Some mathematical and geometrical interpretations of the Sator Square

Paul Dario Toasa Caiza[1]

[1] *Karlsruher Institut für Technologie. Versuchsanstalt für Stahl, Holz und Steine. Otto-Ammann-Platz 1. 76131 Karlsruhe. Germany.*

paul.toasa@kit.edu

Abstract

In 1738, the King of Naples and future King of Spain, Carlos III, commissioned the Spanish military engineer Roque Joaquín de Alcubierre to begin the excavations of the ruins of the ancient Roman city of Pompeii and its surroundings, buried by the terrible explosion of Vesuvius in AD 79. Since that time, archaeologists have brought to light wonderful treasures found in the among ruins. Among them, the Sator Square is one of the most peculiar, apparently simple but mysterious. Supernatural and medicinal powers have been attributed to this object and its use was widespread during the Middle Age. Studies to explain its origin and meaning have been varied. There are theories that relate it to religion, the occult, medicine and music. However, no explanation has been convincing beyond pseudo-scientific sensationalism. In this study, the author intends to eliminate the mystical character of the Sator Square and suggests considering it as a simple palindrome or a game of words with certain symmetrical properties. However, these properties are not exclusive to the Sator Suare but are present in various mathematical and geometric objects.

1 Introduction

I was very intrigued by an article I read about the enigmatic Sator Square on the BBC Spanish service. And the intrigue was because all the mentioned explanations were spiritual and pseudo-scientific. Despite the fact that the object presented certain symmetries, no geometric or mathematical study was mentioned in this regard. So I searched for mathematical studies on this object and only found countless sensational and absurd works. So I asked myself: Is it possible to consider the Sator Square as a mathematical object and study it?

Among the many wonderful historical artifacts found in the ruins of Pompeii caused by the eruption of Vesuvius in A.D. 79, Sator Square is one of the most unique and unusual,

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S	A	T	O	R
A	R	E	P	O
T	E	N	E	T
O	P	E	R	A
R	O	T	A	S

Figure 1: Sator square

see Figure 1. Particularly, during the Middle Age, the square was used as talisman or amulet, against rabies and other diseases and complaints, such as fever, intestinal gout and rose, jaundice and consumption, cramps, headache and toothache, birth complications and worms. It was also regarded as a remedy against thieves, witchcraft and fire, as a spell of exorcism, blessing of bullets and hunting aid, even as a love magic [Bad87].

An immense amount of works have been written in order to explain its origin, meaning and, in the case of being a riddle, its solution. All these works suggest a relationship with Christianity, Judaism, Satanism, philology, numerology, esotericism, magic, mysticism and even medicine [Sel14], [Dar32], [Fis64]. However, none of these works provides a complete and convincing explanation about the origin and meaning of the square.

The magic of this square rest surely on the perfect symmetry of its component letters, which yield the same combinations in four different directions [Fis64]. It has 5 Latin words, which can be read from left to right and from top to bottom, or from left to right and bottom to top and their “message” is the same, see Figure 2

SATOR AREPO TENET OPERA ROTAS

Figure 2: Five words palindrome

In [Fis64], the author analyses how difficult to build a rebus with twenty five letters in five Latin words is. Moreover, it is suggested that the words were written by Latin-speaking Jews.

Other similar squares have been written by considering other words and other languages like Greek and Hebrew see [Sel14].

Even in the pop culture the Sator Square has inspired the creation of Tenet, which is a 2020 science fiction action thriller film directed and written by Christopher Nolan. In this film the Protagonist master the art of “time inversion” as a way of countering the threat that is to come.

In [Gri71] the origin of AREPO is discussed and it is proposed that the square has Egyptian origin.

In [Fer74] the work of Moeller [Moe73] is reviewed. Moeller proposes that Sator is Saturn and assures "I now present these findings in the firm conviction that I have come upon the correct solution". This declaration received a lot of critisism, saying that this work went in the wrong direction. Others called it provocative but no persuasive.
In fact, about half the text deals with numerology and it also suggests that probably the square has Christian origin, fact that Hemer supports in [Hem78]. This is based on the fact that the phrase "Pater Noster" and additionally the letters A and O, standing for Alpha and Omega, the first and the last were obtained by rearrangin the twenty five letters of the square.
In [Hof76], the work of Moeller is compared to a sandcastle.

Even in the classical music the influence of the square can be noted. The Austrian composer Anton Friedrich Wilhelm von Webern[1] considered the square strucuture in its composition "Concerto for Nine Instruments, Op. 24"[2], written in 1934 [Neu80].

An extensive study about several interpretations of the square, with an abundant bibliography is available in [She03].

2 Reading simmetry

The most discussed chracteristic of the Sator square is related with the concept of palyndromes. The Sator square has five words, located in a particular way, which makes possible to obtain several types of reading simmetry.

2.1 Clasic palindrome

The word TENET is a palyndrome, and it is written in the middle column and row of the square. If two perpendicular axis are drawn trought the center of the square, the palindrome TENET coincides with these axes, see Figure 3.

Moreover, by writing consecutively the five words, a longer palindrome is obtained as well, see Figure 2.

2.2 Palindrome in both directions

The four words SATOR, AREPO, OPERA and ROTAS behave like a palindrome. They can be observed and read in the traditional western way from left to right and from top to bottom or from right to left and from bottom to top. Depending on the reading direction, these words are located in the ith-row and ith-column. For example, OPERA is located in the 4th row and the 4th column from Figure 4a and in the 2nd row and 2nd column from Figure 4b, see the corresponding green arrows.

If both squares from Figure 4 are joined, the five words of the Sator square are reduced to three SATOR, AREPO and TENET, see Figure 5. Moreover, it is also possible to

[1] 3.12.1883 – 15.09.1945
[2] Konzert für neun Instrumente

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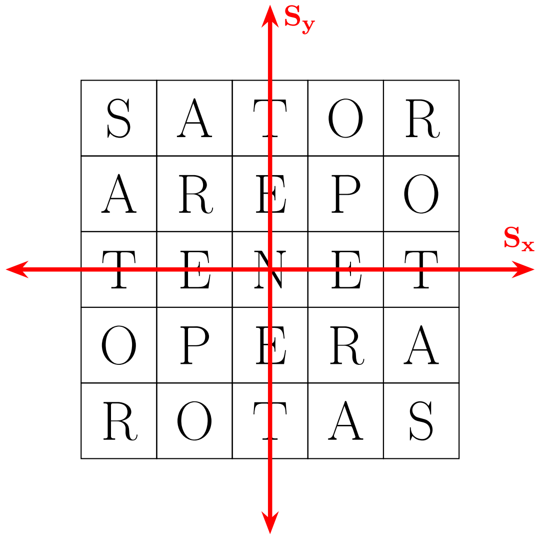
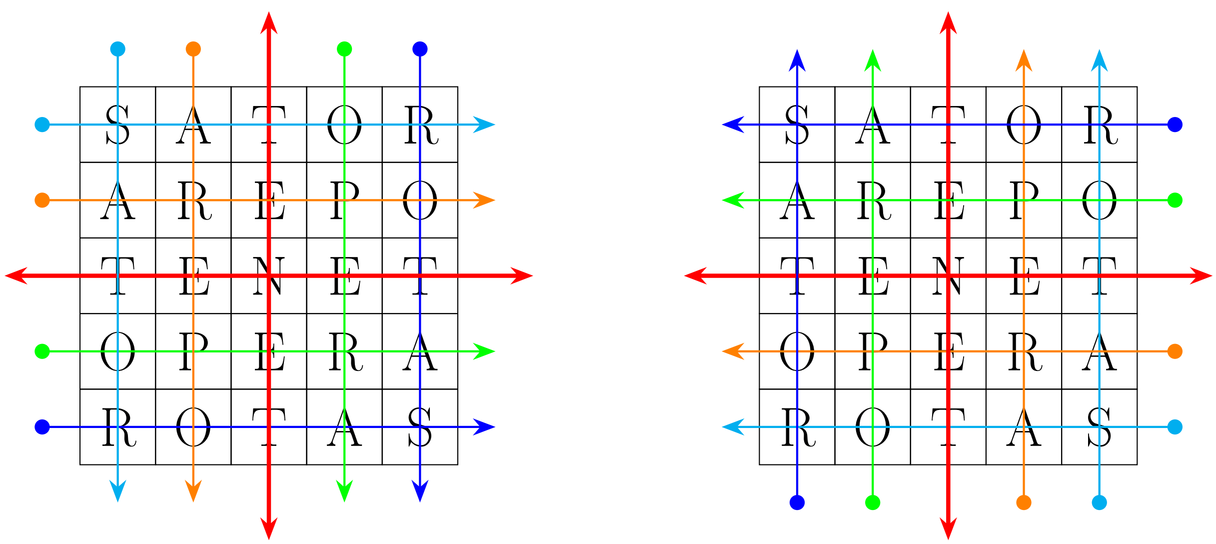


Figure 3: The palindrome TENET of the Sator square



(a) Simmetry from left to right and from top to bottom

(b) Simmetry from right to left and from bottom to top

Figure 4: Reading simmetry of the Sator square in two ways

observe that the Sator square keeps its form after rotating one time around the horizontal axis and one time around the vertical axis. This kind of simmetry will be discussed in the next section.

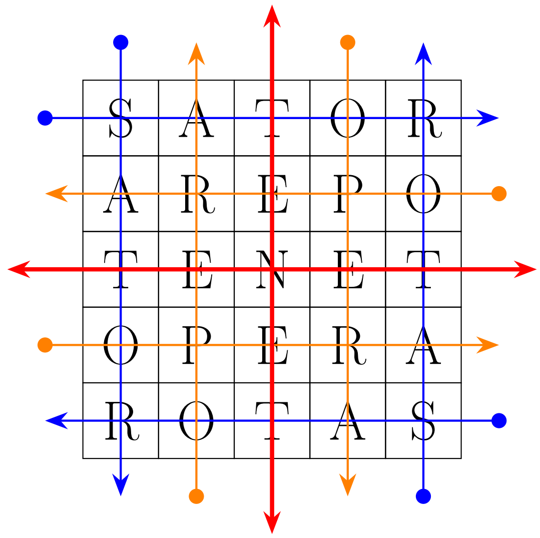


Figure 5: Reading simmetry of the Sator square in four ways

3 Geometric symmetry

The reading symmmetries observed in the previous section can be described trought different types of geometrical symmetries. This description will allow to find a general way to generate Sator Squares of higher dimensions.

3.1 Point symmetry

When every part of an object has a matching part, and both parts have the same distance but in the opposite direction from the centre point, this is known as point symmetry. It's also known as "Order 2 rotational symmetry."
Because the "Origin" is the primary point around which the shape is symmetrical, point symmetry is also known as origin symmetry.
When a figure is rotated 180^o, and it keeps its original shape, it is said that it has point symmetry.
Thus, the palindrome TENET in the Sator Square has a point symmetry located at the letter N, see Figure 3. In other words, the axes S_x and S_y have a point symmetry with respect to the center of the square.

3.2 Reflective or mirror symmetry

Reflective symmetry, also called mirror symmetry, is a type of symmetry where one half of the object reflects the other half of the object. In other words, in this type of symmetry one part of the image or object represents the mirror image of another part of the image. Reflexive Symmetry is described by a line of symmetry.
In the Sator Square two lines of symmetry S_1 and S_2 are present, see Figure 6.

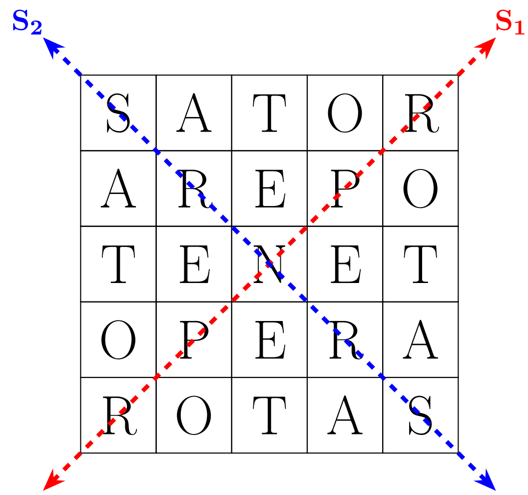
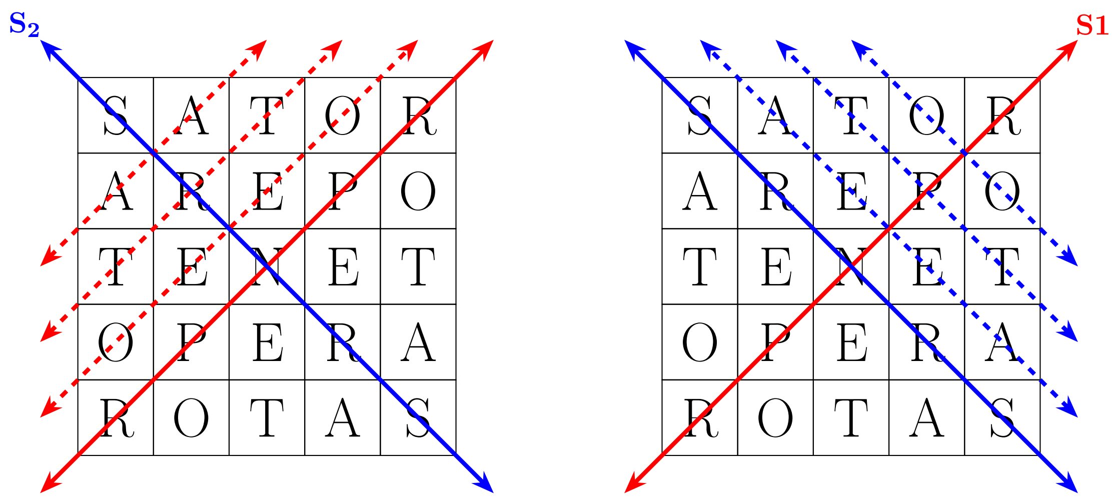


Figure 6: Lines of simmetry of the Sator square

If a mirror is located along the lines of symmetry S_1 and S_2, the letters in the Sator Square will be reflected as it is shown with dotted lines in Figure 7



(a) Reflexions relative to symmetry line S2

(b) Reflexions relative to symmetry line S1

Figure 7: Simmetry lines and their corresponding reflection in the Sator Square

3.3 Rotational simmetry in 3D

Besides the 2D symmetries observed in the Sator Square, a symmetry in 3D is presented as well. If the Sator Squere is rotated $\pm 90^o$, and afterwards it is rotated $\pm 180^o$ by considering as rotation axis the symmetry line S_x or S_y, the Sator Square keeps its original

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form, see Figure 8. Doing these operation in a reverse order, the original form is kept as well.

S	A	T	O	R
A	R	E	P	O
T	E	N	E	T
O	P	E	R	A
R	O	T	A	S

N $\pm 90^o$

R	O	T	A	S
O	P	E	R	A
T	E	N	E	T
A	R	E	P	O
S	A	T	O	R

S_x $\pm 180^o$

S	A	T	O	R
A	R	E	P	O
T	E	N	E	T
O	P	E	R	A
R	O	T	A	S

Figure 8: Rotational simmetry in 3D of the Sator Square

4 Satorian functions and squares

Considering as reference the reading and geometrical simmetries observed in the Sator square, it is possible to find a general discrete way to create Sator squares of higher dimension.

However, before proposing a general definition and for didactic purposes, let us consider the following function, which satisfies the point simmetry along S_x and S_y corresponding to the center of the square and the lines of symmetry S_1 and S_2.

$$f(x,y) = \begin{cases} \sqrt{x^2+y^2} & \text{if } xy \geqslant 0 \\ -\sqrt{x^2+y^2} & \text{else} \end{cases} \tag{1}$$

where $x, y \in \mathbb{Z}$.

Figure 9 shows the application of the Equation (1) in the set given by $|x|, |y| \leqslant 2$. Consequently, a square of dimension $n = 5$ similar to the Sator Square is obtained. In fact, the only difference is that instead of letters, numbers are used. It is worth mentioning that in order to keep the expected simmetries, the dimension of the generated square has to be odd.

By assignng colors to the values of the square in Figure 9b, it is possible to observe the expected simmetries in a easier way, see Figure 10.

The Equation (1) is a particular case of a function, which will be called Satorian function and it will be defined as follows.

Definition 4.1 (Satorian function)**.** *Let (x, y) be a number in $\mathbb{Z}^2$, a function $f(x, y)$ is called Satorian when $f(x, y) = f(y, x)$ and $f(x, y) = f(-y, -x)$.*

The squares shown in Figure 10 are called Satorian squares and are defined as follows.

Definition 4.2 (Satorian square)**.** *The square obtained by applying a Satorian function $f(x, y)$ on a a discrete, centered and symmetric set of numbers of dimension $n \times n$, where n is odd, is called Satorian square of dimension n and it is noted as SS_n.*

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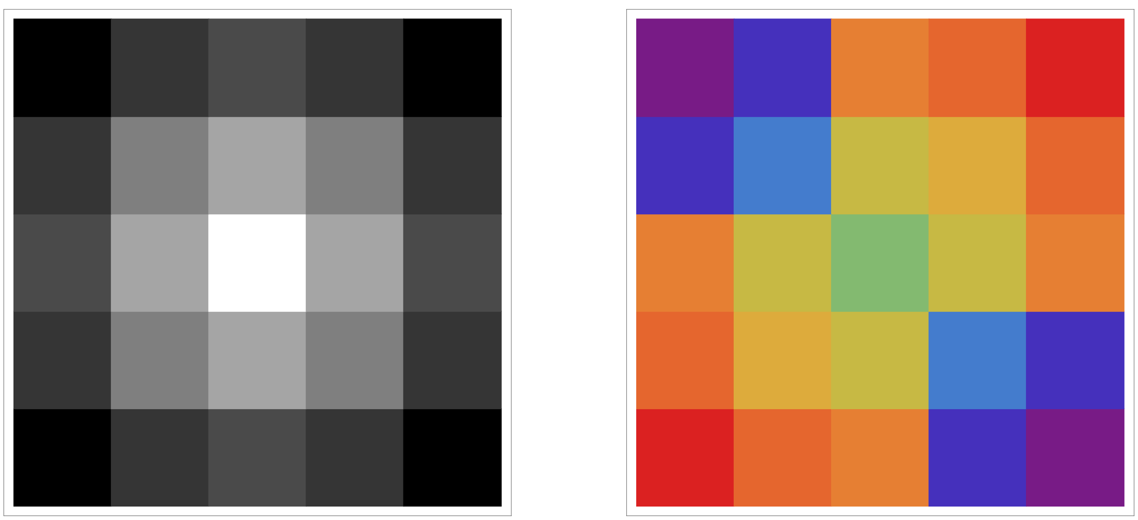
·	·	2	·	·
·	·	1	·	·
-2	-1	0	1	2
·	·	-1	·	·
·	·	-2	·	·

(a) (x, y)

$-\sqrt{8}$	$-\sqrt{5}$	2	$\sqrt{5}$	$\sqrt{8}$
$-\sqrt{5}$	$-\sqrt{2}$	1	$\sqrt{2}$	$\sqrt{5}$
2	1	0	1	2
$\sqrt{5}$	$\sqrt{2}$	1	$-\sqrt{2}$	$-\sqrt{5}$
$\sqrt{8}$	$\sqrt{5}$	2	$-\sqrt{5}$	$-\sqrt{8}$

(b) $f(x, y)$

Figure 9: Application of the Equation (1) for $|x|, |y| \leqslant 2$



(a) SS_5 in gray scale

(b) SS_5 in color scale

Figure 10: Graphical representation of a Satorian square SS_5 in gray scale and color

It is not necessary to define a Satorian function as an explicyt equation, it can be also given by a set of functions or by a set of discrete values. In fact, the archeological Satorian square is given in this way.

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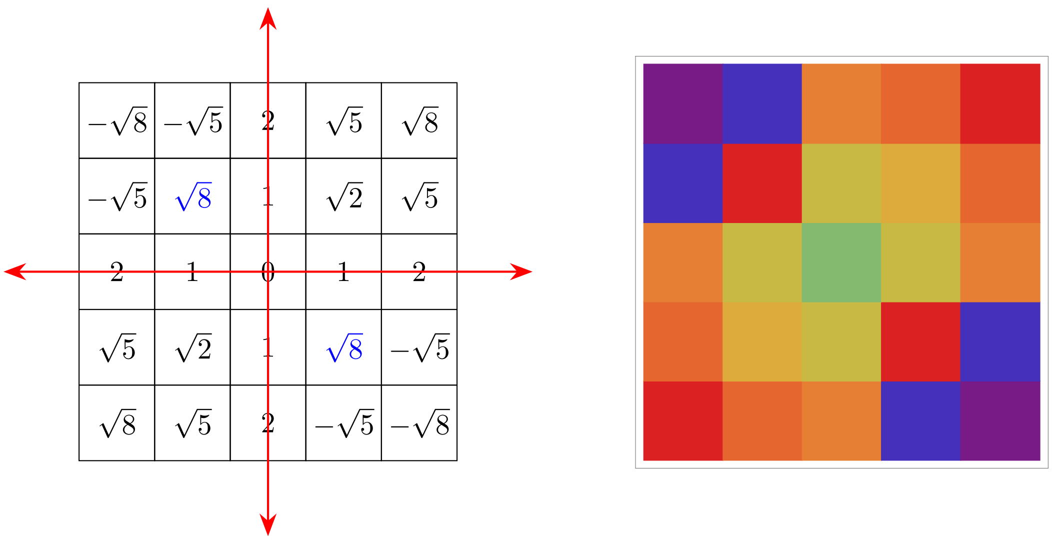
4.1 Archeological Satorian square

Once the definitions of a Satorian function and square have been given, it is possible to realize that the archeological Satorian square is no more than a particular case of a SS_5, where the Satorian function is given by a discrete set. Observe that the difference with the SS_5 given by Equation (1) and shown in Figures 9b and 10b, are the values written in blue in Figure 11a.



(a) $f(x, y)$ (b) Square in color

Figure 11: Archeological Satorian function and square

Generating Satorian squares by using numbers is much easier than using palyndromes or even letters. As example, I tried to build a Sator Square of dimension $n = 7$ by using spanish words. To do this, it is necessary 4 words, one palindrome of 7 letters and 3 words, which satisfy the corresponding simmetries. Unfortunately, I found only three words[1]. The palindrome ANILINA (aniline), ECUADOR, CIANURO (cyanide) and the no meaning word UANIMUD, see Figure 12.

5 Satorian matrices

By considering a Satorian Square as a matrix, it follows that it is a bisymmetric matrix, see [WLL19].

Definition 5.1 (Bisymmetric matrix). *An $n \times n$ matrix $A = (a_{ij}) \in \mathbb{R}$ is a bisymmetric matrix if $a_{i,j} = a_{j,i}$ and $a_{i,j} = a_{n+1-j,n+1-i}$ for all $i, j = 1, 2, ..., n$, that is,*

[1]The author will appreciate if an example with four words is given.

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E	C	U	**A**	D	O	R
C	I	A	**N**	U	R	O
U	A	N	**I**	M	U	D
A	**N**	**I**	**L**	**I**	**N**	**A**
D	U	M	**I**	N	A	U
O	R	U	**N**	A	I	C
R	O	D	**A**	U	C	E

Figure 12: Satorian Square of dimension $n = 7$. It is built with three spanish words, ANILINA(aniline), ECUADOR, CIANURO (cyanide) and one word without meaning UANIMUD.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1,n-2} & a_{1,n-1} & a_{1,n} \\ a_{12} & a_{22} & a_{23} & \cdots & a_{2,n-2} & a_{2,n-1} & a_{1,n-1} \\ a_{13} & a_{23} & a_{33} & \cdots & a_{3,n-2} & a_{2,n-2} & a_{1,n-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{1,n-2} & a_{2,n-2} & a_{3,n-2} & \cdots & a_{33} & a_{23} & a_{13} \\ a_{1,n-1} & a_{2,n-1} & a_{2,n-2} & \cdots & a_{23} & a_{22} & a_{12} \\ a_{1,n} & a_{1,n-1} & a_{1,n-2} & \cdots & a_{13} & a_{12} & a_{11} \end{pmatrix} \tag{2}$$

Bisymmetric matrices have several applications, such as numerical analysis, graph theory or structural mechanics, vibration of beams and eigenavalue problems see [Nou12], [PHZ04], [hPyHZ07], [YWX13].

Definition 5.2 (Satorian matrix)**.** *The matrix defined by the values of a Satorian function or a Satorian square of dimension n is called a Satorian matrix and it will be noted as SM_n.*

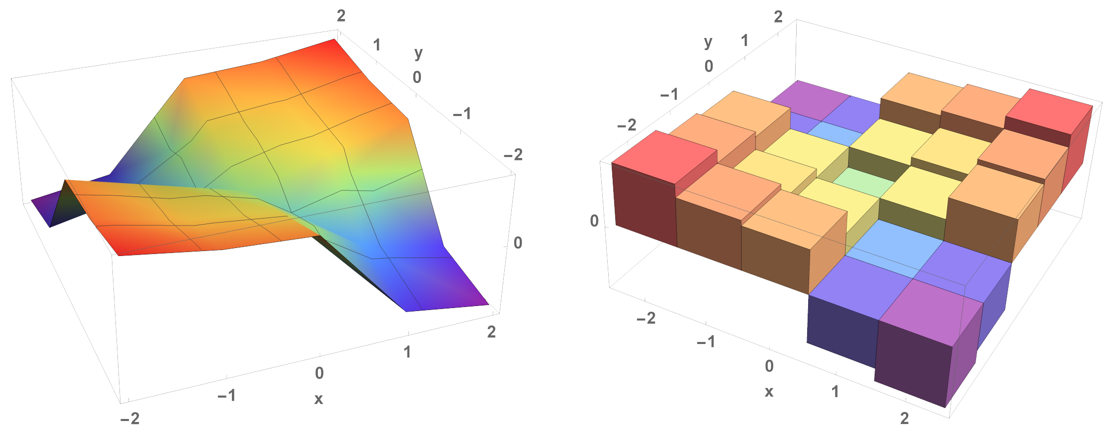
A Satorian matrix SM_n is a bisymmetric matrix.

6 Satorian surfaces and solids

If the values of a Satorian Matrix are considered as the values on the z-axis, symmetric surfaces and solids can be obtained. Thus, the Satorian matrix given by Equation (3) corresponds to the Satorian Square $SS5$ shown in Figure 10.

$$SM_5 = \begin{pmatrix} -\sqrt{8} & -\sqrt{5} & 2 & \sqrt{5} & \sqrt{8} \\ -\sqrt{5} & -\sqrt{2} & 1 & \sqrt{2} & \sqrt{5} \\ 2 & 1 & 0 & 1 & 2 \\ \sqrt{5} & \sqrt{2} & 1 & -\sqrt{2} & -\sqrt{5} \\ \sqrt{8} & \sqrt{5} & 2 & -\sqrt{5} & -\sqrt{8} \end{pmatrix} \tag{3}$$

Observe that the matrix SM_5 depends on the function $f(x, y)$ given by Equation (1) and shown in Figure 9b. The Figure 13 shows the surface and solid corresponding to the Matrix SM_5.

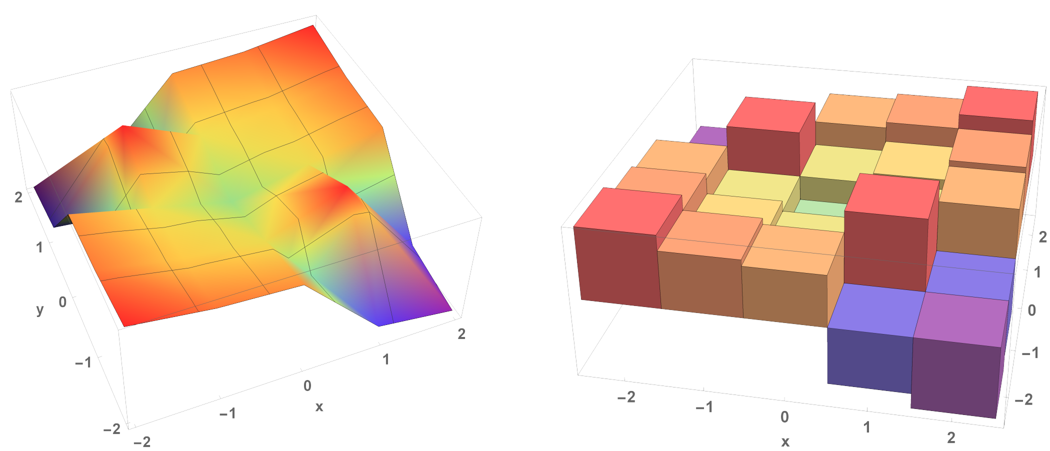


(a) Surface (b) Solid

Figure 13: Satorian surface and solid corresponding to the Satorian matrix SM_5

The matrix corresponding to the Archeological Satorian Square is given in Equation (4), and the surface and solid corresponding to the Archeological Satorian square are shown in Figure 14.

$$ASM_5 = \begin{pmatrix} -\sqrt{8} & -\sqrt{5} & 2 & \sqrt{5} & \sqrt{8} \\ -\sqrt{5} & \sqrt{8} & 1 & \sqrt{2} & \sqrt{5} \\ 2 & 1 & 0 & 1 & 2 \\ \sqrt{5} & \sqrt{2} & 1 & \sqrt{8} & -\sqrt{5} \\ \sqrt{8} & \sqrt{5} & 2 & -\sqrt{5} & -\sqrt{8} \end{pmatrix} \tag{4}$$



(a) Surface (b) Solid

Figure 14: Satorian surface and solid corresponding to the Archeological Satorian matrix ASM_5

As it was mentioned before, the Satorian functions can be defined by a discrete set of points or by a set of real functions. Thus, let us consider an application of a Gaussian function given by

$$f(x,y) = \begin{cases} |x+y| & \text{if } xy = 0, \\ \exp\left(-x^2-y^2\right) & \text{if } xy > 0, \\ -\exp\left(-x^2-y^2\right) & \text{if } xy < 0 \end{cases} \tag{5}$$

By aplying the Satorian function given in Equation (5) in a square of dimension $n = 5$ the corresponding matrix is obtained, see Figure 15.

·	·	1	·	·
·	·	0.5	·	·
-1	-0.5	0	0.5	1
·	·	-0.5	·	·
·	·	-1	·	·

(a) (x, y)

-0.135	-0.287	1	0.287	0.135
-0.287	-0.607	0.5	0.607	0.287
1	0.5	0	0.5	1
0.287	0.607	0.5	-0.607	-0.287
0.135	0.287	1	-0.287	-0.135

(b) $f(x, y)$

Figure 15: Application of the Equation (5) on a discrete square

By assigning colours to the Satorian square shown in Figure 15b and by plotting the values on the z axis, the corresponding square and surface are obtained.

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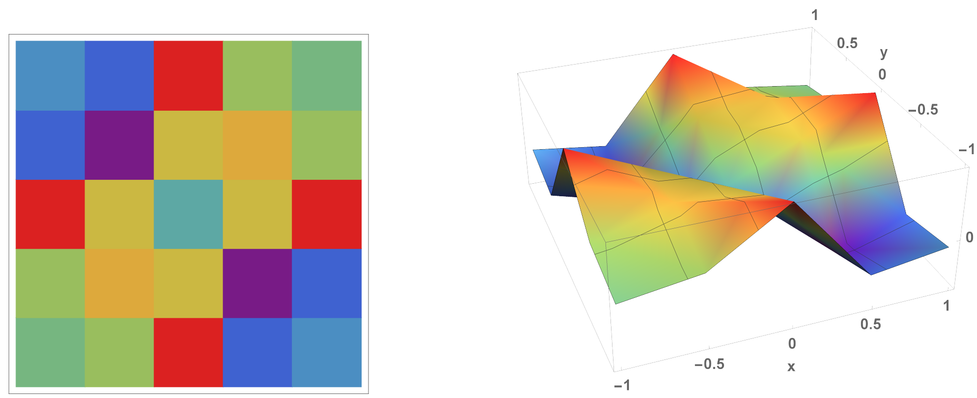
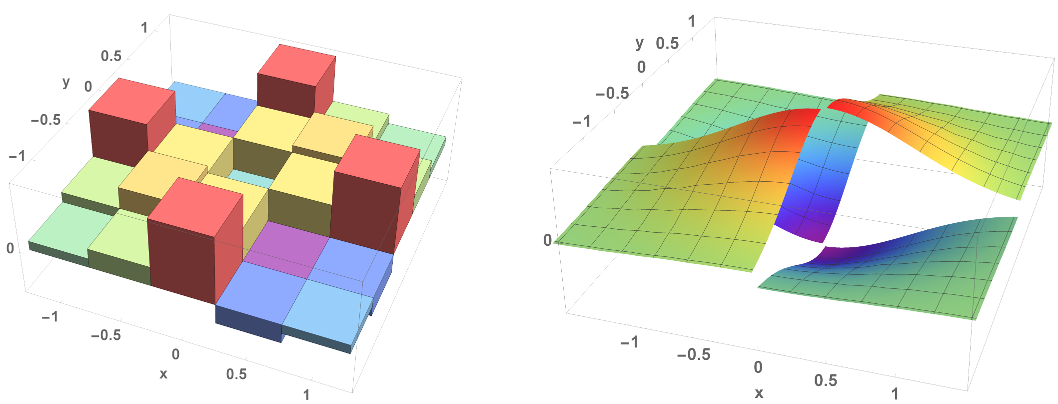


(a) Square (b) Surface

Figure 16: Satorian square and surface corresponding to the function given by Equation (5)

The Figure 17a shows the solid correspondig to the function given by Equation (5) and the Figure 17b shows the surface when the function $f(xy)$ is evaluated in $x, y \in \mathbb{R}$.



(a) Solid (b) Surface in $\mathbb{R}$

Figure 17: Satorian solid and surface corresponding to the function given by Equation (5)

7 Arabic architecture. The Alhambra

The kind of symmetry observed in the Sator Square has been also found in the alicatados[1] used for paving Spanish and Moorish patios but also for wall surfaces. The Figure 18 show how a Satorian mosaic can be creted from three Satorian squares SS_5. To do this, nine squares SS_5 are joined to form a bigger square, which does not satisfy the simmetry S_2 and afterwards, the design is turned 45^o. However, by combining properly nine squares

[1]Mosaic formed of polygonal, coloured glazed tiles. Made up into geometric patterns.

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SS_5, a Satorian square SS_{15} can be obtained as well.
The real picture of the Satorian mosaic in the Alhambra is showed in Figure 19.

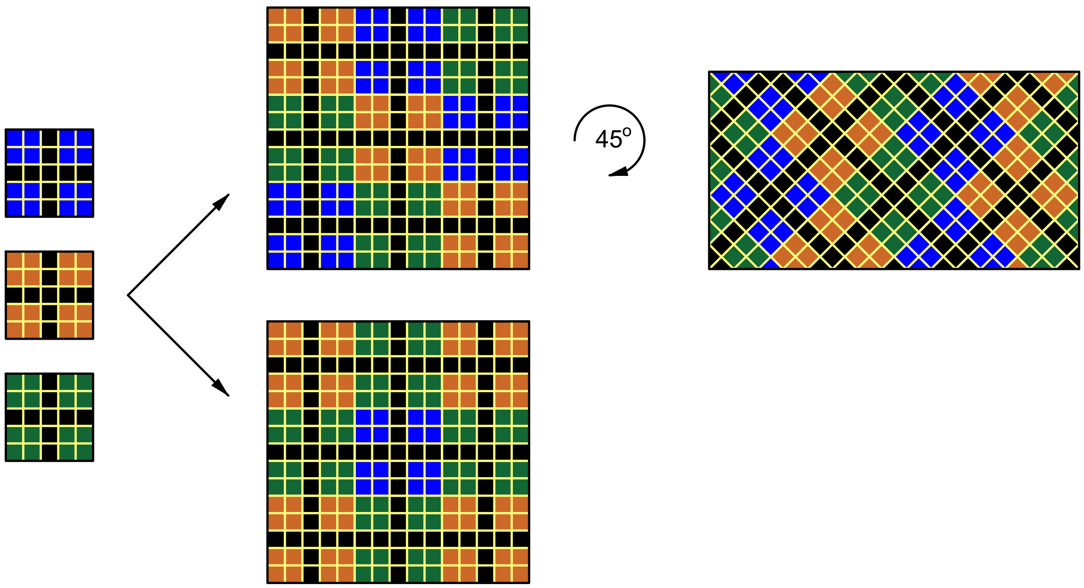

Figure 18: Creation of a Satorian mosaic

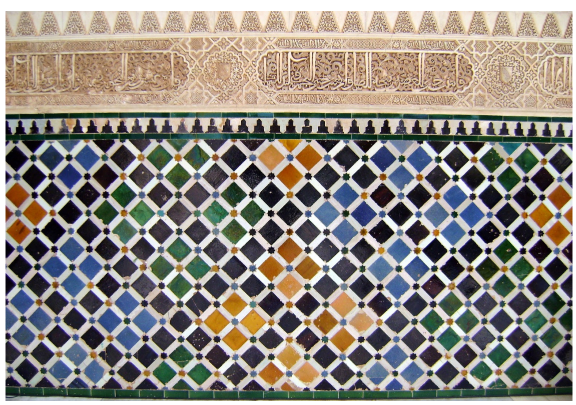

Figure 19: Alicatado on one wall at the Court of the Myrtles from the Comares Palace inside the Alhambra

A fascinating work about simmetry in the Alhambra and other fields can be found in [dS08].

8 Conclusions

For centuries the Sator Square has been considered as a mysterious riddle. Most of the studies about this archeological object have been based on religions or pseudo-sicentific disciplines, such as Christianity, Judaism, Satanism, Philology, esotericism, magic and mysticism. Nobody has been able to propose a rational and convincing explanation about its meaning.

In this work, the author proposed to see this square as a simple symmetric game of words. However, as every symmetric object, its structure also can be related to mathematical objects. In fact, the structure of the Sator Square can be described as a function, a matrix, a surface or a solid. By defining the mathematical properties of a Sator Square, it is possible to create Satorian structures of higher dimension.

Perhaps if the Sator square is analized or studied from an analytical perspective, a reliable explanation about its origin and meaning can be acquired.

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