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MODAL ANALYSIS OF THE FLOATING CALIPER BRAKE ASSEMBLY THROUGH COMPLEX EIGENVALUE ANALYSIS: A WEIGHTED BOLT APPROACH

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Abstract: This study demonstrates the application of Complex Eigenvalue Analysis (CEA) to optimize a weighted bolt that targets unstable frequencies in a floating caliper brake assembly. A commercial automobile brake assembly undergoes experimental modal analysis, and a corresponding finite element model is developed. The experimental and analytical models are compared, and a modal assurance criterion matrix is established based on relevant modes. Mass modifications are implemented, and the results obtained are presented.

KEYWORDS: CEA; brake squeal; brake pad; MAC

1 Introduction

Disc brake squeal is a continuously occurring problem in the automotive industry. It is an unwanted irritant at best and can be an indicator of a bigger problem at worst. Brake squeal is defined as brake noise generated at higher frequencies, typically more than 1 kHz, whereas other noises such as "creep groan" for example occur at a lower frequency and are believed to be caused by the "stick-slip"[1-3] phenomenon, during which additional energy is introduced into the system as the friction force varies with speed. High frequency noise, the interest of this paper, arises during vehicle braking. Under the assumption that the brake pads are in good condition, high frequency squeal is usually attributed to the coupling mechanism of the complex mode shapes. While the mode shapes themselves depend mostly on geometry, boundary conditions, material properties. A large number of literature on this topic suggests that it is exactly this mode coupling phenomenon that is the primary reason for the onset of brake squeal [4-6]. It is therefore paramount to break-up, so to speak, the mode shapes mainly through targeted modification of the part geometry by changing the mass distribution or by the introduction of targeted damping, for example in the form of shims. These procedures are performed in automotive centres in design, before the brakes are installed in the car and appear on the road and are part of NVH (noise, vibration, harshness) testing.

A large amount of research on the topic of mode shapes has already been conducted and dates back at least as far as to 1930[1][8]. At the time the research was focused on experimental methods on brakes with proposed simple discrete numerical models to explain the phenomenon of brake squeal. Later on as increased computational power became readily available, treating the system as a discrete continuum system compromised of many finite elements proved to be an effective way to analyse brake instability.

Working on the basis of the FEM the complex eigenvalue analysis (CEA) and the dynamic transient analysis are the staple way to predict brake squeal in the automotive industry as of today. The CEA being the preferred for its lesser computational requirements and its connection to the modal space where the solution is of the form of linearly independent equations as opposed to the FEM formulation. Using a set of single degree of freedom systems that are connected to the

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multiple degree of freedom system by the modal transformation equation, a set of coupled DOFs can be reduced to a simpler system. One equivalent single-degree of freedom system is created for each mode of the linked physical equations, and they are all linearly independent of one another. This is crucial because it means that any complex system, such a finite element model with a million degrees of freedom, may be broken down into a group of similar systems with just one degree of freedom that are incredibly easy to solve. Among the first published papers on the CEA and its use on brake system was Liles et al. [9]. For the goal of predicting brake instability, this paper employs the complex eigenvalue analysis on an assembly that consists of a brake disc, brake pads, shims and a piston with the goal to detect instabilities in system and hence to predict the potential onset of brake squeal.

The results of the CEA give us information about instabilities in the frequency spectrum and the complex mode shapes. The complete elimination of brake squeal still remains somewhat of an illusive problem as the number of contributing factors is quite substantial. Wear and operating circumstances to the tribology and temperature effects on the brake pads, affect brake squeal, making the prediction of squeal challenging and error-prone. For example, [10] discusses the effects of the tribology between the brake disc and brake pad. For the numerical model to be valid the data needs to be often correlated, and parameter adjustment needs to be made to bring the finite element model in line with measured data. In addition, non-linearities that arise due to part interactions between parts prove to be a frequent source of difficulty.

Compared to a real mode shape, the complex mode has a phase angle as its imaginary part and therefore the nodes that make up the mode shape do not pass through their equilibrium at the same time. The CEA implementation in ABAQUS utilizes the subspace projection method to perform steady-state dynamic analyses. This approach is based on the concept that a selected set of modes of the undamped system that fall within the frequency range of the applied excitation can accurately reflect the steady-state vibration response. To accomplish this, the dynamic equilibrium equations are projected onto a subspace of selected modes, resulting in a smaller, simplified system of equations. The subspace projection method allows for a direct solution of the steady-state dynamic equations. The equation system obtained from the subspace projection method is then solved to obtain modal amplitudes. These modal amplitudes are then used to calculate the nodal displacements, stresses, and other relevant quantities. Although the subspace projection method is considered to be an over-predicting method, meaning that not all of the predicted modes will necessarily generate brake squeal under real-world operating conditions, the number of modes is still significantly lower than in traditional modal analysis. Additionally, one of the advantages of using the CEA implementation in ABAQUS over traditional modal analysis is its ability to take into account friction and damping effects between the different parts, leading to a more comprehensive and accurate analysis.

As mentioned, the main phenomena responsible for brake squeal is mode coupling of the system's mode shapes, also referred to as eigenvectors. These mode shapes occur at the system’s natural frequencies, also called eigenvalues, and are derived from the shape, stiffness, density, damping ability and numerous interactions of the relevant parts of the mechanical system. As a continuous system has an infinite number of eigenvalues and eigenvectors it is only a particular interaction between some of these that produces brake squeal. More specifically the out-of-plane modes in particular tend to be strongly correlated to brake squeal. An out-of-plane mode can get merged with an in-plane one and the resulting complex mode that has a high likelihood to be unstable and to produce brake squeal.

The interaction between parts of the brake assembly has a paramount effect on mode shapes [11]. Factors such as the friction interaction between parts, mainly brake pads and the rotor, have proven to have a significant effect on system stability as shown in [12]. As such Coulomb friction between pads and disc is being introduced, and the CEA is performed for each of the following values μ_i = 0.3,0.5,0.7. It should be noted that the friction coefficient in real scenario varies around 0.4 with higher values starting to become more common as newer lining materials are introduced. In addition, the shape of the pad has a significant impact on system instability on and the CEA results, as such this paper will look at how different pad variations affect system stability.

Vehicle safety is greatly affected by the friction contact between brake pads and discs. The friction generated by the brake pads pressing against the rotating disc slows down or stops the vehicle. The brake pads must have adequate friction material to grip the disc and provide the necessary stopping power. The condition and composition of the brake pads and discs are critical to the vehicle's ability to stop effectively and safely. Worn or damaged brake pads and discs can reduce stopping ability and increase stopping distances, which is a major safety concern. Regular maintenance and inspections of the brake system can ensure the optimal functioning of the brake pads and discs, helping to maintain the overall safety of the vehicle.

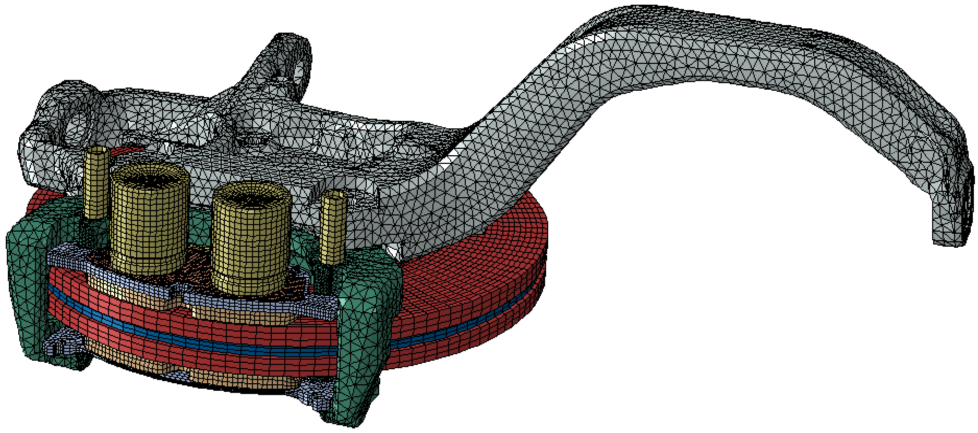

Fig. 1 Finite element model of a vented floating caliper disc brake assembly.

2 Materials and Methods

2.1 Finite Element model

In this study, we aim to analyse and compare the structural behaviour of brake system assembly through the utilization of Finite Element Method (FEM) models. A CAD assembly of a floating caliper disc brake is discretized and a mesh of finite elements is shown on Figure. 1. This design of brake assembly is widely used in many commercial automotive vehicles that are currently in operation. The brake disc is referred to as a "ventilated disc," which has been engineered with ribs that are strategically placed to efficiently dissipate heat during the braking process.

To accurately map the brake assembly, the HYPERMESH pre-processing software was used to generate a structured, fine mesh, primarily made up of hexahedra first-degree elements. In areas that required a more complex geometry, a hybrid mesh, consisting of both hexahedra and pentahedral elements, was employed. After this, the standard implicit ABAQUS solver was used to calculate the finite element solution.

It is important to keep in mind that the material properties of the brake disc play an essential role in determining the vibration mode patterns, as was demonstrated in the study conducted by Kharate et al. [13]. The material properties of this study are shown in Table 1.

Two distinct FEM models will be employed, each serving a specific purpose. The first model focuses on classical modal analysis and correlation with the experimental assembly. It comprises various components such as the knuckle, anchor, bolts, modpin, weight, hub, and outer/inner shell. Notably, the inclusion of weighted bolts attached to the anchor bracket allows for a significant influence on mode shapes primarily affected by the anchor-knuckle setup.

Table 1. Material components

Part Name	Young's Modulus (GPa)	Density (kg/m^3)	Poisson's Ratio
Anchor	177	7200	0.28
Bolt	210	7860	0.30
Disc	120	7200	0.29
Hub	100	7800	0.29
Inner Shell	150	7200	0.31
Outer Shell	160	7250	0.30
Steering Knuckle	70	2700	0.29
Steering Knuckle (cast iron)	120	7200	0.28
Steering Knuckle (steel)	190	7800	0.29
Modpin	210	5600	0.30
Roller	210	7600	0.30
Housing	170	7400	0.32
Brake Pad	70	1550	0.25
Piston	200	7800	0.30

The second FEM model encompasses the entire assembly and incorporates nonlinearities through friction interactions between the disc and brake pads. This model enables a comparative examination of the impact of weighted bolts on the unstable modes obtained from CEA. To ensure accurate representation, it is crucial to apply the FEM boundary conditions to the analytical model.

The assembly itself is affixed to the knuckle connection points of the brake system using the JOINTC Abaqus element. These elements serves to establish a linearly elastic constraint between two nodes, utilizing springs to accurately model the connection's stiffness and behaviour. Through this comprehensive analysis, we aim to enhance our understanding of brake system dynamics and contribute to the advancement of automotive engineering. As mentioned the second model shall also feature piston, pads, located inside a housing, is driven by pressure and begins to push against the brake pads, causing them to clamp the disc.

2.2 Correlation of the Experimental Modal analysis

In this experiment two distinct FEM models will be employed, each serving a specific purpose. The first model focuses on classical modal analysis and correlation with the experimental assembly. It comprises various components such as the knuckle, anchor, bolts, modpin, weight, hub, and outer/inner shell. Notably, the inclusion of weighted bolts attached to the anchor bracket allows for a significant influence on mode shapes primarily affected by the anchor-knuckle setup. The second FEM model encompasses the entire assembly and incorporates nonlinearities through friction interactions between the disc and brake pads. This model enables a comparative examination of the impact of weighted bolts on the unstable modes obtained from CEA. To ensure accurate representation, it is crucial to apply the FEM boundary conditions to the analytical model. The assembly itself is affixed to the knuckle connection points of the brake system using the JOINTC Abaqus element. This element serves to establish a linearly elastic constraint between two nodes, utilizing springs to accurately model the connection's stiffness and behaviour. Through this comprehensive analysis, we aim to enhance our understanding of brake system dynamics and contribute to the advancement of automotive engineering. As mentioned the second model shall also feature piston, pads, located inside a housing, is driven by pressure and begins to push against the brake pads, causing them to clamp the distal study, a modal analysis was performed to explore the dynamic characteristics of a structure. The output response resulting from excitation at different points, as illustrated in Figure. 2 (left), yielded the frequency response function (FRF). FRF acquisition entails measuring the input and output responses of the structure in the frequency domain. By utilizing a small automated hammering system to apply controlled excitation forces and measuring the resulting structural response, the FRFs were derived. These FRFs were obtained by calculating the ratio of the output signals to the corresponding input signals. The overall response of the system was subsequently determined. The response of the system was measured using a 3D laser vibrometer station POLYTEC. Laser interferometry is a sophisticated optical technique used for precise displacement and vibration measurements. It employs a laser beam that is split into two paths, with one directed towards a reference surface and the other towards the target object – in this case, the mechanical floating caliper car disc brake assembly.

Subsequently, a modal assurance criterion (MAC) [14] analysis was conducted to assess the correlation between the experimentally obtained mode shapes and a reference set of mode shapes. MAC analysis is a widely used technique in modal analysis to evaluate the similarity and consistency of mode shapes. By comparing the experimentally derived mode shapes with the reference set, the MAC values were calculated to quantify the degree of modal correlation.

$$MAC(i,j) = \frac{\left|\phi_i^T \cdot \phi_j\right|^2}{(\phi_i^T \cdot \phi_i) \cdot \left(\phi_j^T \cdot \phi_j\right)} \tag{1}$$

The Modal Assurance Criterion (MAC) equation can be modified to incorporate nodal weights. In such cases, the weighted Modal Assurance Criterion (wMAC) is used. The wMAC takes into account the importance or significance of each node in the mode shape comparison. The equation for the weighted Modal Assurance Criterion (wMAC) is as follows:

$$\omega MAC(i,j) = \frac{\left|\phi_i^T \cdot \mathbf{W} \cdot \phi_j\right|^2}{(\phi_i^T \cdot \mathbf{W} \cdot \phi_i) \cdot \left(\phi_j^T \cdot \mathbf{W} \cdot \phi_j\right)} \tag{2}$$

The PRCE (Principal Resonance Curve Extraction) method mathematically fits a sum of complex exponential functions, representing the modes of vibration, to the measured frequency response data Figure 4. This involves optimization techniques, such as least squares or maximum likelihood estimation, to minimize the difference between the measured data and the model predictions. As such a polynomial was fitted to the FRF. By accurately fitting the mathematical model to the data, the modal parameters were extracted, including natural frequencies, damping ratios, and complex mode shapes. Extraction of modal parameters from 6500Hz onwards was not conducted as the higher portions of the frequency domain were deemed too unreliable.

Weighted bolts were employed for this study for. This involved adding physical weights or attachments to increase the mass of the bolts. Suitable weights were selected, such as washers or adhesive weights, and securely attached to the bolts. The modified bolts were then reinstalled in the brake assembly, ensuring proper assembly without overtightening. The modal analysis was subsequently performed, applying appropriate input excitation and measuring the response to examine the impact of the added weight on the mode shapes. It is important to note that this approach introduced an artificial modification to the system, and the results should be interpreted with caution, considering the intended purpose of studying the sensitivity of the mode shapes to changes in mass distribution.

To perform the MAC analysis, the mode shapes and natural frequencies of both the analytical model and experimental data were extracted using appropriate measurement techniques and software tools. These mode shapes represent the vibration patterns of the brake assembly at different frequencies. The MAC correlation method was employed to quantify the similarity between the mode shapes obtained from the analytical model and the experimental data. It measures the consistency of modal shapes by comparing the overlap between the shapes and provides a value ranging from 0 to 1, where 1 indicates a perfect match. By comparing the MAC values for each mode shape, a correlation between the analytical model and experimental data was established. Higher MAC values indicate a stronger correlation, suggesting that the analytical model accurately predicts the dynamic behavior of the brake assembly. Conversely, lower MAC values suggest a discrepancy between the analytical model and experimental data, indicating areas for improvement in the model's predictive capability.

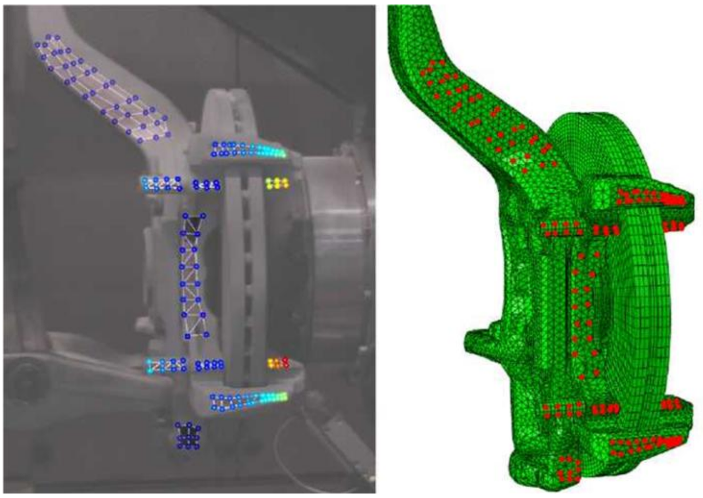

Fig. 2 Correlation of EMA and Analytical FEM model.

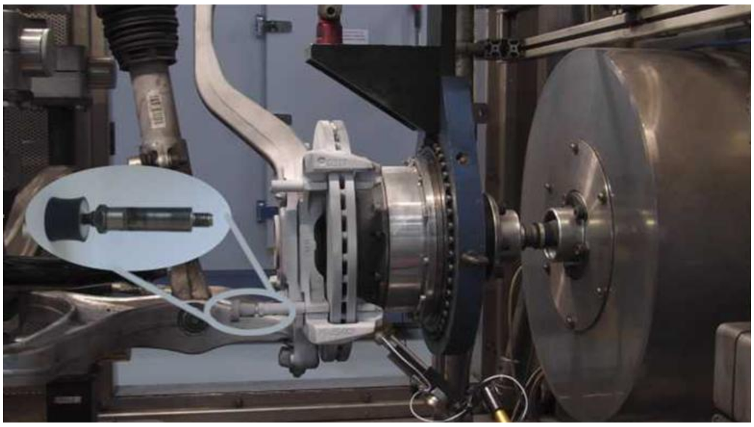

Fig. 3 Experimental modal analysis with weighted bolt.

2.3 Weighted bolt variation

The floating caliper brake assembly is a critical component in modern automotive braking systems, responsible for ensuring efficient and reliable deceleration. In order to improve the performance and stability of this assembly, several variations incorporating weighted bolts have been extensively tested. These weighted bolt variations, weighing 60g, 90g, 120g, 150g, 180g, and 210g, were evaluated to understand their impact on braking effectiveness and overall system dynamics.

Additionally, this study also focuses on investigating the material properties of the steering mechanism, specifically comparing cast iron, steel, and aluminium as potential options for the baseline material. Understanding the behaviour of these materials is crucial for optimizing steering performance, as they directly influence factors such as weight, durability, and corrosion resistance. By comprehensively observing and analysing the properties of cast iron, steel, and aluminium, valuable insights can be gained to inform the selection of the most suitable material for the steering system.

2.4 Complex Eigenvalue analysis

For the CEA in the first step of the simulation process, the surface friction contacts are established and a load is applied to the piston surface in the form of pressure. This ensures that the brake pads clamp the disc properly and that the rotation of the disc is properly controlled. On top of that rotation on the disc is applied. The third step involves a modal analysis, where the natural frequencies are extracted. It is crucial to extract the natural frequencies before performing the extraction of complex eigenvalues with the subspace-projection method, as this is the space where the complex modes are projected. The final step is the complex eigenvalue analysis itself, where the complex eigenvalues of the system were extracted.

- Static analysis for establishing contacts and brake-line pressure.
- Static analysis to impose rotational speed on the disc.
- Modal analysis to extract natural frequency of undamped system.
- Complex eigenvalue analysis that incorporates the effect of friction coupling.

The stability of a system can be analysed by creating a plot that charts the frequency of the system on the ordinate (vertical axis) and the real part of the complex eigenvalue on the abscissa (horizontal axis). The presence of friction within the system leads to an asymmetrical stiffness matrix, resulting in a complex solution.

On the stability chart, the negative modes found on the left side of the plot will be suppressed, while the modes on the right side are considered unstable frequencies. It is important to note that not all of these unstable frequencies will necessarily lead to brake squeal in a real-world scenario, as the complex eigenvalue analysis (CEA) may over-predict the likelihood of this phenomenon. For a system to have a negative damping coefficient, it means that it is not dissipating energy but instead generating energy back into the system. Changing the friction coefficient value will have an impact on the overall stability of the system. The mathematical representation of a multi-degree finite element system that includes mass, stiffness, and damping matrices is given by an equation of the form:

$$[M]\,\{\ddot{x}\} + [C]\,\{\dot{x}\} + [K]\,\{x\} = 0 \tag{3}$$

The Modal Assurance Criterion (MAC) equation can be modified to incorporate nodal weights. In such cases, the weighted Modal Assurance Criterion (wMAC) is used. The wMAC takes into account the importance or significance of each node in the mode shape comparison. The equation for the weighted Modal Assurance Criterion (wMAC) is as follows:

$$\omega MAC(i,j) = \frac{\left|\phi_i^T \cdot \mathbf{W} \cdot \phi_j\right|^2}{(\phi_i^T \cdot \mathbf{W} \cdot \phi_i) \cdot \left(\phi_j^T \cdot \mathbf{W} \cdot \phi_j\right)} \tag{4}$$

The homogeneous, second order matrix differential equation has a complementary solution that has the following shape:

$$\{u\} = \{\phi\} e^{\lambda t} \tag{5}$$

Solving and substituting we get the result in the form of the complex eigenvalue problem.

$$([M]\lambda^2 + [C]\,\lambda + [K])\,\{\phi\} = \{0\} \tag{6}$$

The eigenvalue pair for a given mode is complex. The eigenvalues for under damped systems always appear in complex conjugate pairs in the form:

$$\lambda_i = \sigma_i + j\omega_i \tag{7}$$

The σ denoting the real part of and *w* the imaginary part. σ denoting the damping coefficient and ω the damped natural frequency which describe damped sinusoidal motion. The displacement of the nodes can be rewritten as periodic wave.

$$\{x_i\} = \{\phi_i\} e^{\sigma_i t} cos\omega_i t \tag{8}$$

The following equation is used to define and compute an additional term known as the damping ratio.

$$\xi = \frac{\sigma}{\pi|\omega|} \tag{9}$$

Where ξ is a mode's damping ratio. The brake system is unstable if the damping ratio is negative, and vice versa. The subspace-projection method is being utilized in this study to obtain complex eigenvalues for the analysis of brake squeal. This method assumes that the difference between the real eigenvectors obtained from modal analysis and the ones obtained from CEA is small, and therefore the same subspace can be reused for the CEA. The Complex Eigenvalue Analysis (CEA) is then performed, providing the complex eigenvalues.

3 Results

3.1 EMA

The natural frequencies of the floating caliper brake were determined using experimental modal analysis (EMA). Table 2 presents the identified natural frequencies along with their corresponding mode numbers and damping.

Table 2 EMA modes

Mode n.	Nat. f. (Hz)	Damping	Mode n.	Nat. f. (Hz)	Damping
1	278.5	0.0514	22	2197.4	0.0163
2	381.9	0.0139	23	2306.2	0.0060
3	567.4	0.0185	24	2404.1	0.0097
4	615.9	0.0145	25	2483.4	0.0117
5	748.3	0.0146	26	2566.6	0.0165
6	896.1	0.0200	27	2671.6	0.0077
7	980.8	0.0184	28	2742.0	0.0121
8	1056.6	0.0090	29	3009.7	0.0057
9	1123.6	0.0081	30	3153.2	0.0149
10	1154.6	0.0108	31	3893.8	0.0073
11	1176.4	0.0236	32	4059.0	0.0081
12	1295.9	0.0082	33	4324.0	0.0065
13	1347.5	0.0114	34	4425.8	0.0102
14	1384.8	0.0145	35	4730.6	0.0070
15	1489.7	0.0082	36	5029.4	0.0062
16	1541.7	0.0142	37	5342.2	0.0119
17	1658.5	0.0196	38	5473.4	0.0116
18	1736.9	0.0098	39	5782.2	0.0104
19	1847.3	0.0228	40	5841.8	0.0047
20	2004.1	0.0170	41	6439.5	0.0047
21	2125.1	0.0126			

The natural frequencies measured ranged from 0 Hz to 6000 Hz, indicating a wide range of vibration modes exhibited by the structure. These were obtained by fitting a polynomial on to the FRF Figure 4. The FRFs demonstrated good agreement with the identified natural frequencies and mode shapes, confirming the accuracy of the modal analysis. Of course, not all of the measured natural frequencies will lead to instability, for such the CEA will be used.

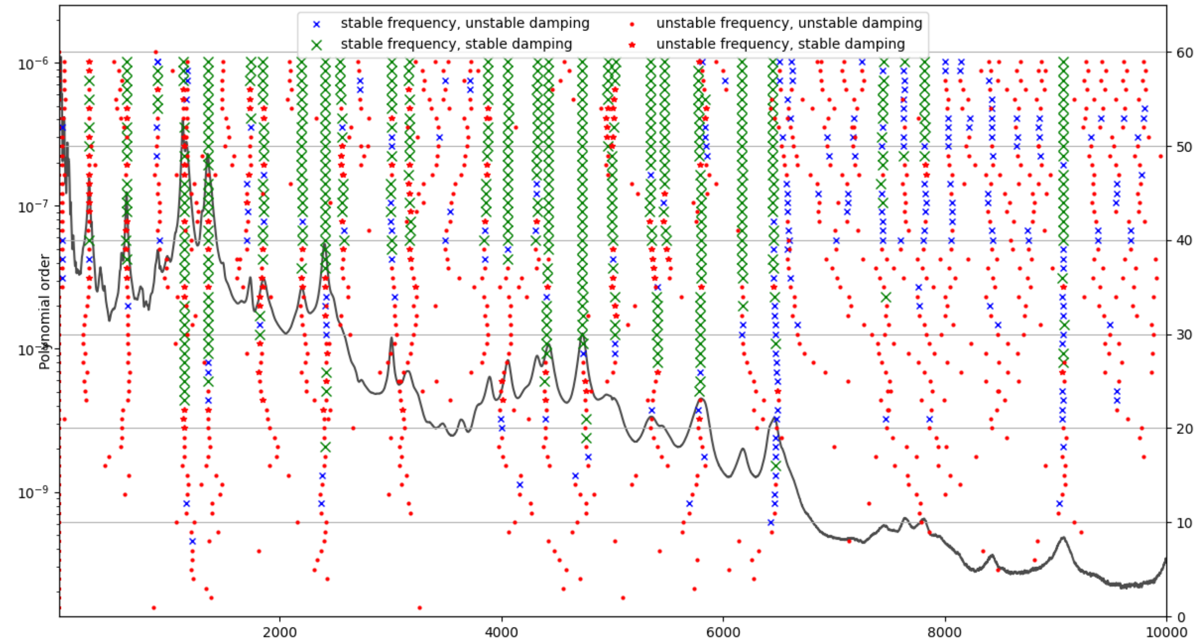

Fig. 4 Response of the experimental system

3.2 CEA

The results of the CEA are similar to that obtained from classical eigenvalue extraction, but with the added benefit of determining system stability. The output consists of frequencies and mode shapes of the system, which have a complex component. This component allows the stability to be determined by analysing the eigenvalues and eigenvectors. The MA found approximately 40 modes. The CEA was able to identify only 8 of those as being unstable. The stability determination is done using the complex component as described in Eq. 7.

The results were generated by performing simulations for three different friction coefficient values of μ = 0.3,0.5, and 0.7 and for an assembly with and without the weighted bolts.

The results of the simulations were presented in the form of stability plots, which are displayed in Figures 5 and 6. These plots represent the unstable modes of the brake system, showing the frequencies at which instability occurs. The degree of instability in the complex eigenvalue analysis stability chart can be determined by considering the location and behavior of the eigenvalues. If the eigenvalues move closer to the unstable region (right-half plane) as the parameters vary, it indicates increasing instability. The degree of instability can be quantified by measuring the distance of the eigenvalues from the imaginary axis or the rate of change of eigenvalues with respect to parameter variations.

In conclusion, the results obtained from the CEA are important in understanding the stability of the system and how it can be impacted by various factors such as friction coefficients and brake pad designs. These findings can then be utilized to improve the design of brakes and enhance the stability of the system.

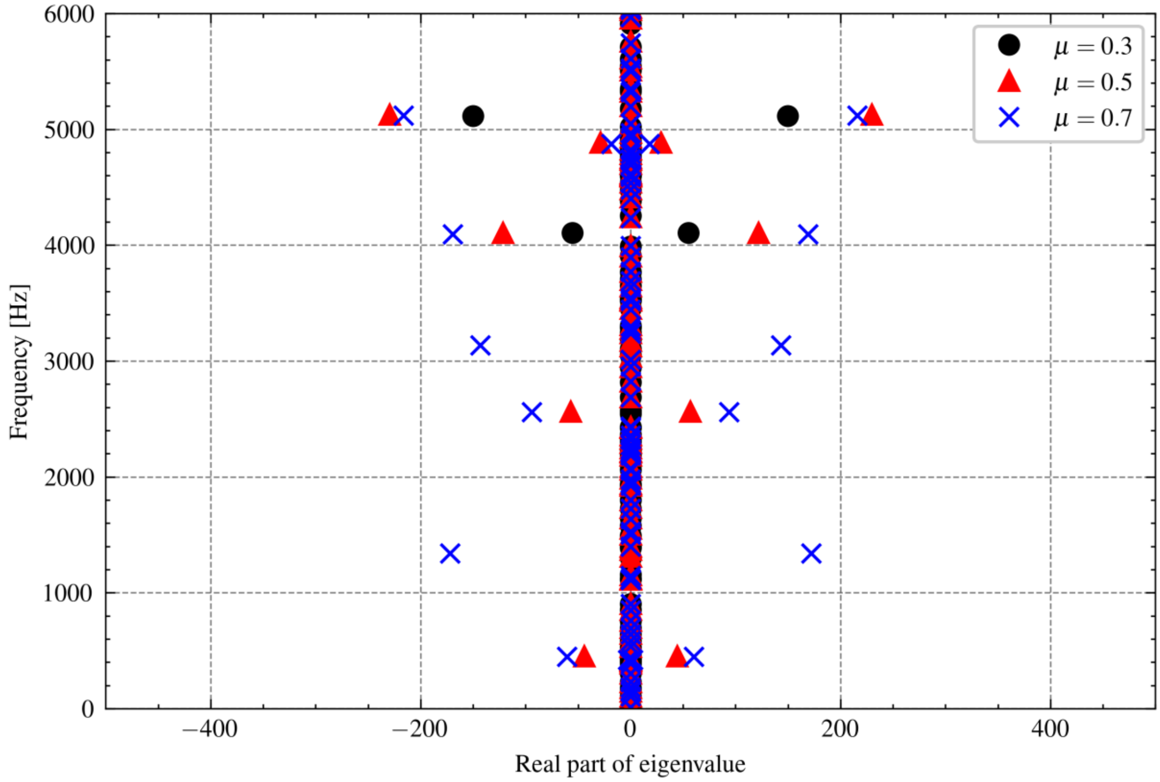

Fig. 5 Stability plot from assembly with the weighted bolt

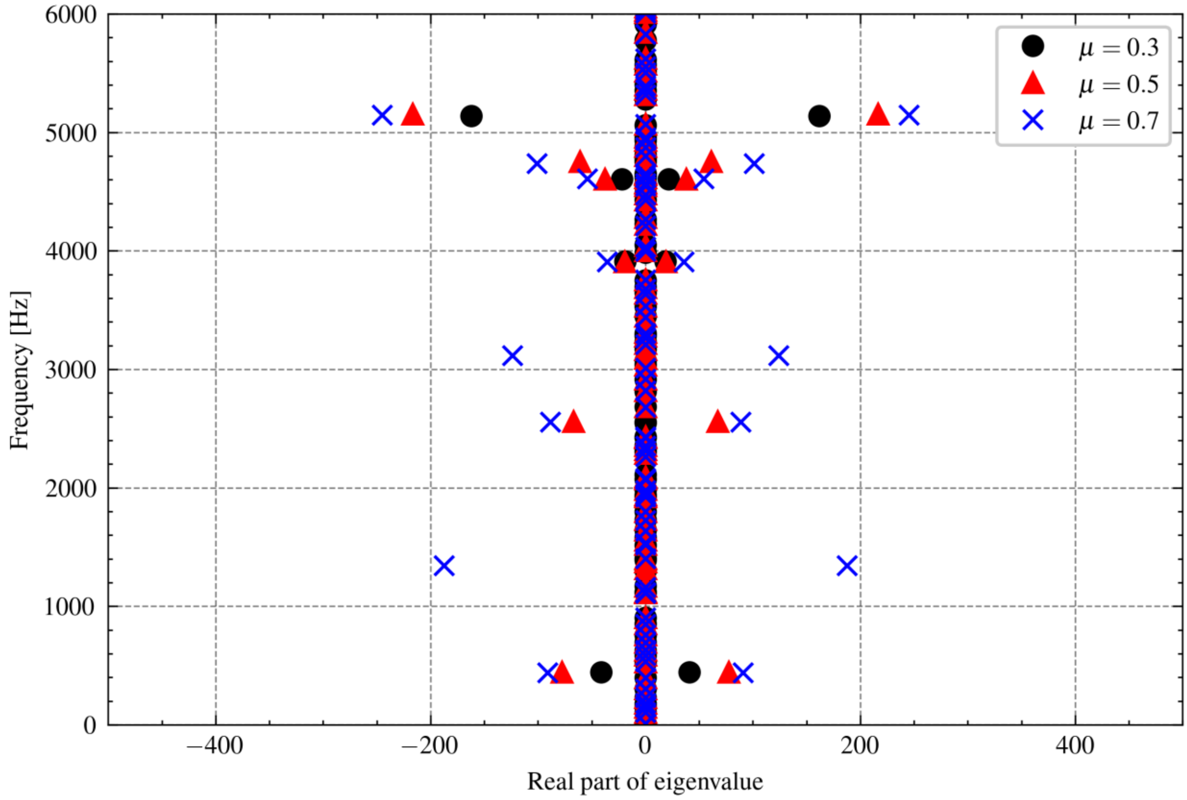

Fig. 6 Stability plot from assembly without the weighted bolts

CONCLUSION

The investigation of the floating caliper brake assembly holds significant importance in comprehending the stability of brake systems, which greatly influences vehicle safety. The primary objective of this study was to assess the effectiveness of altering the modal characteristics of the assembly and breaking up mode shapes by incorporating weighted bolts into the brake assembly's anchor.

The results of the complex eigenvalue analysis showed that the weighted variant changed up the mode shapes of the system. The CEA demonstrates the significance of comprehensively considering all factors that contribute to brake instability, which encompasses aspects such as friction and the shape of the lining material. In the future, conducting transient analysis with a broader range of parameters would be valuable. Transient analysis with a weighted bolt allows for observing the evolution of instability over time and under varying conditions.

The complex eigenvalue analysis employed in this study primarily assumes linear elastic material behaviour, thus it would be for the future to observe the outcomes of similar studies that incorporate non-linear material behaviour. Furthermore, it would be beneficial to explore the effects of environmental conditions, such as temperature and humidity, on brake stability. In summary, future research should include the following aspects:

- The eigenfrequencies exhibited minimal variation among the different weighted bolt variations.
- Also, it was found that just changing the material properties of the steering knuckle alone wasn't significant for variation in natural frequencies to occur.
- The introduction of weighted bolts resulted in modifications to the instability observed in the stability plots derived from the CEA (Complex Eigenvalue Analysis).
- Care should be taken not to introduce additional unstable modes into the system.
- The higher friction coefficient values increase the propensity for unstable frequencies to occur.

The article highlights the importance of considering aspects of brake design when developing brake systems. In short, this study provides a examination of the floating caliper brake assembly and its impact on brake stability, emphasizing the importance of considering all factors when designing brake systems. The results of this study can inform future brake system design and guide ongoing research efforts to improve the safety and reliability of these critical components.

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