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# MECHANISMS OF BOLT PRELOAD RELAXATION ARISING FROM IN-PLANE LOADS AND BOLT BENDING

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**Abstract:** Classical analysis methods have shown that flexural, in-plane, deflections due to shear strain in the flange pack of preloaded bolted joints produce bending moments and tensile loads on bolts within the joint. These bending moments and tensile loads are most noticeable in joints that are required to support large in-plane loads and torsional moments. The shear loads on the bolt associated with the flexural deflections can result in the bolt head, or nut, slipping relative to the flanges. This slippage will in turn result in mechanical loosening of the joint. The bending moments associated shear loads and tensile loads produced by the combination of both out-of-plane and in-plane external loads and torsional moments are in addition to the bolt preload. These additional loads, combined with a high initial bolt preload, can cause yielding of the bolt, which can also result in relaxation of the initial preload. This article reviews earlier work in determining the tensile bolt stresses induced in preloaded bolts by the external loads and presents methods of calculating the amount of preload relaxation that could occur from each mechanism discussed.

**KEYWORDS:** bolt bending, preload relaxation, preload loss, bolt loosening, bolt slip,

## Nomenclature

| | |
|---|---|
| $A_b$ | Tensile area of each bolt |
| $A_f$ | Faying surface area |
| $A_j$ | Total area of joint (Faying surface plus bolts) |
| $A_{s.b}$ | Shear area of each bolt |
| $A_\mu$ | Contact area under the bolt head or nut |
| $d_b$ | Nominal bolt diameter |
| $d_i$ | Inner diameter of bolt head or nut contact area |
| $d_o$ | Outer diameter of bolt head or nut contact area |
| $d_{\mu b}$ | Mean effective friction diameter of the bolt head or nut |
| $d\theta_{slip_{(n)}}$ | Angle of bolt head/nut rotation from transverse slip. |
| $D_b$ | Effective diameter of bolt tensile area |
| $D_{f.e}$ | Effective diameter of Rotscher's pressure cone |
| $D_n$ | Basic effective diameter of bolt thread (pitch diameter) |
| $D_s$ | Minor diameter of bolt thread (root diameter) |
| $E_b$ | Young's Modulus of elasticity for bolt material |
| $F_{b_{(n)}}$ | Total axial bolt load on bolt '$n$' |
| $F_{b.crit_{(n)}}$ | Critical axial bolt load on bolt '$n$' |
| $F_p$ | Preload in each bolt |
| $F_{p.low}$ | Lower bound on bolt preload (based on first yield) |

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| | |
|---|---|
| $F_{p.up}$ | Upper bound on bolt preload (based on fully plastic) |
| $F_{s.b_{(n)}}$ | Shear load on bolt '$n$' |
| $F_{s.b.crit_{(n)}}$ | Upper Limiting Shear Force for bolt '$n$' |
| $F_{t.b_{(n)}}$ | Tensile load on bolt '$n$' due to in-plane external loads |
| $F_{t.b.crit_{(n)}}$ | Critical ensile load on bolt '$n$' due to in-plane external loads |
| $F_{ty}$ | Proof stress of bolts |
| $F_z$ | External axial load in direction of '$z$' axis |
| $G_f$ | Shear Modulus for flange material |
| $I_b$ | Second Moment of Area of the tensile area of each bolt |
| $I_b'$ | Second Moment of Area of the tensile area of a bolt |
| $I_{xx.j}$ | Second Moment of Area of joint about '$x$' axis |
| $I_{yy.j}$ | Second Moment of Area of joint about '$y$' axis |
| $J_s$ | Polar Second Moment of Area at minor diameter (root diameter) |
| $k_{jp}$ | Joint pack stiffness |
| $L_f$ | Through flange thickness |
| $L_g$ | Bolt grip length (including washers plus an allowance of one thread pitch) |
| $M_x$ | External moment acting about the '$x$' axis |
| $M_y$ | External moment acting about the '$y$' axis |
| $N_b$ | Number of bolts in the joint |
| $p_{thrd}$ | Thread pitch |
| $P_b$ | Contact pressure under the bolt head or nut |
| $r$ | Radius within bolt section |
| $R_k$ | A stiffness factor |
| $t_{f.min}$ | Thickness of the thinnest outer flange in flange pack |
| $t_w$ | Thickness of washer |
| $T_M$ | Makeup torque |
| $T_{M.t}$ | Torque in bolt shank (locked in torque) |
| $T_\mu$ | Effective torque required to overcome the friction at the nut |
| $x_{(n)}$ | Coordinate of bolt '$n$' |
| $y_{(n)}$ | Coordinate of bolt '$n$' |
| $\propto$ | Flank angle of thread (half the included angle) |
| $\delta_{b_{(n)}}$ | Displacement of bolt '$n$' bolt head normal to bolt axis |
| $\delta_{b.crit_{(n)}}$ | Critical displacement of bolt '$n$' bolt head normal to bolt axis |
| $\delta'_{b.crit_{(n)}}$ | Approximation of critical displacement $\delta_{b.crit_{(n)}}$ |
| $\delta_{b.lim_{(n)}}$ | Limiting displacement of bolt '$n$' bolt head normal to bolt axis |
| $\delta'_{b.lim_{(n)}}$ | Approximation of limiting displacement $\delta_{b.lim_{(n)}}$ |
| $\delta_{b.slip_{(n)}}$ | Bolt head slip |
| $\delta'_{b.slip_{(n)}}$ | Approximation of bolt head slip $\delta_{b.slip_{(n)}}$ |
| $\varepsilon$ | Strain |

| | |
|---|---|
| $\varphi$ | Rotscher's pressure angle |
| $\theta_{p.low}$ | Lower bound on bolt twist angle (based on first yield) |
| $\theta_{p.up}$ | Upper bound on bolt twist angle (based on fully plastic) |
| $\theta_{slip_{(n)}}$ | Total angle of bolt head/nut rotation under load cycling. |
| $\lambda$ | Thread lead angle |
| $\mu_b$ | Dynamic friction coefficient under bolt head or nut |
| $\mu_{b.static}$ | Static friction coefficient under bolt head |
| $\mu_{eff.t}$ | Effective friction coefficient for the thread and nut |
| $\mu_t$ | Dynamic friction coefficient at thread flank |
| $\sigma$ | Stress |
| $\sigma_{b.1st}$ | Bolt tensile stress at first yield |
| $\sigma_{b.r}$ | Bolt tensile stress at radius '$r$' |
| $\sigma_{a.b_{(n)}}$ | Axial stress in bolt '$n$' |
| $\sigma_{b.1_{(n)}}$ | Axial stress at centreline of bolt '$n$' |
| $\sigma_{b.2_{(n)}}$ | Axial stress at thread root of bolt '$n$' |
| $\sigma_{b.3_{(n)}}$ | Axial stress at thread root of bolt '$n$' |
| $\sigma_p$ | Bolt preload stress |
| $\sigma'_p$ | Bolt preload stress after relaxation |
| $\sigma'_{p.\infty}$ | Bolt preload stress after relaxation (infinite stiffness flanges) |
| $\sigma_{s.b_{(n)}}$ | Bending stress component in bolt '$n$' |
| $\sigma_{VM.c_{(n)}}$ | Equivalent (Von Mises) stress at core of bolt '$n$' |
| $\sigma_{VM.r}$ | Equivalent (Von Mises) stress at radius '$r$' |
| $\tau_{b.r}$ | Shear stress in bolt at radius '$r$' |
| $\tau_p$ | Residual shear stress in each bolt from makeup torque |
| $\tau_{xy_{(n)}}$ | Mean shear stress in flange region surrounding bolt '$n$' |
| $\tau_{xy.f_{(n)}}$ | Mean shear stress at the faying surface surrounding bolt '$n$' |

## 1 Introduction

Preloading of bolts within a joint is particularly important when fluctuation in the externally applied loads could lead to fatigue failure of the bolt or when even small relative movements within the joint could lead to malfunctions. In these cases, it is essential that a suitable level of preload is maintained throughout the joint's life. It is known that a number of factors can cause relaxation, or a reduction, of the initial bolt preload produced during the assembly of the joint. These factors include embedment of the bolt head/nut and washer into the outer surfaces of the joint, creep of the bolt material at elevated temperature, mechanical loosening of the bolt due to vibration or other small/macro movements causing the nut to rotate relative to the bolt, and external loads on the joint causing the bolt material to exceed its limit of proportionality.

Loss of preload from factors such as embedment and creep are events that happen in the medium to long term, although some initial embedment will occur during assembly. However, loss of preload from external loads causing mechanical loosening or plastic deformation usually occurs relatively early in the joint's life cycle. The loss of preload due to the bolt material exceeding its limit of proportionality does not preclude the effect of embedment and it may be the initiation of mechanical loosening.

A method of detailed classical analysis of bolt bending within preloaded bolted joints loaded with out-of-plane external loads and moments plus in-plane external loads and torsional moments has been presented by Welch in the paper "*Analysis of Bolt Bending in Preloaded Bolted Joints*" (2018) [1]. Classical analysis methods were used to provide an understanding of what actually happens to preloaded bolts under in-plane loading conditions and to answer the question; what was the true bolt shear stress? It was found that in-plane loads and moments on preloaded bolted joints would produce flexural deflections of the bolts due to shear strain in the flange pack. These deflections produce a shear load and bending moment on the bolt shank. This in turn would produce tensile loads on the bolts that were in addition to the bolt preload and the tensile loads associated with external out-of-plane loads and moments.

It was possible to show that in-plane loads can induce tensile axial loads and bending moments which are additive to the loads produced by out-of-plane loads. These tensile loads and bending moments could, combined with the other loads and moments, potentially exceeding the bolt proof load and limit of proportionality thus cause yielding, producing permanent deformation and extension of the bolt. This in turn can result in a reduction of the bolt preload. It was argued that this offered some explanation of how bolts with a low bending resistance can be prone to self-loosening under transvers load reversals. Even with nut locking there can be a relaxation in bolt preload. The phenomena of loss of preload in bolts is well known. Indeed, VDI 2230 Part 1 "*Systematic calculation of high duty bolted joints with one cylindrical bolt*" [2] suggests that there could be up to 20% reduction in preload in bolts with a low bending resistance, and possibly more.

The article being presented here provides a method of estimating the amount of preload loss in individual bolts due to in-plane external loads and torsional bending moments.

## 2 Mechanical Bolt Loosening

A study of the effects of external in-plane loads on preloaded bolted joints, which included a method of calculating the additional tensile loads generated by the in-plane external loads has been conducted by Welch (2018) [1]. In the article being presented here it is considered that mechanical loosening of bolts may be initiated under this type of external loading. The bolt head could slip, causing mechanical loosening of the bolt, when the bolt load is not sufficient to prevent transverse slip of the bolt head, or nut, under the bolt shear load from the transverse displacement of the bolt head. This bolt head slip is illustrated in Figure (1).

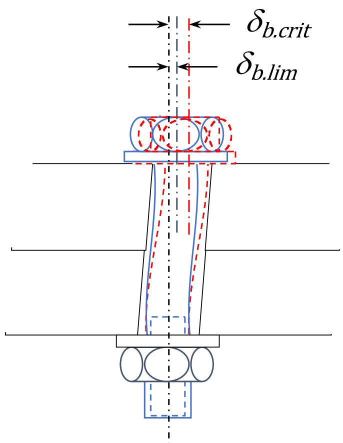


Fig. 1 Bolt head slip

The bolt head transverse deflection, prior to slip, is illustrated as $\delta_{b.crit}$ and the displacement after slip is illustrated as $\delta_{b.lim}$.

Of course, this type of slip is not limited to the bolt head, it could occur at the nut instead. It can be expected that slip would at either the bolt head or the nut, not both. This is because once slip starts to occur at one location the reduction in the friction coefficient as it transitions from static friction to dynamic friction would make this first location of slip the most vulnerable to any further slip. For the purpose of this presentation slip will be regarded as occurring at the bolt head.

## 2.1 Analysis Method for Out-of-Plane Loads

It was shown by Welch (2018) [1] that the total axial load on each bolt of a preloaded bolted joint is given by equation (1):

$$F_{b_{(n)}} = \sigma_{a.b_{(n)}} \cdot A_b + F_{t.b_{(n)}} \tag{1}$$

Where $\sigma_{a.b_{(n)}}$ is the axial stress in a bolt due to the preload and the external out-of-plane loads and moments, $A_b$ is the bolt tensile area, and $F_{t.b_{(n)}}$ is the additional tensile load produced by the external in-plane loads and torsional moments.

Assuming that the theory of bending of beams can be applied to a preloaded bolted flanged connection. the axial stress in any individual bolt due to the out-of-plane loads and moments can be given by equation (2):

$$\sigma_{a.b_{(n)}} = \frac{F_p}{A_b} + \frac{F_z}{A_j} + \frac{M_x}{I_{xx.j}} \cdot y_{(n)} - \frac{M_y}{I_{yy.j}} \cdot x_{(n)} \tag{2}$$

Where $I_{xx.j}$ and $I_{yy.j}$ are the section properties of the joint and $x_{(n)}$ and $y_{(n)}$ are the coordinates of bolt '$n$' defined with respect to the centroid of the bolt group.

Equation (2) follows the "right hand" rule, as illustrated in Figure (2). Loads are positive in the direction of the axes and positive moments act clockwise about the axes when viewed from the origin.

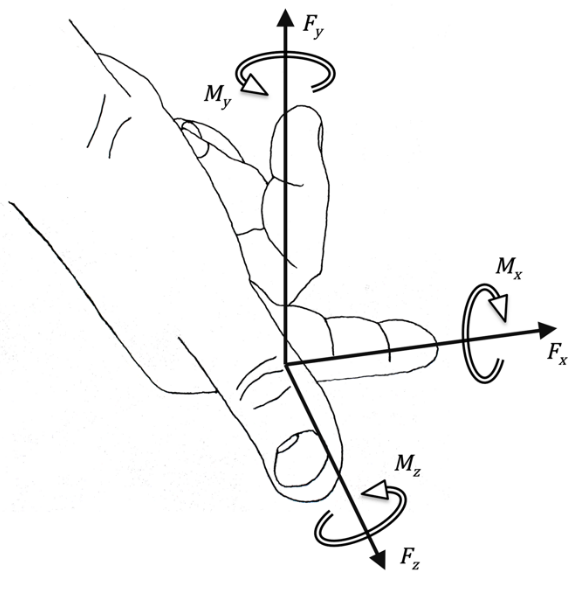


Fig. 2 Right hand rule coordinates

In an Ideal joint, preloading the joint's bolts induces a uniform compressive stress at the faying surface. In practice, the contact pressure will not be uniform across the surface. Each preloaded bolt influences an approximately circular region of the faying surface that surrounds it. The concept of a pressure cone was first proposed by Rotscher (1927) [3]. Figure (3) illustrates Rotscher's pressure cone.

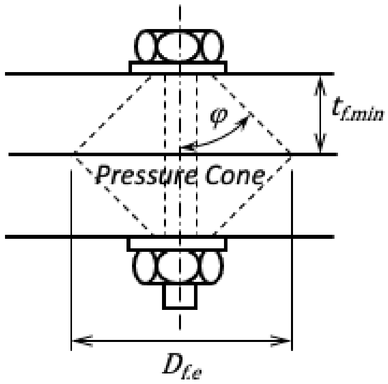


Fig. 3 Rotscher's Pressure Cone

Equation (3) provides an estimate of the diameter of the circular region.

$$D_{f.e} = d_o + 2 \cdot (t_{f.min} + t_w) \cdot tan(\varphi) \tag{3}$$

An experimental study of pressure distributions with the pressure cone have been carried out by Archer (2010) [4]. This shows that a standard washer is not stiff enough to distribute the bolt load across the entire area of the washer. Hence, the washer thickness $t_w$ has been included in equation (3) to account for the restricted load distribution.

Rotscher proposed a half cone angle of $\varphi = 45^o$. However, this has been found to be an overestimate. A more accurate suggestion would be to use a pressure angle of $\varphi = 30^o$.

More detailed discussion of Rotscher's pressure cone is made in sections 3 and 5 of reference [2] and by Budynas and Nisbett (2006) [5].

The bolt axial stresses calculated from equation (2) are influenced by the section properties $I_{xx.j}$ and $I_{yy.j}$. These section properties can be calculated assuming the contact surface consists of the circular regions produced by Rotscher's pressure cone, as described above.

### 2.2 Analysis Method for In-Plane Loads

Figure (4) illustrates the way Welch (2018) [1] assumed the external in-plane loads would be reacted into the bolts. The shear stresses in the flanges produced by the external in-plane loads and moment resulted in transverse deflections at the bolt head, relative to the nut, perpendicular to the bolt axis. The shear loads producing these deflections were transmitted by friction under the bolt head/nut. The through thickness stiffness of the flanges was assumed to be significantly greater than the flexural stiffness of the bolts. Therefore, it was assumed that there was no flexural rotation of the bolt head or nut.

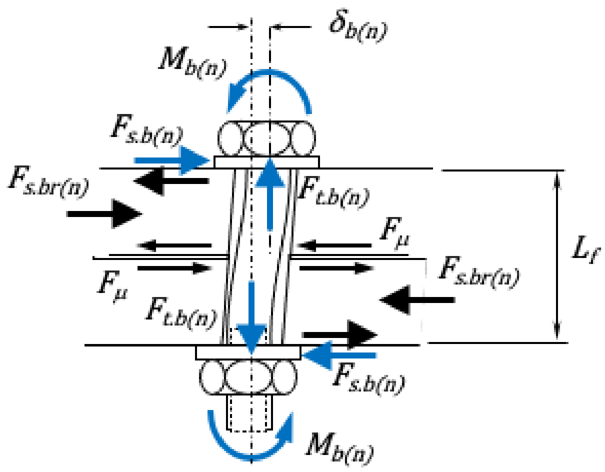


Fig. 4 Joint In-Plane loads reacted into a bolt

The friction loads under the bolt head/nut, $F_{s.b_{(n)}}$, were normal to the axis of the bolt and produce bending moments on the bolt at the head and nut and a shear load on the shanks of each bolt within the joint. These shear loads existed even though the bolts did not bear on the sides of the holes in the flanges.

It was shown by Welch (2018) [1] that the transverse deflection of the bolt head could be given in terms of the mean shear stress acting in the flange material local to the bolt. This is given by equation (4):

$$\delta_{b_{(n)}} = \frac{\tau_{xy_{(n)}} \cdot L_f}{G_f} \tag{4}$$

Where $\tau_{xy_{(n)}}$ is the mean, through thickness, shear stress in flange region surrounding bolt '$n$', $L_f$ is the through flange thickness, and $G_f$ is the Shear Modulus for the flange material.

The through flange thickness $L_f$ comprises the parts of the flange pack that are active in transmitting the shear load through the joint. It does not include the washers.

Using equation (4), it was possible to show that the shear load perpendicular to the bolt shank is given by equation (5):

$$F_{s.b_{(n)}} = \frac{12 \cdot E_b \cdot I_b}{{L_g}^3} \cdot \delta_{b_{(n)}} \tag{5}$$

Where $E_b$, $G_f$ and $I_b$ are Young's Modulus of elasticity for bolt material, the Shear Modulus for the flange material, and the second moment of area of the bolt respectively.

The grip length $L_g$ is the total length of bolt, including washers. It is recommended that an additional allowance of one bolt thread pitch is included as part of the grip length. This additional allowance is to account for flexibility of the thread within the nut.

Welch (2018) [1] used the mean shear stress surrounding the faying surface as the mean, through thickness, flange shear stress, $\tau_{xy_{(n)}}$. This is a conservative assumption that can lead to overestimates of the transverse deflection of the bolt head, $\delta_{b_{(n)}}$, and bolt shear load, $F_{s.b_{(n)}}$. A more accurate value for the mean, through thickness, shear stress would be to use the arithmetic mean of the shear stress at the faying surface local to the bolt and the shear stress in the flange surface under the bolt head, or nut. This arithmetic mean, through thickness, shear stress is given by:

$$\tau_{xy_{(n)}} = \frac{1}{2}\left(\tau_{xy.f_{(n)}} + \frac{F_{s.b_{(n)}}}{A_\mu}\right)$$

Where $\tau_{xy.f_{(n)}}$ is the mean shear stress in the region of faying surface surrounding bolt '$n$', $F_{s.b_{(n)}}$ is the shear load perpendicular to the bolt, as given by equation (5), and $A_\mu$ is the contact area between the flange pack outer surface and the bolt head, or nut.

Using equations (4) and (5) in this function for the mean, through thickness, shear stress it can be shown that:

$$\tau_{xy_{(n)}} = \frac{\tau_{xy.f_{(n)}}}{\left(2 - \frac{12 \cdot E_b \cdot I_b}{L_g{}^3 \cdot A_\mu} \cdot \frac{L_f}{G_f}\right)} \quad (6)$$

Calculating the flange shear stresses is discussed in more detail in Section 2.3 of this article, titled "Flange Shear Stresses".

The second moment of area of a bolt threaded along its full length can be given by equation (7a):

$$I_b = \frac{A_b{}^2}{4 \cdot \pi \cdot} \quad (7a)$$

For bolts with a plain shank and threaded section a mean effective Second Moment of Area could be used. The mean effective Second Moment of Area is given by equation (7b):

$$I'_b = \frac{L_g}{\sum_i \frac{L_{(i)}}{I_{(i)}}} \quad (7b)$$

Where, $I_{(i)}$ and $L_{(i)}$ are the Second Moment of Area and Length of each element of the bolt.

The shear load on the bolt $F_{s.b_{(n)}}$ also produces a bending moment on the bolt which in turn generates a bending stress. The maximum bending stress in the bolt shank can then be calculated using the equation:

$$\sigma_{s.b_{(n)}} = \frac{F_{s.b_{(n)}} \cdot L_g}{2 \cdot I_b} \cdot \frac{D_b}{2} \quad (8)$$

This bending stress would be local to the bolt head and/or nut and can be considered as acting at a location under the surface of the bolt shank, just below the region of residual stress at the thread root.

Using the term for displacement presented in equation (4), Welch (2018) [1] was able to show that, due to the flexure of the bolt shank, an additional tensile bolt load component would be produced by the in-plane external loads. This additional bolt tensile load produced by the in-plane external loads is given by:

$$F_{t.b_{(n)}} = \frac{F_{s.b_{(n)}}^2 \cdot L_g{}^4 \cdot A_b}{240 \cdot E_b \cdot I_b{}^2} \quad (9a)$$

The additional tensile bolt load given by equation (9a) assumes an infinitely stiff joint pack. Allowing for the stiffness of the joint pack the equation can be rewritten as:

$$F_{t.b(n)} = \frac{{F_{s.b}}_{(n)}^{2} \cdot {L_g}^{4} \cdot A_b}{240 \cdot E_b \cdot {I_b}^{2} \cdot \left(1 + \frac{A_b \cdot E_b}{k_{jp} \cdot L_g}\right)} \tag{9b}$$

Where $k_{jp}$ is the through flange thickness stiffness of the joint pack under the bolt head.

Neglecting the effects of the joint pack stiffness, by using equation (9a) to calculate the additional bolt tensile load instead of equation (9b), will result in a slightly conservative (high) tensile load component.

An estimate of the through thickness stiffness of the joint pack under each bolt can be found using equation (10a).

$$k_{jp} = \frac{A_f \cdot E_f}{N_b \cdot L_f} \tag{10a}$$

There is an implied assumption within equation (10a) that the bolts are uniformly distributed within the joint. It also ignores the effect that Rotscher's pressure cone has on the through flange stiffness. A more accurate value for the through thickness stiffness of the joint pack can be calculated using equation (10b).

$$k_{jp} = \frac{\pi \cdot E_f \cdot D_h \cdot tan(\emptyset)}{2 \cdot \ln\left(\frac{(L_f \cdot tan(\emptyset) + D_{o.eff} - D_h) \cdot (D_{o.eff} + D_h)}{(L_f \cdot tan(\emptyset) + D_{o.eff} + D_h) \cdot (D_{o.eff} - D_h)}\right)} \tag{10b}$$

Where $D_{o.eff}$ is the effective contact diameter between the washer and the flange and is given by:

$$D_{o.eff} = d_o + 2 \cdot t_w \cdot tan(\emptyset) \tag{11}$$

Equation (10b) is based on part 3 of section 8.5 of reference [5]. The equation given by Budynas and Nisbett (2006) [5] has been modified to represent the total through flange stiffness, not the stiffness of Rotscher's pressure cone for a single flange.

Other methods of calculating the joint pack stiffness are suggested in section 5.1.2 of reference [2], and ESDU 85021 (2005) [6]. Alternative methods for determining the stiffness of bolted joints have also been proposed by Canyurt and Sekercioglu (2016) [7] and Routh and Das (2016) [8].

### 2.3 Critical Slip Condition Analysis

When preloaded bolted joints are assembled the make-up torque produces a load in the bolt shank that is reacted into the flanges being connected. These loads are highly localised at the bolt head and nut but become defused below the outer surfaces of the flange, as illustrated in Figure (3). Prior to any external loads being applied to the joint, the contact pressure at the faying surface balances the loads at the bolt head and nut.

If out-of-plane external compressive loads, and/or external moments that produce a compressive load in the region of the bolt, are also applied in conjunction with the in-plane loads, the bolt load at the bolt head and at the nut will be reduced. At the same time, the pressure at the faying surface will be increased. It is thought that there is a real possibility that, under a combination of high out-of-plane compressive loads and high in-plane shear stress, the resulting bolt tensile load could reduce to a level where friction at the bolt head is unable to sustain the transvers shear load on the bolt and either the bolt head or nut will slip. This slip at the bolt head/nut would not be accompanied by slip between the elements of the joint, the flanges, because friction and the increased contact pressure at the faying surface will prevent any movement.

The condition to prevent transverse slip that could lead to mechanical loosening can be described by equation (12a).

$$F_{s.b_{(n)}} \leq \mu_{b.static} \cdot F_{b_{(n)}} \tag{12a}$$

Where $F_{s.b_{(n)}}$ is the shear load on bolt '$n$', $F_{b_{(n)}}$ is the total axial bolt load on bolt '$n$' and, $\mu_{b.static}$ is the static friction coefficient under the bolt head.

The critical transverse deflection of the bolt head/nut is at the point when bolt head slip can first occur. Hence, referring to equation (12a), it can be seen that the 'Critical Shear Force' is given by:

$$F_{s.b.crit_{(n)}} = \mu_{b.static} \cdot F_{b_{(n)}} \tag{12b}$$

Note that both the 'Critical Shear Force' $F_{s.b.crit_{(n)}}$ and the total bolt load $F_{b_{(n)}}$ are dependent on a critical displacement $\delta_{b.crit_{(n)}}$.

If the shear force $F_{s.b_{(n)}}$ exceeds the 'Critical Shear Force' $F_{s.b.crit_{(n)}}$, the condition defined by equation (12a) no longer holds true and bolt slip will occur.

Referring to equation (1) to replace the term for $F_{b.crit_{(n)}}$ in the critical condition described by equation (12b):

$$F_{s.b.crit_{(n)}} = \mu_{b.static} \cdot \left(\sigma_{a.b_{(n)}} \cdot A_b + F_{t.b_{(n)}}\right) \tag{13}$$

Again, the tensile load $F_{t.b_{(n)}}$ produced by in-plane shear will be dependent on the critical displacement $\delta_{b.crit_{(n)}}$.

Using equation (9b) in equation (13) and simplifying:

$$F_{s.b.crit_{(n)}} = \mu_{b.static} \cdot A_b \cdot \left(\sigma_{a.b_{(n)}} + \frac{F_{s.b.crit_{(n)}}^{\,2} \cdot L_g^{\,4}}{240 \cdot E_b \cdot I_b^{\,2} \cdot \left(1 + \frac{A_b \cdot E_b}{k_{jp} \cdot L_g}\right)}\right)$$

Notice that the term which represents the component of tensile load on the bolt due to in-plane external loads $F_{t.b_{(n)}}$ has been written in term of the 'Critical Shear Force' $F_{s.b.crit_{(n)}}$.

Referring to equation (5) to replace the term for $F_{s.b.crit_{(n)}}$ and simplifying:

$$\frac{12 \cdot E_b \cdot I_b}{L_g^{\,3}} \cdot \delta_{b.crit_{(n)}} = \mu_{b.static} \cdot A_b \cdot \left(\sigma_{a.b_{(n)}} + \frac{3 \cdot E_b \cdot \delta_{b.crit_{(n)}}^{\,2}}{5 \cdot L_g^{\,2} \cdot R_k}\right) \tag{14}$$

Where $\delta_{b.crit_{(n)}}$ is the critical deflection, the deflection at which slip occurs. Also:

$$R_k = \left(1 + \frac{A_b \cdot E_b}{k_{jp} \cdot L_g}\right) \tag{15}$$

And $k_{jp}$ is the through flange stiffness of the joint. If $k_{jp}$ is infinite, $R_k = 1$.

Rearranging equation (14):

$$0 = {\delta_{b.crit_{(n)}}}^2 - \frac{20 \cdot I_b \cdot R_k}{L_g \cdot A_b \cdot \mu_{b.static}} \cdot \delta_{b.crit_{(n)}} + \frac{5}{3} \cdot {L_g}^2 \cdot R_k \cdot \frac{\sigma_{a.b_{(n)}}}{E_b}$$

Solving the quadratic equation:

$$\delta_{b.crit_{(n)}} = \frac{10 \cdot I_b \cdot R_k}{L_g \cdot A_b \cdot \mu_{b.static}} - \sqrt{\left(\frac{10 \cdot I_b \cdot R_k}{L_g \cdot A_b \cdot \mu_{b.static}}\right)^2 - \frac{5}{3} \cdot {L_g}^2 \cdot R_k \cdot \frac{\sigma_{a.b_{(n)}}}{E_b}} \qquad (16a)$$

If the bolt head/nut slips there is potential for the bolt or nut to rotate and loosen.

As a simplified approximation, it can be assumed that the term for $F_{t.b_{(n)}}$ in equation (13) is small. Hence:

$$F_{s.b.crit_{(n)}} = \mu_{b.static} \cdot \sigma_{a.b_{(n)}} \cdot A_b$$

Again, referring to equation (5) to replace the term for $F_{s.b.crit_{(n)}}$:

$$\frac{12 \cdot E_b \cdot I_b}{{L_g}^3} \cdot {\delta'}_{b.crit_{(n)}} = \mu_{b.static} \cdot \sigma_{a.b_{(n)}} \cdot A_b$$

Where ${\delta'}_{b.crit_{(n)}}$ approximates of the critical deflection $\delta_{b.crit_{(n)}}$. Then, rearranging:

$${\delta'}_{b.crit_{(n)}} = \frac{\mu_{b.static} \cdot \sigma_{a.b_{(n)}} \cdot A_b \cdot {L_g}^3}{12 \cdot E_b \cdot I_b} \qquad (16b)$$

If the critical deflection $\delta_{b.crit_{(n)}}$ (or ${\delta'}_{b.crit_{(n)}}$ for the approximated solution) is reached either the bolt head or nut will start to slip. Once the bolt head or nut starts to slip the dynamic friction coefficient will come into play. The friction coefficient under the bolt head/nut will change from the static friction coefficient $\mu_{b.static}$ to the dynamic friction coefficient $\mu_b$. As a result, the bolt head/nut displacement will reduce to the limiting displacement $\delta_{b.lim_{(n)}}$. The bolt/nut can be expected to slip until the shear force, or bending resistance, associated with the now reduced bolt head transverse displacement equates to the 'Lower Limiting Shear Force'. At that point friction should be able to arrest any further slip.

When the joint was assembled some of the torque required to tighten the joint will have been 'locked' into the bolt shank due to the friction under the nut (or bolt head, depending on which is rotated). During slip the Jost effect could allow some of the 'locked in' torque on the bolt to be relaxed. However, as the bolt was being tightened the dynamic friction coefficient would have been in play. As soon as the bolt tightening procedure was completed the static friction coefficient would 'secure' the bolt. Undoing this 'secured' bolt would require a greater torque than had been used to tighten it. Hence, it could be argued that any loss of locked in torque due to the Jost effect is likely to be negligible.

Referring to equation (16a), the limiting deflection is given by replacing the static friction coefficient $\mu_{b.static}$ with the dynamic friction coefficient $\mu_b$. Hence:

$$\delta_{b.lim_{(n)}} = \frac{10 \cdot I_b \cdot R_k}{L_g \cdot A_b \cdot \mu_b} - \sqrt{\left(\frac{10 \cdot I_b \cdot R_k}{L_g \cdot A_b \cdot \mu_b}\right)^2 - \frac{5}{3} \cdot {L_g}^2 \cdot R_k \cdot \frac{\sigma_{a.b_{(n)}}}{E_b}} \qquad (17a)$$

Similarly, referring to equation (16b) the limiting deflection can be approximated by:

$${\delta'}_{b.lim_{(n)}} = \frac{\mu_b \cdot \sigma_{a.b_{(n)}} \cdot A_b \cdot {L_g}^3}{12 \cdot E_b \cdot I_b} \qquad (17b)$$

Therefore, the amount of bolt slip from the initial position is given by:

$$\delta_{b.slip_{(n)}} = \delta_{b_{(n)}} - \delta_{b.lim_{(n)}} \tag{18a}$$

Or, using the approximated values:

$$\delta'_{b.slip_{(n)}} = \delta_{b_{(n)}} - \delta'_{b.lim_{(n)}} \tag{18b}$$

Where the theoretical maximum displacement $\delta_{b_{(n)}}$ is given by equation (4).

$$\delta_{b_{(n)}} = \frac{\tau_{xy_{(n)}} \cdot L_f}{G_f}$$

The slip given by equation (18a) or equation (18b) could be an initial slip. If in-plane loads are increased, or if out-of-plane loads change and reduce tensile load on a bolt, a second incident of slip could occur. Any analysis of this further slip would need to reflect the reduced shear load from initial slip.

The amount of bolt slip $\delta_{b.slip_{(n)}}$ can be used to make an estimate of the amount of bolt head/nut rotation $d\theta_{slip_{(n)}}$ that could occur.

$$d\theta_{slip_{(n)}} = \arcsin\left(\frac{\delta_{b.slip_{(n)}}}{d_{\mu b}}\right) \tag{19}$$

Where $d_{\mu b}$ is the mean effective diameter of the contact area under the bolt head or nut.

The lateral slip $\delta_{b.slip_{(n)}}$ and the associated rotation $d\theta_{slip_{(n)}}$ can be expected to be on a macroscopic scale. However, under cyclic loading, where the direction of the lateral slip $\delta_{b.slip_{(n)}}$ could be reversed, the rotation of the bolt, relative to the nut, would be incremental and the total rotation would increase with each load cycle. This increasing total rotation $\theta_{slip_{(n)}}$ would result in gradual loosening of the bolt and loss of preload.

The effective torque required to overcome the friction at the nut, or bolt head, can be given by:

$$T_\mu = \mu_b \cdot P_b \cdot A_\mu \cdot \frac{d_{\mu b}}{2}$$

Where $P_b$ is the contact pressure under the nut or bolt head, $A_\mu$ is the annular contact area of the nut or bolt head.

Similarly, the torque required to overcome the friction at the nut or bolt head can also be given by:

$$T_\mu = \int_{\frac{d_i}{2}}^{\frac{d_o}{2}} 2 \cdot \pi \cdot r^2 \cdot \mu_b \cdot P_b \cdot dr$$

Where $d_o$ and $d_i$ are the outer and inner diameters respectively of the bolt head or nut contact area and $\mu_b$ is the dynamic friction coefficient for the bolt head/nut.

Equating the two terms for $T_\mu$ and simplifying.

$$A_\mu \cdot \frac{d_{\mu b}}{2} = 2 \cdot \pi \int_{\frac{d_i}{2}}^{\frac{d_o}{2}} r^2 \cdot dr$$

Then:

$$d_{\mu b} = \frac{\pi}{6 \cdot A_\mu}\left({d_o}^3 - {d_i}^3\right)$$

And using $A_\mu = \frac{\pi}{4} \cdot \left({d_o}^2 - {d_i}^2\right)$

$$d_{\mu b} = \frac{2}{3}\left(\frac{{d_o}^3 - {d_i}^3}{{d_o}^2 - {d_i}^2}\right) \tag{20}$$

The loss of preload resulting from mechanical loosening can be mitigated by the use of locking systems that prevent the bolt head and nut from rotating relative to each other.

## 3 Flange Shear Stresses

The torsional stiffness of the joint is dominated by the second polar moment of area of the flange cross-section. The shear stresses in the bolt shanks are as a result of bending stiffness of the bolts. Hence, the shear stresses in the bolt shanks are significantly less than the shear stresses at the regions of flange face surrounding the bolts. Therefore, the influences of the bolt cross sections have on the torsional stiffness of the joint are limited by the bending stiffness of the bolts.

Welch (2018) [1] assumes an average shear stress distribution for the flange pack. He assumes that the in-plane loads are reacted into the flanges across the entire outer surface of the flange pack. he also assumes the shear stress component produced by torsional moments are determined by the second polar moment of area of the joint and the radial distance from the centroid of the joint. In-plane side, or shear, loads are assumed to produce a uniform shear stress component. Again, these are assumed to be reacted into the flange across the entire outer surface of the flange.

In reality the in-plane loads at the outer surfaces of the flanges will be concentrated at the regions where the flanges are attached to the structure. This concentration will result in highly localised shear stresses at these connections. This localised shear stress concentration will be diffused through the thickness of the flange, tending towards the value of the assumed average shear stress.

The Rotscher's pressure cones will also influence the way the shear stress is dispersed, helping to produce a more uniform shear stress distribution at the faying surface. This dispersion of the shear stress is illustrated in Figure (5).

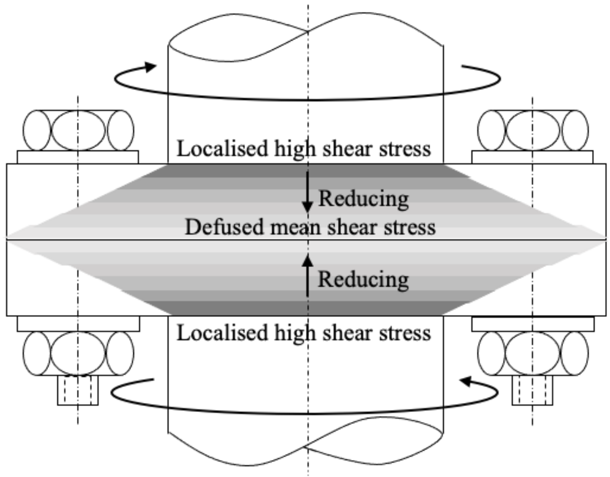

Fig. 5 Shear stress dispersion

Generally, torsional shear stresses are usually calculated using the second polar moment of area of the joint and the radial distance from the centroid of the section. However, it is known that for non-circular sections the ‘torsional constant’ is always less than the second polar moment of area and that the maximum shear stress is not necessarily at the maximum distance from the centroid. For example, for a rectangular cross-section the maximum shear stress is at the mid-positions of the longest sides, as illustrated in Figure (6).

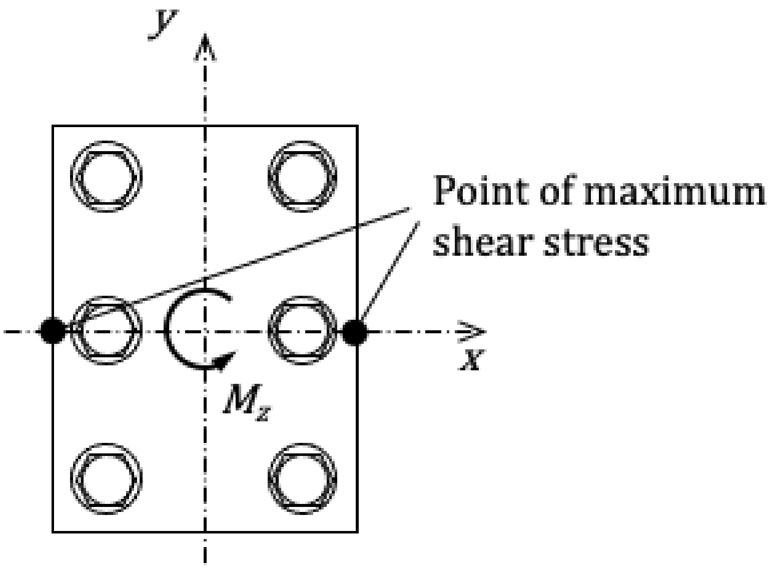

Fig. 6 Position of maximum shear stress from torsion of a rectangular section

Also, the maximum shear stress for non-circular sections is always greater than that predicted using the second polar moment of area. Hence, it is recommended that the calculation of the bolt head flexural displacement, the resulting bending moment and additional tensile stress in the bolt should be calculated using an appropriate torsional constant for the flange geometry. The shear stresses used in this calculation should also include the addition of the shear stress component produced by the direct in-plane loads.

## 4 Loss of Preload from Plastic Deformation

When bolts are assembled by 'Yield Controlled Tightening' or 'Angle controlled Tightening' external loads, applied after tightening, could cause the bolt loads to exceed the bolt material proof stress. If this occurs there will be some loss of the bolt's initial preload. Figure (7) illustrates a method of estimating the loss of bolt preload.

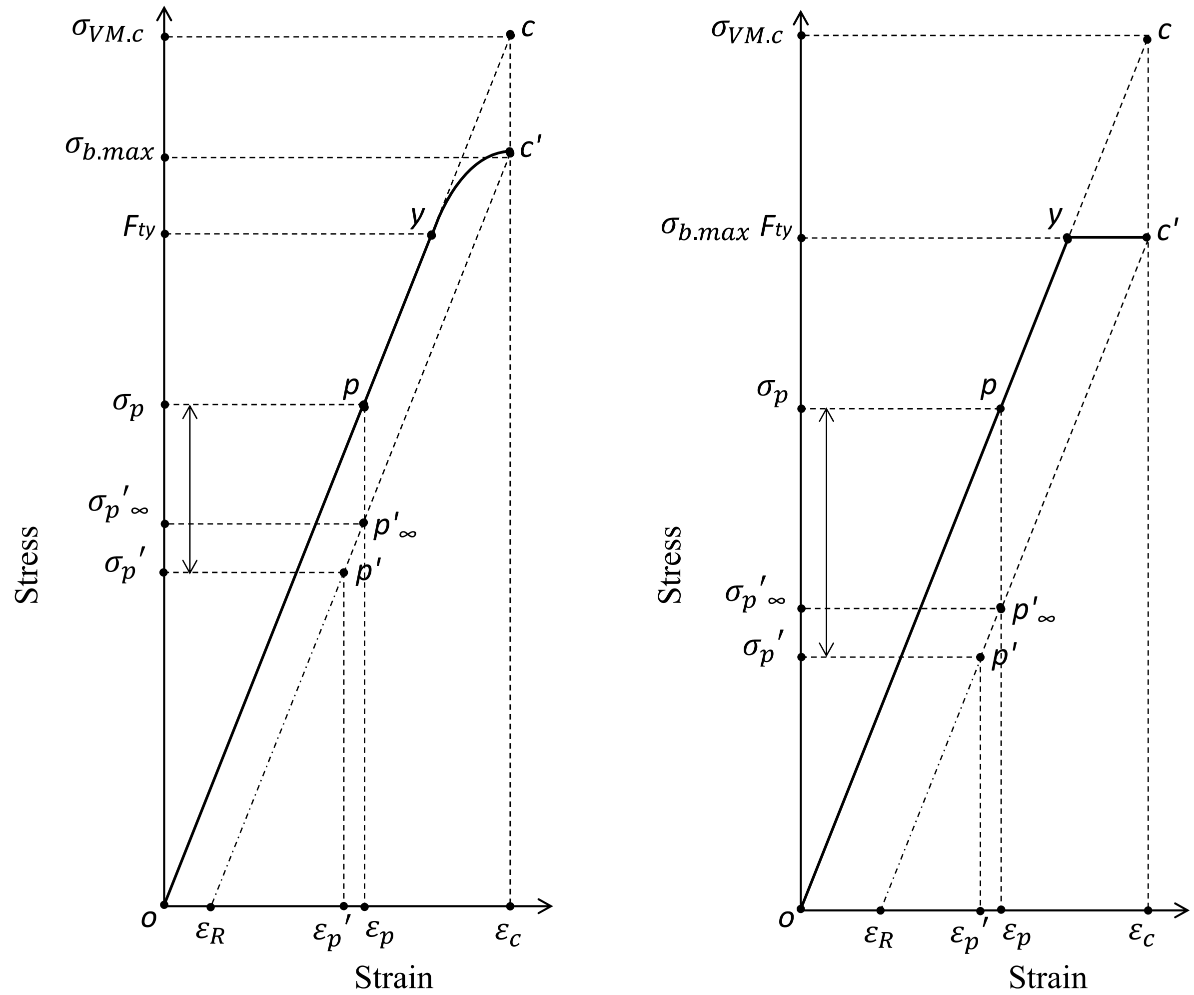


Figure 7a Figure 7b

Fig. 7 Graphical estimate of loss of bolt preload

The solid curve, $o\,p\,y\,c'$, shown in Figure (7a), follows the stress-strain curve for the bolt material. Point $p$ indicates the bolt preload, prior to external loads being applied, where $\sigma_p$ and $\varepsilon_p$ are the bolt tensile stress and strain respectively resulting from the bolt preload. Similarly, the point $y$ indicates where, as the external loads are applied, the bolt stress reaches the proof stress of the bolt material $F_{ty}$ and the bolt material enters the non-linear, plastic, region of the curve. When Yield Controlled Tightening is used the points $p$ and $y$ are coincident.

In his analysis Welch (2018) [1] assumes a linear elastic solution and his equation predicts an axial load for the bolt. The axial stress, calculated from the axial load, and bolt shear stress are then used to calculate the Von Mises stress $\sigma_{VM.c}$ which is described/indicated by point $c$ in Figure (7a). If the Von Mises stress $\sigma_{VM.c}$ exceeds the bolt material proof stress $F_{ty}$ the bolt will enter the plastic range and behave non-linearly. Hence, the bolt material will follow the stress-strain curve until it reaches the condition described/indicated by point $c'$. The compressive stresses within the joint flanges will remain within the elastic range of the flange material. Therefore, the bolt displacement, under plastic deformation, is controlled by the thickness of the flange pack. The points $c$ and $c'$ are both associated with $\varepsilon_c$ which is the strain at the bolt core. When the external loads are removed the bolt material will behave elastically and follow the line $c'p'_{\infty}\,p'$. Again, the bolt strain is controlled by the flange pack. The point $p'_{\infty}$ is described by $\varepsilon_p$ which is the bolt's initial preload condition. The bolt stress $\sigma'_{p.\infty}$ associated with the point $p'_{\infty}$ will be less than the bolt stress $\sigma_p$ from the initial preload. If the through flange stiffness were infinite then the point $p'_{\infty}$ would be the new, reduced, bolt preload condition and the stress $\sigma'_{p.\infty}$ would be the corresponding bolt preload stress. However, if the effect of flange pack stiffness is taken into consideration the actual reduced preload condition would be described by point $p'$ and the actual bolt reduced preload stress $\sigma'_p$ would be given by equation (21):

$$\sigma'_p = \frac{\sigma'_{p.\infty}}{\left(1 + \frac{A_b \cdot E_b}{k_{jp} \cdot L_g}\right)} \tag{21}$$

Where $\sigma'_{p.\infty}$ can be found graphically using a stress-strain curve for the bolt material.

The strain $\varepsilon_R$ shown in Figure (7a) represents the permanent set in the bolt.

An idealised stress-strain curve is illustrated in Figure (7b). This idealised curve neglects the effects of work hardening and will predict a greater loss of preload than would actually occur. Using Figure (7b) it is possible to show value of the stress $\sigma'_{p.\infty}$ can be given by equation (22):

$$\sigma'_{p.\infty} = F_{ty} - \left(\sigma_{VM.c_{(n)}} - \sigma_p\right) \tag{22}$$

The final bolt reduced preload stress $\sigma'_p$ would then be approximated by using the term for $\sigma'_{p.\infty}$ in equation (21).

Alternatively. the stress-strain curve for bolts conforming with BS EN ISO 898-1 [9] can be approximated using equation (23):

$$\varepsilon = \frac{\sigma}{E_b} + K \cdot \left(\frac{\sigma}{E_b}\right)^n \tag{23}$$

Where the constants *K* and *n* are given in Table (1).

| | **Grade 8.8** | **Grade 9.8** | **Grade 10.9** | **Grade 12.9** |
|---|---|---|---|---|
| ***K*** | $3.302 \times 10^{36}$ | $1.105 \times 10^{40}$ | $6.719 \times 10^{84}$ | $4.381 \times 10^{77}$ |
| ***n*** | 15.72 | 17.49 | 37.60 | 35.55 |

Table 1 Constants for Stress-Strain Relationship

Plots of equation (23) for each bolt grade are presented in Figure (8).

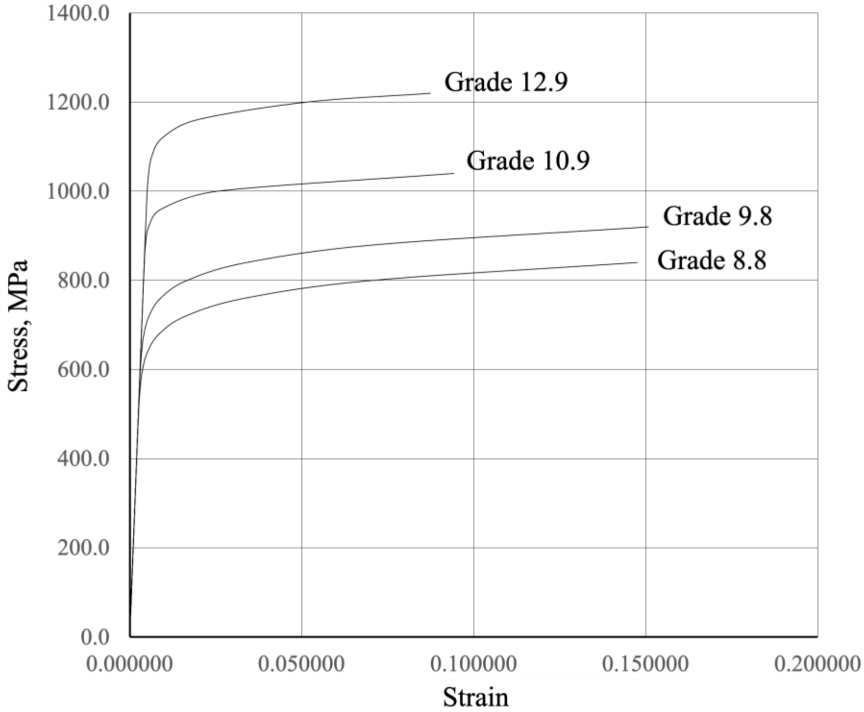


Fig. 8 Approximate Stress-Strain Curves for a Range of Bolt Grades

### 4.1 Von Mises Stress

There are three potential locations within a bolt where the Von Mises stress could indicate the onset of plastic deformation within the bolt that could result in relaxation of the bolt preload. The first location is along the centreline of the bolt, line 1 in Figure (9), where the bolt shear stress from in-plane loads is at its maximum.

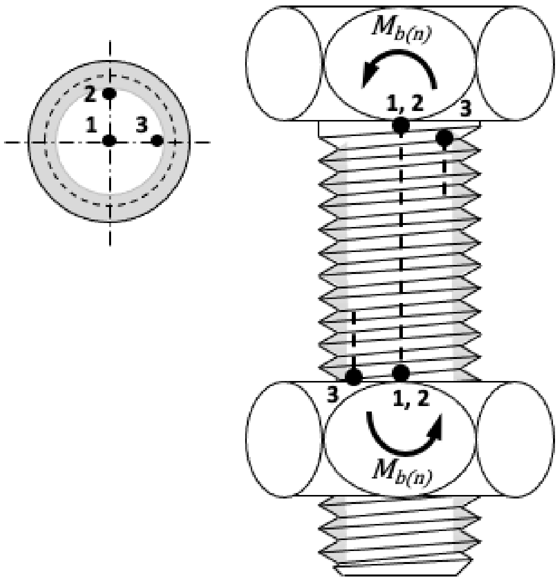


Fig. 9 Locations of High Von Mises Stresses

The Von Mises stress is based on both the axial and shear stresses in the bolt. The shear stress will comprise of the component of shear stress produced by the in-plane external loads.

Hence, the Von Mises stress at the core of the bolt, along line 1 of Figure (9), can be given by equation (24a).

$$\sigma_{VM.c_{(n)}} = \sqrt{{\sigma_{b.1_{(n)}}}^2 + 3 \cdot \left(1.5 \cdot \frac{F_{s.b_{(n)}}}{A_{s.b}}\right)^2} \tag{24a}$$

Where $\sigma_{b.1_{(n)}}$ is the tensile stress in the bolt at point (1), along the line of the first location.

The term $F_{s.b_{(n)}}/A_{s.b}$ represents the mean, or nominal, shear stress in the bolt. Where $A_{s.b}$ is the shear area of the bolt, equivalent to the cross-sectional-area of the thread at the thread root.

Equation (24a) assumes that the peak shear stress at the centre of the bolt section will be 1.5 times the mean shear stress. Where the factor of 1.5 is based on the circular form of the bolt cross-section.

The second location for the Von Mises stress is near the thread root, just below the region of residual stress at the thread root, as illustrated by line 2 in Figure (9). In this instance, the shear stress will comprise of the component of shear stress produced by the in-plane external loads plus the 'locked in' component of shear stress $\tau_p$ produced from pre-tensioning the bolt. The Von Mises stress under the thread root, along the full active length of the bolt, along line 2, can be given by equation (24b).

$$\sigma_{VM.c_{(n)}} = \sqrt{{\sigma_{b.2_{(n)}}}^2 + 3 \cdot \left(1.5 \cdot \frac{F_{s.b_{(n)}}}{A_{s.b}} + \tau_p\right)^2} \tag{24b}$$

Where $\sigma_{b.2_{(n)}}$ is the tensile stress in the bolt at point (2), the second location, and $\tau_p$ is the locked in shear stress produced from the makeup torque when the bolt was pre-tensioned.

The third location for the Von Mises stress is again near the thread root, just below the region of residual stress at the thread root, as illustrated by line 3 in Figure (9). In this instance the Von Mises stress is based on the axial tensile plus bending stresses in the bolt. The shear stress will comprise of only the component of shear stress produced from pre-tensioning the bolt. The effect of the bending stress will reduce with distance from the underside of the bolt head, or nut face, hence any plastic deformation of the bolt is likely to be localized to the point of maximum bending stress. The Von Mises stress under the thread root, for a short distance away from the bolt head and nut face, along line 3, can be given by equation (24c).

$$\sigma_{VM.c_{(n)}} = \sqrt{\left(\sigma_{b.3_{(n)}} + \sigma_{s.b_{(n)}}\right)^2 + 3 \cdot {\tau_p}^2} \tag{24c}$$

Again, where $\sigma_{b.3_{(n)}}$ is the tensile stress in the bolt at point (3), the third location. Also, $\sigma_{s.b_{(n)}}$ is the maximum bending stress component in bolt '$n$'.

Note that equation (24b) is for point (2), perpendicular to the action of the shear load and equation (24c) is for point (3) on the line of the shear load.

## 4.2 Yield Controlled Tightening

The relationship between torque and shear stress is complex, particularly when the bolt is being tightened into the plastic range. Initially, all of the bolt's core will behave elastically. When the Von Mises stress at the outer edge of the core reach yield condition, plastic deformation will start to occur. The principal direction of 'flow' of the plastic deformation being determined by a relationship of the shear and axial stresses. As the applied torque increase the area of the bolt subjected to plastic deformation will spread inwardly, following what can be regarded as a ‘plastic wave front’. At the same time, the areas behind the plastic wave front will work harden. At the plastic wave front, or at a point just in front of it, the increasing applied torque will increase the tensile stress in the bolt. That means the Von Mises criteria will utilise a higher tensile stress component than was previously needed behind the plastic wave front. Therefore, the elastic shear stress will be less than that when plastic deformation had been initiated at points now behind the plastic wave front. Hence, the relationship between shear and axial stresses will have changed and the principal direction of ‘flow’ of the plastic deformation will be different. As the plastic front approached the centre of the bolt, the elastic shear stress component will be approaching zero and the tensile stress component will be the proof stress of the bolt material. The yield strength of the areas that have been work hardening as the applied torque has been increased will have been increasing through the procedure. If the process of work hardening is thought of as a continuous application of Von Meses criteria then it can be assumed the shear stresses in the work hardening regions have also been increasing. From this, it is reasonable to assume that when tightened into the plastic state, the bolt core has a non-linear shear stress distribution ranging from zero at the centreline to a maximum at the outer edge. Similarly, it can also be assumed that the tensile stress in the bolt has a non-linear distribution with the maximum tensile stress being at the centreline and the minimum tensile stress being at the outer edge.

The torsional stiffness of the bolt is significantly less than the torsional stiffness of the active region of flanges surrounding it hence there will be very little relaxation of the shear stress when the torque is no longer being applied. Therefore, it can also be assumed that the shear stress distribution is locked into the bolt shank after the bolt has been preload.

If the bolted joint is assembled by ‘yield-controlled tightening’ then the bolts will be loaded up to, and beyond, the point where the bolt material just enters the yield state. If either equation (24b) or equation (24c) are applied to the bolts just as first yield occurs they can both be written as:

$$F_{ty} = \sqrt{\sigma_{b.1st}{}^2 + 3 \cdot \tau_p{}^2} \tag{25}$$

Where $\sigma_{b.1st}$ and $\tau_p$ are the bolt tensile stress and shear stress respectively, and are located at, or just inside, the root (or minor) diameter $D_s$.

Note that in equation (25) the bolt tensile stress $\sigma_{b.1st}$ occurs at the first instance of the bolt material entering the plastic condition, which occurs just inside the root diameter away from the residual stress. During make-up of the joint the term for bending stress $\sigma_{s.b_{(n)}}$ in equation (24c) is zero.

### 4.3 Make-up Torque Equation

Appendix B of British Standard BS 3580:1964 [10] gives an equation which can be used to calculate the required make-up torque for the bolts.

$$T_M = F_p \cdot \left(\frac{D_n}{2} \cdot tan(\lambda) + \frac{D_n}{2} \cdot \frac{\mu_t}{cos(\propto)} + \frac{d_o + d_i}{4} \cdot \mu_b\right) \tag{26}$$

Where $\lambda$ is the thread lead angle, $\propto$ is the thread flank angle, $D_n$ is the thread pitch diameter, and $d_o$ and $d_i$ are the outer and inner diameters respectively of the nut or bolt head contact area, depending upon which is turned during the tightening process. The friction coefficients, $\mu_t$ and $\mu_b$ are the dynamic friction coefficients for the thread and bolt head/nut respectively.

When assembling joints, it is usual practice is to turn the nut. This practice ensures that any resistance to rotating the bolt shank within the bolt hole does not influence the bolt preload. Such resistance to rotation could arise from misalignment of the bolt holes, resulting in bolt bearing on the side of the hole. The bolt head is turned only when it is not practical to turn the nut. Hence, when this article refers to turning the nut, it can be taken that it also applies to situations where the bolt head is turned.

The term $F_p \cdot \frac{D_n}{2} \cdot tan(\curlywedge)$ in equation (26) represents the torque component required to produce the load $F_p$ in a frictionless system. Similarly, the term $F_p \cdot \frac{D_n}{2} \cdot \frac{\mu_t}{cos(\propto)}$ represents the torque component required to overcome friction at the thread, and the term $F_p \cdot \frac{d_o+d_i}{4} \cdot \mu_b$ represents the torque component required to overcome friction under the nut or bolt head.

A more accurate term for the torque required to overcome the friction at the nut can be found by replacing the arithmetic mean diameter of the friction area $\frac{d_o+d_i}{2}$ with the area weighted mean diameter $d_{\mu b}$ given by equation (20).

Hence, equation (26), can be rewritten as:

$$T_M = F_p \cdot \left(\frac{D_n}{2} \cdot tan(\curlywedge) + \frac{D_n}{2} \cdot \frac{\mu_t}{cos(\propto)} + \frac{1}{3} \cdot \left(\frac{{d_o}^3 - {d_i}^3}{{d_o}^2 - {d_i}^2}\right) \cdot \mu_b\right) \tag{27}$$

The torque required at the thread to tighten the bolt to the preload condition comprises the first two terms of equation (26) and hence is given by:

$$T_{M.t} = F_p \cdot \left(\frac{D_n}{2} \cdot tan(\curlywedge) + \frac{D_n}{2} \cdot \frac{\mu_t}{cos(\propto)}\right)$$

The lead angle $\curlywedge$ is given by $\curlywedge = arctan\left(\frac{p_{thrd}}{\pi \cdot D_n}\right)$ where $p_{thrd}$ is the thread pitch.

Hence:

$$T_{M.t} = F_p \cdot \left(\frac{p_{thrd}}{\pi \cdot D_n} + \frac{\mu_t}{cos(\propto)}\right) \cdot \frac{D_n}{2}$$

This equation can be simplified by introducing the concept of an effective friction coefficient:

$$T_{M.t} = F_p \cdot \mu_{eff.t} \cdot \frac{D_n}{2} \tag{28}$$

Where the effective friction coefficient for the thread and nut (or bolt head) is given by:

$$\mu_{eff.t} = \frac{p_{thrd}}{\pi \cdot D_n} + \frac{\mu_t}{cos(\propto)} \tag{29}$$

Hence, the equation for the make-up torque, equation (27) can be written as:

$$T_M = F_p \cdot \left(\frac{D_n}{2} \cdot \mu_{eff.t} + \frac{1}{3} \cdot \left(\frac{{d_o}^3 - {d_i}^3}{{d_o}^2 - {d_i}^2}\right) \cdot \mu_b\right) \tag{30}$$

### 4.4 Bolt Preload at First Yield

During the initial phase of tightening the bolt core will behave elastically. The 'bolt core' is that part of the bolt section under/inside the region influenced by the residual stress produced by thread rolling. The initial elastic shear stress in the bolt core due to the torque will have a typical linear elastic distribution, maximum at the root (or minor) diameter and reducing to zero at the bolt centreline. There will also be a uniform tensile stress due to the bolt having been elongated. The bolt load at the instance of first yield can be considered as a 'lower bound' for the bolt preload. Assuming the bolt core material is within its elastic range, the shear stress $\tau_p$ used in equations (24b) and (24c) can be given by:

$$\tau_p = \frac{T_{M.t}}{J_s} \cdot \frac{D_s}{2} \tag{31}$$

Where $D_s$ is the thread minor (or root) diameter, $T_{M.t}$ is the torque in the bolt shank produced when the bolt is preloaded to the onset of plastic deformation, and $J_s$ is the polar second moment of area of the bolt core.

$$J_s = \frac{\pi \cdot {D_s}^4}{32} \tag{32}$$

Using equation (28), with a value of bolt preload at first yield, in equation (31):

$$\tau_p = \frac{F_{p.low} \cdot \mu_{eff.t} \cdot D_n \cdot D_s}{4 \cdot J_s} \tag{33}$$

Where $F_{p.low}$ is the bolt load that relates to the first onset of yielding of the bolt, which occurs at the bolt root (minor) diameter. Therefore, $F_{p.low}$ represents a 'lower bound' on the bolt preload that would be achieved by yield-controlled tightening of the bolt. Notice that this lower bound value is influenced by the friction coefficient at the thread flank. Hence, even with yield-controlled tightening, good friction control is desirable.

Using equation (33) with equation (25), again using the 'Lower Bound Preload' $F_{p.low}$ at first yield, leads to:

$$F_{ty} = \sqrt{{\sigma_{b.1st}}^2 + 3 \cdot \left(\frac{F_{p.low} \cdot \mu_{eff.t} \cdot D_n \cdot D_s}{4 \cdot J_s}\right)^2}$$

Since the bolt load condition is just at the point of transition from elastic to plastic deformation, the tensile stress of $\sigma_{b.1st}$ will be acting uniformly across the bolt cross-section and the elastic relationship of $F_{p.low} = \sigma_{b.1st} \cdot A_b$ will still hold true. Hence, it is possible to write:

$$F_{ty} = \sqrt{{\sigma_{b.1st}}^2 + 3 \cdot \left(\frac{\sigma_{b.1st} \cdot A_b \cdot \mu_{eff.t} \cdot D_n \cdot D_s}{4 \cdot J_s}\right)^2}$$

Rearranging:

$$\sigma_{b.1st} = \frac{F_{ty}}{\sqrt{1 + 3 \cdot \left(\frac{A_b \cdot \mu_{eff.t} \cdot D_n \cdot D_s}{4 \cdot J_s}\right)^2}} \tag{34}$$

Hence:

$$F_{p.low} = \frac{F_{ty} \cdot A_b}{\sqrt{1 + 3 \cdot \left(\frac{A_b \cdot \mu_{eff.t} \cdot D_n \cdot D_s}{4 \cdot J_s}\right)^2}} \quad (35)$$

## 4.5 Bolt Analysis Beyond First Yield

The bolt shear stress at the root (minor) diameter when first yield occurs (excluding residual stresses) can be found by using equation (35) in equation (33).

$$\tau_p = \frac{F_{ty} \cdot \mu_{eff.t} \cdot D_n \cdot D_s}{\sqrt{\left(\frac{4 \cdot J_s}{A_b}\right)^2 + 3 \cdot \left(\mu_{eff.t} \cdot D_n \cdot D_s\right)^2}} \quad (36)$$

At the time of first yielding, the core of the bolt is acting elastically and the shear stress at any radius '$r$' can be given in terms of the shear stress at the radius $D_s/2$ by the equation:

$$\tau_{b.r} = \frac{2 \cdot r}{D_s} \cdot \tau_p \quad (37)$$

The Von Mises stress at any radius '$r$' is given by an equation of similar form to equation (25):

$$\sigma_{VM.r} = \sqrt{\sigma_{b.r}{}^2 + 3 \cdot \tau_{b.r}{}^2} \quad (38)$$

Since the bolt core acts elastically up until first yield, the tensile stress has a uniform value of $\sigma_{b.1st}$ across the bolt cross-section. Therefore, the Von Mises stress at any radius when the bolt reaches first yield (at radius $D_s/2$) can be given by using equation (37) in equation (38) with $\sigma_{b.r} = \sigma_{b.1st}$ for all values of radius '$r$'.

$$\sigma_{VM.r} = \sqrt{\sigma_{b.1st}{}^2 + 3 \cdot \left(\frac{2 \cdot r}{D_s} \cdot \tau_p\right)^2} \quad (39)$$

Since the condition described by equation (39) is still in the elastic range, the ratio of the elastic Von Mises stress with the proof stress, $F_{ty}$ can be regarded as an indication of how close the Von Mises stress is to the point of plastic deformation.

$$f_r = \frac{\sigma_{VM.r}}{F_{ty}} \quad (40)$$

It can then be assumed that the material a radius '$r$' will enter the plastic range when the shear stress at radius '$r$' reaches:

$$\tau_{b.r} = \frac{\tau_p}{f_r} \quad (41)$$

Using equation (37):

$$\tau_{b.r} = \frac{2 \cdot r}{D_s} \cdot \frac{\tau_p}{f_r} \quad (42)$$

The progressive growth of the amount of the bolt core entering into plastic deformation can be described by equation (38), replacing the term for $\sigma_{VM.r}$ with the proof stress $F_{ty}$ to reflect the transition from elastic to plastic deformation:

$$F_{ty} = \sqrt{\sigma_{b.r}{}^2 + 3 \cdot \tau_{b.r}{}^2} \quad (43)$$

Substituting equation (42) into equation (43):

$$F_{ty} = \sqrt{\sigma_{b.r}{}^2 + 3 \cdot \left(\frac{2 \cdot r}{D_s} \cdot \frac{\tau_p}{f_r}\right)^2} \tag{44}$$

Equation (42) implies that the shear stress for first yield $\tau_p$, which occurs at $r = D_s/2$, remains constant beyond first yield. This is an implied assumption that no significant work hardening occurs, or that any work hardening is negligible.

Rearranging equation (44) and using equation (40) to substitute for the term $f_r$ the tensile stress at any radius '$r$' is given by:

$$\sigma_{b.r} = \sqrt{F_{ty}{}^2 - 3 \cdot \left(\frac{2 \cdot r}{D_s} \cdot \frac{F_{ty}}{\sigma_{VM.r}} \cdot \tau_p\right)^2}$$

Then using equation (39) to substitute for the term $\sigma_{VM.r}$ and simplifying:

$$\sigma_{b.r} = \frac{F_{ty}}{\sqrt{1 + 3 \cdot \left(\frac{2 \cdot r}{D_s} \cdot \frac{\tau_p}{\sigma_{b.1st}}\right)^2}} \tag{45}$$

The progressive growth of the amount of the bolt core entering into plastic deformation can be described by equation (43). This equation is derived by substituting equation (42) into equation (38) and replacing the term for $\sigma_{VM.r}$ with the proof stress $F_{ty}$ to reflect the transition from elastic to plastic deformation.

Similarly, by rearranging equation (43), the shear stress at any radius '$r$' can be given by:

$$\tau_{b.r} = \sqrt{\frac{F_{ty}{}^2 - \sigma_{b.r}{}^2}{3}} \tag{46}$$

Typical plots of the tensile stress and shear stress distribution across a bolt section are illustrated in Figure (10) and Figure (11) respectively.

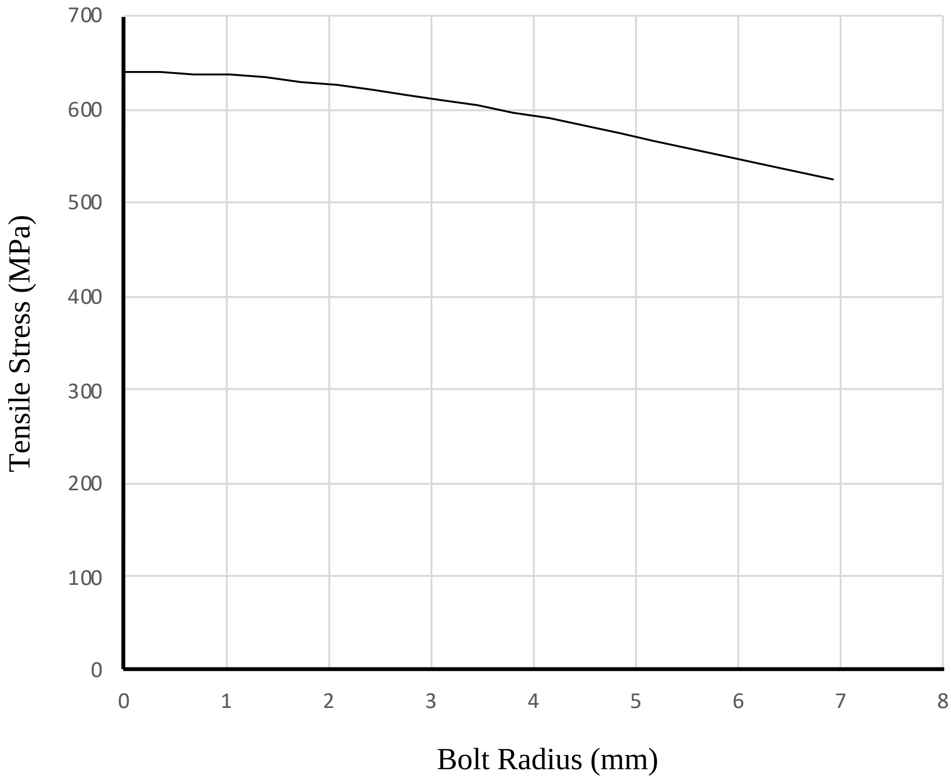

Fig. 10 Bolt Tensile Stress Distribution

Figure (10) shows the plot of the tensile stress distribution across the section of a grade 8.8 M16 bolt when tightened to the fully plastic condition. Figure (10) does not include the effects of work hardening.

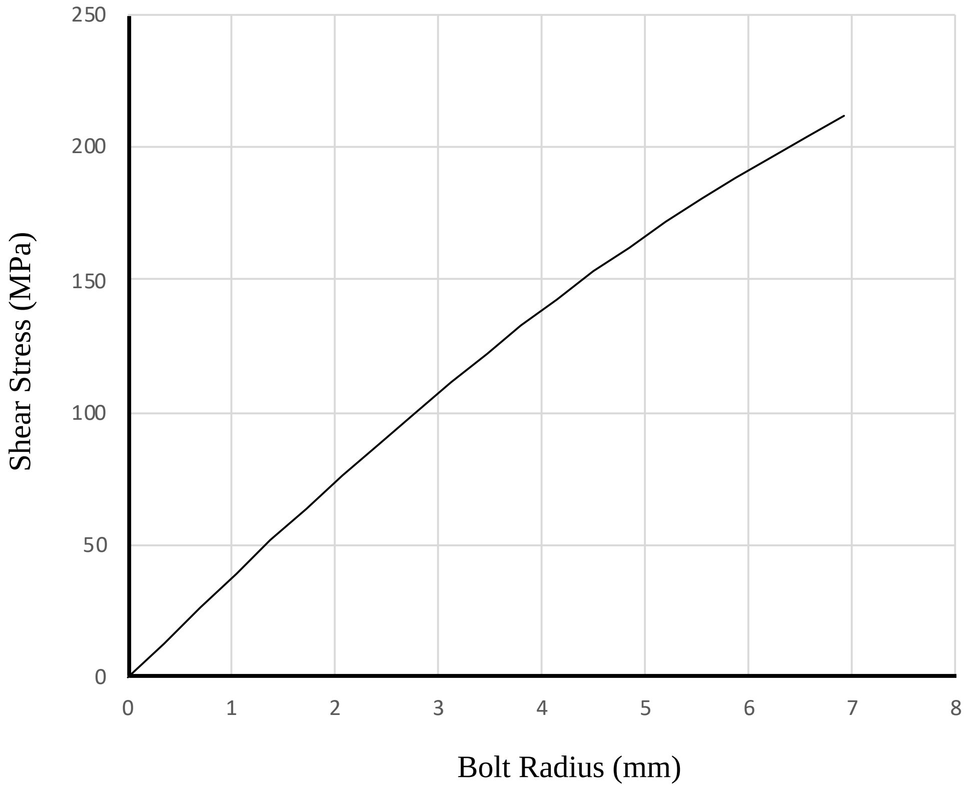

Fig. 11 Bolt Shear Stress Distribution

Similarly, Figure (11) shows the plot of the shear stress distribution across the section for the same bolt. Again, Figure (11) does not include the effects of work hardening.

### 4.6 Bolt Preload When Bolt Core is Fully Plastic

The upper limit on the bolt preload occurs when all of the bolt's core is in the plastic region. Hence:

$$F_{p.up} = \int_0^{\frac{D_S}{2}} 2 \cdot \pi \cdot r \cdot \sigma_{b.r} \cdot dr$$

Using equation (45)

$$F_{p.up} = 2 \cdot \pi \cdot F_{ty} \int_0^{\frac{D_S}{2}} \frac{r}{\sqrt{1 + 3 \cdot \left(\frac{2 \cdot r}{D_s} \cdot \frac{\tau_p}{\sigma_{b.1st}}\right)^2}} \cdot dr \tag{47}$$

The derivation of equation (47) assumes there is no work hardening.

Integrating within the limits:

$$F_{p.up} = \frac{2 \cdot \pi \cdot F_{ty}}{3} \cdot \left(\frac{D_s}{2} \cdot \frac{\sigma_{b.1st}}{\tau_p}\right)^2 \cdot \left(\sqrt{1 + 3 \cdot \left(\frac{\tau_p}{\sigma_{b.1st}}\right)^2} - 1\right) \tag{48}$$

This analysis of the bolt preload neglects the effect of work hardening.

### 4.7 Bolt Shear Stress and Torque when Tightened into the Plastic Range

If the effects of work hardening are ignored the shear stress at the bolt root at the fully plastic condition will be the same as the shear stress at first yield. Hence, equation (36) can be assumed to apply throughout the bolt tightening process. Equation (36) is:

$$\tau_p = \frac{F_{ty} \cdot \mu_{eff.t} \cdot D_n \cdot D_s}{\sqrt{\left(\frac{4 \cdot J_s}{A_b}\right)^2 + 3 \cdot \left(\mu_{eff.t} \cdot D_n \cdot D_s\right)^2}}$$

Referring to equation (28), the torque required at the thread to tighten the bolt to a preload within the range of the 'Lower Bound' $F_{p.low}$ and the 'Upper Bound' $F_{p.up}$

$$\begin{bmatrix} T_{M.t.low} \\ T_{M.t.up} \end{bmatrix} = \begin{bmatrix} F_{p.low} \\ F_{p.up} \end{bmatrix} \cdot \mu_{eff.t} \cdot \frac{D_n}{2} \tag{49}$$

The analysis presented calculates a 'Lower Bound' $F_{p.low}$ and an 'Upper Bound' $F_{p.up}$ for the bolt preload when the bolt is tightened using Yield Controlled Tightening methods. The actual bolt loads will fall within the range of these 'bounded' loads depending upon the method, and consistency, of dynamically monitoring the bolt during tightening. In practice, they will probably tend to be towards the 'Upper Bound' limit. Hence, the torques, $T_{M.t.low}$ and $T_{M.t.up}$, are the lower and upper bound for the torque in the bolt shank at the preload condition.

Similarly, referring to equation (30), the makeup torque to tighten the bolt to a preload within the range of the 'Lower Bound' $F_{p.low}$ and the 'Upper Bound' $F_{p.up}$, is:

$$\begin{bmatrix} T_{M.low} \\ T_{M.up} \end{bmatrix} = \begin{bmatrix} F_{p.low} \\ F_{p.up} \end{bmatrix} \cdot \left( \frac{D_n}{2} \cdot \mu_{eff.t} + \frac{1}{3} \cdot \left( \frac{{d_o}^3 - {d_i}^3}{{d_o}^2 - {d_i}^2} \right) \cdot \mu_b \right) \tag{50}$$

Again, the torques, $T_{M.low}$ and $T_{M.up}$, are the lower and upper bound for the make-up torque at the preload condition.

Assuming the torque at the thread, as given by equation (49) is 'locked in' after preloading the bolt, then the 'locked in' angle of twist of the bolt shank due to the locked in torsion is given by:

$$\begin{bmatrix} \theta_{p.low} \\ \theta_{p.up} \end{bmatrix} = \begin{bmatrix} T_{M.t.low} \\ T_{M.t.up} \end{bmatrix} \cdot \frac{L_g}{J_b \cdot G_b} \tag{51}$$

Similarly, the angles, $\theta_{p.low}$ and $\theta_{p.up}$, are the lower and upper bound for the angle of twist for the bolt shank at the preload condition.

## CONCLUSIONS

If out-of-plane external loads, and/or external moments result in a compressive load in the region of the bolt, the tensile load in the bolt will be reduced. Under this condition, external in-plane loadings could cause the bolt head to slip, causing mechanical loosening of the bolt. This slip results from the shear load on the bolt caused by the transvers displacement of the bolt head/nut. It would occur when the bolt tensile load and friction are not sufficient to prevent transverse slip of the bolt head, or nut. This slip at the bolt head/nut would not be accompanied by slip between the elements of the joint, the flanges, because friction and the increased contact pressure at the faying surface will prevent any movement. Slip at the bolt head or nut could lead to rotation of the nut, relative to the bolt, causing a reduction in the bolt preload.

The lateral, transverse, slip of the bolt and resulting relative nut rotation will be on a macroscopic scale. Under cyclic loading the loss of preload resulting from slip would increase incrementally with each load cycle. The amount of total bolt head/nut laretal slip could still remain within a macroscopic scale up until the point sufficient bolt preload had been lost to allow joint slip at the faying surface.

The torsional stiffness of the joint depends on the second polar moment of area of the flange cross-section and is dominated by friction at the faying surface, which arises from the bolt preload. The bolt shear and bending stresses have only a small influence on the torsional stiffness of the joint.

There will be localised shear stresses, higher than the mean shear stress, at the interface between the flanges and the structural elements to which they are attached. The 'intensity' of this shear stress will be dispersed through the thickness of the flange, tending towards the value of the assumed average shear stress.

Generally analyses uses the second polar moment of area of the joint to calculate the shear stresses produced by in-plane torsional loads. For non-circular sections the 'torsional constant' is always less than the second polar moment of area hence, the true shear stress is always greater than that predicted by using the second polar moment of area. It is recommended that an appropriate torsional constant for the flange geometry is used for the calculation of the bolt head flexural displacement, the resulting bending moment and additional tensile stress.

If the bolt total maximum tensile stress, including the calculated additional tensile stress from the sideways flexural deflection of the bolt head, exceeds the bolt material proof stress then there will be some loss of the initial preload.

If the Von Mises stress exceeds the bolt material proof stress the bolt material will no longer act elastically but instead will enter the plastic range and behave non-linearly. Hence, the bolt material will follow the stress-strain curve for the material.

There are three potential locations where plastic deformation within the bolt could be initiated. The first is along the centreline of the bolt where the bolt shear stress from in-plane loads is at its maximum. The second and third locations are both near the thread root, just below the region of residual stress at the thread root. The second is at a point perpendicular to the line of action of the shear load and the third is at a point on the line of the shear load.

When ‘Yield Controlled Tightening’ is applied, the bolt core has a non-linear shear stress distribution ranging from zero at the centreline to a maximum at the outer edge. Similarly, the tensile stress in the bolt has a non-linear distribution with the maximum tensile stress being at the centreline and the minimum tensile stress being at the outer edge. The shear stress distribution will be locked into the bolt shank after the bolt has been preload, prior to the application of external loads on the joint.

The joint flange material remains within the elastic range, the bolt displacement under plastic deformation is controlled by the thickness of the flange pack.

Even with yield-controlled tightening, good friction control is desirable.

When bolts are tightened using ‘Yield Controlled Tightening’ the bolt preloads will fall within a ‘Lower Bound’ and an ‘Upper Bound’. The actual preload will depend upon the sensitivity of the system used to detect plastic deformation. In practice, they will probably tend to be towards the ‘Upper Bound’ limit.

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