POLISH MARITIME RESEARCH 4 (128) 2025 Vol. 32; pp. 150-156
10.2478/pomr-2025-0058

Searching for Determinism in Mobile Gravimetry Data

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ABSTRACT

The accuracy of gravimetric measurements is essential in various fields, from the safety of the navigation of unmanned autonomous vessels to searching for natural resources to the level of underground water to the accuracy of geodetic data. Usually, we have to deal with measurements contaminated by environmental noise, as well as noise generated by different mechanical devices, city transportation systems, and human beings; some of those sources have a periodical nature. In the presented article, we consider the problem of the influence of noise on registered data. A gravimeter is, in fact, a vibration analyzer, so most of the artificial noise caused by engines, machines, and other technical systems is included in the final recorded data. By testing a statistical hypothesis, we try to convince the reader that in recorded time series, there is other information that is deterministic in nature and may have an important impact on the analysis of gravimetric data.

Keywords: mobile gravimeter, time series analysis, hypothesis testing

INTRODUCTION

Modern gravimeters are able to monitor the delicate balance of the Earth's gravitational force. A relative gravimeter performs measurements by tracking the deformation of the active element, which is a spring loaded with a probe mass. In theory, these deflections are proportional to the magnitude of the gravity field itself. Changes that can be observed in the gravity field are caused by the ellipsoidal shape of the Earth and the nonuniform density distribution in the crust and core. These are the main factors influencing gravimetric measurements. Other factors that also affect the Earth's gravity field come from other celestial bodies such as the moon, the sun, and other planets. Thus far, we have only mentioned the factors that generate the useful part of the signal, which is the magnitude of the gravity field; the other part of the signal is noise. Noise in gravimetric signals has many sources, and it cannot be easily eliminated. Some noise is always present, even if the gravimeter is stationary and settled in a place isolated from significant micro-seismic noise.

The gravimeters that have been used up till now are mechanical devices, where the measuring sensor, in the form of a spring, is loaded with a unit mass and the stretching of the spring is converted into the acceleration of the force of gravity. Modern devices whose principle of operation is based on the fundamental laws of nature, i.e. quantum effects, are in the early stages of development, although there are already models being sold on the world market; however, these devices are still in the experimental phase. Classic gravimeters are susceptible to all kinds of vibrations, especially in the case of mobile gravimetry; these vibrations are the main source of noise and make it

difficult to analyze gravimetric signals. The aim of the paper is to demonstrate that the share of deterministic elements in the gravimetric signal is significant and affects the methods of filtering these signals. Gravimetric signal filtering, in addition to measurement accuracy, is critical for mapping gravitational anomalies. Comparative navigation systems based on these types of maps can be of great importance when autonomous underwater vehicles are used to monitor critical infrastructure facilities at sea. Currently, in the case of the Baltic Sea, we are dealing with the permanent interference of GNSS (Global Navigation Satellite System) signals; this could be an effective method of navigation in such difficult conditions. Work on such systems is essential for the safety of navigation, which is directly related to the efficiency and effectiveness of maritime transport and affects the exploitation of the coastal zone, including the extraction of natural resources, the construction of hydrotechnical structures, and finally recreation. Traditional hydrographic methods are not able to provide data on the geophysical side of the coastal zone as detailed as the data provided by mobile gravimetry.

Precise gravimetric measurements are becoming more popular around the world. They are particularly important in the case of research on alternatives to GNSS positioning methods.

An example of changes in the magnitude of the gravity field is shown in Fig. 1. These measurements were collected during a static measurement session. The instrument was settled in the basement in the main building of Gdańsk University of Technology. The location of the gravimeter let us neglect all acceleration noise generated by movement. The desired part of the signal is generated by changes in the gravity in the chosen location. The rest of the signal is instrumental noise mixed with the signals coming from ground vibrations. In the paper, we present an attempt to describe the nature of gravimetric signals through the verification of a statistical hypothesis.

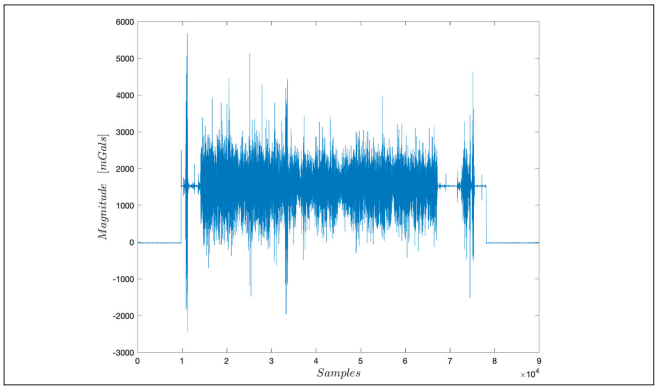

Fig. 1. An example time series recorded in the experiment with mobile gravimeter MGS-6

In recent years, many new approaches for performing and analyzing measurements using gravimeters considered to be mobile have been developed. The term 'mobile', however, does not necessarily mean that a given gravimeter is adapted to work on a mobile platform. The term 'mobile' is used even for gravimeters whose movement is a complex procedure, and the places where they are installed must fulfil many conditions [1]. According to this principle, we will adopt a more restrictive division into gravimeters requiring fixed installation and gravimeters that can carry out measurements while being installed on mobile platforms. Measurements made by a gravimeter mounted on a mobile platform (maritime and air) are more complex than those taken by a gravimeter mounted on a fixed substrate[2], [3]. There are efficient techniques for providing accurate data with a high spatial resolution of 20 m or better [4].

Currently, gravimeters can perform measurements with a frequency of up to 20 Hz and can achieve a resolution of 2 m. The recorded measurements, taking into account the array, can be classified as micro-gravimetric. It would be good if, in terms of measurement precision, those recordings could also be classified as micro-gravimetric rather than milli-gravimetric. Taking into account signals recorded by a mobile gravimeter, we must also think about the external factors, resulting from platform movement, that interfere with the conducted data collection. The development of an effective method of filtering out a useful signal from interfering noise is the basis for micro-gravimetric measurements, and this is not an easy task [5]. The elimination of disturbances accompanying measurements requires the measurement of individual external factors and their separation. The second problem is the proper analysis of the signal recorded during the measurement. In the following, we would like to present a rather simple method; however, if it works, then it opens up new possibilities concerning the time series recorded by a gravimeter. Our working hypothesis is that all signals recorded by the mobile gravimeter MGS-6 can be well described by a Gaussian process.

The paper is organized as follows: in Section 2, we describe the procedure for doing gravimetric measurements in shallow water; then, we describe the device used for our laboratory experiment, as well as the conditions in which the measurements were made. The theory of the statistical tools used for this test is also presented. In Section 3, the results of the employed statistical test are presented and discussed. The article ends with the conclusions section.

METHODS AND MATERIALS

The constant development of gravimetric systems, such as the ZLS, Dynamic Meter, Micro-g, LaCoste MGS-6, Elektropribor Checkan, and Canadian Micro Gravity AM GT-2M, is evidence of the high demand for such measurements [6], [7], [8]. Our device, the LaCoste MGS-6, represents cutting-edge technology in the field of gravimetric devices. This class of devices can be used in areas that do not have good data coverage or are difficult to access. Such measurements are called dynamical gravimetry and are the method of choice for areas in which stationary gravity measurements cannot be made [9]. The crucial part of these kinds of measurements is the separation of the useful signal from noise. For that reason, before we utilize the new statistical analysis method on the high-noise data collected on a moving platform, it is useful to perform a stationary laboratory test.

The main building of Gdansk University of Technology has been chosen as a laboratory to perform stationary tests; all the measurements considered in this paper were recorded in this building. As we described in the introduction, even a signal

registered by a stationary relative gravimeter is a convolution of many signals and noises. Thus, we can also test our method on a static signal. The usage of static registration for the purpose of analysis method testing has the advantage of allowing us to control the level of the perturbing signals, which is not the case during real-world examples, for example, during marine campaigning.

Conducting measurements in places where the depth is very small or where ground and sea areas connect, especially when the gravimeter is in motion onboard a small vessel or RIB (Rigid InflaTable Boat) boat, is very difficult. The device is mounted on a mechanically stabilized platform due to the necessity of keeping the sensor sensitivity axes aligned with the plumb line. In the MGS-6 gravimeter, sensor stabilization is achieved in several stages. A frame supports the gimbal and sensor, and isolates them from vibrations with suspension cords, air dampers, and air-filled vibration mounts. The above-mentioned system is supported by computer software [10]. The optimization of the stabilized reference parameters is superior to the solutions applied to the first versions of this type of gravimeter [11], [12]. The gravity meter is equipped with an advanced data collection system, synchronized with the 'rubidium atomic' clock. The clock can be used to synchronize the measurements of other sensors installed on the mobile platform. The second important issue is to maintain the fixed thermodynamic parameters of the sensor and its immediate surroundings. As one can see, the experiment procedure is complicated; therefore, we decided to start our research with a laboratory test to have control over all aspects of the experiment. To give necessary background, we introduce the reader to some details of our experiment, starting with some basic concepts from time series analysis. Let us consider a purely deterministic system. When its initial condition is determined, all future states are also determined. Therefore, it is important to establish a space, also called phase space, in which by indicating a point, we may determine the state of the system, and vice versa [13]. In this way, we can study the dynamics of the system by examining the dynamics of points in the phase space. Usually, dynamic systems are described by a vector differential equation (i.e. a system of ordinary differential equations, usually ordinary first-order differential equations) [14]:

$$\frac{d\mathrm{x(t)}}{dt} = \mathbf{f}(\mathbf{x}(t)), \tag{1}$$

where

$$\mathbf{x(t)} = [x_1(t), x_2(t), \ldots, x_n(t)] \tag{2}$$

is the studied vector function of a numerical variable (time), called the phase vector. Then $\mathbf{f} = (f_1, f_2, \ldots, f_n)$ is a known vector function of a vector argument (phase variables, running through the phase space). In discrete time $t = n \cdot \Delta t$ the vector function is a mapping of $\boldsymbol{R}^d$ onto itself:

$$\mathbf{x_{n+1}} = \mathbf{f}(\mathbf{x_n}). \tag{3}$$

Therefore, we can treat a time series as a sequence of observations $s_n = s(\mathbf{x_n})$, represented by some measurement function $s(^\star)$.

Using these observations, we are able to calculate the trajectories along which the dynamic system moves in phase space for specific initial conditions. It is worth emphasizing that in the case of real systems, even non-deterministic ones, the concept of the system state is still useful. Emphasizing the importance of phase space for studying systems with deterministic properties, we must face the first problem we observe during the experiment: this is not an object in phase space, but a time series, i.e. a sequence of measurements. Therefore, we need to convert the observations into a state vector. This is the basic problem of phase space reconstruction, which can be solved by the so-called *delay method*, expressed in:

$$s(t) = \Big(s(t-(m-1)\tau, s\big(t-(m-2)\tau, \ldots, s(t)\big)\Big), \tag{4}$$

where τ is the time delay (or lag) and m is a dimension of reconstructed space (embedding dimension). According to the embedding theorem, when this is done properly, the dynamics reconstructed using this formula are equivalent to the dynamics on an attractor in the origin phase space in the sense that characteristic invariants of the system are conserved. The time delay method and related aspects are described in the literature.

Usually, we observe the system under consideration through some measurement function , as presented in:

$$s_n = s(x_n) + \xi_n, x_n = f(x_{n-1}) + \eta_n \tag{5}$$

where η_n represents the measurement noise, due to fact that there is no system that is really isolated, and ξ_n is a random component (random variable). In the case of the measurements recorded by the MGS-6 device in the lab at Gdansk University of Technology, the influence of the noise, considering the specific location of Gdańsk, is reduced by a few orders of magnitude in comparison to the dynamical measurements, but noise is still not neglected. The magnitude of the recorded signals represents a part of the gravitational acceleration and the influence of the different sources of vibrations, such as the micro-movements of ground masses, changes in underground water, the interaction between freshwater in the harbour canals and incoming saltwater from Gdańska Bay due to extreme weather phenomena caused by climate change, the everyday life of the city, and public transport; all these sources of vibration inevitably influenced the useful signal.

Given the above difficulties, the effective filtering of the signal recorded by the gravimeter seems to be impossible. The presented analyses are only an introduction, the first step on the long journey to identify the aforementioned components that contain important information about, for example, the behaviour of groundwater, which is vital for the Old City of Gdańsk.

The delay reconstruction of phase space in s_n dimensions is performed using the vectors, as presented in.

As we are not able to completely restore $\mathbf{x_n}$ from s_n, the best we can do is to use some kind of embedding s_n and look for a mapping that works on the embedding vectors. If the original phase space is d-dimensional, the mapping may need to be defined in dimensions up to $2d$ according to the theory of embeddings.

All the concepts presented above have become the basis for testing the working hypothesis that signals recorded by a mobile gravimeter can be well described by Gaussian noise [15]. Accepting the null hypothesis means that the signals from the gravimeter are random. If this hypothesis can be rejected (we accept the alternative hypothesis), this means that there are deterministic components in the gravimetric signal, which in turn opens the way to the use of non-linear methods of signal analysis. One of the motivations of the authors was the belief that this signal, after filtering methods are applied, can be used to monitor the status of groundwater, e.g. in the Old Town of Gdańsk.

Theoretical considerations regarding the study of the behaviour of phenomena occurring in the real world require data sets of unlimited length. Solving differential equations describing the dynamics of the studied systems can provide such data.

However, it is extremely difficult to accomplish this when it comes to data from real-world observations.

For noise-free data of unlimited length, we may assume the existence of a diffeomorphism between the original attractor and the reconstructed one. This is true for almost any choice of delay and a sufficient embedding dimension. However, when considering real-world systems, the experimental data represented by time series may span a small time period and be influenced by noise. Therefore, the quality of the reconstruction of the phase space and the attractor depends on the choices of the embedding dimension and time delay. A time delay that is too small leads to redundancy.

When analyzing various real systems, we only have observations collected over a short period of time, which are often very difficult to repeat. The data collected during observations of seismic shocks or volcanic eruptions are a specific example of this. It should be mentioned here that one cannot assume the presence of deterministic chaos in any time series recorded during the observation of natural phenomena. However, this does not prevent us from using certain concepts derived from this theory, because they give us the appropriate skills to solve specific problems. The question of how much information we can extract using the time-delay method from a limited-length noisy data set is still open. It is very difficult to make any assumptions about the presence of noise in the measurement data. The nature of this noise may result from measurement errors, but it can also be caused by a phenomenon that underpins what is reflected in the recorded time series. An example of an achievement in this field is the method presented by H. Kantz and T. Schreiber [16], [17]. The reconstruction of the phase space makes it possible to recreate the trajectory of the dynamics of the examined system and to eliminate noise.

RESEARCH RESULTS – TESTING OF STATISTICAL HYPOTHESES

In the case of analyzing the signals in this paper, hypothesis testing takes on special significance because the calculated results determine further actions. In this case, we deal with the following assumption: the null hypothesis assumes that the analyzed signals can be well described by the Gaussian linear stochastic process. To verify this hypothesis, artificial data (surrogate data) must be generated. There are two types of such data:

1. FT (Fourier Transform surrogates);
2. ARMA (Autoregressive-Moving Average surrogates).

The Test

To test statistical hypotheses, we have to make certain assumptions. Namely, it should be checked whether the tested waveforms can be described by a Gaussian linear process.

We are going to show that the signals at our disposal contain linear or non-linear elements. To this end, we have to prepare a data set that will contain the original signal and the artificial signals generated using the FT or ARMA methods. The next step is to choose the proper statistics, i.e. a certain non-linear quantity whose values are then calculated for all the signals in a given set. Since the probability distribution of the selected non-linear statistics remains unknown, some non-parametric tests can be used. The most appropriate one seems to be the non-parametric test suggested in [18], which proposes a more general null hypothesis allowing non-linear rescalings of a Gaussian linear process. If the results for the original data differ from those obtained for artificially generated data, we reject the null hypothesis. Taking into account the confidence level (here), we have created artificial data sets. Then, we calculated the selected statistics for the original data and all artificially generated data sets.

In short, in the presence of random components in the original data, the non-linearity test will not be reliable [19], [20].

There are many examples of different statistical methods [18], [21], [22], [23], [24], [25], [26], [27] that can be used to discover the nature of the time series being considered. To test the working hypothesis, two approaches were used:

1. The first method is based on a well-known statistic $t = t(x_n)$ called the time-reversibility, which is defined as follows:

$$t^{rev}(\tau) = \langle (x_n - x_{n-\tau})^3 \rangle. \tag{6}$$

2. The second one is based on using the delay vector variance (DVV) method, which is one of the most popular methods for detecting deterministic behavior in measurement data.

Both of these methods are based on the generation of surrogate data according to the procedure proposed by Schreiber and Schmitz [18].

The DVV method was recently introduced by Gautama et al. [28], and it is focused on revealing the nature of data represented by certain time series through the estimation of predictability and determinism. The method is based on the time delay embedding representation of the considered time series and uses the above-mentioned method of *phase space reconstruction* described in detail by Kantz [16]. The core of the DVV method is calculating the average of the target variances σ_k over all sets divided by the variance of time series σ_x^2, as presented in:

$$\sigma^{*2} = \frac{(1/n)\sum_{k=1}^{n} \sigma_n^{*2}}{\sigma_x^2}. \tag{7}$$

DISCUSSION

In the above experiment, we compared time series recorded by MGS-6 devices and those generated as a realization of our working hypothesis.

A significant share of the noise in the frequency spectrum of the analyzed signal, as presented in Fig. 2, indicates the chaotic nature of the phenomenon we are trying to understand. From the visual inspection of Fig. 1, we can conclude that the time series we are dealing with represents a non-stationary process, which in turn is a significant problem in the data analysis. The time series represented in Fig. 3 is stationary in the sense of second-order statistical moments. This means that we can use these data in further analysis. In order to be able to apply statistical methods, it is necessary to divide the signal into blocks that will be stationary. An example of such a block is presented in Fig. 3. We prepared 10 blocks.

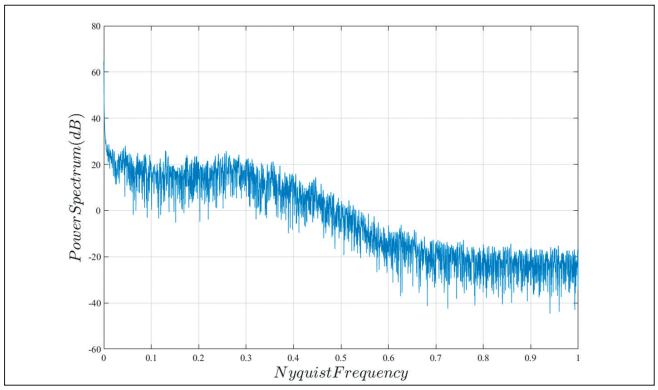

Fig. 2. Frequency spectrum of the signal recorded by mobile gravimeter MGS-6. We cannot see dominant frequencies because everything is covered by noise

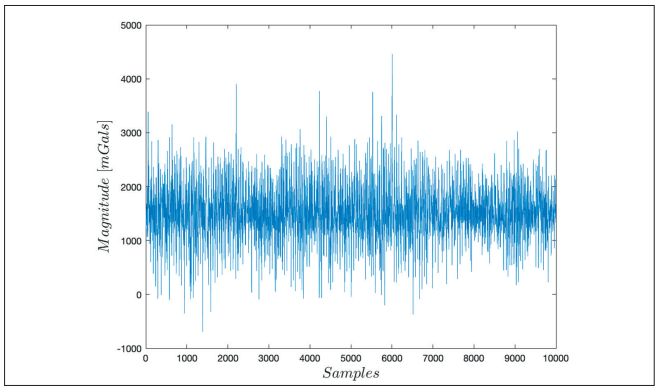

Fig. 3. Segment – an example of the data – that is stationary in the sense of second-order statistical moments

In the case of the first statistic, we expected that the original data (segment) would deviate from the values calculated for the surrogate data. Taking into account the time reversal statistic, this value may fall above or below the values for the surrogate files. We applied the method proposed by Theiller [29]. To achieve a significance level of 95%, we need 39 surrogates. We have constructed 10 sets each of 1 segment from the original data and 39 artificially generated files representing our null hypothesis. The results of using the first method of checking our hypothesis (time reversal symmetry) on one of the segments of the data are presented in Fig. 4.

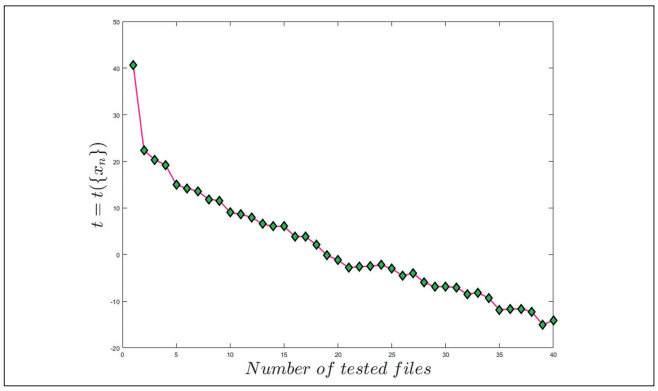

Fig. 4. Time reversal statistics results calculated for one of the set of 10 segments

Therefore, it may be said that the null hypothesis about the randomized character of the original data can be rejected. Taking into account the second method (DVV), if the determinism in the time series is strong, then we should have lower values of variance at smaller spans. The variance should converge to unity for a larger span, as we may observe in Fig. 6. As shown in Fig. 5, the variance of the original time series and its surrogates will diverge from the bisector line. All these results are evidence of the non-linearity in the considered time series coming from measurements recorded by MGS-6.

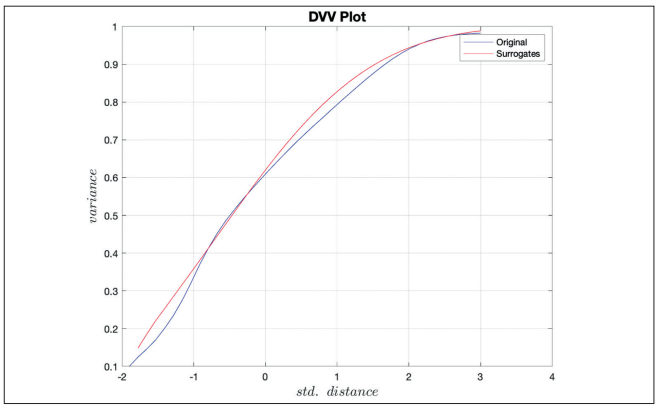

Fig. 5. Target variance – results calculated for the same segment of the data considered in Fig. 4

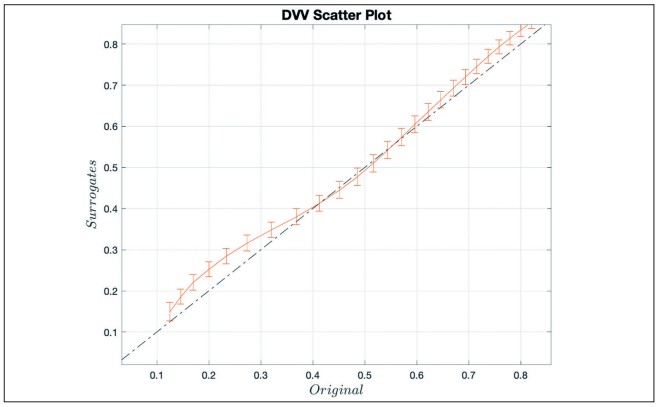

Fig. 6. DVV scatter figures for the same segment of the data considered in Fig. 4

CONCLUSIONS

The question of whether the signal recorded by the MGS-6 gravimeter may contain deterministic signatures was the subject

of the investigations presented in our article. Taking into account the way in which the measurement is made (change in the spring elongation), it is possible that the useful signal is superimposed by various environmental signals, which may be deterministic, e.g. the sound of a tram moving on rails, nearby mechanical devices, passing ships, moving cars, etc. These signals certainly overlap with the performed measurement, and therefore they can be detected as deterministic signatures in the data recorded by MGS-6. At the moment, we cannot clearly determine the source of these signatures, and we do not want to presume that changes taking place in the structures of the Earth's mantle, from ground structures to the liquid core, are deterministic; however, undoubtedly the presented results open up interesting possibilities for analyzing the data recorded by the MGS-6 gravimeter.

By applying tests for non-linearity, taking into account the stationary segments of the whole signal recorded by the mobile gravimeter, we tested the null hypothesis concerning the assumption that the signal presented in Fig. 1 can be well described by a Gaussian process. In fact, this hypothesis is the simplest one; however, if this simplest assumption works, then we may say that the phenomenon will also follow the same rules. We applied the time reversal statistic and DVV and analyzed the results of calculating the prediction error for the set of real and artificial files representing the null hypothesis. Rejecting the null hypothesis opens the way for non-linear time series analysis to reveal the true nature of signals recorded by a mobile gravimeter. As we mentioned above, the problem of filtering these types of signals is a complex task. However, in the context of research on underground water, including its movement and interactions with the ground and saltwater from the sea, we assumed that the signal generated by those phenomena seems to be very weak, hidden in the measurement noise and covered by other possibly recorded signals, such as noise generated by everyday life in the city, public transportation, and other vibrations in the broadly understood city structures.

Mobile gravimetry is a source of data necessary to create the so-called maps of gravitational anomalies, which, in addition to providing a specific geophysical description of the shape of the Earth, can also be the basis for building a comparative navigation system. The first research conducted to determine the gravimetric geoid was carried out in the years 2022–2024, the result of which is the development of a geoid for the area of the territorial waters of the Republic of Poland. Our future research will focus on the possibilities of extracting well-known sources of vibrations and trying to find out how the useful signal generated may reflect the fragile balance between water and the layers of the ground in historical cities like Gdańsk, but the long-term goal is to start research on the development of Polish solutions in the field of modern quantum sensors for geophysical applications.

ACKNOWLEDGEMENTS

This research was co-funded by the European Union from the European Regional Development Fund under the 2014–2020 Operational Programme Smart Growth. The project entitled 'Development of technology for acquisition and exploration of gravimetric data of foreshore and seashore of Polish maritime areas' was implemented as part of the National Centre for Research and Development competition: 1/4.1.4/2018 Application projects.

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