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PARAMETER IDENTIFICATION OF LATCHING SOLENOID INCLUDING PERMANENT MAGNETS

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Abstract: This article deals with the measurement and modelling of a latching solenoid containing permanent magnets. The aim of the research is to determine the flux linkage provided by the permanent magnets and the damping, which slows down the movement of the iron core. The differential equations of the electromechanical system are formulated from energy-based approach. In the proposed method, parameters to be identified are determined by drop tests and iterative simulations. The effect of hysteresis is also taken into account.

KEYWORDS: latching solenoid, permanent magnet, parameter identification, electromechanical model, hysteresis

1 Introduction

Latching solenoids including permanent magnets are used in relays in order to minimize the bouncing of the contacts. To establish an electromechanical model of this type of solenoid, it is necessary to determine the initial flux linkage generated by the permanent magnets as a function of the iron core position. In addition, the damping parameter governing the core motion and the time constant of the hysteresis related to magnetization voltage decay must be identified.

The electromechanical model of a latching solenoid including permanent magnets can be derived using energy-based considerations [1]-[3]. The magnetic co-energy function can be obtained as a function of the position of the iron core and the current flowing through the coil by integrating the measured values of the force acting on the iron core [4].

Being aware of the magnetic and kinetic co-energies, and the potential function, the electrical and mechanical differential equations describing the system can be derived using the Lagrange's equation of the second kind [1], [5], [6].

The initially unknown coefficients in the system of equations can be determined by parameter identification. The identification is considered an optimization task, which is mostly based on an iterative process [7], [8]. The identification procedure searches for the best possible values by comparing the measured and simulated results. This article proposes an iterative procedure that minimizes the errors between the measured and simulated values of the peak voltage and drop time obtained from tests also considering the effect of hysteresis in an intuitive way [9].

This paper is organised as follows: Section 2 deals with the theoretical backgrounds of the analysis; Section 3 presents the description of the measurement related to drop tests; Section 4 describes the algorithm of the proposed parameter identification. Finally, the paper is closed with concluding remarks.

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2 Theoretical backgrounds of the analysis

Built-in permanent magnets included in a latching solenoid can hold the iron core in closed position. The flux linkage of permanent magnets induces a voltage through the motion of the iron core even when no external voltage source is connected to the solenoid. This phenomenon is analysed by simulation and experimentally.

The iron core is dropped from a given height into the vertically held solenoid housing while the coil is not energized. At this point, the iron core is only affected by the force of gravity and the pull of the permanent magnet. At the end of the fall, the iron core impacts the base plate and remains in place. The induced voltage resulting from the movement can be measured. The aim of the experiment is to determine the flux of the permanent magnet as a function of the position of the iron core and the resistance that slows down the movement.

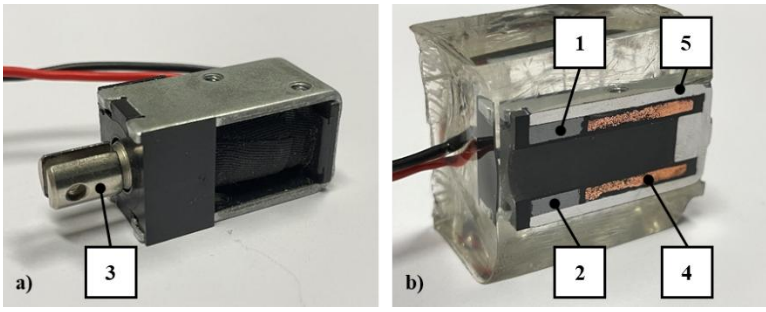

Fig. 1 Latching solenoid containing permanent magnets: a) structure b) cross-section

Fig. 1 presents the investigated latching solenoid with permanent magnets (item 1 and 2) and movable iron core (item 3). Coil of electromagnet (item 4) is winded around the plastic cylinder part which navigates iron core to close magnetic circuit through metal frame (item 5).

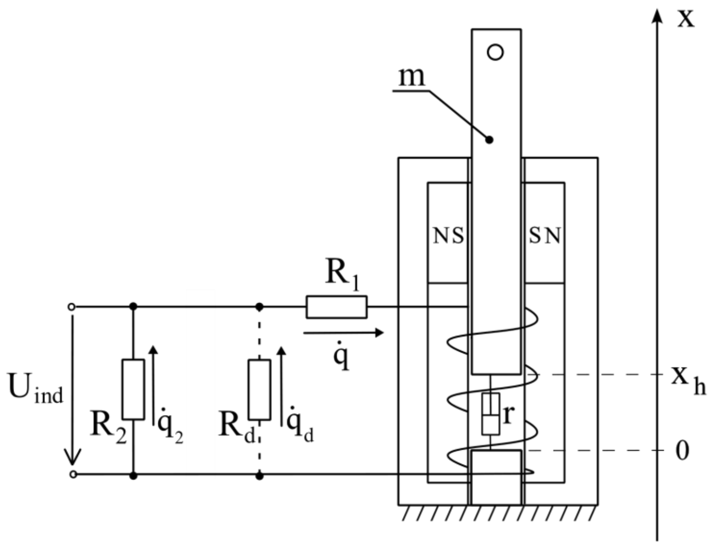

Fig. 2 The electromechanical model of latching solenoid

Fig. 2 shows the sketch of the drop test, where m is the mass of the iron core, x_h is its initial position, r represents the damping of the motion, R_1 is the Ohmic resistance of the coil, R_2 represents the Ohmic resistance of an oscilloscope in the electric circuit on which motion-induced voltage U_{ind} can be measured, permanent magnets are denoted by poles NS. Hysteresis originates from magnetization losses that cause energy dissipation in the system. In the model, a latent resistor R_d connected in parallel with the solenoid forms a dissipative path, allowing the stored magnetic energy to dissipate even when the circuit is open. Electric currents are denoted by $\dot{q}$, $\dot{q}_2$, $\dot{q}_d$.

The voltage generated due to motion-induced electromagnetic induction in accordance with the Faraday–Lenz law. When the resistive losses of the coil are neglected, the sum of mechanical and electrical powers equals to zero ($P = 0$):

$$P = F_{ind}\dot{x} - U_{ind}\dot{q} = 0, \tag{1}$$

where U_{ind} is motion-induced voltage:

$$U_{ind} = \frac{d\lambda_p(x)}{dt} = \frac{d\lambda_p(x)}{dx}\frac{dx}{dt} = \lambda'_p(x)\dot{x}, \tag{2}$$

where $\lambda_p(x)$ is the flux-linkage provided by permanent magnets, and $\lambda'_p(x)$ is the derivative of the flux with respect to position x, the $\dot{q}$ is induced electric current. Substituting (2) into (1) and simplifying by $\dot{x}$, the force resulting from induction is:

$$F_{ind} = \lambda'_p(x)\dot{q}. \tag{3}$$

In the literature [1], [9] known the force acting on the iron core is a function of the square of flux linkage:

$$F_p(x) = \beta\, \lambda_p^2(x). \tag{4}$$

The magnetic force $F_p(x)$ can be measured from which, being aware of parameter β, the flux as a function of the position of the iron core is obtained:

$$\lambda_p(x) = \sqrt{\frac{F_p(x)}{\beta}}. \tag{5}$$

The mathematical model of the latching solenoid can be written using an energy-based approach where, given the magnetic co-energy of the electromagnet and the assumed flux linkage of the permanent magnet. The conservative energy components of the system are contained in the Lagrangian depending on displacement x, velocity $\dot{x}$ and current $\dot{q}$:

$$\mathcal{L}(x, \dot{x}, \dot{q}) = T^*(\dot{x}) + W^*_{m\Sigma}(x, \dot{q}) - V_p(x), \tag{6}$$

where $T^*(\dot{x})$ is the kinetic co-energy, which can be expressed as

$$T^*(\dot{x}) = \frac{1}{2}m\dot{x}^2, \tag{7}$$

and the gravitational potential energy is written as

$$V_p(x) = mgx, \tag{8}$$

furthermore, the magnetic co-energy $W^*_{m\Sigma}(x, \dot{q})$ can be determined experimentally. The current flowing in the system consists of two components (see Fig. 2) in accordance with Kirchhoff's first rule:

$$\dot{q}_{\mathrm{d}} = \dot{q} - \dot{q}_2\,. \tag{9}$$

The non-conservative elements are included in the virtual work increment:

$$\overline{\delta W_{nc}} = -\big(R_1\dot{q} + R_d(\dot{q} - \dot{q}_2) + \lambda'_p(x)\dot{x}\big)\delta q - \big(R_2\dot{q}_2 - R_d(\dot{q} - \dot{q}_2)\big)\delta q_2 + (\lambda'_p(x)\dot{q} - r\dot{x})\delta x. \tag{10}$$

where δq and δx are the virtual increments in the charge and displacements, respectively. Being aware of the Lagrangian and virtual work increment the following Lagrange's equations of the second kind can be obtained:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = -\left(R_1\dot{q} + R_d(\dot{q} - \dot{q}_2) + \lambda_p'(x)\dot{x}\right), \tag{11}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} = \lambda_p'(x)\dot{q} - r\dot{x}, \tag{12}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_2}\right) - \frac{\partial \mathcal{L}}{\partial q_2} = -R_2\dot{q}_2 + R_d(\dot{q} - \dot{q}_2). \tag{13}$$

Substituting (6), (10) into (11), (12), (13) and performing the derivations the following equations have resulted in case of falling iron core:

$$\frac{\partial^2 W_{m\Sigma}^*(x,\dot{q})}{\partial \dot{q}^2}\frac{d\dot{q}}{dt} + \frac{\partial^2 W_{m\Sigma}^*(x,\dot{q})}{\partial x \partial \dot{q}}\frac{dx}{dt} + (R_1 + R_d)\dot{q} = R_d\dot{q}_2 - \lambda_p'(x)\dot{x}, \tag{14}$$

$$m\ddot{x} + r\dot{x} = \frac{\partial W_{m\Sigma}^*(x,\dot{q})}{\partial x} + \lambda_p'(x)\dot{q} - mg, \tag{15}$$

$$0 = -R_2\dot{q}_2 + R_d(\dot{q} - \dot{q}_2). \tag{16}$$

Equation (15) is modified with the contact force F_{cont} when the iron core is impacting with the base of the house:

$$m\ddot{x} + r\dot{x} = \frac{\partial W_{m\Sigma}^*(x,\dot{q})}{\partial x} + \lambda_p'(x)\dot{q} - mg + F_{cont}, \tag{17}$$

where the contact force is expressed as:

$$F_{cont} = -\frac{m\dot{x}(t_n)}{\Delta t} - \alpha g(x). \tag{18}$$

The first term of right-hand side of (18) represents the perfectly inelastic impact force, which effectively stops the motion of the iron core. The second one is a penalty term with penalty parameter α and gap function $g(x) = x$, which maintains the iron core in stationary state.

The attraction force F_p of the permanent magnets (4) acting on the iron core was measured and is shown in Fig. 3 a) as a function of the iron core position. The magnetic co-energy was obtained using a force-measurement-based approach [4] and is shown in Fig. 3 b) as a function of the current and position of the iron core.

After the impact of the iron core, its velocity theoretically becomes zero, causing the motion-induced voltage to vanish. However, according to the experiment, the magnetic co-energy stored at the instant of the voltage peak continues to sustain a decaying current. The time constant τ_d of this current decay is associated with the hysteresis. This parameter is obtained from the voltage decay after the impact of the iron core.

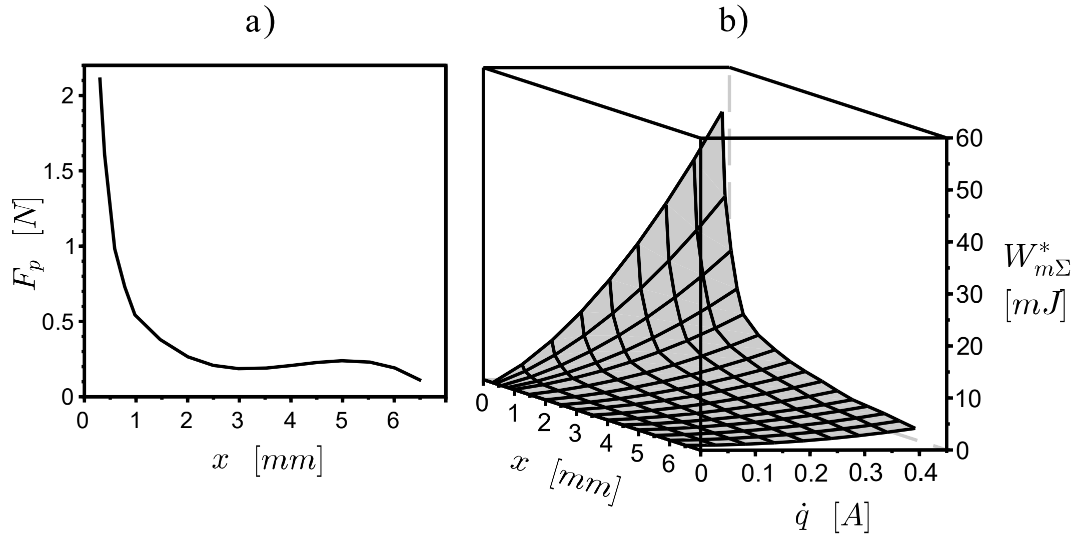

Fig. 3 Measurement setup of drop test

The resistor R_d is chosen such that the circuit's time constant τ_d corresponds to the measured voltage decay curve:

$$R_d = \frac{L - \tau_d R_1}{\tau_d}. \tag{19}$$

Being aware of the magnetic co-energy, a system of equations (14), (15) and (17) can be solved numerically. The voltage drop on Ohmic resistance R_2 is calculated as:

$$U_{ind} = \dot{q}_2 R_2 \, , \tag{20}$$

as well as the displacement of the iron core.

3 Description of the measurement

During a drop, the iron core fell freely from a given height due to gravity and the attraction force of permanent magnets. Fig. 4 shows the setup of the drop experiment. LeCroy 44XI digital oscilloscope with an internal resistance $R_2 = 1M\Omega$ can measure voltage and displacement at the same time (item 1), the Micro-Epsilon OptoNCDT2220 laser distance measuring sensor that monitors the position of the iron core (item 2), the solenoid housing is fixed in a camping unit (item 3).

The scope screen simultaneously shows the measured voltage in yellow and the displacement of the iron core in purple. It can be seen the time variation of the two quantities is significantly different, the voltage changes significantly before the impact. The drop was repeated from different values of height x_h. The drop time t_f and the maximum values of the induced voltage U_{ind} are collected in Table 1.

Table 1 Maximum values of drop tests

No.	x_h $[mm]$	t_f $[ms]$	max (U_{ind}) $[V]$
I	6.808	31.6	9.712
II	5.593	26.32	9.953
III	4.672	23.86	9.539

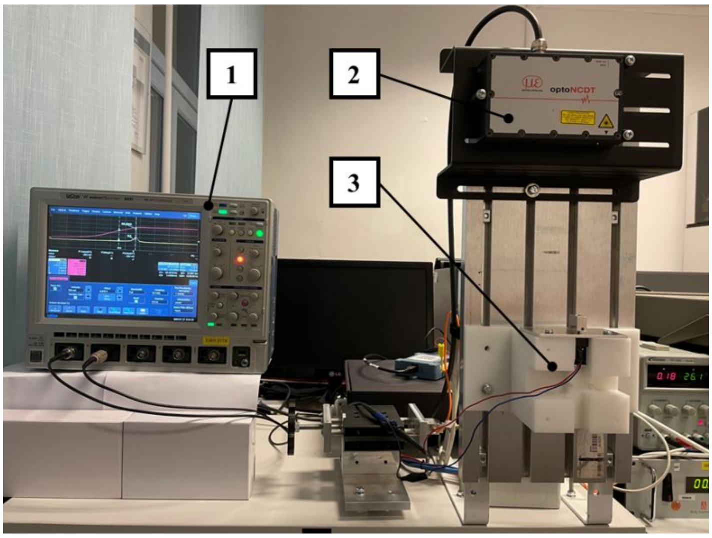

Fig. 4 Measurement setup of drop test

4 Parameter identification

The drop time and the induced voltage are primarily influenced by the damping factor and the β parameter, respectively. After the initial selection of parameters, the values obtained either underestimate or overestimate the measured phenomenon. Applied in two successive iterations, the value of r is found incrementally with a given tolerances ($tol_r = 0.0005, tol_U = 0.00005$) while keeping the value of β constant. The other iteration cycle aims to determine β by the same method while keeping the value of r constant. Then, the two iterations are repeated until both parameters simultaneously satisfy the prescribed conditions. The flowchart of parameter identification is shown in Fig. 5.

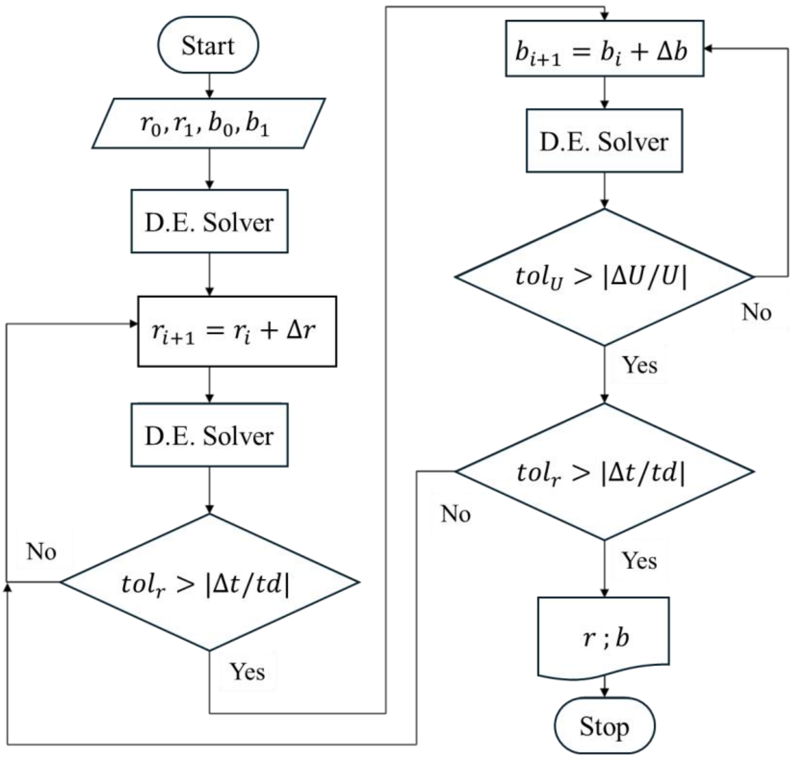

Fig. 5 Flowchart of parameter identification.

Starting the iteration assumes that the results for the drop time and induced voltage have been determined for the value pairs (r_0, b_0) and (r_1, b_1). Then the increments in the damping and the beta parameter can be determined in Fig. 5:

$$\Delta r = r_i \frac{t_m - t_i}{t_i - t_0}, \tag{21}$$

$$\Delta b = b_i \frac{U_m - U_i}{U_i}, \tag{22}$$

where parameter b_i related to β_i as:

$$\beta_i = \frac{1}{{b_i}^2}, \tag{23}$$

provides the magnetic flux linkage in (5). Based on the flowchart Fig. 5, a special purpose computer code was developed in Scilab software. Parameter identification was performed based on the data in Table 1 and the time constant τ_d of current decay, the damping parameter r, parameter β and number of iterations N_{iter} have been determined.

Table 2 Results of the parameter identification

Drop test	$\tau_d [ms]$	$r \left[\frac{Ns}{\mathrm{m}}\right]$	$\beta \left[\frac{N}{V^2 s^2}\right]$	N_{iter}
I	0.382	0.7298	13 053.9	9
II	0.415	0.8478	10 813.9	9
III	0.388	0.8746	11 201.4	9
Average	0.395	0.8174	11 689.7	9

In each case, the solution was found in 9 iteration steps. The parameters identified from three drops at different heights show negligible variation; hence, their average is recommended for simulations.

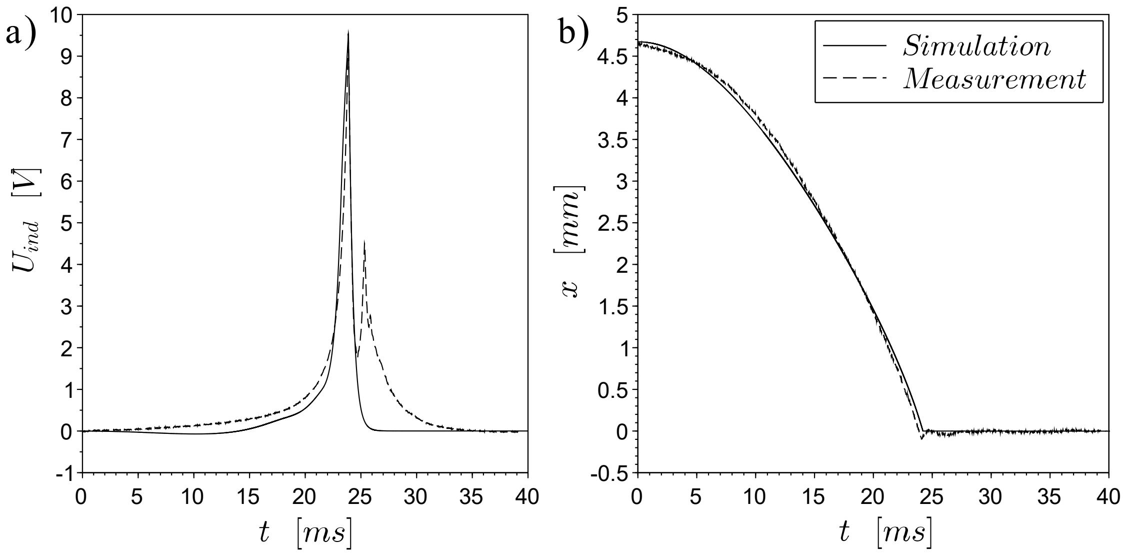

Fig. 6 Drop test. a) Voltage versus time, b) Displacement versus time

In case problem III, the simulated and measured induced voltage and the function of the iron core displacement are illustrated in Fig. 6 a) and b), respectively, where the initial position of the iron core was $4.672\ mm$. Both the voltage and displacement functions are approximately the same and are suitable for parameter identification purposes.

After the iron core impacts in experiment, a small amplitude bounce is observed in the displacement function in Fig. 6 b), which also affects the decaying induced voltage in Fig. 6 a).

In the simulation, the impact was assumed to be perfectly inelastic, therefore the decaying induced voltage is monotonically decreasing in Fig. 6 a).

CONCLUSION

A novel method for parameter identification of latching solenoid with permanent magnets is developed. The differential equations of the electrotechnical system are derived by energy-based approach using the magnetic co-energy as a function of current and the position of iron core. The derived differential equations are complemented by the equation for the dissipation current, which accounts for hysteresis in an intuitive way.

The flux linkage of the permanent magnets of the latching solenoid and the damping of the iron core are obtained by means of drop tests and simulations. The procedure of the parameter identification consists of two consecutive iterations, which provides a convergent process for determining the induced voltage peak value and the occurrence of collision in accordance with the measurements. The functions of induced voltage and the displacements show good agreement comparing experiments and simulations. The average parameters obtained from the proposed method are recommended for use in various simulations of the investigated latching solenoid. The proposed iterative parameter identification procedure demonstrates fast convergence.

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