Strojnícky časopis – *Journal of* MECHANICAL ENGINEERING,
VOL 74 (2024), NO 1, 83 - 90

SIMULATION OF THE WATER BALANCE OF A GREEN/BLUE ROOF SUPPORTING AN ECOLOGICAL CYCLE

MENYHÁRTNÉ DR BARACSKAI Melinda[1], HADAS-RAPI Ádám[1*], DR. ÍRÓ Béla[1]

[1] *Széchenyi I*stván *University in Győr, Faculty of Mechanical Mechanical Engineering, Electrical Engineering, Department of Machine Design, 9026 Győr, Egyetem tér 1, Hungary, e-mail: hadas.adam@ga.sze.hu*

Abstract: In our article, we set out the inspection of the control shafts of the green or blue roof drainage equipment aim. The height of the water formed on the roof through the differently shaped holes on the devices we want to specify the amount of water flowing through per unit of time. In the first case, the water level is constant, and in the second, the height of the water decreases or increases continuously.

KEYWORDS: fluid mechanics, green roof, rain drainage, flooding examination, outflow ratio

1 Introduction

The expansion of cities requires the incorporation of more and more natural areas. Therefore, more and more insects have become endangered. Blue / green roofs are suitable for supporting biodiversity by functioning as a habitat. To achieve this, the roof is planted with plants that require a medium thickness of 6-20 cm or 40 cm, which tolerate well the 'humus-free' planting medium with a low organic matter content, continuous sunlight, and the absence of shade. Of course, it is necessary to ensure that the plants are watered, and it is also very important to ensure the appropriate living conditions even in the event of rain. Which requires us to determine the influence of roof gutters. We consider it our task to determine the amount of liquid flowing through the openings on the roof confluence objects (Figure 1), given the height of the water column specified there, in the case that the height of the water level is constant (VEK) and in the case that the height of the water level decreases continuously (LK). (In the case of making calculations, the roof is considered to be a simple vessel, which is not affected by other flow-influencing factors.

The amount of water flowing through an holes per time unit '$\dot{V}$'˙ can be written as the product of the velocity of the water flowing through the elements and the cross-section of the opening, i.e. the following general surface integral for the cross-section of the opening:

$$\dot{V} = \phi \cdot (2 \cdot g)^{0,5} \cdot \int_{(A)} z^{0,5} \cdot dA = \phi \cdot (2 \cdot g)^{0,5} \cdot \int_{(A)} z^{0,5} \cdot S(z) \cdot dz \qquad (1)$$

In the context, 'g' is the gravitational acceleration and 'ϕ' is the so-called discharge coefficient (value less than 1). This coefficient takes into account that, in the case of a finite-sized hole, the actual flow rate at different points of the cross-section is different and differs from the theoretically determinable value. The discharge coefficient can only be determined through experiments, and its value depends on the design and shape of the opening and whether the hole is entirely below the water level, or only partially, etc. Later, for the specific calculations, we take into account a constant value selected based on the recommendations in the literature.

DOI: 10.2478/scjme-2024-0009, Print ISSN 0039-2472, On-line ISSN 2450-5471

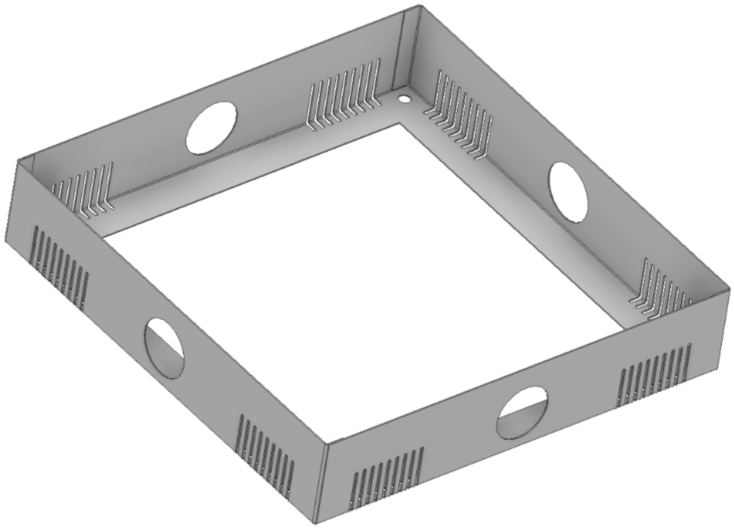

Fig. 1 Outflow element

Solving the relation is simple in cases where the chord width (S) of the opening is either constant or can be written as a first-order function of the height of the opening at most.

2 Determining the amount of water discharged per unit of time at a constant water level height

With a constant water level height, the amount of water discharged per unit of time of a water drainage can obviously be given by summing the values of the amount of water discharged per unit of time from the openings formed on the roof drainage. It also follows from this that an opening of any shape can be divided into several parts (ideally parts for which the chord width of the bore is either constant or can be written as a first-order function of the height of the opening) and the parts can be treated separately.

During the preparation of the study, we distinguished the following types of openings on the water drainage:

a) A right-angled rectangular opening formed in a vertical surface, which
   - one of its sides is horizontal,
   - neither side is horizontal,
b) circular opening,
c) complex-shaped opening, which with a good approximation can be broken down into the right-angled rectangular openings mentioned in point a),
d) a right-angled rectangle, circle or other shaped opening on a horizontal surface.

2.1 The solution of equation (1) for a right-angled opening, one of whose sides is horizontal

This type of opening is characterized by a constant chord width (S). From that case, the equation (1) is significantly simplified, the constant chord width can be highlighted and the surface integral is replaced by the integral according to the 'z' coordinate.

$$\dot{V} = \phi \cdot (2 \cdot g)^{0,5} \cdot S \cdot \int z^{0,5} \cdot d\,z \qquad (2)$$

Omitting the details, the correlation obtained after integration, after an expedient expansion and rearrangement, is the following:

$$\dot{V} = \left[\phi \cdot (2 \cdot g)^{0,5} \cdot \frac{2}{3} \cdot \frac{z_f^{1,5} - z_a^{1,5}}{\Delta z}\right] \cdot A = c_{ik} \cdot A \tag{3}$$

So, the amount of water flowing through the opening per unit of time is the product of the average velocity 'c_{ik}' determined as an integral mean value for the lower and upper edges of the opening and the cross section.

It is important to mention that the obtained relation is mathematically exact and can be applied even in the case of any water column height.

The solution of equation (1) is for the case of a right-angled rectangular opening formed in a vertical surface, none of the sides of which is horizontal.

The opening belonging to this type can be clearly divided into three parts by two horizontal lines. One of the two horizontal lines, one from the upper part of the right-angled quadrilateral and the other from its lower part, cuts a right-angled triangle-shaped opening part (of the same size, but symmetrically located at the center) with a horizontal cross-section, as well as a parallelogram-shaped opening part with a constant chord width.

The parallelogram-shaped opening part can be treated analogously to relation (2) and all the statements made there are also true for this opening part.

In the case of two right-angled triangular opening parts, two cases need to be examined depending on whether the right-angled triangular opening

- is completely below the water level,
- it is partially below the water level.

2.2 The right-angled triangular opening part is completely below the water level

There is no exact correlation. As an approximate solution, as an analogy to equation (2), the following simple relationship can be used, where 'c_{sp}' is the speed at the center of gravity of the right-angled triangle entirely below the water level

$$\dot{V} = c_{sp} \cdot A \tag{4}$$

Omitting the details, it can be shown that in the case of this calculation method, the maximum error occurs when the water level coincides with the highest point of the right-angled triangle, and only approx. 2%, which is less than 8th of the maximum error that we would make in case of direct application of equation (2).

For the sake of completeness, two things should be mentioned here

- the error decreases rapidly if the distance between the water level and the upper edge of the triangle is larger (e.g. in the case of 5 mm, the error in relation (4) is only 0.01%),
- in both cases, the error is independent of the dimensions of the right-angled square opening from which the right-angled triangle 'comes' and it is also indifferent to the degree of the slant of the right-angled opening.

2.3 The right-angled triangular opening part is partially below the water level

An exact relation can be obtained by transforming equation (1), using the fact that the length of the side of a right triangle is a linear function of the height of the triangle.

By omitting the derivation, in the case of the "upper" right-angled triangle, the result of the integration, which otherwise directly and accurately gives the amount of water flowing through, is the following:

$$\dot{V} = \phi \cdot (2 \cdot g)^{0,5} \cdot \frac{a}{m} \cdot \frac{2}{5} \cdot z^{2,5} \tag{5}$$

In equation (5), 'a' and 'm' are the hypotenuse and corresponding height of the right-angled triangle, and 'z' is the distance between the water level and the span.

In the case of the "lower" right-angled triangle, the result of the integration is different, since this right-angled triangle is congruent with the "upper" one, but it is in a mirrored position. Again omitting the derivation, the relationship, which also accurately specifies the amount of water flowing through, is the following:

$$\dot{V} = \phi \cdot (2 \cdot g)^{0,5} \cdot a \cdot \left[\frac{2}{3} \cdot z^{1,5} - \frac{1}{m} \cdot \frac{2}{5} \cdot z^{2,5}\right] \tag{6}$$

2.4 The solution of equation (1) for the case of a circular opening formed in a vertical surface

This time, substituting the length of the chord width of the circle into relation (1), the form of the relation will be as follows:

$$\begin{aligned}\dot{V} &= \phi \cdot (2 \cdot g)^{0,5} \cdot \int z^{0,5} \cdot S(z) \cdot dz \\ &= \phi \cdot (2 \cdot g)^{0,5} \cdot \int z^{0,5} \cdot 2 \cdot (r^2 - (z - r)^2)^{0,5} \cdot dz\end{aligned} \tag{7}$$

This relationship simply cannot be integrated, so only an approximate calculation is possible, either with the speed calculated as the integral mean or with the center of gravity speed.

In the case of a circular opening, two cases must be examined, depending on whether the opening is circular

- is completely below the water level,
- it is partially below the water level.

2.5 The circular opening is completely below the water level

Equation (3) can be used directly as an approximate solution.

Omitting the details, it can be shown that in the case of this calculation method, the maximum error occurs when the water level coincides with the upper tangent of the circular opening, and its maximum value is less than 2%. The application of the center of gravity speed could arise, but in this case the maximum error is approx. it would be twice as big.

For the sake of completeness, two things should be mentioned here

- the error decreases rapidly if the distance between the water level and the upper edge of the circular opening is greater (e.g. in the case of 5 mm, the error is only 0.27%); at this point, the error is already below 1%, but still approx. three times higher than in the case of using the speed calculated with the integral mean value,
- the error is independent of the diameter.

2.6 The circular opening is partially below the water level

There is no exact correlation. Equation (3) can be used as an approximate solution here as well.

Omitting the details, it can be shown that with this calculation method, the average error is around 8.5% and this time is also independent of the diameter. Compensation for this average error has been incorporated into the part of the calculation method that applies to this case.

There is also a more accurate calculation method, but it would significantly increase the calculation demand and thus the required time. However, with attention to the fact that the error is only significant if the water level is within the circular cross-section, and in the case of roof drainages.

- a circular opening rarely occurs and a
- the total cross-section of circular openings is only a fraction of the total cross-section of other openings,

therefore, the approximate calculation is acceptable.

2.7 The solution of equation (1) for the case of a complex-shaped opening formed on a vertical surface

From the point of view of the applicable calculation method, it does not count as a special case, since by opening the compound opening, the resulting opening parts apply to the case of the right-angled opening formed on a vertical surface.

2.8 The solution of equation (1) in the case of a right-angled rectangle, circle or other shaped opening in a horizontal surface

For this type of opening, the relationship (1) changes to a trivial form, since all points of the opening - regardless of its shape - are at the same distance from the water level, i.e. the speed at all points is the same, which clearly depends on the water level height

$$\dot{V} = \phi \cdot (2 \cdot g)^{0,5} \cdot \int_{(A)} z^{0,5} \cdot dA = \phi \cdot (2 \cdot g)^{0,5} \cdot A \cdot \int z^{0,5} \cdot dz \qquad (8)$$

3 Determination of the *LK* and *VEK* value of a drainage element

To determine the *LK* value of an arbitrary drainage opening from the initial value of the water column height (z_0) until reaching a selected final water column height (z), the sum of the elementary times interpreted as the ratio of the elementary water volume (*dV*) and the *VEK* value corresponding to the given water column height must be determined, i.e.:

$$LK = \int_{z=Z_0}^{z=Z} \frac{dV}{VEK(z)} \qquad (9)$$

Given that, based on what was described in chapter 2.1, the integration of equation (9) is generally not possible, a numerical solution is required.

$$LK = \sum_{j=0}^{j=m} \frac{\Delta V_e}{VEK_j} \qquad (10)$$

In Equation

- 'ΔV_e' is the elementary volume, which is the product of the reduction steps 'Δz' obtained by dividing the difference in the water column heights belonging to the

selected initial and final state into 'j' equal parts and the also selected area to be drained (A),
- 'VEK_j' is the VEK value belonging to the j^{th} step.

In the case of an 'n' product made with different openings, to determine the LK value, it must be taken into account that

- the individual openings used on the Product realize water drainage (drainage) in parallel with each other,
- the elemental volume to be emptied (ΔVe) can be determined by dividing the distance between the initial state and the lowest point of the "deepest" opening of the openings formed on the Product, as the physical emptying limit that can be interpreted in the case of the given product, into 'j' parts.

Based on all this, the value of the LK for a product.

$$LK = \frac{1}{\sum_{i=1}^{n} \frac{1}{\sum_{j=1}^{j=m} \frac{\Delta V_e}{VEK_j}}} \tag{11}$$

Based on equation (9), the LK value is automatically set to an infinitely high value if we assign a zero water level height to the end state of the discharge. In the case of the implemented numerical solution, this does not occur, but it is worth considering that the smaller steps the water column height is reduced, the closer we get to this unmanageable situation, which in short, in the extreme case, means that regardless of the openings used, the LK value of all products is the same, namely infinite.

In order to be able to make comparisons/analyses, it is advisable

- the introduction of a value called the permanent water column height of a finite size, which is a chosen water column height interpreted above the physical discharge limit of the product, at which the given examined area can be treated as practically drained,
- examine the function curve that shows the decrease of the water column height as a function of time (discharge diagram).

4 Simulation of the drainage element

In the following, we would like to present the drainage diagram of the roof drain element shown in Figure 1, subject to the following conditions.

The emptying object has 72 right-angled openings in the vertical plane, the longest side of which is 40 mm. The shorter side is 3 mm. The angle of the longer side with the horizontal is 90°. The lowest point of the flow opening is located 0mm above the base. The geometry includes four openings with a circular cross-section, which are also located in the vertical plane, with a flow diameter of 50 mm.

During our present calculations, the value of the measuring liquid column height above the base is z=110 mm. The standard size of the surface to be emptied is 100 m^2. The standard discharge coefficient is 0.661.

Draining is considered complete when the height of the water column above the lower point of the flow opening is not greater than 1 mm.

4.1 Results

In the case of liquid drainage through openings formed in the vertical plane

Through the openings with a circular cross-section In stationary operating conditions (in the case of a constant water column height, the amount of water theoretically transmitted by 20823 l/h

The amount of water theoretically transmitted through the openings with a rectangular cross-section in stationary operating conditions (in the case of a constant water column height) is 25160 l/h

If we place 72 openings 3 mm wide and 10 mm long on the horizontal plane, in that case the amount of liquid passed through the flow openings per hour is 6970 l/h.

Based on the above values, we can quickly come to the conclusion that we can drain the largest volume of medium through the openings located in the vertical plane of the roof gutter.

In Figure 2, it can be observed that through the openings placed in the vertical plane of the roof drain, the reduction of the 110 mm column of liquid collected on the roof to a height of 40 mm is realized in 22 minutes, but if we only place the drainage openings in the horizontal plane, it takes 1 hour and 25 minutes to drain the same amount of medium. It can be seen from the figure that for the elements placed in the vertical plane, the first 22 minutes is the most intense and steepest section, after which we can observe some slowdown.

Of course, the fastest draining can be observed if drain openings are placed both in the vertical plane and in the horizontal plane. In this case, we can empty the entire surface in 1 hour.

In the Figure 2 the ‘*VP*’ is the vertical plane and the ‘*HP*’ is the horizontal plane.

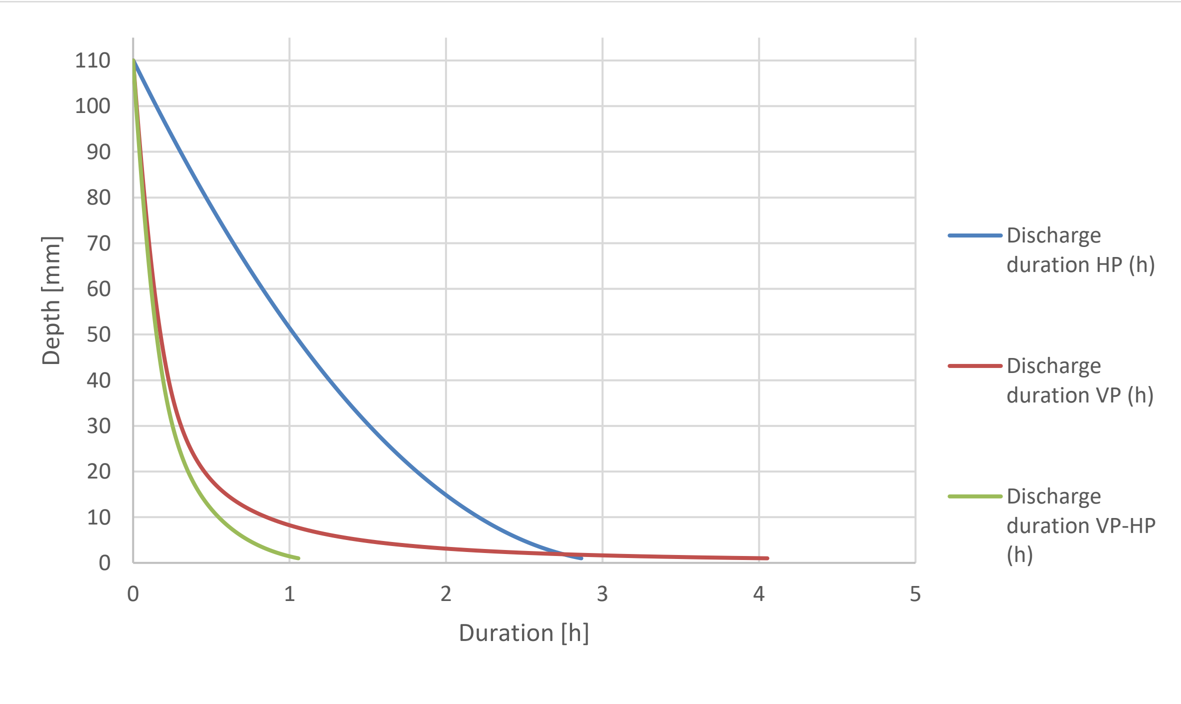

Fig 2 Discharge diagram for the outflow product

Conclusion

To support the existence of biodiversity, it is essential to drain the amount of liquid collected on the roof as a result of rain. In the study, we write down calculations to determine the volume flow that can be discharged for a given roof drain in the case of using various through-flow openings. it was determined that the openings placed in the vertical plane of the roof drain enable the drainage of medium with a larger volume flow than the openings placed in the horizontal plane. We specify how quickly the roof drain can be expected in the case of openings located in individual planes, or by using them together. As a further development of the article, we aim to support the results of our prescribed calculations with measurements. Also, we design a roof drain that can drain the largest amount of liquid in a unit of time.

REFERENCES

[1] Lajos, T. “Az áramlástan alapjai”, Műegyetemi kiadó, Budapest, **2000**. (in Hungarian)

[2] Gruber, J. “Folyadékok mechanikája”, Tankönyvkiadó, **1973**. (in Hungarian)

[3] Idel´chik, I. E. “Handbook of Hydraulic Resistance, Coefficients of Local Resistance and of Friction”, U.S. DEPARTMENT OF COMMERCE, **1960**.

[4] Çengel Y. A., Cimbala, J. M, Turner R. H. “Fundamentals of Thermal-Fluid Sciences”, 5^{th} Edition, MC Graw Hill Education, **2016**. ISBN 978-0-07-802768-0

[5] Rodi W., Martelli F. “Engineering Turbulence Modelling and Experiments”, Proceedings of the Second International Symposium on Engineering Turbulence Modelling and Measurements, Florence, Italy, pp. 991 – 993, **1993**. DOI: 10.1016/C2009-0-10243-X

[6] Huebsch W., Gray D., Thompson G., Spaur P. J. “Discharge Coefficients for Irregular Orifices”, PS Division of Fluid Dynamics (Fall) 2013, abstract id. R10.002, **2013**.

[7] Valkovic, V., Janco, R., Frydrysek, K. "The effect of landside on gas pipeline", Strojnícky časopis – Journal of Mechanical Engineering 66 (2), pp. 95 – 100, 2016. DOI: 10.1515/scjme-2016-0023

[8] Benrahmoune, M., Hafaifa, A., Guemana, M., Chen, X. "Detection and Modeling Vibrational Behavior of a Gas Turbine Based on Dynamic Neural Networks Approach", Strojnícky časopis – Journal of Mechanical Engineering 66 (2), pp. 95 – 100, 2016. DOI: 10.2478/scjme-2018-0032