, t \models \varphi \to \psi \quad \equiv \quad$ if $\mathfrak{M}, t \models \varphi$, then $\mathfrak{M}, t \models \psi$,

d) $\mathfrak{M}, t \models G\varphi \quad \equiv \quad$ for all t' such that tRt', $\mathfrak{M}, t' \models \varphi$,

e) $\mathfrak{M}, t \models H\varphi \quad \equiv \quad$ for all t' such that $t'Rt$, $\mathfrak{M}, t' \models \varphi$.

Axiomatization

0. φ, if φ is a tautology of classical propositional logic in the language of K_t.
1. $G(\varphi \to \psi) \to (G\varphi \to G\psi)$,
2. $H(\varphi \to \psi) \to (H\varphi \to H\psi)$,
3. $\varphi \to GP\varphi$,
4. $\varphi \to HF\varphi$.

Rules

R1: Modus Ponens, R2: $\dfrac{\varphi}{G\varphi}$, R3: $\dfrac{\varphi}{H\varphi}$.

Specific axioms of K_t are axioms 1–4. φ and ψ appearing in the formulations of axioms and inference rules of K_t can encode temporal expressions.

The concept of a temporal expression is defined as follows:

Definition 2

A **temporal expression** is an expression in a given language that, when used in a sentence, leads to the relativization of the truth value of that sentence (judgment that the sentence indicates at the time of its use) to the truth values of judgments (or a judgment) referring to facts that took place at the time indicated by that expression[25].

K_t can be axiomatized in many ways. The completeness of the K_t system under the given axiomatization was shown by J. F. A. K. van Benthem[26]. K_t is a minimal logic system. It was intended as a formal system encoding reasoning about the world in terms of operators G, H, F, P, taking into account its temporal aspect. In the semantics of K_t, no assumptions are made about the structure of time.

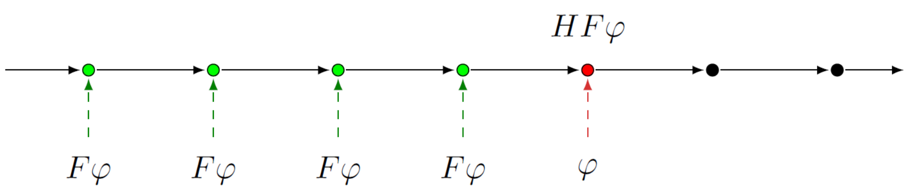
Usually, it is assumed that real time is linear, continuous, and without a beginning or end moment. Such a structure is adopted due to the fact that many physical and philosophical considerations that take into account the temporal aspect of the world are greatly simplified with such a time structure. However, this does not apply to all considerations. Some considerations become complicated with such a structure of real time, especially when arguments for determinism are rejected. Certain axioms that do not express determinism in minimal tense logic express the logical determinism thesis in linear tense logic without end[27]. One such axiom is:

$$\varphi \to HF\varphi.$$

According to the understanding of tense operators adopted in the K_t system (and in its extensions, where the semantic time is linear), the expression $\varphi \to HF\varphi$ is read as:

if φ is true, then it has always been the case that φ will be true.

Assuming the linearity of time, from the truth of the sentence *in the future, φ will be true*, one can conclude that indeed φ will occur in the future[28].



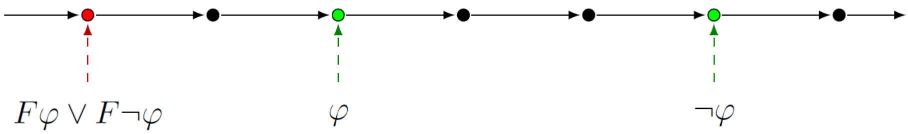
The sentence $\varphi \to HF\varphi$ states that if an event Z has occurred currently, which is the semantic correlate of the sentence φ, then it has always been the case in the past that the event Z will occur in the future. Since the future occurrence of the event Z was predetermined at any moment in the past, one can say that the event Z is determined.

The tautology, built upon classical logic, of linear time tense logic without end is the sentence

$$F\varphi \vee F\neg\varphi.$$

According to the accepted understanding of the F operator, this expression is read as follows:

For any sentence φ, in the future, either φ will be true or it will be true that φ is false.



The sentence $F\varphi \vee F\neg\varphi$ in the language of linear temporal logic without end is therefore a statement of determinism, because its consequence is the assertion that all future events are determined and there are no accidental future events. Using the language of linear temporal logic beyond classical logic, it is possible to describe only a world in which the future is determined.

Indeterminists typically assume that it is the past, not the future, that is determined. What has already happened is not subject to change. The past may be unknown at a certain point in the development of the world, but there is only one past. The future development of the world, on the other hand, is indetermined and can unfold in many ways. This leads to considerations of the branching structure of time.

The idea of branching time logic comes from A.N. Prior[29]. One of the primary motivations for constructing such logic was to create an indeterministic temporal logic. Arguments for determinism were not to be rejected by accepting more than two truth values, but by modifying the temporal structure. This modification involves recognizing that time has a branching structure, rather than a linear one as is usually assumed. Considering time as having a branching structure means that moments of time with a status similar to those in a linear structure are considered, as well as moments whose status is fundamentally different, i.e., moments where time branches. In these moments, alternative possibilities for the realization of the world arise. Because the realization of the world occurs according to only one of the alternatives, taking different actions may contribute to the future being realized according to a different branch of the branching temporal structure.

2.2. Branching tense logic

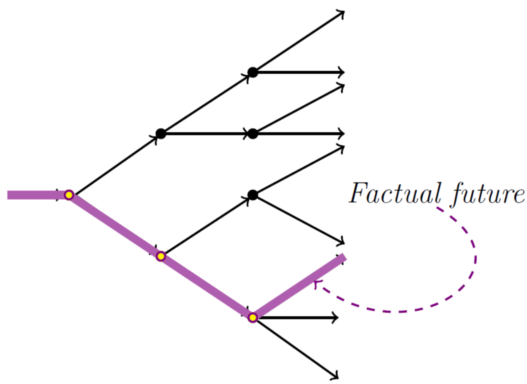
The basic system of branching tense logic is the system constructed by Nino Cocchiarella and denoted as "CR"[30]. In this system, the only condition imposed on the "earlier-later" relation is transitivity. The lack of additional assumptions about the properties of time allows for symmetry of the future and the past[31]. To avoid this symmetry, an additional condition of left-linear linearity of the "earlier-later" relation is usually imposed in branching tense logics[32].

To formally express the discussed issues, we need to provide a definition of one of the auxiliary concepts of branching tense logic, namely the concept of a tree.

Definition 3

Let T be a set of time moments and R be a "before-after" relation defined on the set T. A **tree** is a non-reflexive and transitive ordered structure $\langle T, R\rangle$ in which any two elements have a common predecessor, and the set of all predecessors is linearly ordered by the relation R.

In such a structure, the past has no alternatives and is determined, while the realization of the future can occur in many ways. This is called **possible futures**. However, it is usually assumed that only one of the possible futures is realized. This is the **factual future**.



To better express the intuitions related to the factual future and to properly define the truth of sentences containing temporal operators in branching time models, it is necessary to provide a definition of a branch.

Definition 4

Let $\mathcal{T}$ ($\langle T, R\rangle$) be a tree. b ($\subseteq T$) is a **branch** in $\mathcal{T}$ if and only if b is a maximal linearly ordered subset of T.

Definition 5

Let $\mathcal{T}(= \langle T, R\rangle)$ be a tree. For any $t \in T$, b_t ($\subseteq T$) is a $t-$**branch** in $\mathcal{T}$ if and only if b_t is a branch and $t \in b_t$.

Let B_t denote the set of all and only branches containing moment t, where $t \in T$. The standard definition of the truth of a sentence $F\varphi$ is as follows[33]:

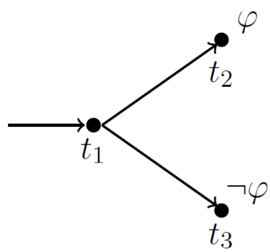
$$\mathfrak{M}, t \models F\varphi \ \equiv \ \mathfrak{M}, t' \models \varphi, \text{ for some } t', \text{ such that } tRt'.$$

In the case of linear time, it is consistent with intuitions and does not raise objections. In the case of branching time, adopting the standard understanding of the truth of the sentence $F\varphi$ can lead to results that raise objections. Richmond Thomason provides an example of such a result[34] by considering the consequences of accepting the standard definition of truth for the metric tensed operator Fh (i.e., an operator such that the sentence $Fh\varphi$ is read as: "in an hour, φ will be true") . Thomason considers the following situation:

Let the definition of the truth of the sentence $Fh\varphi$, consistent with the intuitions presented above (i.e., without reference to branches), be as follows:

$\mathfrak{M}, t \models Fh\varphi \ \equiv$ for some t', which will be an hour later than t, $\mathfrak{M}, t' \models \varphi$

Consider the temporal structure presented in the figure below:



Assuming that the time intervals between t_1 and t_2 and between t_1 and t_3 are equal and exactly one hour. Let φ be true at t_2 and false at t_3. Therefore, in the context of tense logic, which extends classical logic with branching time, at t_1, $Fh\varphi$ is true and at the same time, $Fh\neg\varphi$ is true. This means that at t_1, it is true that "in one hour φ will be true" and at the same time, it is true that "in one hour it will be false that φ is true."

To avoid situations like the one described above, for models based on branching time $\mathcal{T}(= \langle T, R\rangle)$, the truth of a sentence φ at time t is relativized to pairs $[t, b_t]$, where $t \in T$. Let $\mathfrak{X}$ be the set of propositional letters, and let $V : \mathfrak{X} \to 2^{[t,b_t]:\ t\in T,,b_t\in B_t}$.

Definition 6

The definition of the truth of a sentence φ relativized to the pair $[t, b_t]$ (we write "$\mathfrak{M} \models \varphi[t, b_t]$" which is read as "the sentence φ is true in the model $\mathfrak{M}$ at time t belonging to branch b_t") is as follows:

a) $\mathfrak{M} \models \varphi[t, b_t] \quad \equiv \quad [t, b_t] \in V(\varphi)$, if $\varphi \in \mathfrak{X}$, it is not true that $\mathfrak{M} \models \varphi[t, b_t]$,

c) $\mathfrak{M} \models (\varphi \rightarrow \psi)[t, b_t] \quad \equiv$ if $\mathfrak{M} \models \varphi[t, b_t]$, then $\mathfrak{M} \models \psi[t, b_t]$,

d) $\mathfrak{M} \models P\varphi[t, b_t] \quad \equiv$ there exists $t_1 (\in b_t)$ such that ($t_1 R t$ and $\mathfrak{M} \models \varphi[t_1, b_t]$),

e) $\mathfrak{M} \models F\varphi[t, b_t] \quad \equiv$ there exists $t_1 (\in b_t)$ such that ($t R t_1$ and $\mathfrak{M} \models \varphi[t_1, b_t]$).

However, not all logicians consider it sufficient to explain that due to the inability to predict which branch of the branching time structure will be the actual future, branching-time tense logics can be considered indeterministic logics. For example, P. Yourgran argues that regardless of which branch of the branching time structure will be the factual future, events occur at specific moments in time on individual branches, and the structure of real time takes the form of a broken line. If the factual future is chosen, then there is only one branch, and the structure of real time is reduced to a linear structure. If the structure of the factual future is reduced to a linear structure, then events occur in a deterministic manner[35].

The solution to the problem of reducing a tree-like structure to a linear structure – which occurs when choosing the actual future from the tree of possible futures – seems to be the modalization of tense operators.

NOTES

[1] I would like to express my heartfelt thanks to Professor Kazimierz Trzęsicki for inspiring me to research the issues of tense logic and for his many years of scientific supervision. The discussions I had with the Professor on various research topics were always an exceptional intellectual feast.

[2] Only the main tense logic systems related to the issues discussed in this text have been discussed. Other systems are, in a sense, derivative systems, so they are omitted.

[3] This observation does not apply to enriching the language with modal-time operators. The logic with modal-time operators seems to be indeterministic. One drawback of this solution is the multitude of formal-logical tools used in its construction (applying modal-time operators to tense logic of branching time).

[4] The term "logical determinism" was introduced by M. Schlick. (cf. M. Schlick, *Die Kausalität in der gegenwärtigen Phisik*, Die Naturwissenschaften 19 J., Heft 7, 1931, pp. 145–162).

[5] In addition to logical determinism, there is physical determinism. **Physical determinism** is the view that everything happens necessarily, and every fact has eternal causes in other facts that occurred earlier. Physical determinism is associated with the principle of causality. The principle of causality establishes the form of the causal bond, while physical determinism states that whatever happens, happens in accordance with this principle.

[6] Cf. Aristotle, *Hermeneutics*, The Complete Works of Aristotle, Volume II, Princeton University Press, 1991, pp. 139–140.

[7] Cf. for example A.S. Karpienko, *Fatalism and the randomness of the future*, Moscow 1990.

[8] The symbol $\dot{\vee}$ denotes the logical disjunction, also known as exclusive or. A sentence constructed using this connective is true if and only if exactly one of the component sentences is true.

[9] See J. Łukasiewicz, *Aristotle's Syllogistic from the Standpoint of Modern Logic*, Second Edition, Clarendon Press, Oxford 1957, pp. 133–134, 152–153.

[10] T. Kotarbiński points out the non-equivalence in one logical system of two concepts of 'possibility'. On the one hand, 'possibility' is expressed by assigning the truth value "$\frac{1}{2}$" to the sentence. This kind of 'possibility' refers to sentences about future accidental events. On the other hand, 'possibility' is characterized by the properties of the M operator. However, these properties indicate that modal expressions do not refer to future accidental events because the truth values assigned to these expressions are: "1" and "0". (See T. Kotarbiński, *Selected Writings*, Vol. 1, Warsaw 1957, pp. 494-495).

[11] This is a stronger formulation of the principle of causality, which can be found, among others, in Jan Łukasiewicz's *On determinism*.

[12] This formulation of the principle of causality can be found, among others, in K. Trzęsicki's *Zasada przyczynowosci i skutków (związki logiczne)*, Studia Filozoficzne, 1988, No. 6-7, 1988, p. 190.

[13] We say that an event is **inevitable** only when in every possible course of events it must occur sooner or later.

[14] See K. Trzęsicki, *Zasada przyczynowosci i skutków (związki logiczne)*, Studia Filozoficzne, 1988, No. 6–7, 1988, p. 195.

[15] Aristotle argued that causes cannot form an infinite chain and that there must be a first cause, thus infinity is unthinkable. (See Aristotle's *Metaphysics*, Book II, Chapter II, 994a). This argument, however, is unsustainable unless one adopts an extremely empiricist (e.g. Locke's) conception of knowledge.

[16] The causal relationship is dense only if for any two events Z_1 and Z_2 belonging to the same causal chain there exists an event Z_3 belonging to this chain such that Z_3 is the effect of Z_1 and the cause of Z_2.

[17] See J. Łukasiewicz, *On Determinism.*

[18] Causal chains may either have a beginning or not. If it is assumed that causal chains have an absolute beginning, then this also assumes the action of the first cause. Assuming the first cause does not negate the principle of causality, it only limits the scope of its validity. This assumption is not internally inconsistent, but it cannot be reconciled with the principle of sufficient reason. According to this principle, a reason is required for the existence of the first cause.

If we assume that causal chains do not have a beginning, then since every event in a causal chain is situated at some point in time, we must assume the absence of a beginning of time.

[19] The issue of logically describing reality with regard to its temporal aspect has engaged philosophers since ancient times. We can find this subject matter in the reflections of philosophers from the Megarian school, the Stoics, and Aristotle. In the late 1940s, there was a significant increase in interest in temporal issues. Nicolas Rescher identifies three sources of this increase:

– Studies on historical material, mainly on the logic of the Stoics (see, e.g., Martha Hurst Kneale, *Implication in the Fourth Century B.C.*, Mind, vol. 44, 1935, pp. 485–

495) and medieval logic (see, e.g., Ernest Moody, *Truth and Consequence in Medieval Logic*, Amsterdam, 1953).
– Logical analysis of grammatical tenses (Hans Reichenbach, *Elements of Symbolic Logic*, New York, 1947).
– Attempts by the Polish logician Jerzy Łoś to create a formal system of temporal logic and apply it to the analysis of certain issues in the philosophy of science, especially Mill's canons of induction (see Jerzy Łoś, *Podstawy Analizy Metodologicznej Kanonów Milla*, Uniwersytet Marii Curie Skłodowskiej, Sekcja F, vol. 2, 1948, pp. 269-301).

The ideas of J. Łoś were then thoroughly analyzed and developed by A.N. Prior.

[20] The mutual definability of the F and G as well as the P and H operators occurs in tense logics based on classical logic. In intuitionistic and many-valued tense logics, this mutual definability of the operators is usually not the case.

[21] It is the counterpart of the minimal deductive system K for modal logics.

[22] There are also tense logic systems in which time is assumed to have an interval structure in semantic considerations. (See, e.g., J. F. A. K. van Benthem, *The Logic of Time*, D. Reidel Publishing Company, Dordrecht, Holland, 1983, pp. 193-218).

[23] This function assigns to propositional letters subsets of the set of time moments T. Intuitively, propositional letters are interpreted as elementary events. Therefore, $V(p)$ (where p is a propositional letter) is the set of time moments in which the event p occurs (elementary event).

[24] We will also use the notation "$\mathfrak{M}, t \not\models \varphi$" as a shortcut for "it is not true that $\mathfrak{M}, t \models \varphi$". Therefore, "$\mathfrak{M}, t \not\models \varphi$" means that the sentence φ is false in the model $\mathfrak{M}$ at the time moment t.

[25] See K. Trzęsicki, *Logika operatorów czasów gramatycznych...*, 1986, p. 117.

[26] See J. F. A. K. van Benthem, *The Logic of Time*, D. Reidel Publishing Company, Dordrecht, Holland, 1983.

[27] Examples of linear tense logic include CL – tense logic of linear and transitive time (cf. R. P. McArthur, *Tense Logic*, 1976, p. 24) and SL – tense logic of linear, transitive, beginningless, and endless time (cf. R. P. McArthur, *Tense Logic*, 1976, p. 30). These systems are extensions of logic K_t.

[28] However, such an inference may not hold in non-linear time structures, as shown later in the text.

[29] See A. N. Prior, *Past, Present and Future*, Oxford University Press, 1967. (It is reported that Prior was inspired to construct such a logic by solving the problem of God's omniscience proposed by Ockham and the views of C. Pierce on expressions about future events. See, for example, A. Karpienko, *Fatalism and the chance of the future*, Moscow 1990, p. 154).

[30] Cf. A. N. Prior, *Past, Present and Future*, Oxford University Press, 1967, Appendix A.

[31] The system CR is cited as a model for contemporary views in physics on relativistic time. (Cf. e.g. R.P. McArthur, *Tense logic*, Dordrecht 1976, p. 39).

[32] This property is required, for example, in the system K_b. In tense logics whose semantics is based on transitive and left-linear time structures, as opposed to systems such as CR, the so-called **mirror image property** is not preserved, i.e., the property that when we replace in any thesis every occurrence of the operator F with the operator P and replace every occurrence of the operator P with the operator F (since operators G and H are defined using operators F and P, we replace G with H accordingly), we obtain the thesis. (Cf. N. Rescher, A. Urquhart, *Temporal Logic*, Wien, New York 1971, Chapter 4).

[33] Assuming semantics as in K_t or extensions of K_t for linear time.

[34] Cf. R. H. Thomason, *Indeterminist time and truth-value gaps*, Theoria No. 36, 1970, pp. 264–284.

[35] See P. Yourgran, *On the logic of indeterministic time*, The Journal of Philosophy, Vol. 82, 1985, pp. 548–559.

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