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Using an isopleth map to determine the population of any area

Abstract. The aim of this article is to present and verify a method of estimating population numbers based on an isopleth map that meets the volumetric condition. This boils down to a situation where the volume of a solid enclosed by a statistical surface corresponds to the number of people living in that area. To verify this method, an isopleth map of population density was created based on census data from the Central Statistical Office (in Polish: Główny Urząd Statystyczny, GUS) from 2021, referenced to a grid of 1×1 km squares. Population estimates were made for municipalities in the Lublin Province and then compared with the actual population recorded in these municipalities. The results of the comparison showed that the MPE (mean percentage error) was -2.69 and the MAPE (mean absolute percentage error) was 4.2%. As a result, the proposed method can be considered an effective tool for estimating the population in areas for which no systematic statistical surveys are conducted. These may be, for example, protected areas (national parks, landscape parks) or physiographic units. The method is so versatile that it allows working with official data and data contained in global grid models of population distribution.

Keywords: population data, isopleth map, map volume, population calculation

1. Introduction

The answer to the question of how the population is distributed is of fundamental importance for many areas of the functioning of the state and its society. Information of this type is used in spatial planning, influencing the optimization of infrastructure and the location of public services. It plays a key role in social and economic policy planning and in crisis management during natural disasters. An equally important area of application for such data is science, allowing processes to be tracked and their future trajectory to be estimated. The conclusions drawn from such analyses influence the accuracy of decisions made, affecting the well-being of citizens.

In order to provide information on spatial distribution, data is collected by statistical services established for this purpose. In Poland, for example, this work is carried out by the Central Statistical Office (GUS). As a rule, population data are referred to administrative units, and the method of obtaining these data has many drawbacks (Śleszyński, 2011). Such data organization is very useful, but it does not allow for determining the number of people living, for example, within the boundaries of a protected area or an area affected by a natural disaster. Therefore, there is a need to develop more flexible estimation methods that would provide reliable and accurate population estimates.

The article aims to present an original method of estimating population numbers using an isopleth map, which allows for better results than the methods used so far. An example of this is estimates based on assigning part of the population of administrative units in proportion to the area within another area unit. The originality of the proposed method lies in the use of an isopleth map that preserves the volume of the phenomenon as a database of population distribution.

The proposed method allows the population of an arbitrarily defined area to be determined. The method is based on a flexible approach, which also allows historical distributions to be created. The proposed method is based on a consistent methodology that allows a high level of objectivity to be maintained in the analysis of changes in population distribution. As a result, the new methodological approach to studying population distribution may contribute to improving the state of knowledge and increasing the accuracy of population estimates in arbitrarily defined areas.

2. Literature review

Information on population distribution and numbers in the form of cartographic models has been with us since the beginning of statistical research. The first nationwide census was conducted in Sweden in 1686 (Spallek, 2011). Subsequent censuses took place in the 18th century. However, it was in the 19th century that this field flourished and became an impetus for the development of methods for visualizing collected data. In 1826, Charles Dupin constructed the first cartogram (Korycka-Skorupa & Pasławski, 2017), and in 1830, the first dot map was created by the Frenchman Frère de Montizon (Pasławski, 2015), and in 1843, L. Lalanne laid the methodological foundations for isoline population maps. The first isopleth map appeared only in 1857 and depicted the population of Denmark; its author was N.F. Ravn (Mościbroda, 1975; Robinson, 1982). Earlier, in 1837, an atlas by H.D. Harness included pie charts to illustrate the number of inhabitants of cities in Ireland (Spallek, 2011). These examples show that already in the first half of the 19th century, all currently known quantitative methods of cartographic presentation were used to visualize population distribution. They fulfilled their role in conveying a general picture of distribution, but over time, there was an increasing need for more detailed information that was not strictly related to the areas – most often administrative – for which the data was aggregated. The pursuit of accurate representation of distribution had two aspects, the first of which was to increase the detail of the data. An example of this are dot maps with increasingly less weight per dot. This approach has the greatest potential in terms of topographical representation of distribution (Kimerling, 2009). A major advantage in the context of population estimation is the ability to overlay any boundaries on a dot map and count the number of dots within them. Such a dot map can be treated not only as an image of a phenomenon but also as a source of data. The obstacle to their greater popularity is the large amount of work required to produce them (Pasławski, 2015). As a result, maps produced using this method are few and far between, although they are considered to be very valuable studies – e.g., the 1919 map of population distribution in Sweden produced by S. de Geer (Hey & Bill, 2014). Another example that highlights the usefulness of such maps for various analyses is the large-scale dot map of the population distribution in Lublin in 2005, created by T. Frąś (Cebrykow, 2017).

The second aspect involved improving the visualization methodology by introducing additional information that influenced population distribution. Information about uninhabited areas, such as forests, was introduced. This approach can be found in the dasymetric method, which allows for the separation of areas with similar population densities (Spallek, 2002).

Estimating the population based on distribution maps was laborious and required knowledge of cartographic presentation methodology. When performed for larger areas, the results were very general. Therefore, such analyses were performed sporadically.

The introduction of computer techniques opened up new possibilities in the analysis of population distribution. Parallel to the development of new technology, previously unused data sources, such as satellite imagery, also appeared. The use of additional information related to land cover or transport infrastructure allowed for increased accuracy of analysis while improving spatial resolution. An additional direction of change in the fields for which the population was estimated was the pursuit of their geometrization and unification in terms of surface area. This was not an entirely new direction, as such attempts were made in Japan and Sweden in the pre-computer era as part of the development of national atlases (Balk et al., 2006). New tools enabled the introduction of global reference grids based on a geographic grid, with the first solutions appearing in the US (Goodchild et al., 1993; Tobler et al., 1997).

Contemporary methods of estimating population numbers within grid reference fields can be divided into two groups: "top-down" and "bottom-up." The "top-down" approach is based on the use of high-quality data referenced, for example, to census fields, which are disaggregated into grid cells.

Within the group of top-down methods, three methodological approaches can be distinguished: unmodeled, weakly modeled, and highly modeled. An example of the first is the Gridded Population of the World (GPW), which is currently available in version 4, with version 5 planned for 2025 (CIESIN, 2017). Lightly modeled open-access datasets include the Global Rural-Urban Mapping Project (CIESIN, 2017) and the Global Human Settlement Layer Population (Całka & Bielecka, 2020; Schiavina et al., 2023), while examples of heavily modeled datasets include the LandScan Global Population Dataset (Bhaduri et al., 2007) and WorldPop (Tatem, 2017). The modeling process uses auxiliary data that narrows down the area of the ecumene in accordance with the idea of a dasymetric map. The data used includes land cover maps, nighttime lighting, communication networks, water areas, etc. Solutions are also proposed that limit the use of data to satellite images and rough population data, an example of which is the POPCORN project (Metzger et al., 2024).

The second approach – "bottom-up" – is based on a model defined during empirical research, which estimates the population in other fields based on their similarity to reference polygons (Lloyd et al., 2019; Stevens, 2023). The evolution of the method has led to increased accuracy in estimating population distribution, limiting itself to urban areas using tools such as the Settlement Mapping Tool (SMT). Compared to the top-down approach, the bottom-up approach uses very high spatial resolution (up to 10 m) and works well for smaller areas (Stevens, 2023).

The literature also mentions other methodological solutions based on registration data and building records. Verification of census data improves its reliability, but its use is limited to small areas, e.g., urban areas (Kuzara & Szmytkie, 2022, 2024; Śleszyński, 2011). Estimates based on so-called big data from mobile telephony are also of similar importance (Przeworska & Turzyński, 2023). The latter two examples are the basis for another discussion, which clearly creates the need to develop a new definition of a resident of a specific area.

The outlined overview of examples of population estimation methods shows the entire spectrum of solutions that are used in small areas as well as on a global scale. The accuracy of estimates varies, and there is certainly no universal method. There is still room for research on alternative solutions.

3. Data and research methodology

The proposed method for determining the population of any given area involves the use of an isopleth map, which meets the condition of preserving the statistical volume of the phenomenon. In the case of maps showing the spatial distribution of population density values, the preservation of this volume can be equated with compliance with the statistical data on the number of inhabitants recorded on the map (Cebrykow, 2004; Mościbroda, 1999). Continuing, it can be said that a map understood as a statistical surface encloses a solid with a base defined by the boundaries of the mapped area. A solid constructed in this way has a volume that can be interpreted as the number of inhabitants. From the total volume, it is possible to separate partial volumes, which include prisms defined as the projection of the boundaries of any area (e.g., it may be a physiographic division unit, protected area or area affected by a natural disaster) onto the horizontal surface that is the base of the map (Figure 1). The volume of the solid defined in this way will correspond to the population living in the designated area. The assumption presented is simple but requires appropriate preparation of the measurement base in the form of a geometric construction of an isopleth map.

The entire process of estimating the population can be divided into three stages:

1. Preparation of a spatial database on population size
2. Developing an isopleth map in the form of a geometric model
3. Estimating the population

The first stage determines the spatial resolution of the population database. In order to achieve greater accuracy, it is necessary to use data with the highest degree of reliability, referring to the smallest possible surface units. In countries with well-developed statistics, census data, usually prepared at 10-year intervals and related to administrative units, meet these conditions. In Poland, population data from the last two censuses (NSP 2011 and NSP 2021) have also been made available in relation to 1×1 km square grid fields (GUS, 2024). In the presented procedure, the use of such fields is a good solution, as it ensures the possibility of maintaining the volume of the phenomenon in relatively small areas similar in size to the surface of the grid element. The selected grid is regular and consists of square fields. In order to perform interpolation, points representing each grid field must be entered. In the case of square fields, the selection of the point location is simple because the center of gravity of the field coincides with the geometric center. Each point representing a grid field inherits the population density index value assigned to it. The result is a set of points with specific X, Y, Z coordinates. All data preparation operations were performed using QGIS 3.42.

Once the data has been prepared, the second stage of the procedure can be started, which involves creating a continuous statistical surface in the form of an isopleth map. To do this, an interpolation method must be selected that will preserve the volume of the phenomenon. There are many interpolation methods, the most commonly used of which are IDW and kriging (Li & Heap, 2011). These two methods can give positive results in the case of data organized in a regular dense network. However, it is important to remember to select appropriate parameters, such as the number of points taken into account when estimating each new value. This is to limit the influence on the new value to only points in the immediate vicinity. In the case of the IDW method, the weight of influence should also be set to 1, which in practice aims to establish linear relationships in the interpolated surface. Therefore, it is worth using the linear interpolation method directly, which has simple rules and does not require the specification of additional parameters, as is the case with other interpolation methods. Moreover, with a dense regular data network, it ensures that the volume of the phenomenon is preserved (Mościbroda, 1999). The constructed statistical surface acts as a volume model here, and its mathematical properties are a priority. Methods such as IDW and kriging lose their advantages in this context due to the better visual properties of the surfaces obtained with their help (Cebrykow, 2025).

The third stage of the described procedure is to estimate the population within a freely defined area. To do this, measure the volume of the prism cut out of the closed solid from the top with the calculated statistical surface. The prism of interest will have its base at the level of the base of the entire map in the form of a plane determined by the projected boundary of the area designated on the interpolated statistical surface (Figure 1). On the map, the elevation of the statistical area depends on the population density index and is expressed in units of length. If a square field with a side length of 1 km is raised to 1 km, we obtain a cube with a volume equal to 1 km^3. This volume can be interpreted as 1 person. According to this principle, by measuring the volume of the entire solid enclosed by the surface of, for example, a landscape park, we can determine the number of people living in that area.

The last two of the three stages of the procedure were performed in Surfer 24.

4. Research results

The presented method of estimating the population within arbitrarily defined areas was verified using an example that provides a reliable basis for comparison. For this purpose, data on the population in the municipalities of the Lublin Province (NSP 2021) were used as reference data. These data were compared with the calculated values based on an isopleth map made using data from the same census but reported only for fields in a 1×1km grid. In total, the comparison covered 213 municipalities with a reference population ranging from 1,524 to 332,852 inhabitants.

The following were used to determine errors in population calculations: ME (mean error), MPE (mean percentage error), MAE (mean absolute error), and MAPE (mean absolute percentage error). The error statistics are included in Table 1.

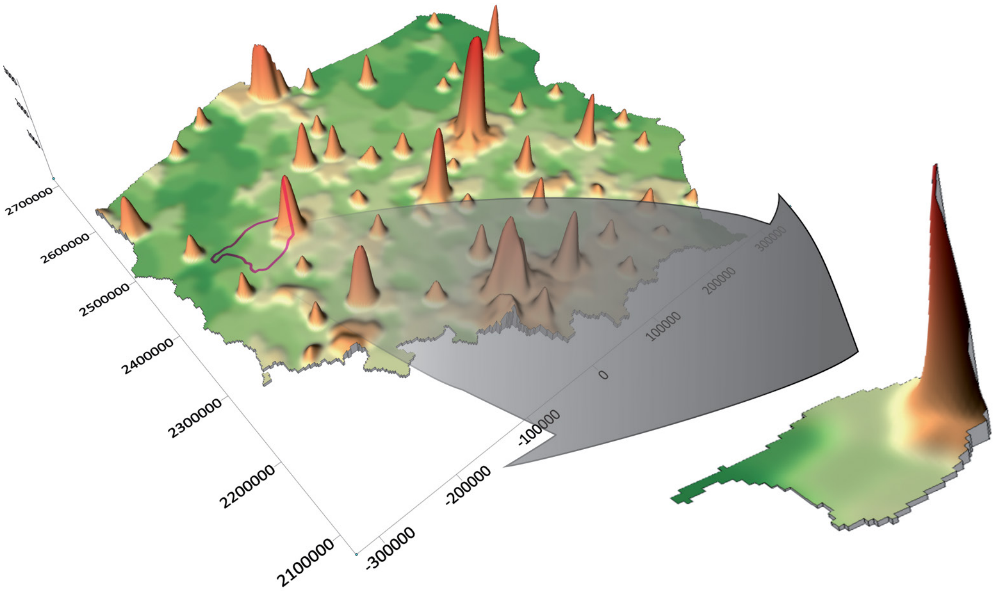

Figure 1. Visualization of the separation of a partial volume from a solid containing the total volume of the phenomenon. In this case, the geometric solid was defined by the extent of the Poznań Lake District, and the surface represented the distribution of population density values in 2012 (own study)

Table 1. Statistical parameters comparing estimated values with reference data (own study)

Type of indicator	Population	%
Max underestimation (Tomaszów Lub. – town)	-3843	-21.01
Max overestimation (Lublin – town)	9688.5	2.91
Max overestimation % (Tomaszów Lub. – village)	2739.7	24.78
Median	–	-2.88
Standard deviation	–	4.83
ME (mean error)	-306	–
MPE (mean percentage error)	–	-2.69
MAE (mean absolute error)	438.1	–
MAPE (mean absolute percentage error)	–	4.20

The error values range from -21.01% to 24.78% and their distribution corresponds to a normal distribution. A graphical interpretation of the distribution is presented in the histogram in Figure 2. The mean percentage error (MPE) is -2.69% and the median is -2.88%. The concentration of error values is determined by the standard deviation, which is 4.83%. Out of a total of 213 municipalities, 178 were within the error range <MPE-St. Dev. – MPE+St. Dev.>, which accounts for 83.57% of the results. The absolute average error reached 4.2%. The results indicate an underestimation of the population in municipalities, as error values lower than 0 were observed in 174 municipalities. Overestimated results concerned only 39 municipalities.

The greatest differences from the reference data mainly concerned urban municipalities and adjacent rural areas, as can be seen on

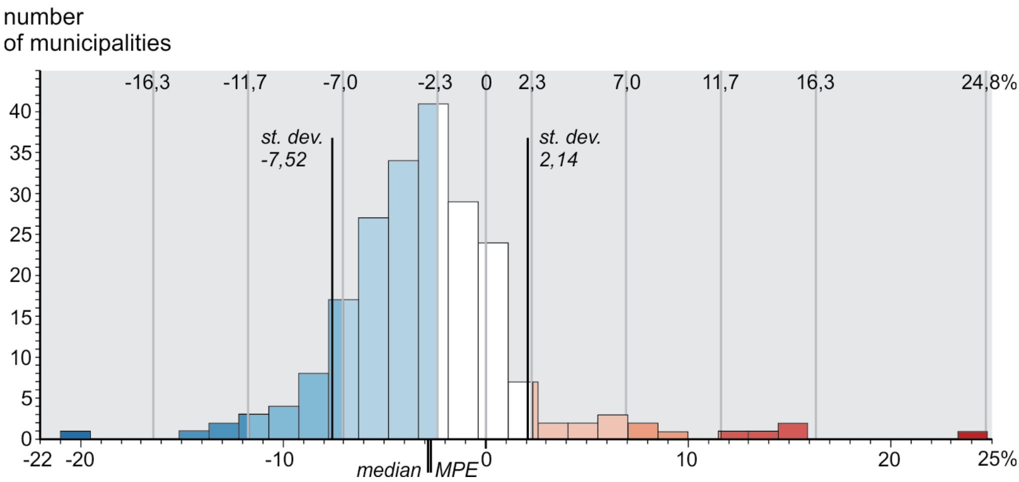

Figure 2. Distribution of errors in estimating population numbers based on the partial volume of the isopleth map (own study)

the map in Figure 3. There is a systematic underestimation of the population of cities, while the number of inhabitants of adjacent areas is overestimated.

5. Discussion

The proposed method allows for determining the population size in areas with arbitrarily defined boundaries. There are many such areas that require information about population size for their efficient management. Examples include protected areas such as landscape parks, landscape parks, etc. Population size is one of the key factors influencing the level of anthropogenic pressure. The described method has been verified by comparing the estimation results with data referring to municipalities. This raises the question of the selection of reference data, which are essentially the same data (NSP 2021). However, the difference is significant and consists in referring the data to different basic fields. An important feature of such a comparison is the result, which limits the factors influencing the errors to those resulting from the proposed method itself.

The error analysis showed that the MPE was -2.69 and the MAPE was 4.2%. The results obtained indicate that the proposed method is not perfect, but the error value may be acceptable in many applications, such as infrastructure planning or crisis management. Population estimates based on currently functioning population distribution models, such as WorldPop or Global Human Settlement GHS, achieve a MAPE for similar spatial units of 5-15% (Całka & Bielecka, 2020; Eichhorn, 2020). The better result of the proposed method does not prove its superiority, as it is largely dictated by the use of input data from the census. However, it is an argument in favor of accepting the obtained results, and thus the proposed method.

For a better understanding of the mechanisms affecting the value of errors, it should be noted that the data used to develop the isopleth map on which the population estimate was made was organized in the form of a grid. The grid consisted of square fields measuring 1×1km. From a cartographer's perspective, this field size provides a very accurate representation of population distribution. However, the perspective outlined above assumes analysis at a higher level of generalization. When estimates are needed for smaller areas, such as physiographic units or protected areas, the models should provide a higher level of accuracy. Certainly, a denser grid would have a positive impact on the accuracy of the proposed method. However, such accurate statistical data are not available. Although this gap

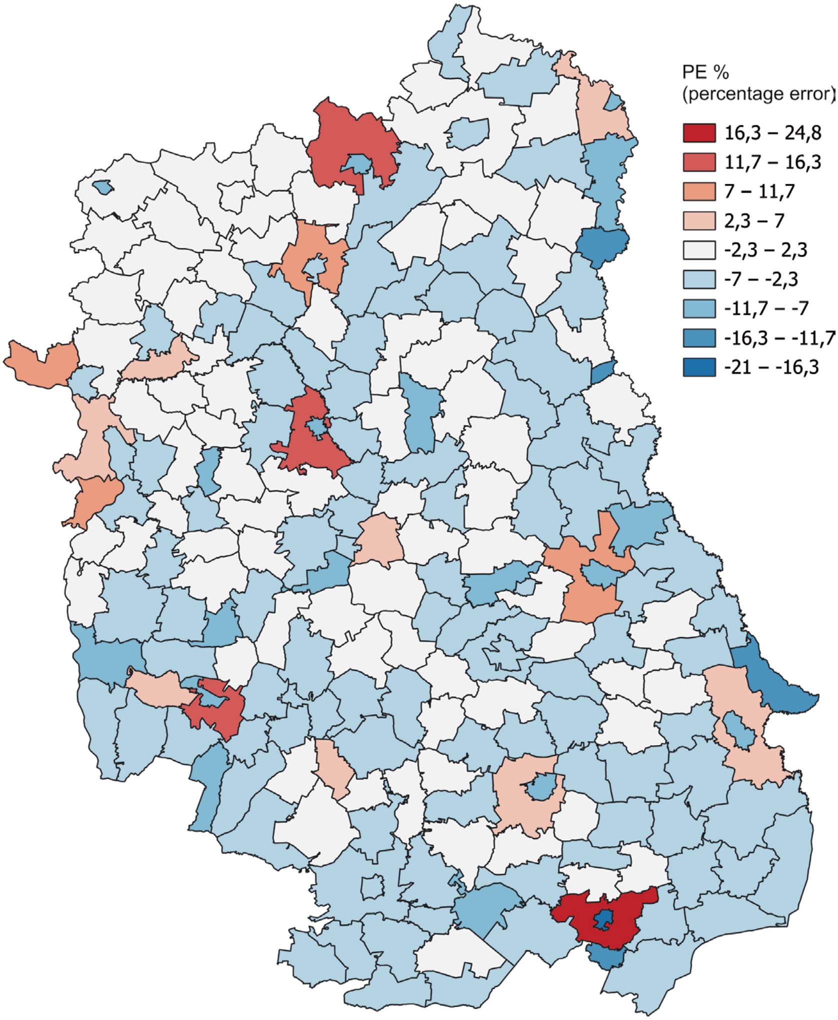

Figure 3. Spatial distribution of errors in population estimates in municipalities (own study)

is filled by grid models such as WorldPop and GHS, as already mentioned, they do not provide a higher level of data reliability.

The problem of errors in reproducing the exact number of inhabitants of the smallest administrative units based on a grid model stems from the inability to reproduce the boundaries of these units in accordance with the boundaries of the grid fields. The largest errors occur when the administrative boundary separates densely populated areas from uninhabited areas. In such a situation, part of the population is transferred to a neighboring unit, whose population increases. This situation is most often observed in the case of urban municipalities and adjacent rural municipalities. In such a pair, the population of the city usually decreases and increases in rural municipalities. The described relationships are perfectly illustrated in Figure 4, where the western and southern borders of Tomaszów Lubelski separate settlements from uninhabited areas.

The analyzed method is characterized by high versatility in terms of the use of various

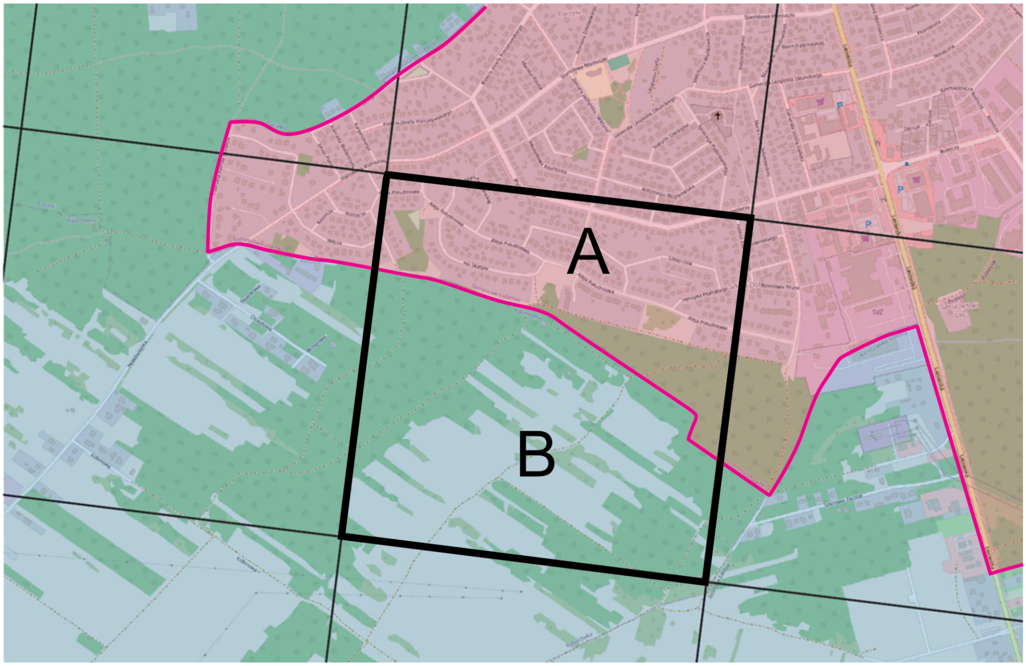

Figure 4. An example of a square field covering a part of the town of Tomaszów Lubelski with residential buildings (A – pink area) and a part of the adjacent rural municipality without residential buildings (B). The town boundary runs along the boundary of the built-up area, and the superimposed grid of squares is not consistent with the town boundaries (own study based on OSM)

types of data. The most easily usable data are those organized in regular grids (GUS data, WorldPop, GHS), which are becoming increasingly widely available. Referencing data to small square fields of equal area facilitates the creation of an isopleth map with the volume of the phenomenon preserved. As a result, the proposed method becomes much easier to use. It can also be used to create historical images of population distribution. Models developed in this way, through the use of compatible basic fields in the form of a grid, can be more comparable to distributions developed on the basis of contemporary data. Unfortunately, this approach requires time-consuming data preparation. The biggest challenge is to modify the data to take into account the anecumene, which can be determined, for example, on the basis of old topographic maps or aerial photographs. As a result, it becomes possible to perform analyses illustrating spatial and temporal changes.

6. Conclusions

The presented method of estimating population numbers based on an isopleth map showed moderately high accuracy. The obtained MAPE and MPE error indicators suggest that the method is reliable, especially in regional applications.

The method tends to underestimate, most often in cities. The study shows this example of Tomaszów Lubelski. The main cause of errors is the mismatch between administrative unit boundaries and the regular grid. Despite the limitations indicated, the accuracy of the estimates obtained is sufficient for many applications such as spatial planning, infrastructure management, environmental protection, and crisis management.

When verifying the method, there is a problem with finding reference data that would enable a broader examination of its reliability.

This is due to the lack of reliable population data related to areas such as landscape parks. Determining such data independently is labour-intensive and requires separate research.

The advantages of the method should be emphasized, namely its versatility, expressed in the possibility of using various data sources (NSP, WorldPop, GHS), and its potential for creating historical maps of population distribution. It is possible to increase the accuracy of the method by increasing the spatial resolution of the data in grid form.

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