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INFLUENCE OF THE TRACK MODEL IN THE SIMULATION OF THE VERTICAL VIBRATION BEHAVIOUR OF THE RAILWAY VEHICLE CARBODY

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Rezumat

În lucrare se analizează influenţa modelului căii de rulare în simularea regimului de vibrații verticale al cutiei vehiculului feroviar. Studiul este dezvoltat pe baza a două modele ale sistemului vehicul – cale de rulare, respectiv un model de tip „cale rigidă” și un model de tip „cale elastică”. Modelul „cale rigidă” este un model cu 7 grade de liberate, care include un model de tip cuplat rigid – flexibil al vehiculului și un model de tip rigid al căii. Modelul „cale elastică” este un model cu 15 grade de libertate, care include același model de tip cuplat rigid-flexibil al vehiculului și un model echivalent cu parametri concentrați pentru cale. Evaluarea regimului de vibrații al cutiei vehiculului se realizează pentru ambele modele ale sistemului vehicul – cale de rulare, utilizând rezultatele obținute prin simulări numerice privind funcțiile de răspuns în frecvență, densitatea spectrală de putere și abaterea medie pătratică a accelerației în trei puncte relevante pentru regimul de vibrații al cutiei – un punct poziționat la mijlocul cutiei și două puncte situate în dreptul boghiurilor. Concluziile sintetizate pe baza acestor rezultate evidențiază în general, influența modelului căii asupra regimului de vibrații al cutiei și, în particular, influența modelului căii asupra modurilor dominante de vibrație și nivelului de vibrații al cutiei.

Cuvinte cheie: vehicul feroviar, vibrații, model „cale rigidă”, model „cale elastică”

Abstract

The paper analyses the influence of the track model in the simulation of the vertical vibration behaviour of the railway vehicle carbody. The study is developed based on two models of the vehicle-roadway system, respectively a "rigid track" model and an "elastic track" model. The "rigid track" model is a 7-degree-of-freedom model, which includes a rigid-flexible coupled vehicle model and a rigid track model. The “elastic track” model is a 15-degree-of-freedom model that includes the same rigid-flexible coupled model of the vehicle and an equivalent lumped-parameter model for the track. The evaluation of the vibration behaviour of the vehicle carbody is carried out for both models of the vehicle - track system, using the results obtained through numerical simulations regarding the frequency response functions, the power spectral density and the root mean square of the acceleration at three points relevant to the vibration of the carbody - one point positioned at the middle of the carbody and two points located above the bogies. The conclusions synthesized based on these results highlight, in general, the influence of the track model on the vibration behaviour of the carbody and, in particular, the influence of the track model on the dominant vibration modes and the vibration level of the carbody.

Keywords: railway vehicle, vibrations, "rigid track" model, "elastic track" model

1. INTRODUCTION

Railway transport has an important role in today's society, where mobility is a pressing necessity both economically and socially. In the general domain of transports, railway transport stands out by satisfying the requirement of the mobility of passengers and goods in safety conditions, economic efficiency and with a low impact on the environment. All of these contribute to the creation of an attractive, efficient and sustainable transport system, competitive on the transport market marked by strong competition from the air and road transport system [1]. The general trend, which characterizes the development of the railway transport system, is directed towards increasing speed, reducing energy consumption and manufacturing costs. These desired goals can be achieved through a permanent research-innovation activity, ever more refined and complex, which requires a systemic approach in identifying the best technical solutions to ensure at the same time the dynamic performance of railway vehicles – ride comfort, ride quality and safety, at high speeds.

In the context of the above, the railway vehicle was and is constantly subject to innovations in order to maintain dynamic performances it must ensure in order to respond to the constantly changing requirements on the transport market. The innovation activity starts from the careful study of the phenomena that occur during the running of the vehicle, in this way it is possible to establish the important factors that influence the behaviour of the vehicle and the theoretical foundation of the proposed technical solutions. One of the phenomena that manifests permanently during the running of the railway vehicle is vibrations, which are a result of the interaction between the vehicle and the track [2, 3]. Vibrations of the railway vehicles affect the traffic safety, ride quality and ride comfort [4 - 6], the traction behaviour of the motor vehicles [7, 8] and cause wear of the rolling surfaces and fatigue of the vehicle structure [9, 10].

Approached from the perspective of a theoretical study, the analysis of a vibration problem assumes as an important stage the modelling of the vehicle - track system [11]. In principle, the mechanical model of the vehicle-track system is obtained by a simplified representation of the real system, in which the important elements are considered, in accordance with the specifics of the problem studied. In general, the degree of complexity of the mechanical model is established depending on the accuracy required for the results [12]. The more complex the model, the closer the results will be to reality, but drawing general conclusions regarding the characteristics of the railway vehicle's vibration behaviour will be more difficult.

In general, in the mechanical models of the vehicle-track system, adopted for the study of vertical vibrations of passenger vehicle carbodies, the track is considered rigid. Based on such models, numerous researches have been developed that can be grouped, depending on the issue addressed, into several important directions that have as objectives: analysis and control of vertical vibrations of flexible carbodies of high-speed railway vehicles [13, 14]; identification of methods for reducing vertical bending vibrations of the carbody [15 - 17]; study of the influence of the flexibility of the carbody on the ride quality and ride comfort [18 - 19]; improvement of ride comfort to vertical vibrations [20 - 22]; analysis and evaluation of ride comfort to vertical vibrations [23 - 25].

Complex mechanical models of the track are usually used for the study of rail vibrations [26, 27]. Depending on the structure of the model, two types of track models are distinguished - models with continuous elastic support with one elastic stage or with two elastic stages and models with discrete elastic support [28]. Such complex track models have been integrated into models for investigating problems such as vertical vibrations of the vehicle–track system [29, 30], train–track [31], passenger–train–track [12], train–track–bridge [32, 33], the dynamic response of the carbody to vehicle movement on a ballasted track [34, 35], and ride comfort when crossing bridges [36, 37].

The paper analyses the influence of the track model on the evaluation of the vertical vibration behaviour of the railway vehicle carbody based on the results obtained through numerical simulations. Numerical simulation applications are developed based on two models of the vehicle-track system, namely a "rigid track" model and an "elastic track" model. The "rigid track" model brings together the track model, considered rigid, and the rigid-flexible coupled model of the vehicle, with 7 degrees of freedom, corresponding to the vibration modes of the vehicle's sprung masses - the bounce, pitch and the first vertical bending mode of the carbody, respectively the bounce and pitch of the bogies. The “elastic track” model includes the same rigid-flexible coupled model of the vehicle and the track, which is represented by an equivalent lumped parameter model. Assuming the hypothesis of linear Hertzian contact between the wheel and the rail, the connection between the vehicle model and the track model, respectively the elasticity of the contact between the wheel and the rail, is considered by a Hertzian-type elastic element with linear characteristics. The "elastic track" model is a model with 15 degrees of freedom, corresponding to the vertical vibration modes of the carbody, bogies and the vertical displacements of the wheels and rails. To evaluate the vertical vibration behaviour of the carbody, the frequency response functions of the acceleration, the power spectral density of the acceleration and the root mean square deviation of the acceleration are analysed in parallel for the two models, at three points relevant to the carbody vibration, positioned at the middle of the carbody and above the bogies.

2. MECHANICAL MODELS OF THE VEHICLE-TRACK SYSTEM

2.1. Description of mechanical models of the vehicle-track system

For the proposed study, the case of a passenger vehicle on bogies, with two suspension stages, is considered, which is running at constant speed V on a track in alignment with vertical irregularities. The vehicle-track system is represented by two different mechanical models which are presented in figure 1 and figure 2. The differences between the two models are found in the model of the track and the wheel-rail contact, as will be shown below. In both models, the railway vehicle is represented by a rigid-flexible coupled model, which comprises a flexible body for the carbody and six rigid bodies for the chassis of the two bogies and the four wheelsets. These bodies are connected to each other by Kelvin-Voigt systems, through which the two suspension stages of the vehicle are represented, respectively the secondary suspension and the primary suspension.

The vehicle carbody is modelled by a free-free equivalent beam of constant section and uniformly distributed mass, Euler-Bernoulli type. The rigid vibration modes of the carbody are bounce and pitch, and the flexible vibration mode is the first vertical bending mode. The inertia of the carbody with respect to the rigid vibration modes is represented by the mass m_c and the inertia moment J_c.

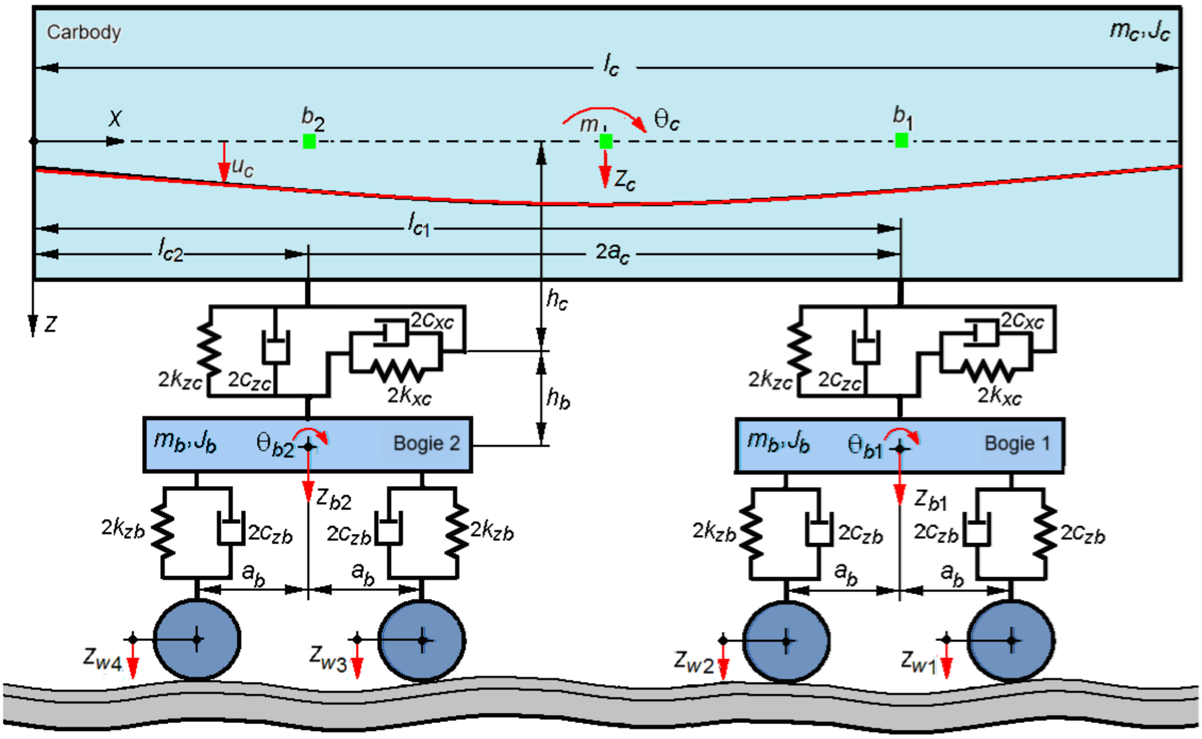

Figure 1. Model of the vehicle – rigid track system (the "rigid track" model).

On the figures 1 and 2 they are highlighted the vertical displacement of the carbody corresponding to the bounce, denoted by z_c, and the angle of rotation at the pitch, denoted by θ_c. The vertical displacement of a section of the carbody located at a distance x from the origin of the reference system, denoted by u_c, is the result of the superposition of the three vibration modes – the bounce, pitch, and the first vertical bending mode. At some time t, the vertical displacement of the carbody is given by the relation

$$u_c(x,t) = z_c(t) + \left(x - l_c/2\right)\theta_c(t) + Y_c(x)T_c(t), \tag{1}$$

where l_c represents the length of the carbody, T_c is the time coordinate of the first vertical bending mode, and Y_c is the eigenfunction of the first vertical bending mode,

$$Y_c(x) = \sin\beta x + \sinh\beta x - \frac{\sin\beta l_c - \sinh\beta l_c}{\cos\beta l_c - \cosh\beta l_c}(\cos\beta x + \cosh\beta x) \tag{2}$$

with $$\beta = \sqrt[4]{\omega_c^2 \rho_c / E_c I_c} \tag{3}$$

which verifies the characteristic equation

$$\cos\beta l_c \cosh\beta l_c - 1 = 0, \tag{4}$$

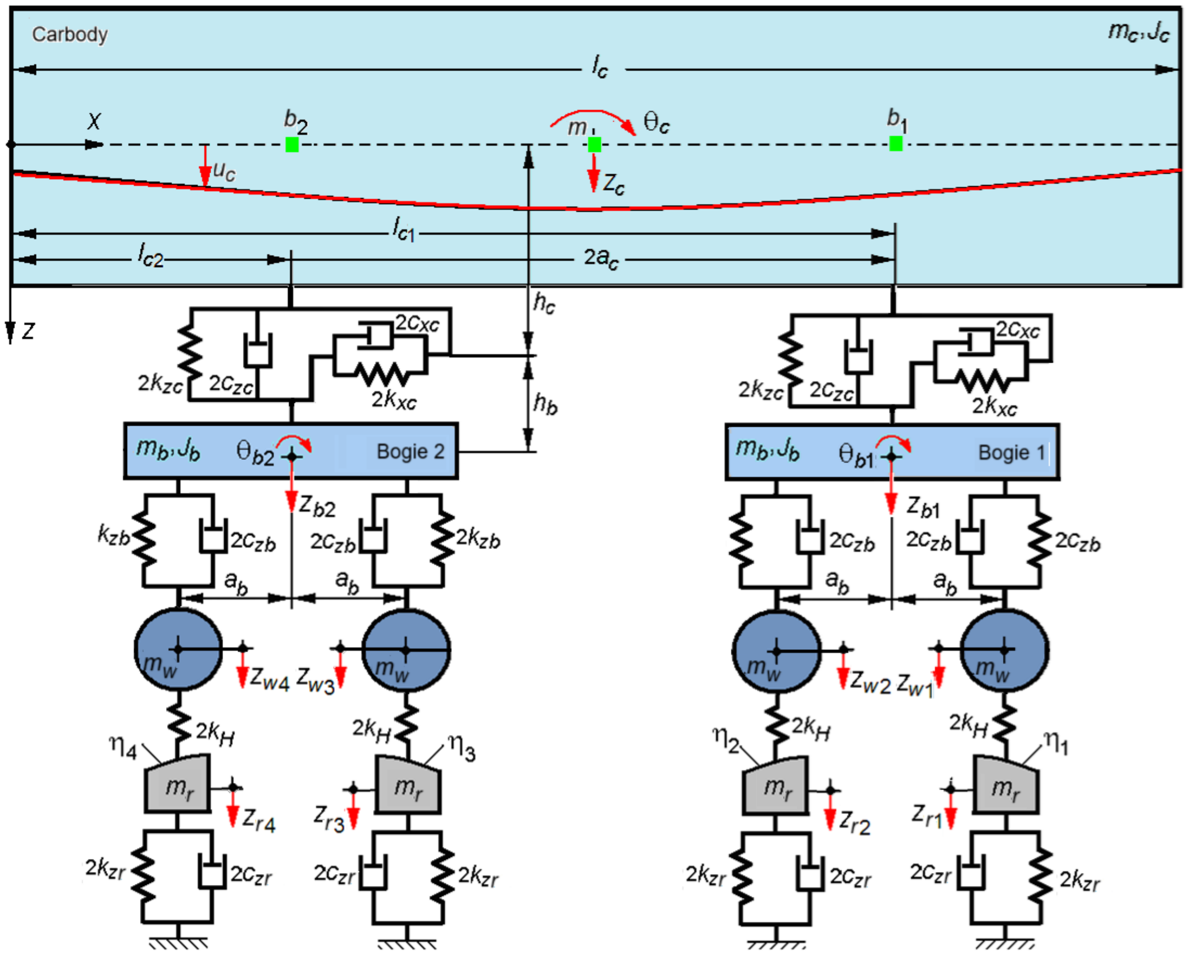

Figure 2. Model of the vehicle – elastic track system (the "elastic track" model).

where: ω_c is natural angular frequency of the first vertical bending mode of the carbody; $\rho_c = m_c/l_c$ represents the mass of the carbody per unit length; E_cI_c is the bending stiffness (E_c - longitudinal modulus of elasticity, I_c - the moment of inertia of the cross section of the carbody).

The rigid vibration modes of the bogies are the bounce and pitch. The vertical displacement corresponding to the bounce is denoted by $z_{b1,2}$, and the pitch angle is denoted by $\theta_{b1,2}$. The inertia of the bogies with respect to the rigid vibration modes is represented by the mass m_b and the moment of inertia J_b.

The primary suspension corresponding to a wheelset is modelled by a single Kelvin-Voigt system, for translation in the vertical direction, having the elastic constant $2k_{zb}$ and the damping constant $2c_{zb}$. The secondary suspension corresponding to a bogie is modelled by two Kelvin-Voigt systems for translation. The Kelvin-Voigt system for vertical translation models the stiffness and vertical damping of the suspension; the elastic constant is denoted by $2k_{zc}$ and the damping constant by $2c_{zc}$.The Kelvin-Voigt system for longitudinal translation models the system for transmitting longitudinal forces between the bogies and the carbody. This system is positioned at a distance h_c from the mean fibre of the carbody and at a distance h_b from the centre of mass of the bogie and has the elastic constant $2k_{xc}$ and the damping constant $2c_{xc}$.

Other common elements of the two models, marked on figures 1 and 2, are the carbody wheelbase $2a_c$, the bogie wheelbase $2a_b$ and the distances $l_{c1,2}$ which fix the position of the carbody support points on the secondary suspension. Also, three points are marked on the carbody model, at the middle of the carbody – point m, and two others above the bogies, respectively above the carbody support points on the secondary suspension – points b_1 and b_2. These points are relevant for the vertical vibration of the carbody.

In the vehicle-track system model presented in Figure 1, the track is considered rigid, and in this hypothesis the vertical irregularities of the track are transmitted directly to the wheelsets, which impose vertical displacements denoted by $z_{w1...w4}$. Unlike the first model, in the case of the vehicle-track system model presented in Figure 2, the elastic contact between the wheel and the rail is considered, which is modelled by elastic elements with a linear Hertzian characteristic of stiffness $2k_H$. The wheelsets, of mass m_w, have vertical displacements, denoted by $z_{w1...w4}$. Within the same model, neglecting the coupling effects between the wheels caused by the propagation of bending waves through the rails, an equivalent model with lumped parameters is adopted for the track. At each wheelset, the track is represented by a system with one degree of freedom in the vertical direction, the corresponding displacement being denoted by $z_{r1...r4}$. The equivalent track model has mass m_r, stiffness $2k_{zr}$ and damping constant $2c_{zr}$. The vertical track irregularities at each wheel are denoted by $\eta_{1...4}$.

To simplify the way of presentation, hereafter the two models of the vehicle-track system will be called the "rigid track" model (figure 1) and the "elastic track" model (figure 2).

2.2. The equations of motion

In the case of the "rigid track" model, the vertical motions of the vehicle - track system are described by seven equations, corresponding to the rigid vibration modes of the carbody and bogies – bounce and pitch, and by the equation of the first vertical bending mode of the carbody. For the "elastic track" model, four more equations of motion for the vertical displacements of the four wheelsets and four equations of motion for the vertical displacements of the rails are added to these equations.

The equations of motion of the carbody corresponding to the bounce, pitch and vertical bending are common to both models, and their derivation starts from the general form of the equation of motion

$$E_c I_c \frac{\partial^4 u_c(x,t)}{\partial x^4} + \mu_c I_c \frac{\partial^5 u_c(x,t)}{\partial x^4 \partial t} + \rho_c \frac{\partial^2 u_c(x,t)}{\partial t^2} = \sum_{i=1}^{2} F_{zci} \delta(x - l_{ci}), \tag{5}$$

where $\delta(.)$ is the Dirac delta function, μ_c represents the structural damping coefficient, and $F_{zc1,2}$ and $F_{xc1,2}$ represent the forces of the secondary suspension,

$$F_{zc1,2} = -2c_{zc} \left(\frac{\partial u_c(l_{c1,2},t)}{\partial t} - \dot{z}_{b1,2} \right) - 2k_{zc} \left[u_c(l_{c1,2},t) - z_{b1,2} \right], \tag{6}$$

$$F_{xc1,2} = 2c_{xc} \left(h_c \frac{\partial^2 u_c(l_{c1,2},t)}{\partial x \partial t} + h_b \dot{\theta}_{b1,2} \right) + 2k_{xc} \left(h_c \frac{\partial u_c(l_{c1,2},t)}{\partial x} + h_b \theta_{b1,2} \right). \tag{7}$$

Through the modal analysis method, the equation of motion (5) is transformed into three second-order differential equations with ordinary derivatives, which describe the three vibration modes of the carbody – bounce, pitch and vertical bending:

$$m_c \ddot{z}_c = \sum_{i=1}^{2} F_{zci} \tag{8}$$

$$J_c \ddot{\theta}_c = \sum_{i=1}^{2} F_{zci} \left(l_{ci} - \frac{l_c}{2} \right) - h_c \sum_{i=1}^{2} F_{xci}, \tag{9}$$

$$m_{mc} \ddot{T}_c + c_{mc} \dot{T}_c + k_{mc} T_c = \sum_{i=1}^{2} F_{zci} Y_c(l_i) - h_c \sum_{i=1}^{2} F_{xci} \frac{\mathrm{d}Y_c(l_{ci})}{\mathrm{d}x}. \tag{10}$$

In equation (10) it was denoted by m_{mc} - the modal mass of the carbody, by c_{mc} - the modal damping of the carbody and by k_{mc} - the modal stiffness of the carbody, these being defined as follows:

$$m_{mc} = \rho_c \int_0^{l_c} Y_c^2 \mathrm{d}x, \; c_{mc} = \mu_c I_c \int_0^{l_c} \left(\frac{\mathrm{d}^2 Y_c}{\mathrm{d}x^2} \right)^2 \mathrm{d}x, \; k_{mc} = E_c I_c \int_0^{l_c} \left(\frac{\mathrm{d}^2 Y_c}{\mathrm{d}x^2} \right)^2 \mathrm{d}x. \tag{11}$$

In the equations of motion of the carbody (8 – 10) the expressions of the forces of the secondary suspension are replaced, according to relations (6) and (7). Also, based on the symmetry properties of the eigenfunction $Y_c(x)$, the notations are introduced

$$Y_c(l_{c1}) = Y_c(l_{c2}) = \varepsilon\,, \tag{12}$$

$$\frac{\mathrm{d}Y_c(l_{c1})}{\mathrm{d}x} = -\frac{\mathrm{d}Y_c(l_{c2})}{\mathrm{d}x} = \lambda\,. \tag{13}$$

The equations of motion of the carbody are reduced to the following form:

$$m_c\ddot{z}_c + 2c_{zc}[2\dot{z}_c + 2\varepsilon\dot{T}_c - (\dot{z}_{b1} + \dot{z}_{b2})] + 2k_{zc}[2z_c + 2\varepsilon T_c - (z_{b1} + z_{b2})] = 0 \tag{14}$$

$$\begin{aligned} &J_c\ddot{\theta}_c + 2c_{zc}a_c[2a_c\dot{\theta}_c - (\dot{z}_{b1} - \dot{z}_{b2})] + 2k_{zc}a_c[2a_c\theta_c - (z_{b1} - z_{b2})] + \\ &+2c_{xc}h_c[2h_c\dot{\theta}_c + h_b(\dot{\theta}_{b1} + \dot{\theta}_{b2})] + 2k_{xc}h_c[2h_c\theta_c + h_b(\theta_{b1} + \theta_{b2})] = 0 \end{aligned} \tag{15}$$

$$\begin{aligned} &m_{mc}\ddot{T}_c + c_{mc}\dot{T}_c + k_{mc}T_c + \\ &+2c_{zc}\varepsilon[2\dot{z}_c + 2\varepsilon\dot{T}_c - (\dot{z}_{b1} + \dot{z}_{b2})] + 2k_{zc}\varepsilon[2z_c + 2\varepsilon T_c - (z_{b1} + z_{b2})] + \\ &+2c_{xc}h_c\lambda[2h_c\lambda\dot{T}_c + h_b(\dot{\theta}_{b1} - \dot{\theta}_{b2})] + 2k_{xc}h_c\lambda[2h_c\lambda T_c + h_b(\theta_{b1} - \theta_{b2})] = 0 \end{aligned} \tag{16}$$

The bounce and pitch equations of the two bogies are derived using the fundamental laws of mechanics, considering that they are acted upon by the inertia forces when bounce and pitch, the forces of the secondary suspension corresponding to the two bogies ($F_{zc1,2}$ and $F_{xc1,2}$) and the forces of the primary suspension corresponding to the four wheelsets ($F_{zb1...4}$).

In the case of both models of the vehicle-track system, the equations describing the bounce and pitch movements of the bogies are of the form:

$$m_b\ddot{z}_{b1} = \sum_{j=1}^{2} F_{zbj} - F_{zc1}, \quad m_b\ddot{z}_{b2} = \sum_{j=3}^{4} F_{zbj} - F_{zc2}\,, \tag{17}$$

$$J_b\ddot{\theta}_{b1} = a_b\sum_{j=1}^{2}(-1)^{j+1}F_{zbj} - h_bF_{xc1}, \quad J_b\ddot{\theta}_{b2} = a_b\sum_{j=3}^{4}(-1)^{j+1}F_{zbj} - h_bF_{xc2}\,, \tag{18}$$

in which the forces in the primary suspension are given by the relations:

$$F_{zb1,2} = -2c_{zb}(\dot{z}_{b1} \pm a_b\dot{\theta}_{b1} - \dot{z}_{w1,2}) - 2k_{zb}(z_{b1} \pm a_b\theta_{b1} - z_{w1,2})\,, \tag{19}$$

$$F_{zb3,4} = -2c_{zb}(\dot{z}_{b2} \pm a_b\dot{\theta}_{b2} - \dot{z}_{w3,4}) - 2k_{zb}(z_{b2} \pm a_b\theta_{b2} - z_{w3,4})\,. \tag{20}$$

After processing, equations (17) - (18) become:

$$\begin{aligned} &m_b\ddot{z}_{b1} + 2c_{zb}[\dot{z}_{b1} - (\dot{z}_{w1} + \dot{z}_{w2})] + 2k_{zb}[z_{b1} - (z_{w1} + z_{w2})] + \\ &+2c_{zc}(\dot{z}_{b1} - \dot{z}_c - a_c\dot{\theta}_c - \varepsilon\dot{T}_c) + 2k_{zc}(z_{b1} - z_c - a_c\theta_c - \varepsilon T_c) = 0 \end{aligned} \tag{21}$$

$$\begin{aligned} & m_b\ddot{z}_{b2} + 2c_{zb}[\dot{z}_{b2} - (\dot{z}_{w3} + \dot{z}_{w4})] + 2k_{zb}[z_{b2} - (z_{w3} + z_{w4})] + \\ & + 2c_{zc}(\dot{z}_{b2} - \dot{z}_c + a_c\dot{\theta}_c - \varepsilon\dot{T}_c) + 2k_{zc}(z_{b2} - z_c + a_c\theta_c - \varepsilon T_c) = 0 \end{aligned} \tag{22}$$

$$\begin{aligned} & J_b\ddot{\theta}_{b1} + 2c_{zb}a_b[2a_b\dot{\theta}_{b1} - (\dot{z}_{w1} - \dot{z}_{w2})] + 2k_{zb}a_b[2a_b\theta_{b1} - (z_{w1} - z_{w2})] + \\ & + 2c_{xc}h_b[h_b\dot{\theta}_{b1} + h_c(\dot{\theta}_c + \lambda\dot{T}_c)] + 2k_{xc}h_b[h_b\theta_{b2} + h_c(\theta_c + \lambda T_c)] = 0 \end{aligned} \tag{23}$$

$$\begin{aligned} & J_b\ddot{\theta}_{b2} + 2c_{zb}a_b[2a_b\dot{\theta}_{b2} - (\dot{z}_{w3} - \dot{z}_{w4})] + 2k_{zb}a_b[2a_b\theta_{b2} - (z_{w3} - z_{w4})] + \\ & + 2c_{xc}h_b[h_b\dot{\theta}_{b2} + h_c(\dot{\theta}_c - \lambda\dot{T}_c)] + 2k_{xc}h_b[h_b\theta_{b2} + h_c(\theta_c - \lambda T_c)] = 0 \end{aligned} \tag{24}$$

Next, the equations of motion of the wheelsets and rails for the "elastic track" model are derived.

To deduce the equations of motion of the wheelsets, it is taken into account that each wheelset is acted upon by the vertical force from its corresponding primary suspension, described by equations (19) – (20) and the vertical dynamic force F_j, with j = 1...4, which, under the assumption of Hertzian contact between the wheel and the rail, is given by the relation:

$$F_j = k_H(z_{wj} - z_{rj} - \eta_j)\,. \tag{25}$$

The equations of vertical motion of the wheelsets have the general form:

$$m_w\ddot{z}_{wj} = 2F_j - F_{zbj}\,. \tag{26}$$

After replacing the expressions for the forces of the primary suspension and the vertical dynamic forces, the equations of motion of the wheelsets are reduced to the form:

$$\begin{aligned} & m_w\ddot{z}_{w1,2} + 2c_{zb}(\dot{z}_{w1,2} - \dot{z}_{b1} \mp a_b\dot{\theta}_{b1}) + 2k_{zb}(z_{w1,2} - z_{b1} \mp a_b\theta_{b1}) + \\ & + 2k_H(z_{w1,2} - z_{w1,2} - \eta_{1,2}) = 0 \end{aligned} \tag{27}$$

$$\begin{aligned} & m_w\ddot{z}_{w3,4} + 2c_{zb}(\dot{z}_{w3,4} - \dot{z}_{b2} \mp a_b\dot{\theta}_{b2}) + 2k_{zb}(z_{w3,4} - z_{b2} \mp a_b\theta_{b2}) + \\ & + 2k_H(z_{w3,4} - z_{r3,4} - \eta_{3,4}) = 0 \end{aligned} \tag{28}$$

Each rail is subjected to the vertical dynamic force F_j (eq. 25) and the vertical force F_{zrj} given by the relation:

$$F_{zrj} = -2c_{zr}\dot{z}_{sj} - 2k_{zr}z_{rj}\,. \tag{29}$$

The equations of vertical displacements of the rails corresponding to wheelset *j* are

$$m_r \ddot{z}_{rj} = F_{zrj} - 2F_j \,. \tag{30}$$

Equations (30) take the form:

$$m_r \ddot{z}_{r1,2} + 2c_{zr}\dot{z}_{r1,2} + 2k_{zr} z_{r1,2} + 2k_H (z_{r1,2} - z_{w1,2} + \eta_{1,2}) = 0 \,, \tag{31}$$

$$m_r \ddot{z}_{r3,4} + 2c_{zr}\dot{z}_{r3,4} + 2k_{zr} z_{r3,4} + 2k_H (z_{r3,4} - z_{w3,4} + \eta_{3,4}) = 0 \,. \tag{32}$$

The system formed by the 7 equations of motion of the carbody and bogies (eq. 15 – 17 and 21 - 24) corresponding to the "rigid track" model can be written in matrix form,

$$\mathbf{M_r}\ddot{\mathbf{q}} + \mathbf{C_r}\dot{\mathbf{q}} + \mathbf{K_r}\mathbf{q} = \mathbf{P}\dot{\mathbf{z}}_w + \mathbf{R}\mathbf{z}_w \,, \tag{33}$$

where **q** represents the vector of displacement coordinates, and $\mathbf{z}_w$ is the vector of inhomogeneous terms; $\mathbf{M_r}$ is the inertia matrix, $\mathbf{C_r}$ is the damping matrix, and $\mathbf{K_r}$ is the stiffness matrix.

The system formed by the 15 equations of motion of the carbody, bogies, wheelsets and rails (eq. 15 – 17, 21 – 24, 27 – 28 and 31 - 32), corresponding to the "elastic track" model, is written in matrix form,

$$\mathbf{M_e}\ddot{\mathbf{p}} + \mathbf{C_e}\dot{\mathbf{p}} + \mathbf{K_e}\mathbf{p} = \mathbf{Q}\boldsymbol{\eta} \,, \tag{34}$$

where **p** represents the vector of displacement coordinates, and $\boldsymbol{\eta}$ is the vector of inhomogeneous terms; $\mathbf{M_e}$ is the inertia matrix, $\mathbf{C_e}$ is the damping matrix, and $\mathbf{K_e}$ is the stiffness matrix.

The equations of motion of the vehicle-track system written in matrix form according to equations (33) and (34) are used to calculate the frequency response functions of the vehicle carbody.

3. FREQUENCY RESPONSE FUNCTIONS OF THE CARBODY

The analysis of the vertical vibration regime of the carbody will be done in the next section based on the frequency response functions of the acceleration, the power spectral density of the acceleration and the root mean square of the acceleration, at the relevant points of the carbody, respectively at point *m* located at the middle of the carbody and at points b_1 and b_2 located above the two bogies.

The frequency response functions of the carbody acceleration are calculated based on the partial response functions of the carbody displacement corresponding to its vibration modes, respectively for bounce $\bar{H}_{z_c}(\omega)$, pitch $\bar{H}_{\theta_c}(\omega)$ and vertical bending $\bar{H}_{T_c}(\omega)$, according to the relationships:

- at the middle of the carbody,

$$\bar{H}_{cm}(\omega) = -\omega^2 \left[\bar{H}_{z_c}(\omega) + Y_c\left(\frac{l_c}{2}\right)\bar{H}_{T_c}(\omega) \right]; \tag{35}$$

- above the bogies,

$$\bar{H}_{cb_{1,2}}(\omega) = -\omega^2 \left[\bar{H}_{z_c}(\omega) \pm a_c \bar{H}_{\theta_c}(\omega) + Y_c(l_{c1,2})\bar{H}_{T_c}(\omega) \right], \tag{36}$$

where ω is the pulsation induced by the vertical irregularities of the track.

The calculation relations of the power spectral density of the acceleration at the reference points of the carbody are of the form:

- at the middle of the carbody,

$$G_{cm}(\omega) = G(\omega)\left|\bar{H}_{cm}(\omega)\right|^2 = \omega^4 G(\omega)\left|\bar{H}_{z_c}(\omega) + Y_c\left(\frac{l_c}{2}\right)\bar{H}_{T_c}(\omega)\right|^2; \tag{37}$$

- above the bogies,

$$G_{cb_{1,2}}(\omega) = G(\omega)\left|\bar{H}_{cb_{1,2}}(\omega)\right|^2 = \omega^4 G(\omega)\left|\bar{H}_{z_c}(\omega) \pm a_c \bar{H}_{\theta_c}(\omega) + Y_c\left(l_{c1,2}\right)\bar{H}_{T_c}(\omega)\right|^2 \tag{38}$$

where $G(\omega)$ represents the power spectral density of the vertical irregularities of the track. For the power spectral density of the vertical irregularities of the track, the relation [38]

$$S(\Omega) = \frac{A\Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)}, \tag{39}$$

is considered representative of the average statistical properties of European railways. In relation (39) there are: Ω - wave number; A - constant that depends on the quality of the track (A = 2.119·10-7 radm, for good-quality track; A = 6.124·10-7 radm, for low-quality track); Ωc = 0.8246 rad/m, Ωr = 0.0206 rad/m.

If the power spectral density of the vertical irregularities of the track is expressed as a function of the angular frequency ($\omega = V\Omega$),

$$G(\omega) = \frac{S(\omega / V)}{V}, \tag{40}$$

after substitutions, relation (42) becomes:

$$G(\omega) = \frac{A\Omega_c^2 V^3}{[\omega^2 + (V\Omega_c)^2][\omega^2 + (V\Omega_r)^2]}. \tag{41}$$

Based on the power spectral density of the carbody acceleration, the root mean square of the carbody's vertical acceleration is calculated, according to the relationships:

- at the middle of the carbody,

$$a_{cm} = \sqrt{\frac{1}{\pi}\int_0^{\infty} G_{cm}(\omega)\mathrm{d}\omega}\,; \tag{42}$$

- above the bogies,

$$a_{cb1,2} = \sqrt{\frac{1}{\pi}\int_0^{\infty} G_{cb1,2}(\omega)\mathrm{d}\omega}\,. \tag{43}$$

4. ANALYSIS OF THE VERTICAL VIBRATION BEHAVIOUR OF THE RAILWAY VEHICLE CARBODY

In this section, the influence of the track model on the vertical vibration behaviour of the railway vehicle carbody is analysed, using for this the results of numerical simulations developed based on the two models of the vehicle - track system presented previously, respectively the "rigid track" model and the "elastic track" model. The vertical vibration behaviour of the vehicle carbody is analysed based on the frequency response functions of the acceleration (eq. 35 - 36), the power spectral density of the acceleration (eq. 37 - 38) and the root mean square of the acceleration (eq. 42 - 43) calculated at point *m* located at the middle of the carbody and at points b_1 and b_2 located above the two bogies. The reference values of the parameters of the numerical model of the vehicle - track system used in the numerical simulations are presented in Table 1. For these values, the degree damping of the track has the value ξ_{zs} = 0.18.

Table 1. Reference parameters of the numerical model of the vehicle-track system.

Common parameters for the "rigid track" model and the "elastic track" model	
m_c = 34000 kg	E_cI_c = 3.107·10^{9}Nm^2
m_b = 3200 kg	2k_{zc} = 1.2 MN/m
m_{mc} = 35224 kg	2k_{xc} = 4 MN/m
J_c = 1963840 kg·m^2	k_{mc} = 88.998 MN/
J_b = 2048 kg·m^2	2c_{zc} = 34.28 kNs/m
l_c = 26.4 m	2c_{xc} = 50 kNs/m
2a_b = 19 m; 2a_b = 2.56 m	c_{mc} = 53.117 kNm/s

h_c = 1.3 m	$4k_{zb}$ = 4.4 MN/m
h_b = 0.2 m	$4c_{zb}$ = 52.21 kNs/m
Specific parameters of the "elastic track" model	
m_w = 1650 kg	$2k_H$ = 3000 MN/m
m_r = 175 kg	$2k_{zr}$ = 1.4 MN/m
	$2c_{zr}$ = 40 kNs/m

The eigenfrequencies of the vertical vibration modes of the carbody and the bogies corresponding to the reference values of the parameters of the numerical model of the vehicle – track system are summarized in Table 2.

Table 2. Eigenfrequencies of the vibration modes of the carbody and bogies.

Vibration mode	Frequency [Hz]
Bounce carbody	1.17
Pitch carbody	1.73
Vertical bending of the carbody	8.00
Bounce bogie	6.65
Pitch bogie	9.63

The eigenfrequencies of the vehicle are also highlighted in figures 3 and 4, which represent the response functions of the carbody acceleration at its middle and above to the two bogies, for the two models of the vehicle - track system, respectively the "rigid track" model (diagram a) and the "elastic track" model (diagram b).

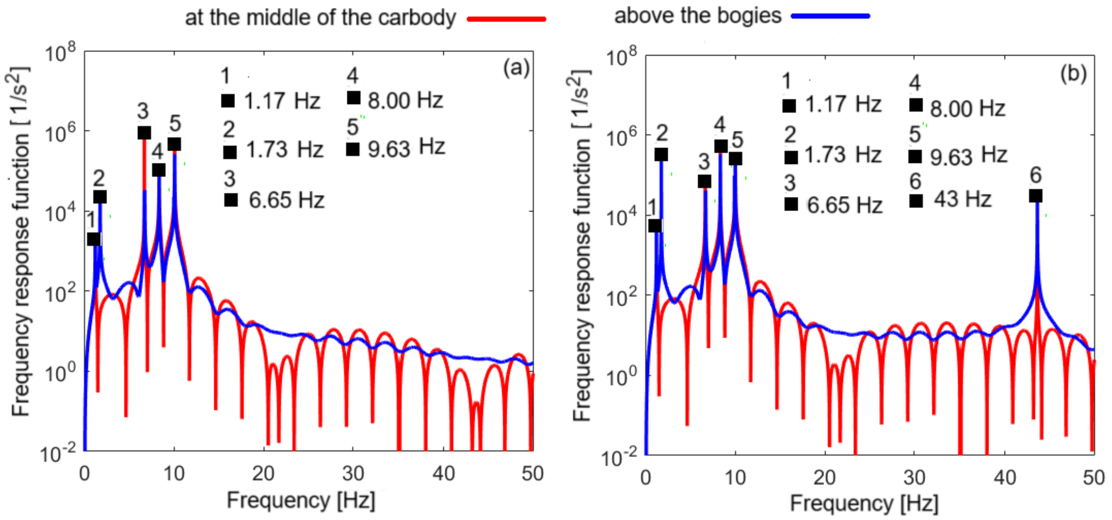

Figure 3. Frequency response functions of the carbody acceleration for undamped system: (a) for the "rigid track" model; (b) for the "elastic track" model.

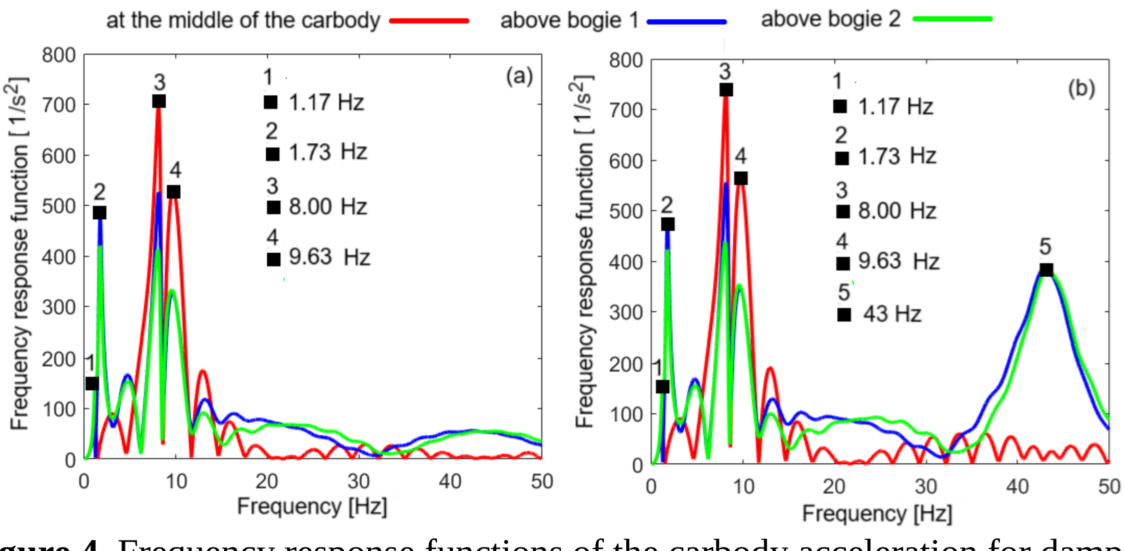

Figure 4. Frequency response functions of the carbody acceleration for damped system: (a) for the "rigid track" model; (b) for the "elastic track" model.

In figure 3, the frequency response functions of the carbody acceleration are obtained for the undamped system, respectively for $c_{zc} = 0$, $c_{xc} = 0$, $c_{zb} = 0$, $c_{mc} = 0$ și $c_{zr} = 0$ (only for diagram b), and in figure 4 for the reference values of the system damping constants. On diagrams (b) of the two figures, the peak of the response functions of the carbody acceleration calculated at the points located above front of the two bogies (points b_1 and b_2), corresponding to the eigenfrequency of the vertical vibrations of the wheelset, at 43 Hz, is marked.

In addition to the peaks corresponding to the resonance frequencies, several minima appear on the curves of the carbody's response functions in the form of anti-resonance frequencies that are due to the geometric filtering effect. This effect is an important characteristic of the vertical vibration behaviour of railway vehicles. In short, the geometric filtering effect is the result of the phase shifts between the vertical movements of the wheelsets generated by track irregularities, phase shifts that are due to the distance between the wheelsets (the carbody wheelbase and the bogie wheelbase) and the speed of the vehicle. If the anti-resonance frequencies coincide with the eigenfrequency of one of the vehicle's vibration modes, then its influence is greatly diminished. According to several studies [39 - 41], the geometric filtering has a selective character, depending on the speed, and a differentiated efficiency along the vehicle carbody. As seen in the diagrams in figures 3 and 4, the frequency response function at the middle of the carbody exhibits more anti-resonant frequencies than the frequency response functions above the bogies, which means that the geometric filtering effect is more effective at the middle of the carbody. This observation is explainable given that of the middle of the carbody, the geometric filtering is produced by the bogie wheelbase and the carbody wheelbase, while above the bogies, the geometric filtering is only due to the bogie wheelbase.

Based on the diagrams in figure 3, it is observed that, in the absence of system damping, the frequency response of the carbody above the two bogies is symmetrical. Introducing damping into the system determines the asymmetry of the carbody response above the bogies (figure 4).

Figure 5 shows the response functions of the acceleration for the "elastic track" model, considering the reference values of the parameters of the numerical model of the vehicle - track system, respectively ξ_{zs} = 0.18, and two analysis values of the track damping ratio ξ_{zs} = 0.10 and ξ_{zs} = 0.30.

It is observed, on the one hand, the reduced influence of the track model on the vibration behaviour of the at its middle (diagram a) and, on the other hand, the significant weight it has on the vibration behaviour of the carbody above the bogie (diagram b). However, it is observed that the influence of the track model on the carbody response manifests itself around the frequency of 43 Hz (the eigenfrequency of the vertical vibrations of the wheelsets on the track), which is outside the frequency range specific to the vertical vibrations of the vehicle carbody, the upper limit of which is not higher than 10 Hz. In the present case, the frequency range of the vertical vibrations of the vehicle is between 1.17 Hz – the eigenfrequency of the carbody's bounce vibrations, and 9.63 Hz – the eigenfrequency of the bogie's pitch vibrations. Regarding the influence of the track damping on the carbody's response, this is very well highlighted in diagram (b), in the 43 Hz frequency area, where significant increases in the response function of the carbody acceleration are observed when the track damping ratio decreases, in the 43 Hz frequency area. For example, by increasing the damping ratio of the track from 0.10 to 0.18, the response function of the acceleration decreases by 35.6%, and by increasing ξ_{zs} from 0.18 to 0.30, the response function of the acceleration decreases by 32.8%.

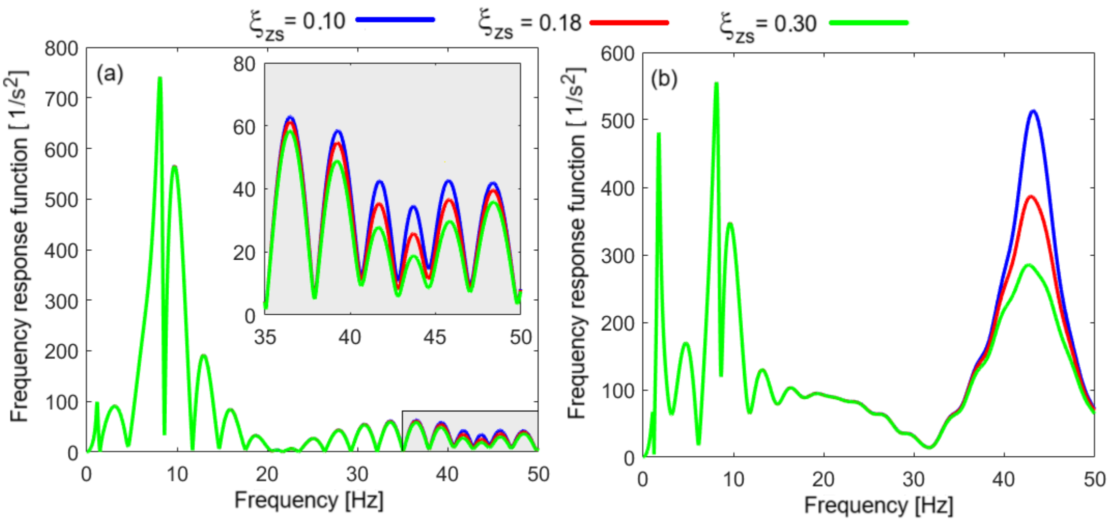

Figure 5. Influence of the track damping ratio on the frequency response function of the carbody acceleration at a speed of 200 km/h:
(a) at the middle of the carbody; (b) above the bogie 1.

Figure 6 shows the power spectral density of the acceleration calculated at the points relevant to the carbody vibration – diagram (a) at the point located at the middle of the carbody and diagram (b) at the points located above the bogies, for the two models of the vehicle-track system, namely the "rigid track" model and the "elastic track" model. The first observation refers to the fact that the track model does not influence the dominant vibration modes at the relevant points of the carbody. At the middle of the carbody, the dominant vibration mode is bounce, but vertical bending also has an important share. Above the bogies, the carbody vibration is dominated by pitch vibrations. The second observation refers to the influence of the track model on the carbody vibration level, which is visible in the high frequency range, above 20 Hz, especially above the bogies. For example, at the eigenfrequency of vertical vibrations of the wheelset on the track (43 Hz), the power spectral density of the acceleration of the carbody at the bogies calculated for the "elastic track" model is 47.6% higher than that calculated for the "rigid track" model.

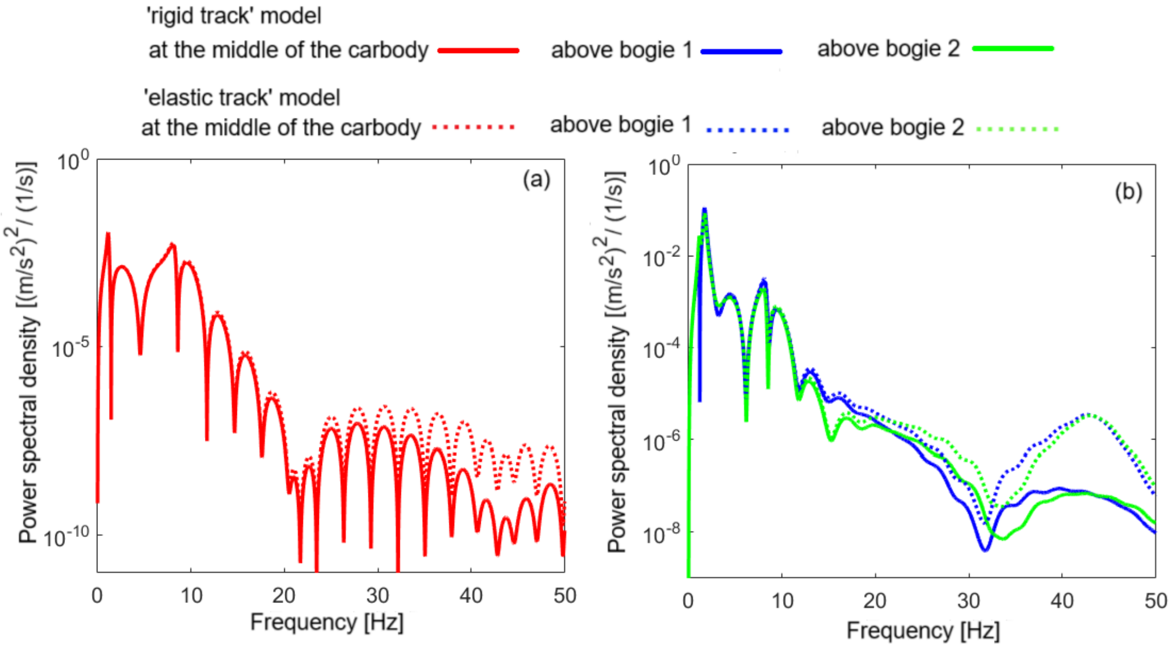

Figure 6. Power spectral density of the acceleration at 200 km/h:
(a) at the middle of the carbody; (b) above the bogies.

The diagrams in figure 7 represent the power spectral density of the acceleration at the eigenfrequencies of the vibration modes of the carbody, over the speed range 10 – 250 km/h, for the two models of the vehicle – track system. These highlight that at the eigenfrequencies of the carbody's bounce and pitch vibrations (diagrams a and b), the track model does not influence the results regarding the carbody's vibration level at any of its relevant points. At the eigenfrequency of the carbody bending vibrations (diagram c) there are differences between the results obtained based on the two models of the vehicle-track system, which are more important at high speeds. For example, at the speed of 237 km/h, the power spectral density of the acceleration at the middle of the carbody calculated for the "elastic track" model is 8.4% higher than that obtained for the "rigid track" model.

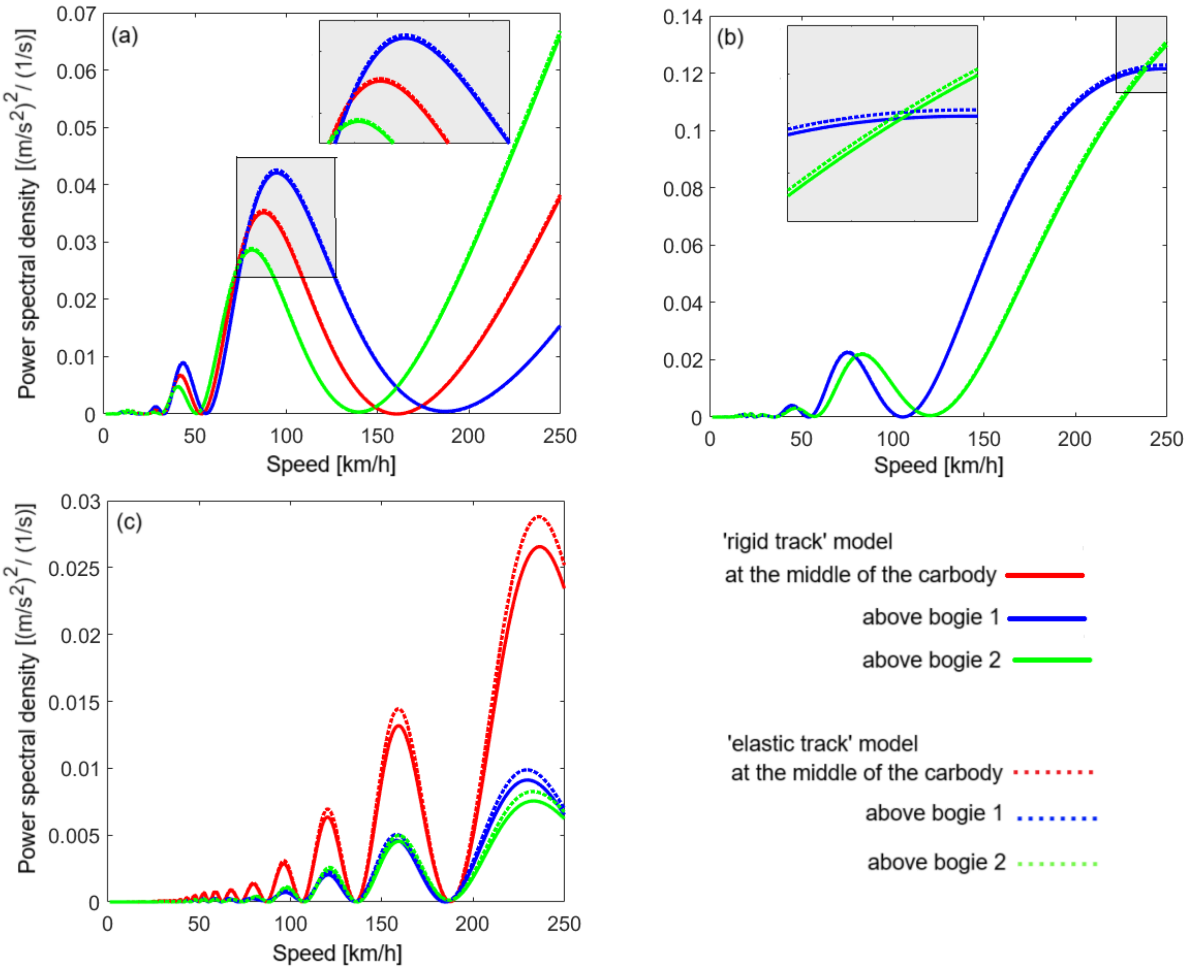

Figure 7. Power spectral density of the acceleration at the eigenfrequencies of the vibration modes of the carbody: (a) at the eigenfrequency of bounce vibrations (1.17 Hz); (b) at the eigenfrequency of pitch vibrations (1.73 Hz); (c) at the eigenfrequency of bending vibrations (8 Hz).

Figure 8 shows the influence of the track model on the root mean square of the acceleration at the middle of the carbody (diagram a) and above the bogies (diagram b). It is observed that the track model does not change the dependence of acceleration on speed. In general, the root mean square of the acceleration increases with velocity, but this increase is not continuous due to the geometric filtering effect. For the "elastic track" model, acceleration increases are obtained only at the middle of the carbody, but these increases are not greater than 4% (see figure 9).

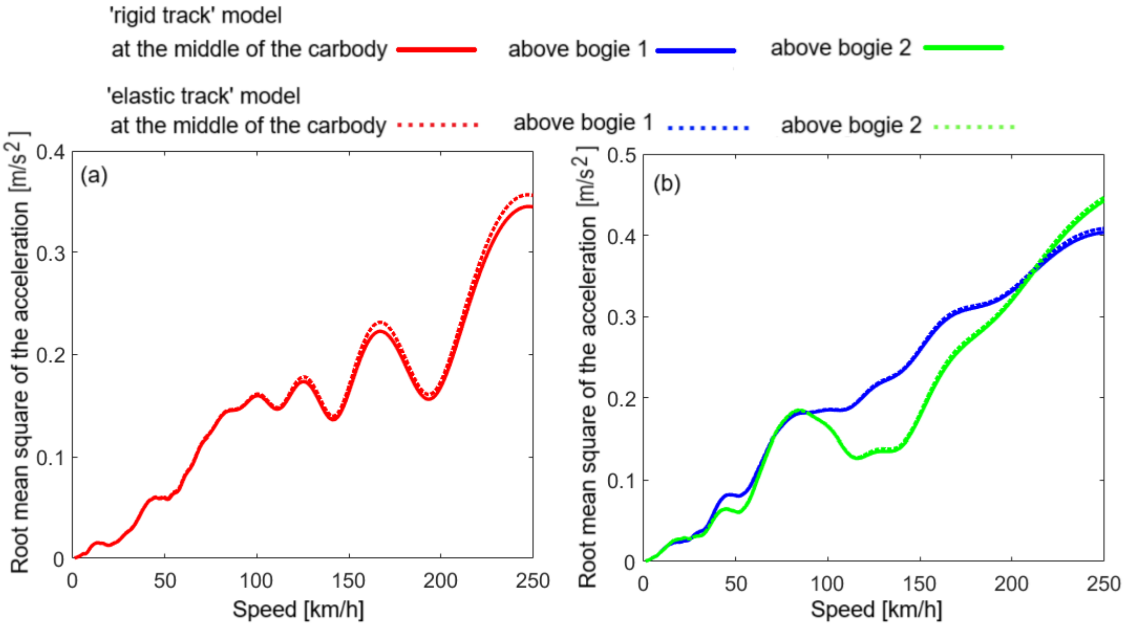

Figure 8. The root mean square of the acceleration:
(a) at the middle of the carbody; (b) above the bogies.

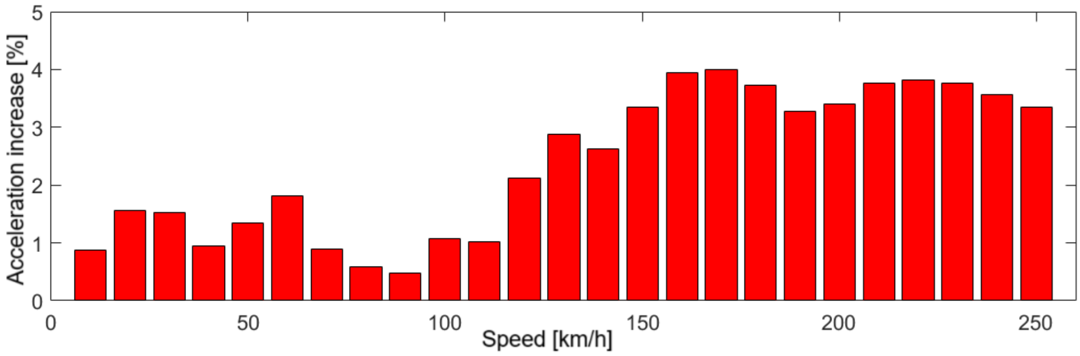

Figure 9. Increase in the root mean square of the acceleration at the middle of the carbody for the "elastic track" model relative to the "rigid track" model.

6. CONCLUSIONS

The paper analyses the influence of the track model on the vertical vibration behaviour of the railway vehicle carbody evaluated based on the results obtained through numerical simulations for two models of the vehicle - track system, namely the "rigid track" model and the "elastic track" model. For this, the frequency response functions, the power spectral density and the root mean square of the acceleration at three relevant points of the carbody are compared – one point positioned at the middle of the carbody, and two points located above the bogies. The conclusions drawn from the study are grouped according to the problems treated as follows:

1. The influence of the track model on the vibration behaviour of the carbody. The track model has little influence on the vibration behaviour of the carbody in its middle but has an important weight above the bogies at frequencies higher than 20 Hz, respectively outside the frequency range of interest for vertical vibrations of the vehicle carbody.

2. The influence of track damping on the vibration behaviour of the carbody is manifested above the bogies, at the eigenfrequency of the vertical vibrations of the wheelset on the track.

3. Influence of the track model on the dominant vibration modes of the carbody. The track model does not influence the dominant modes of the carbody. At the middle of the carbody, the dominant vibration mode is bounce, but vertical bending also has an important share. Above the bogies, the vibration behaviour of the carbody is dominated by pitch vibrations.

4. The influence of the track model on the vibration level of the carbody. At the eigenfrequencies of the bounce and pitch vibrations of the carbody, the track model does not influence the results regarding the vibration level of the carbody among its relevant points. At the eigenfrequency of the carbody bending vibrations there are differences between the results obtained based on the two models of the vehicle-track system, which are more important at high speeds. For the "elastic track" model, acceleration increases are obtained only at the middle of the carbody, but these increases are not greater than 4% and do not modify the speed dependence.

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