

CONTROL OF ACTIVE MAGNETIC BEARINGS

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Abstract: This article presents a model of rigid rotor supported by active magnetic bearings (AMB). Mathematical formulation of magnetic forces generated by AMB has been developed. Equations of motion of rigid rotor supported by controllable AMB with and without control law are presented. Eigen structure assignment method is used to change the eigenfrequency of mechatronic system. Numerical simulations are compared with experimental measured data. The simulations presented in this paper correspond with measurement.

KEYWORDS: active magnetic bearing, eigen structure assignment, mechatronic system

1 Introduction

We classify an active magnetic bearing among the elements of mechatronic systems, where the properties of mechanical systems can be changed by means of control. The first magnetic bearing was realized in the middle of the last century. In the early 1980s, interest in magnetic bearings rose sharply, especially in connection with high-speed rotary machines. The most important works were carried out at the ETH in Zurich (Prof. Schweitzer and Dr. Traxler), at the Technical University in Munich (Prof. Ulbrich), in the USA at the University of Virginia (Prof. Allair) and in commercial companies in Japan. Recently, research centers - departments dedicated to the research and development of magnetic bearings - have been established at the ETH Institute of Robotics in Zurich and at the University of Virginia in the USA. The most important works in this area include work dealing with sensorless magnetic bearings [9] and work in the field of technical applications of magnetic bearings [7, 8, 11, 12]. Work in the field of experimental identification of rotors using magnetic bearings is also important [8, 10]. New control algorithms (robust control [6, 9, 13], use of neural networks and fuzzy logic) are being sought in all areas in order to reduce rotor oscillations from external loads [7, 13, 14].

The aim of the paper is to present the active vibration control of a mechanical structure. For control, the method of assigning eigenvalues and eigenvectors was applied, which was applied to an experimental model of a rotor mounted in magnetic bearings. By applying the mentioned method, decentralized control is changed to centralized - the action intervention in one deposit depends not only on the deviations in the given deposit, but also on the deviations in the other deposit.

2 Magnetic bearing

This part describes the types of magnetic bearings and their functional principle, as well as experimental equipment - a rigid rotor mounted in magnetic bearings, developed at the FME SUT in Bratislava.

We divide magnetic bearings according to their magnetic properties into active and passive ones. Passive bearings are made of permanent magnetic materials and their properties cannot be changed during operation. Active magnetic bearings can be controlled to change their properties during operation according to specified requirements. In terms of force transmission, magnetic bearings are divided into radial and axial types. The functional principle of an active magnetic bearing is shown in Figure 1. The forces acting on the rotor are generated by an electromagnetic field, the magnitude of which depends on the current in the windings of the electromagnets and on the magnitude of the air gap. During operation, the rotor floats without contact in the air gap.

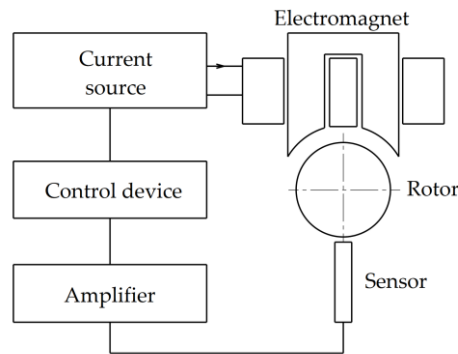


Fig. 1 Active magnetic bearing principle
- radial magnetic bearing

The radial magnetic bearing consists of a stator and a rotor ring. The stator consists of horseshoe-shaped electromagnets. At least three electromagnets are required to generate magnetic forces. To increase the bearing capacity, a variant of four electromagnets with four independent coils is mostly used Figure 2.

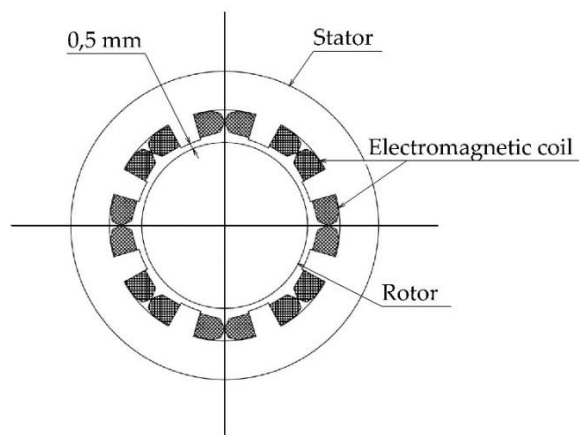


Fig. 2 Model of radial magnetic bearing
- Axial magnetic bearing

Only two opposing electromagnets are required for the radial bearing. The diagram of the axial bearing is shown in Figure 3.

The entire experimental equipment was developed at the Department of Technical Mechanics, Faculty of Mechanical Engineering, STU in Bratislava. The model of the device is shown in Figure 4.

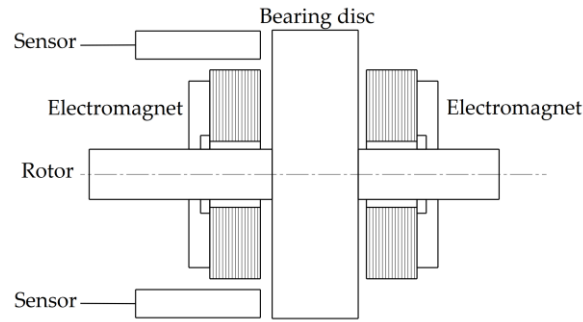


Fig. 3 Model of axial magnetic bearing - Experimental device
- rigid rotor mounted in magnetic bearings

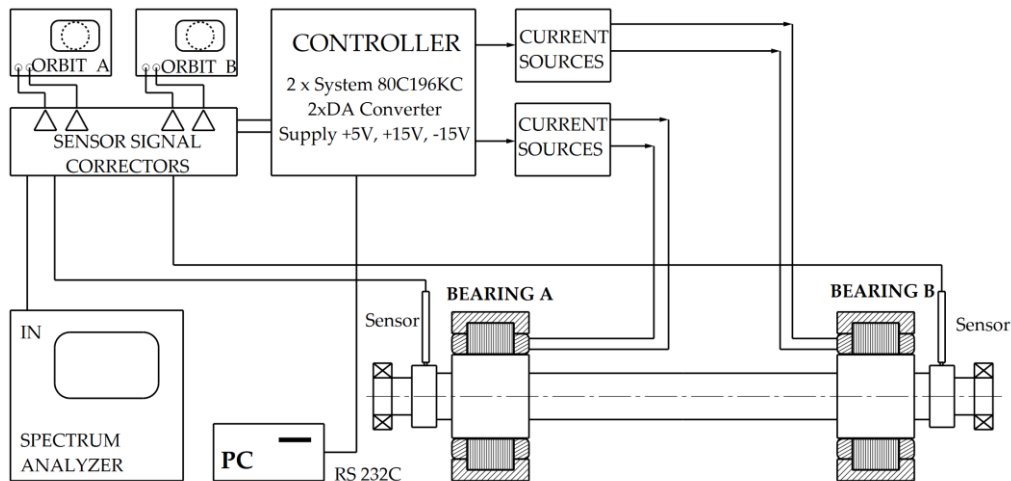


Fig. 4 Experimental test rig block scheme

3 Mathematical model of a magnetic bearing

This section analyzes the force effects of a magnetic bearing on a ferromagnetic core rotor ring. The case of small rotor deflections in the bearing is considered. The relationships describing the interaction are then linearized by the Taylor series.

The conditions in the air gap are decisive for the transmission of force between the rotor ring of the bearing and the stator core of the electromagnetic bearing. The total magnetic flux in the air gap after neglecting small scattering fluxes is given by Eq. (1).

$$\Phi = \frac{\mu_0 A N i_v}{h_v} \quad (1)$$

where A is the area through which the magnetic flux flows, N is the number of turns of the solenoid coil winding, i_v is the current flowing through the coil winding, h_v is the size of the air gap and μ_0 is the permeability of the medium (Figure 5). The force generated by magnetic flux is eq. (2)

$$F = \frac{\Phi^2}{2\mu_0 A} \quad (2)$$

Considering the two poles of the electromagnet, we get the force exerted by the "horseshoe" of the electromagnet on the rotor at the distance h_v , and the current i_v , in the coil (Figure 5).

$$F_h = 2F = \frac{\mu_0 AN^2 i_v^2}{h_v^2} \quad (3)$$

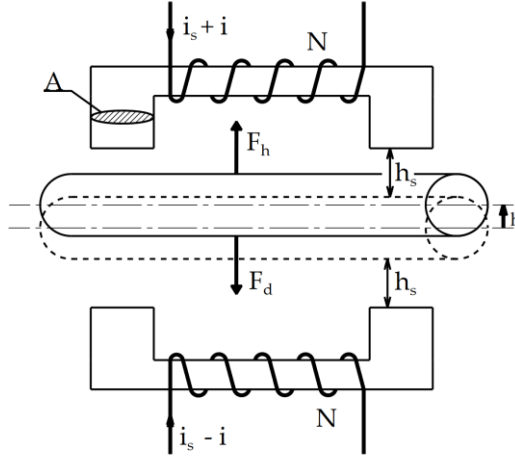


Fig. 5 Electromagnetic force effect

In practice, the most common case of the geometric configuration of the magnets according to Figure 5. The stationary position assumes the same distance of the rotor h_s , from the electromagnets at the stationary current i_s , in the stationary position-geometric center of the bearing. For the magnitude of the current i_v , and the distance h_v , let eq. (4)

$$\begin{aligned} i_v &= i_s + i \\ h_v &= h_s - h \end{aligned} \quad (4)$$

where the total current i_v is formed of a constant component i_s and a variable component i , the magnitude of which is given by the law of control. The resulting value of the air gap size h_v is formed from the value h_s - constant size of the nominal air gap and of the variable value h - value of the change in the equilibrium position.

Then the forces in the individual directions of the axes can be broken down as follows:

For the direction of the y-axis

$$F_{yv} = F_{yh} + F_{yd} = \mu_0 AN^2 \left[\frac{(i_s + i_g + i_y)^2}{(h_s - h_y)^2} - \frac{(i_s - i_y)^2}{(h_s + h_y)^2} \right] \quad (5)$$

For the direction of the z-axis

$$F_{zv} = F_{zl} + F_{zr} = \mu_0 AN^2 \left[\frac{(i_s + i_z)^2}{(h_s - h_z)^2} - \frac{(i_s - i_z)^2}{(h_s + h_z)^2} \right] \quad (6)$$

where h_y , h_z are the deviations from the equilibrium position in the y and z directions; i_y , i_z are the magnitudes of the control currents for the y and z directions; F_{yh} , F_{yd} are the forces in the electromagnets in the direction y - h is upper and d is lower; F_{zl} , F_{zr} are forces in electromagnets in direction z - l is left and r is right.

These relations apply only under the condition that the electromagnetic bearing is rotationally symmetrical and the static currents in both directions are the same. In equation (5) is i_g the magnitude of the current required to overcome the weight of the rotor.

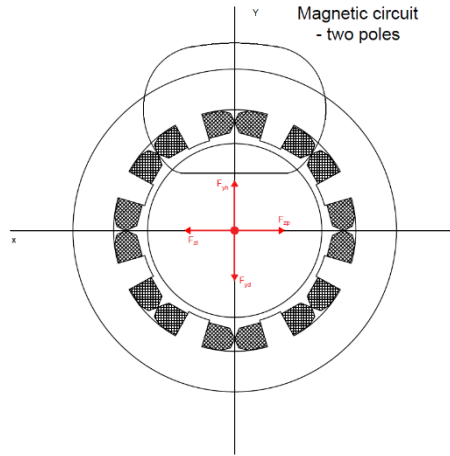


Fig. 6 Geometric configuration of electromagnets

Assuming small rotor deflections in the magnetic bearing, it is possible to linearize the relations (5) and (6) around the equilibrium position by development to the Taylor series. If we use only the first two members of this development, we get

$$F_{yv} = \mu_0 AN^2 \left(\frac{2(2i_s + i_g)}{h_s^2} \right) i_y + \mu_0 AN^2 \left(2 \frac{(i_s + i_g)^2 + i_s^2}{h_s^3} \right) h_y \quad (7)$$

$$F_{zv} = \mu_0 AN^2 \left(4 \frac{i_s}{h_s^2} \right) i_z + \mu_0 AN^2 \left(4 \frac{i_s^2}{h_s^3} \right) h_z \quad (8)$$

The constants for the individual members of the control currents are called *current stiffnesses* and for the members of the positional *stiffness deviations*. The value of h_s is the nominal size of the air gap, the value of i_s is the pre-magnetizing current in the coil windings.

Equations (7) and (8) can be written in the form [1]

$$F_{yv} = k_{iy} i_y + k_{sy} h_y \quad (9)$$

$$F_{zv} = k_{iz} i_z + k_{sz} h_z \quad (10)$$

where the meaning of k_{iy} , k_{iz} , k_{sy} , k_{sz} , is obvious from equations (7), (8), (9) and (10). The constant k_s is also called the negative-destabilizing stiffness, or also the stiffness of the open control loop. Without feedback, the system is unstable and therefore a feedback controller must be included in the magnetic bearing circuit. The feedback model is determined based on the requirements of the operation and can be changed programmatically.

4 Mathematical model of a rigid rotor in magnetic bearings

In this part, a mathematical model of a mechatronic system-rigid rotor mounted in magnetic bearings is created. The relationships derived in the previous section were used as the effects of magnetic bearings. The equations of motion of the rigid rotor were derived based on Figure 7.

The equation of motion has the form

$$\mathbf{M}_1 \ddot{\mathbf{z}}_1 = \mathbf{T}_2 \mathbf{f}_{amb} + \mathbf{G} \quad (11)$$

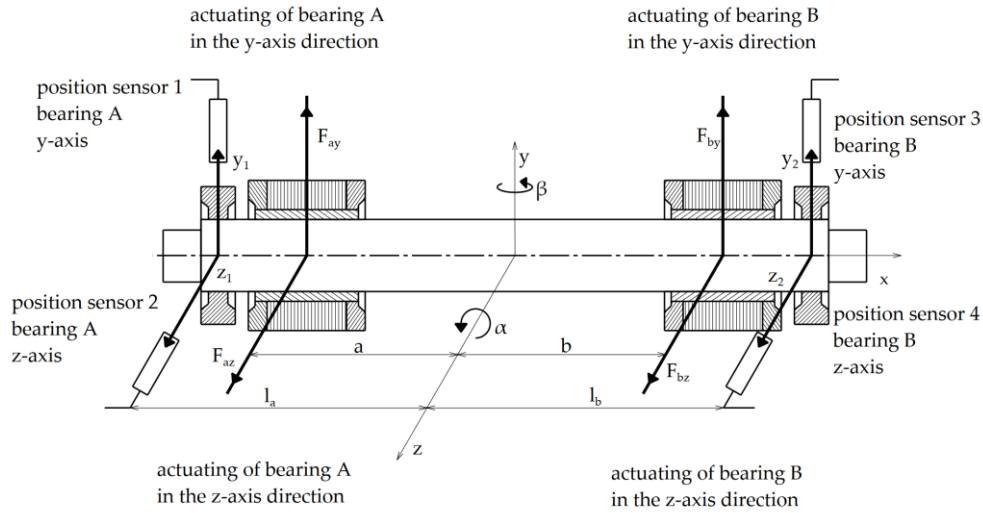


Fig. 7 Force effect on rigid rotor

where:

$$\mathbf{M}_1 = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & J_y & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & J_z \end{bmatrix}, \mathbf{T}_2 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & a & 0 & -b \\ 1 & 0 & 1 & 0 \\ -a & 0 & b & 0 \end{bmatrix}, \mathbf{G} = \begin{bmatrix} 0 \\ 0 \\ -mg \\ 0 \end{bmatrix}, \quad (12)$$

$$\mathbf{f}_{amb} = \begin{bmatrix} F_{ay} \\ F_{az} \\ F_{by} \\ F_{bz} \end{bmatrix}, \quad \mathbf{z}_1 = \begin{bmatrix} z \\ \beta \\ y \\ \alpha \end{bmatrix}$$

m is the mass of the shaft, J_y , J_z are the moments of inertia of the rotor about the x-axis, respectively y ; a , b are the distances of the magnetic field forces from the origin of the coordinate system, and $\mathbf{T}_2$ is the transformation matrix of the force and moment effects of the electromagnetic forces.

Since the deviations are measured at the shaft ends, it is necessary to introduce the transformation of the vector $\mathbf{z}_1$, to the vector $\mathbf{z}$ using the transformation matrix $\mathbf{T}_1$. Then it applies to the vector $\mathbf{z}_1$

$$\mathbf{z}_1 = \begin{bmatrix} z \\ \beta \\ y \\ \alpha \end{bmatrix} = \frac{1}{l_a + l_b} \begin{bmatrix} 0 & l_b & 0 & l_a \\ 0 & 1 & 0 & -1 \\ l_b & 0 & l_a & 0 \\ -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ z_1 \\ y_2 \\ z_2 \end{bmatrix} = \mathbf{T}_1 \mathbf{z} \quad (13)$$

then the equation (11) is transformed

$$\mathbf{M}_1 \mathbf{T}_1 \ddot{\mathbf{z}} = \mathbf{T}_2 \mathbf{f}_{amb} + \mathbf{G} \quad (14)$$

Based on equations (9) and (10), the force effects of the magnetic field can be written in the form

$$\begin{bmatrix} f_{ay} \\ f_{az} \\ f_{by} \\ f_{bz} \end{bmatrix} = \begin{bmatrix} k_{iay} & 0 & 0 & 0 \\ 0 & k_{iaz} & 0 & 0 \\ 0 & 0 & k_{iby} & 0 \\ 0 & 0 & 0 & k_{ibz} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} + \begin{bmatrix} k_{say} & 0 & 0 & 0 \\ 0 & k_{saz} & 0 & 0 \\ 0 & 0 & k_{sby} & 0 \\ 0 & 0 & 0 & k_{sbz} \end{bmatrix} \begin{bmatrix} y_1 \\ z_1 \\ y_2 \\ z_2 \end{bmatrix} \quad (15)$$

or in matrix form

$$f_{amb} = K_i i + K_s z \quad (16)$$

The equation of motion of the rigid rotor mounted in the magnetic bearings is then

$$M_1 T_1 \ddot{z} - T_2 K_s z = T_2 K_i i + G \quad (17)$$

It can be seen from equation (17) that the term at z is negative, the matrix K_s is also called the negative stiffness matrix. Equation (17) can be converted to a state space:

$$\begin{bmatrix} \dot{z} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 & I \\ (M_1 T_1)^{-1} T_2 K_s & 0 \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix} + \begin{bmatrix} 0 \\ (M_1 T_1)^{-1} T_2 K_i \end{bmatrix} i + \begin{bmatrix} 0 \\ (M_1 T_1)^{-1} G \end{bmatrix} \quad (18)$$

or

$$\begin{aligned} \dot{x} &= Ax + Di + G_s \\ y &= Cx + Ei \end{aligned} \quad (19)$$

where the meaning of matrices A , D , G_s is obvious and equation (18). Matrix C is the matrix of output quantities and matrix E is the weight matrix of input. When the rotor moves in magnetic bearings, we measure the deflections in the bearings, therefore the second equation of state notation (19) can be written in the form

$$\begin{bmatrix} y_{y1} \\ y_{z1} \\ y_{y2} \\ y_{z2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} [y_1 \ z_1 \ y_2 \ z_2 \ \dot{y}_1 \ \dot{z}_1 \ \dot{y}_2 \ \dot{z}_2]^T \quad (20)$$

Solving the problem of eigenvalues of matrix A gives results from which it is clear that the system is unstable: The system can be stabilized by suitable feedback. Let the feedback be defined as follows:

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} c_1 & 0 & 0 & 0 & v_1 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 & 0 & v_2 & 0 & 0 \\ 0 & 0 & c_3 & 0 & 0 & 0 & v_3 & 0 \\ 0 & 0 & 0 & c_4 & 0 & 0 & 0 & v_4 \end{bmatrix} [y_1 \ z_1 \ y_2 \ z_2 \ \dot{y}_1 \ \dot{z}_1 \ \dot{y}_2 \ \dot{z}_2]^T \quad (21)$$

or

$$\begin{aligned} i &= F_{inp} x \\ F_{inp} &= [C_F \ V_F] \end{aligned} \quad (22)$$

where the matrices C_F and V_F are apparent from equations (21) and (22). The movement of the rigid rotor can be described in the form

$$\dot{x} = (A + DF_{inp})x + G_s \quad (23)$$

or

$$\begin{aligned} \begin{bmatrix} \dot{z} \\ \ddot{z} \end{bmatrix} &= \begin{bmatrix} 0 & I \\ (M_1 T_1)^{-1} T_2 (K_i C_F + K_s) & (M_1 T_1)^{-1} T_2 K_i V_F \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix} + \\ &+ \begin{bmatrix} 0 \\ (M_1 T_1)^{-1} T_2 K_i \end{bmatrix} i + \begin{bmatrix} 0 \\ (M_1 T_1)^{-1} G \end{bmatrix} \end{aligned} \quad (24)$$

The problem of eigenvalues of a mechatronic system is then defined as follows:

$$\det[A + DF_{inp} - \lambda I] = 0 \quad (25)$$

5 Feedback design

In this part, the method of assigning eigenvalues and eigenvectors applied to an experimental model of a rotor mounted in magnetic bearings was used. Method is described in detail in the literature [2, 5].

Let the rotor be mounted in magnetic bearings, the feedback matrix of the system is $\mathbf{F}_{old}$ and the working range of the system is determined by the speed from $\omega = 0$ to $\omega = \omega_0$. If ω_0 is less than half of the first natural elastic frequency of the rotor, it can be considered as rigid [4]. Let there be a requirement to change the natural frequencies of the system. Then it is necessary to determine the new feedback matrix $\mathbf{F}_{new}$ so that the system has the required eigenvalues. A method for determining an additional feedback matrix is described in the literature [2, 5]. We consider the system with feedback $\mathbf{F}_{old}$ as the original system and we look for a new feedback matrix $\mathbf{F}_{add}$ for it so that we control the original system with feedback $\mathbf{F}_{old}$ with another feedback with the feedback matrix $\mathbf{F}_{add}$. Considering the solution in the linear domain, the resulting feedback matrix $\mathbf{F}_{new}$ is formed by superposition of the old feedback matrix $\mathbf{F}_{old}$ and the new feedback matrix $\mathbf{F}_{add}$, which we have determined to meet the specified requirements for the attenuation size and natural frequencies of the system [2, 5].

As an application, we present the calculation of the feedback supplementary matrix $\mathbf{F}_{add}$ for the real case. The system parameters are as follows:

Shaft weight	$m = 8,36 \text{ kg}$	
Moments of inertia	$J_y = 0,1556 \text{ kg.m}^2$	$J_z = 0,1556 \text{ kg.m}^2$
Position of magnetic bearings	$a = 0,2 \text{ m}$	$b = 0,2 \text{ m}$
Sensor position	$l_a = 0,25 \text{ m}$	$l_b = 0,25 \text{ m}$
Magnetic bearing current constants	$k_{iaz} = 346,1 \text{ N.A}^{-1}$ $k_{ibz} = 346,1 \text{ N.A}^{-1}$	$k_{iay} = 432,6 \text{ N.A}^{-1}$ $k_{iby} = 432,6 \text{ N.A}^{-1}$
Position constants of the magnetic bearing	$k_{saz} = 6,92 \cdot 10^5 \text{ N.m}^{-1}$ $k_{sbz} = 6,92 \cdot 10^5 \text{ N.m}^{-1}$	$k_{say} = 1,125 \cdot 10^6 \text{ N.m}^{-1}$ $k_{sby} = 1,125 \cdot 10^6 \text{ N.m}^{-1}$
Pre-magnetizing current	$i_s = 1 \text{ A}$	
Current to overcome gravity	$i_g = 0,5 \text{ A}$	
Feedback constants	$c_1 = -7500$ $c_3 = -7500$ $v_1 = -2,5$ $v_3 = -2,5$	$c_2 = -7500$ $c_4 = -7500$ $v_2 = -2,5$ $v_4 = -2,5$

For the original system, the eigenvalues are:

$$\lambda = 10^3 \begin{bmatrix} -0,1035 + 0,6668i \\ -0,1035 - 0,6668i \\ -0,1294 + 0,7003i \\ -0,1294 - 0,7003i \\ -0,2780 + 1,0705i \\ -0,2780 - 1,0705i \\ -0,3475 + 1,1142i \\ -0,3475 - 1,1142i \end{bmatrix} \quad (26)$$

Let the new required eigenvalues be changed so that the real part of the eigenvalue is unchanged, and the imaginary part increases in absolute value by 10%. Then the new required eigenvalues are λ_p :

$$\lambda_p = 10^3 \begin{bmatrix} -0,1035 + 0,7335i \\ -0,1035 - 0,7335i \\ -0,1294 + 0,7703i \\ -0,1294 - 0,7703i \\ -0,2780 + 1,1776i \\ -0,2780 - 1,1776i \\ -0,3475 + 1,2257i \\ -0,3475 - 1,2257i \end{bmatrix} \quad (27)$$

Using the method of assigning eigenvalues and eigenvectors, mentioned for example in articles [2, 5], we determine the feedback matrix $\mathbf{F}_{add}$. The new control will be

$$\mathbf{F}_{new} = \mathbf{F}_{old} + \mathbf{F}_{add} \quad (28)$$

The new form of feedback matrix is:

$$\mathbf{F}_{new} = \begin{bmatrix} -8466,4 & 0 & -28,6 & 0 & -2,5 & 0 & 0 & 0 \\ 0 & -8604,9 & 0 & -22,9 & 0 & -2,5 & 0 & 0 \\ -28,6 & 0 & -8466,4 & 0 & 0 & 0 & -2,5 & 0 \\ 0 & -22,9 & 0 & -8604,9 & 0 & 0 & 0 & -2,5 \end{bmatrix} \quad (29)$$

The new matrix of the $\mathbf{F}_{new}$ feedback system in the form $\mathbf{A} + \mathbf{F}_{new}\mathbf{D}$ then has its own numbers specified.

6 Experimental verification

The aim of the experimental measurements was to verify the change in the spectral properties of the mechatronic system. The original system with eigenvalues given in relation (26) was transformed into the system with eigenvalues given in relation (27) by means of a new feedback matrix (29).

To verify this change, we compared computer simulations of the amplitude characteristic and experimentally measured data of the system response to harmonic excitation for selected excitation frequencies. The system was excited by a harmonic force that generated an active magnetic bearing. The control signal for the bearing electromagnets consisted of a part corresponding to the harmonic excitation and a part corresponding to the control according to the control law described in the previous chapter. The part of the signal that corresponds to the bearing excitation was programmed as a harmonic function. The part of the signal that corresponds to the bearing control was also independently programmed, and the resulting signal that was transmitted from the control electronics to the current sources was the algebraic sum of the two data at a given time. The control algorithm was implemented as a discrete digital controller. Since the frequency of the controller (loop A-D conversion - calculation of D-A conversion) was an order of magnitude higher (10 kHz) as the frequency range in the experiment (up to 500 Hz), the controller was considered continuous. The bearing thus functioned as an integrated system - driver and control element. The constant value of the discrete controller frequency was ensured by interrupting the microprocessor timer. The scheme of experimental verification of the excitation system is shown in Figure 8. A comparison of computer simulations and experimentally measured values is shown in Figure 9 and Figure 10.

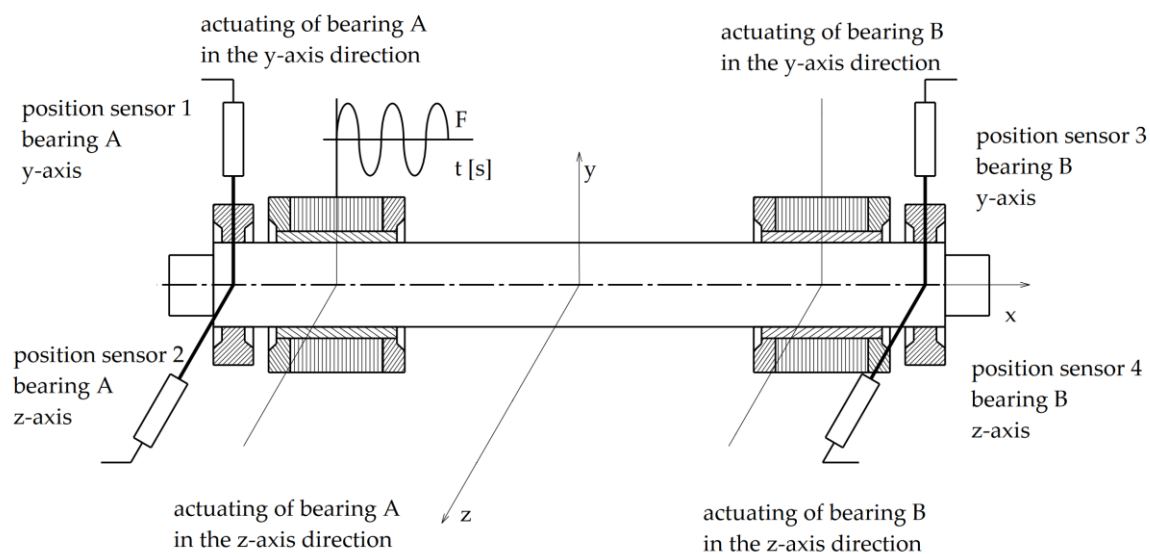


Fig. 8 Experimental examination scheme of excited system

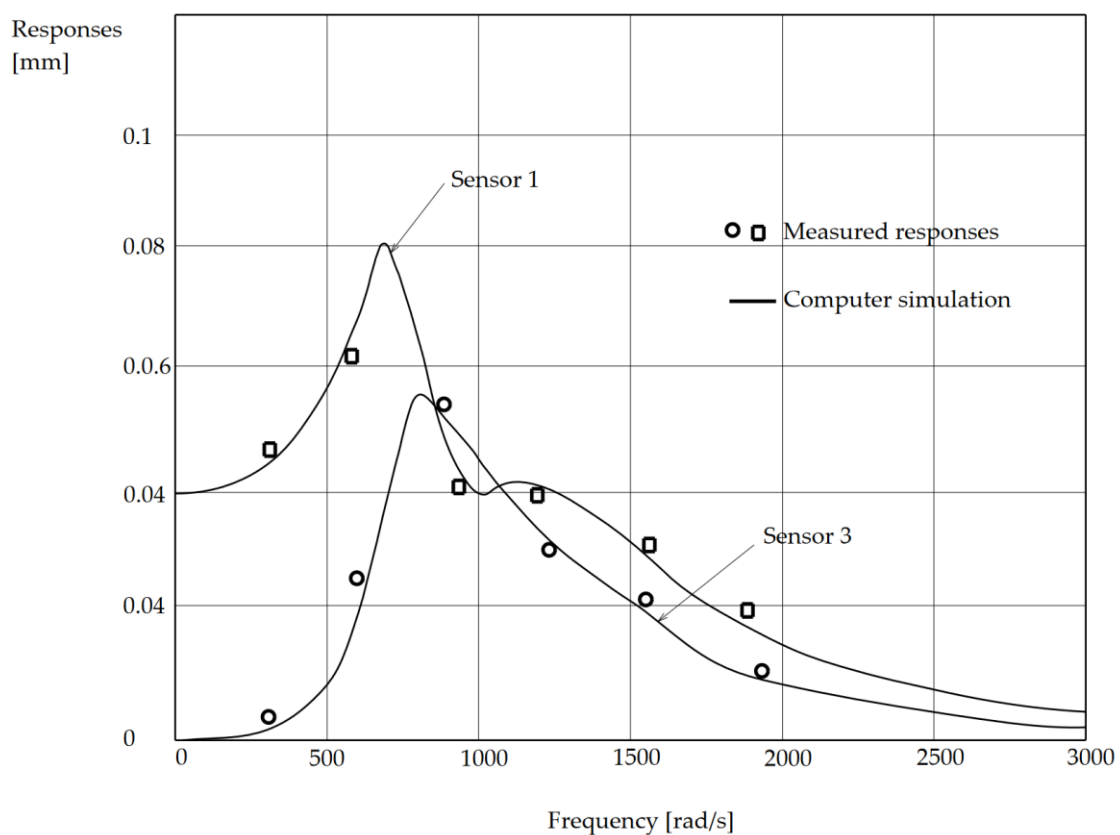


Fig. 9 Frequency response of the system:
feedback matrix $\mathbf{F}_{old}$ - decentralized control

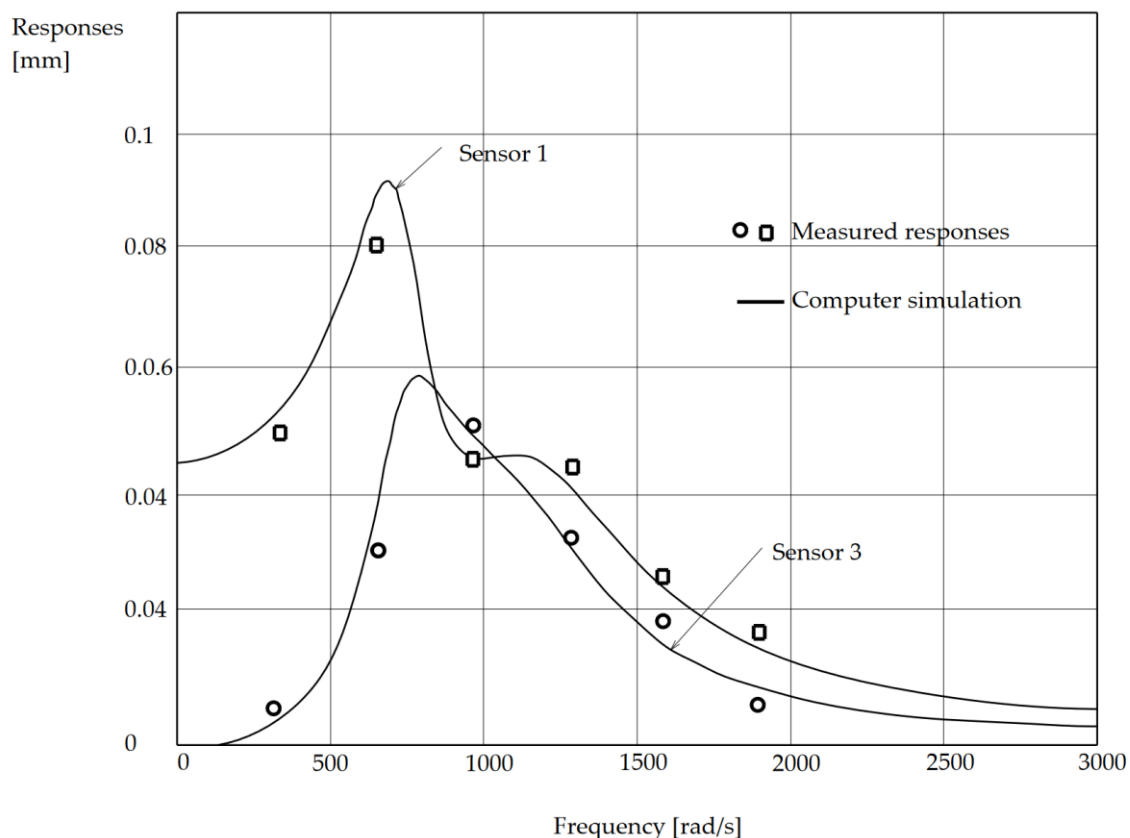


Fig. 10 Frequency response of the system:
feedback matrix $\mathbf{F}_{old}$ - centralized control

It is clear from the figures that the experimental measurements are in good correlation with the computer simulations. Both experiments - with the original feedback matrix and with the new feedback matrix - were excited at the same frequencies. When applying the new feedback matrix, the system was retuned, which was reflected in a decrease in the amplitudes of the system response. It can be stated that by applying said control, the system can be retuned, and thus the oscillation in the system can be reduced.

Conclusion

The aim of the paper is to present the active vibration control of the mechanical system. The method of assigning eigenvalues and eigenvectors, applied to an experimental model of a rotor mounted in magnetic bearings, was used for control.

The first two parts describe the magnetic bearings and an overview of the current state. The third part deals with the issue of force effects of the electromagnetic field in a magnetic bearing. In the fourth part, a mechatronic model of a rigid rotor mounted in magnetic bearings is created. In the fifth part, the method of assigning eigenvalues and eigenvectors according to the methodology given in the literature is applied for the real case of the experimental device. Experimental results verifying the theoretical assumptions are presented in the sixth part.

The main benefit of the article is the use of the method of assigning eigenvalues and eigenshapes in a mechatronic system. The possibility of implementing the law of control on a mechanical system makes it possible to change its spectral and modal properties by means of electromagnetic interaction. Decentralized management is changing to centralized. When using decentralized control, influencing the behavior of the entire rotor depends only on the response at the point of action. In centralized control, the magnitude of the action at the location of the magnetic bearing also depends on the response in the second magnetic bearing.

Thus, centralized control makes it possible to control the oscillation of the rotor based on the response of the entire rotor. Therefore, the possibility of negatively influencing the behavior of the entire rotor system in magnetic bearings, especially in the second bearing, is considerably lower during operation. The choice of new own system numbers depends on the technical application. The variability of the method used based on the requirements for the behavior of the system will allow such choices of the control feedback matrix that it is also practically feasible and the system has a satisfactory response for the given external loads. The control methods applied so far in the literature applied to magnetic bearing systems have always been decentralized.

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