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## Assessment of Effective and Minimal Slab Widths for Evaluating the Shear Capacity of RC Overhang Slabs



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### ABSTRACT

The effective width is a relevant parameter for the design of bridge overhang slabs under concentrated loads. Experimental tests have been used to assess expressions for its calculation. However, the load capacity increases with the width until a transition area is reached. Test specimens may have lacked enough width to reach full shear capacity, affecting thus the evaluation of the results. On the other hand, within the transition area, a threshold value has been

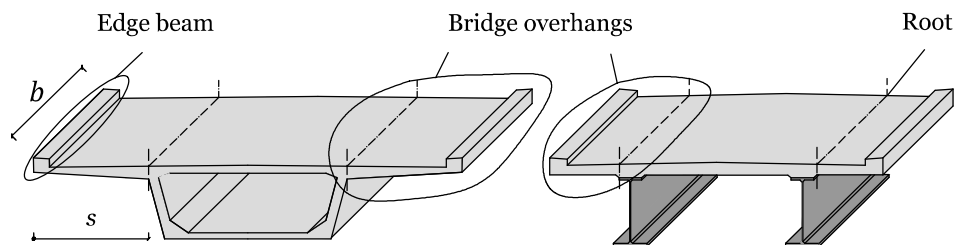
hypothesized to match the effective width. This paper aims to provide recommendations for minimal widths that guarantee the full capacity of overhang slabs and to assess the calculation of the effective width by means of the threshold value and other formulations. The effect of the edge beam is also considered. A campaign of validated non-linear FE-simulations based on experiments on range of width-span ratios was performed. The results suggest using a width-span ratio of at least 4.0 for slabs without an edge beam and 5.3 for slabs with an edge beam for the experimental practice. The efficiency of the formulation for the effective widths is diffuse and the use of threshold value leads to unsafe predictions. Instead, linear-elastic FE-analyses are recommended for the design practice.

**Key words:** effective width; minimal width; threshold width; shear failure; overhang slab; FE-modelling; bridge edge beam; experimental test.

## 1. INTRODUCTION

### 1.1 General

RC bridge overhang slabs as illustrated in Figure 1 without shear reinforcement may be designed under concentrated loads following a one-way shear approach [1, 2, 3]. Using this approach, a relevant parameter for the calculation of the design shear force is the effective width  $b_{\text{eff}}$ . To the authors' knowledge, there is no expression for the effective width  $b_{\text{eff}}$  based on theoretical models. Instead, simplified expressions, described later in this paper, are predominant in practice. These are generally based on a geometric approach but lack theoretical background.



*Figure 1 – Overhang slabs with an edge beam in a concrete box-girder bridge and a steel-concrete composite bridge.*

The suitability of such simplified formulae for RC overhang slabs has lately been assessed through tests [4, 5, 6, 7, 8, 9]. It is common to use experiments to investigate the shear behaviour of these structures and to evaluate or to develop design equations. Therefore, it is vital that the tests can form a reliable basis that guarantees an appropriate assessment. This is associated with the decision-making concerning the dimensions of the specimen. The budget for the experimental campaign and the available space in the laboratory may also be determinant limiting factors.

In this context, this paper aims to draw attention to the choice of the width  $b$  of the overhang slabs for test specimens. This paper introduces the concept of minimal width  $b_{\text{min}}$  which is the least  $b$  of a slab to reach the full load capacity  $Q_u$  that is representative for the real bridge behaviour. A benchmark for  $b_{\text{min}}$  has been the definition of slab according to Eurocode 2 [10], which states that the minimum panel dimension has to be at least 5 times the overall thickness  $t$ . In addition, values of  $b_{\text{eff}}$  computed using various expressions were also used as a lowest threshold for  $b_{\text{min}}$ .

However, the values of  $b_{\min}$  determined in such a way may not be enough. A recent study proposed and verified that the load capacity  $Q_u$  increases with  $b$  until a transition area is reached according to Figure 2 [11]. Starting from this region, the increase smoothens. The study was based on validated non-linear FE-simulations from experiments. Results from previous research sustain the suggested hypothesis of the relationship  $Q_u - b$  [5, 6, 7]. In the studied experimental tests, the increase of  $Q_u$  went beyond the original  $b$  of a specimen. This highlighted that, although the tests fulfilled the definition of slab and were wider than the calculated  $b_{\text{eff}}$ , they had not been wide enough to represent the real structural behaviour in bridges.

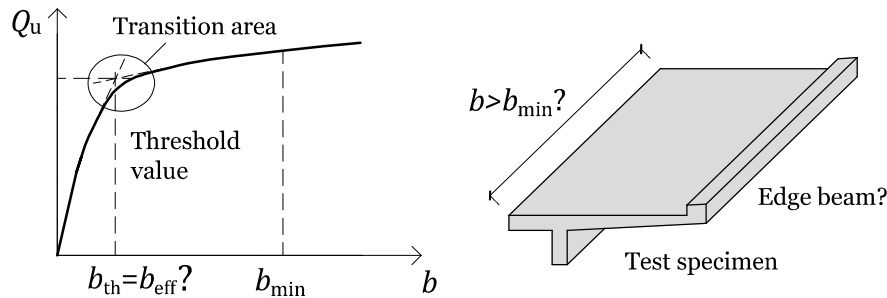


Figure 2 – Increasing  $Q_u$  with  $b$  until a threshold value  $b_{\text{th}}$  ( $b_{\text{eff}}?$ ) in a transition area is identified and minimal width  $b_{\min}$ .

Additionally, considering in the simulations overhang slabs that are wide enough, the results of this study showed that the current expressions for  $b_{\text{eff}}$  were too conservative. Therefore, a revisit was recommended. The location of the transition area in the curve  $Q_u - b$  was of interest. The authors suggested that within this area a threshold value  $b_{\text{th}}$  could match the real value of  $b_{\text{eff}}$ . A similar proposition had been presented in previous studies on concrete slabs [5].

The influence of  $b$  is not only restricted to the value of  $Q_u$  but it may affect the failure mode as well [11]. Narrower overhang slabs may fail in flexure whereas wider ones may fail in shear. Within shear failure, there may also be a shift in the failure section. Typically, the failure will be similar to one-way although for loads far enough from the root it may resemble more two-way shear (punching), especially for tapered overhangs.

Moreover, a member with a relevant influence on  $Q_u$  is the edge beam, which could lead to an increase of up to 22 % compared with overhang slabs without it [3, 11]. This effect was coupled to the increase of  $b$  because wide enough slabs allowed the edge beam to deflect and thus contribute to the load distribution. This impact should not be disregarded since bridges typically include an edge beam system integrated into the deck. However, experimental tests have usually been performed without this member or considering it merely as a permanent line load, which implies, yet again, a limitation.

Consequently, to develop or validate design equations based on tests, the aforementioned facts must be born in mind. Certain experimental tests might have been carried out on overhang slabs of insufficient width  $b$  and/or without considering the edge beam. This implies that the experimental results should have been interpreted more carefully considering these limitations.

Thus, an adequate guidance on  $b_{\min}$  is needed for tests to capture the real behaviour of bridge overhang slabs and give a suitable basis to assess the different approaches to calculate  $b_{\text{eff}}$ . For this purpose,  $Q_u - b$  curves where the transition area is enclosed must be developed. However,

nowadays there is a lack of experimental tests that cover such comprehensive variation of  $b$  in overhang slabs. Undeniably, such experimental program with full-scaled tests would be not only time consuming but also very expensive. To overcome these limitations, the strategy used in this paper is based on non-linear FE-simulations.

The paper aims to contribute to the research of the structural behaviour of RC overhang slabs with no shear reinforcement under concentrated loads. The goal is to provide recommendations for reasonable values of  $b_{\min}$  that guarantee the full capacity of specimens in the experimental practice and discuss the adequacy of the use of  $b_{\text{eff}}$  in the design practice. To reach this goal the objectives are to:

1. Validate non-linear FE-models based on selected experimental tests used as a reference.
2. Perform a campaign of non-linear FE-simulations with a variation of  $b$  to draw  $Q_u - b$  curves, also considering an edge beam.
3. Once enclosed the transition area, quantify the minimal width  $b_{\min}$  and the effective width based on the threshold value  $b_{\text{th}}$  according to a proposed definition by the authors.
4. Determine if the approaches to calculate  $b_{\text{eff}}$ , including  $b_{\text{th}}$ , lead to a reliable estimation of  $Q_u$ .
5. Based on the results, propose recommendations for  $b_{\min}$  for the experimental practice, and discuss the role of  $b_{\text{eff}}$  with respect to the load position and the control section.

## 2. BACKGROUND

### 2.1 Effective widths for shear

A one-way shear approach may be used for the design of RC bridge overhang slabs as illustrated in Figure 1. By using hand calculations, the verification criteria to derive the design shear force per unit meter  $v_{\text{Ed}}$  at the studied control section has to meet Equation (1), where  $v_Q$  is the shear force from the concentrated load(s),  $v_q$  is the shear force from uniform distributed loads,  $v_{\text{SW}}$  is the self-weight,  $v_{\text{pav}}$  is the pavement load,  $v_{\text{perm}}$  is the permanent load, and  $v_{\text{Rd}}$  is the shear capacity of the concrete section.

$$v_{\text{Ed}} = v_Q + v_q + v_{\text{SW}} + v_{\text{pav}} + v_{\text{perm}} \leq v_{\text{Rd}} \text{ [kN/m]} \quad (1)$$

The effective width  $b_{\text{eff}}$  serves to calculate the shear force  $v_Q$  from the concentrated load(s) according to Equation (2), where  $Q_d$  is the design load. Hence, given a shear force distribution  $v(x)$ , the effective width  $b_{\text{eff}}$  may be defined as the width that is effectively distributing the shear forces from the concentrated load(s) at a certain cross section, as Figure 3 illustrates. The maximum value of the distribution  $v_{\text{max}}$  is then identified with  $v_Q$ .

$$v_Q = Q_d / b_{\text{eff}} = v_{\text{max}} \text{ [kN/m]} \quad (2)$$

Table 1 presents formulae for  $b_{\text{eff}}$  (Equations 3-8) that are generally used for design purposes, with a schematic representation in Figure 4. Table 2 summarizes the features of each expression and the factors considered. The French  $b_F$ , intermediate  $b_K$  and Dutch  $b_Z$  are based on a geometrical distribution of  $45^\circ$  from different points of the load towards the root, which is usually considered as the control section (S0). The Model Code [12] proposes to adjust  $b_F$  by assuming a control section at a distance of  $d$  from the root to account for a more precise location of the failure (S0\*).

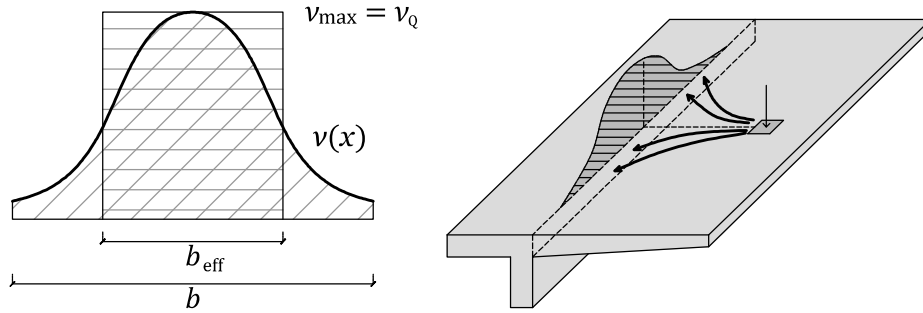
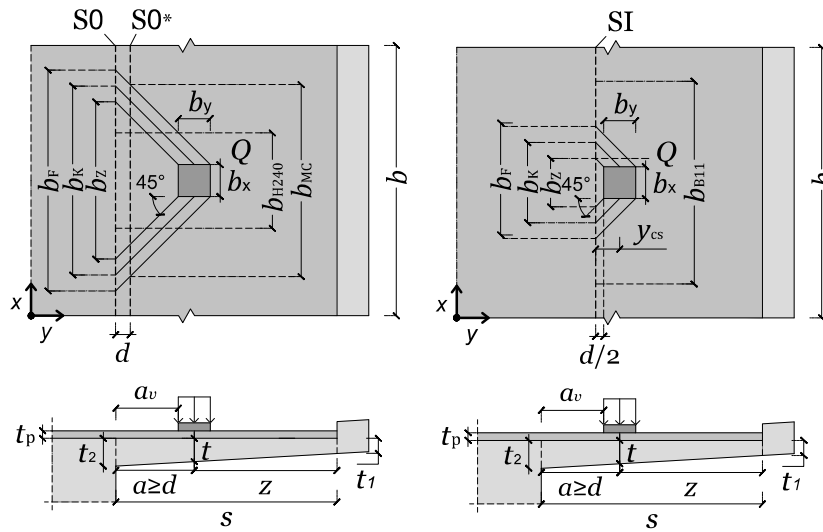

 Figure 3 – Effective width  $b_{\text{eff}}$  for a shear force distribution  $v(x)$  from a concentrated load.

 Figure 4 – Schematic representation of shear effective widths  $b_{\text{eff}}$  for different expressions and control sections.

 Table 1 – Expressions for the calculation of  $b_{\text{eff}}$ 

| Denomination | Control section | Expression                                                                                     |
|--------------|-----------------|------------------------------------------------------------------------------------------------|
| French       | S0, SI          | $b_F = 2a_v + 2b_y + b_x \leq b$ (3)                                                           |
| Intermediate | S0, SI          | $b_K = 2a_v + b_y + b_x \leq b$ (4)                                                            |
| Dutch        | S0, SI          | $b_Z = 2a_v + b_x \leq b$ (5)                                                                  |
| MC 2010      | S0*             | $b_{MC} = 2a_v + 2b_y + b_x - 2d \leq b$ (6)                                                   |
| German       | S0, S0*         | $b_{H240} = t_x + 0,3 (a_v + b_y/2) \leq b$ (7)                                                |
|              |                 | $t_y = b_y + 2t_p + t; t_x = b_x + 2t_p + t$                                                   |
|              |                 | $0,2s < a_v < s,$                                                                              |
|              |                 | $0,2s < t_x \leq 0,4a; t_y \leq 0,2s$                                                          |
| Swedish      | SI              | $b_{B11} = \max \begin{cases} 7d + b_x + t_p \leq b \\ 10d + 1,3y_{cs} \leq b \end{cases}$ (8) |

 Table 2 – Main features behind each expression to calculate  $b_{\text{eff}}$ 

|                      | French   | Interm.  | Dutch    | MC2020    | German                  | Swedish      |
|----------------------|----------|----------|----------|-----------|-------------------------|--------------|
| Background           | Geometry | Geometry | Geometry | Geometry  | Geometry, static system | Empiric      |
| Control section      | S0, SI   | S0, SI   | S0, SI   | S0*: S0-d | S0, S0*                 | SI ( $d/2$ ) |
| Considers $b_x$      | X        | X        | X        | X         | X                       | X            |
| Considers $b_y$      | X        | X        |          | X         | X                       | X            |
| Considers $t$ or $d$ |          |          |          |           | X                       | X            |
| Considers $t_p$      |          |          |          |           | X                       | X            |

The German  $b_{H240}$  [13] not only considers a horizontal distribution but a vertical one as well through a parameter  $t_x$  from the load application to the centroidal axis of the slab, including the thickness of the pavement  $t_p$ . The horizontal distribution is treated by means of the distance from the midpoint of the load to the root, but with a reduction factor of 0.3 to account for the static system. However, this expression was shown to be too conservative [8] and should not be used in bridge deck under concentrated loads. Therefore, it is not considered further in this study.

Another control section studied is at the proximity of the load (SI), commonly at a distance of  $d/2$  or  $2d$ . The aforementioned formulae are not very suitable in this case because of their geometric approach with very conservative results. Instead, the Swedish  $b_{B11}$  [14] may be adequate because it applies to a control section at  $d/2$ . It has an empirical background based on experimental tests [15], and is calibrated to consider a set of geometrical parameters as shown in Figure 4.  $y_{cs}$  is the distance from the load application to the control section.

## 2.2 Experimental tests

The suitability of the simplified expressions for  $b_{eff}$  may be evaluated through experimental tests. For RC overhang slabs without shear reinforcement, very few experimental tests have been performed, as discussed in previous studies [1, 2, 4]. Table 3 shows an overview of a few of the most recent ones. The specimens of Vaz Rodrigues & Muttoni [15] had the highest width-span ratio ( $b/s$ ) of 3.6, whereas the rest were below 2.4. All cases were considered slabs according to the Eurocode 2 since  $b > 5t$  and  $s > 5t$  [10].

*Table 3 – Main features of experimental tests on RC overhang slabs.  $\rho_{top}$  stands for top transversal reinforcement ratio.  $b_x$  and  $b_y$  are the dimensions of the load application*

| Authors                                              | Rodrigues<br>& Muttoni | Natário<br>Ruiz & Muttoni |       | Rombach<br>& Latte     | Henze &<br>Rombach | Reissen & Hegger |           | Vida &<br>Halvonik |
|------------------------------------------------------|------------------------|---------------------------|-------|------------------------|--------------------|------------------|-----------|--------------------|
|                                                      | [16]                   | [17]                      | [18]  | [19]                   | [20]               | [8]              | [21]      | [9]                |
| <b>Year</b>                                          | 2005                   | 2014                      | 2015  | 2008                   | 2017               | 2013             | 2016      | 2018               |
| <b>Web</b>                                           | Concrete               | Steel                     | Steel | Concrete               | Concrete           | Concrete         | Steel     | Concrete           |
| <b><i>b</i> (m)</b>                                  | 10                     | 3                         | 3     | 2.4                    | 4.5                | 0.5 / 3.5        | 0.5 / 3.5 | 0.5 / 2.5          |
| <b><i>s</i> (m)</b>                                  | 2.78                   | 1.25                      | 1.46  | 1.65                   | 1.90               | 1.80             | 1.80      | 1.30               |
| <b><i>b/s</i></b>                                    | 3.6                    | 2.4                       | 2.1   | 1.5                    | 2.4                | 1.9              | 1.9       | 1.9                |
| <b><i>t</i> (m)</b>                                  | -                      | 0.18                      | 0.25  | 0.25                   | 0.25               | 0.28             | 0.28      | -                  |
| <b><i>t</i><sub>1</sub>- <i>t</i><sub>2</sub>(m)</b> | 0.38-0.19              | -                         | -     | 0.30-0.19<br>0.25-0.14 | -                  | 0.28-0.16        | 0.28-0.16 | 0.20-0.15          |
| <b>5 <i>t</i><sub>max</sub></b>                      | 1.90                   | 0.90                      | 1.25  | 1.50                   | 1.25               | 1.40             | 1.40      | 1.00               |
| <b><i>ρ</i><sub>top</sub> (%)</b>                    | 0.96/0.73              | 1.32                      | 1.00  | 1.16                   | 1.16               | 0.98             | 0.98      | 0.73-1.69          |
| <b>N° loads</b>                                      | 1, 2 and 4             | 1                         | 1     | 1                      | 1 and 2            | 1                | 1         | 1                  |
| <b>Preload</b>                                       |                        |                           |       | X                      |                    | X                | X         |                    |
| <b>Cycling</b>                                       | X                      |                           | X     |                        |                    |                  |           |                    |
| <b><i>b<sub>x</sub></i> (m)</b>                      | 0.3                    | 0.4                       | 0.4   | 0.4                    | 0.4                | 0.4              | 0.4       | 0.25               |
| <b><i>b<sub>y</sub></i> (m)</b>                      | 0.3                    | 0.4                       | 0.4   | 0.4                    | 0.4                | 0.4              | 0.4       | 0.25               |

### 2.3 Assessment of effective widths

For the assessment of the formulae for  $b_{\text{eff}}$  a back calculation from experimental tests denoted as  $b_{\text{exp}}$  can be done according to Equation (9), where  $Q_{\text{exp}}$  is the failure load in the test and  $V_G$  the self-weight of the load structure. Another approach is to compare experimental and calculated load-bearing capacities ( $Q_{\text{exp}}$  vs  $Q_{\text{Rd}}$ ), as Reissen and Hegger [8] did for some of the tests in Table 3 (Figure 5).  $Q_{\text{Rd}}$  was derived according to Equation (10).

$$b_{\text{exp}} = (Q_{\text{exp}} + V_G) / (v_{\text{Rd}} - v_g) \quad (9)$$

$$Q_{\text{Rd}} = b_{\text{eff}} (v_{\text{Rd}} - v_g) - V_G \quad (10)$$

The intermediate  $b_K$  had a good agreement although in some cases it fell on the unsafe side. Note, however, that the RC overhang slabs in Rombach & Latte [19] and Reissen & Hegger [8] might not have been wide enough to reach full capacity. Moreover, the test by Vaz Rodrigues & Muttoni [16] consisted of one and two loads both applied close to the free edges of the specimen, which might be another limitation. These aspects should be considered in the interpretation of the results.

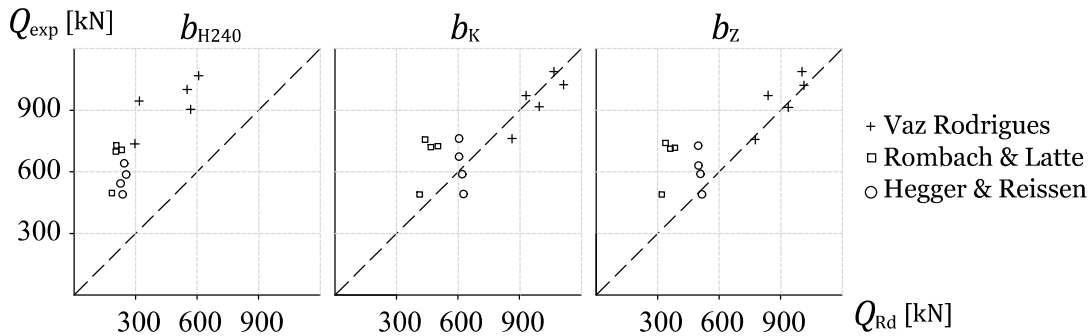


Figure 5 – Comparison of experimental and calculated load-bearing capacities to evaluate  $b_{\text{eff}}$ . Adapted from Reissen and Hegger [8].

### 2.4 Minimal width

Experimental tests presented in Table 3 may present a size limitation that should be considered in the assessment of  $b_{\text{eff}}$ . Figure 2 illustrates a  $Q_u - b$  curve according to [11]. The load capacity  $Q_u$  increases with  $b$  until a transition area is reached. Starting from this region, the increase smoothens. The mechanisms behind the increase of  $Q_u$  with  $b$  were identified as (1) the enhanced distribution of shear forces sideways, and (2) the increase of shear strength capacity. The former is important for loads close to the free edge whereas the latter is important for loads near the root. A redistribution of the shear forces was detected.

In the aforementioned study, three different values of  $b$  were analysed through validated non-linear FE-simulations based on experiments. For one of the experiments, the results showed that slabs wider than  $b$  of the original test specimen reached higher values of  $Q_u$ . Therefore, the original specimen had not been wide enough to replicate the real structural behaviour in bridges although it was wider than the definition of slab and the calculated  $b_{\text{eff}}$ . Therefore, it is important that there are recommendations for reasonable values of minimal widths  $b_{\text{min}}$  that guarantee the full capacity of specimens in the experimental practice.

## 2.5 Threshold width

Within the transition area represented in Figure 2, the authors suggested that a threshold value  $b_{th}$  could match the real value of  $b_{eff}$  [11]. This would constitute another approach to evaluate the expressions of  $b_{eff}$  from experimental tests or simulations. Lantsoght et al. [15] used a similar proposition for the assessment of  $b_{eff}$  in one-way simply and continuously supported concrete slabs. In these experiments,  $b$  varied from 0.5 to 2.5 m and different values of  $a/d$  were tested. Two trend lines were drawn: one represented the initial linear increase of  $Q_{exp}$  with  $b$  for short specimens, and the other one intended to show an averaged maximum load capacity of the widest specimens (Figure 6).  $b_{th}$  was derived as the intersection between both lines and was compared to  $b_{eff}$  from Equations (3) and (5). The French  $b_F$  was assessed to be adequate. However, it should be considered that higher  $a/d$  might result in unsafe values because of the geometric nature of the expression.

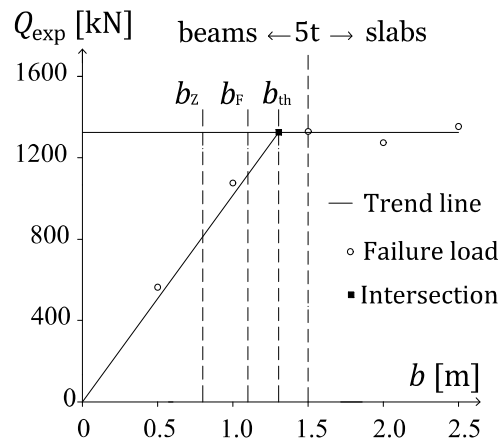


Figure 6 –  $Q_{exp} - b$  for a simple supported slab, derivation of  $b_{th}$  intersection points and comparison with  $b_z$  and  $b_F$ . Adapted from [5].

## 3. NUMERICAL STUDY

The procedure used here includes the following steps:

### Step 0: Choice of experimental tests for the analysis

The experimental tests used as a reference in this paper were 4d-1, 5d and 6d by Rombach and Henze [20], VK2V1 by Rombach and Latte [19], and DR1a by Vaz Rodrigues et al. [16]. All of them had concrete web supports. Test DR1a was not monolithically connected to the web. For simplicity, test 4d-1 will be referred to just as 4d. VK2V1, 4d, 5d and 6d were flat slabs under a single concentrated load at difference positions whereas DR1a was a tapered slab under four concentrated loads. None of them had an edge beam. An additional case named VK2V4 and used in previous studies was also considered. It had a similar layout as VK2V1 but with the load moved 0.4 m towards the free edge. Figure 7 shows an overview of all the tests.

### Step 1: Validation of non-linear FE-models and experimental campaign

The behaviour at ULS was investigated using a 3D non-linear FE-model with solid elements. ABAQUS CAE and Explicit were used to generate the input file and to perform the non-linear FE-analyses, respectively. First, the numerical results were validated against the original experimental tests. Based on these models, an experimental campaign of non-linear FE-models

was performed by widening the original overhangs within a range of  $b$  as presented in Table 4. The value of  $b$  was chosen to keep the same ratio  $b/s$  across the studied experimental tests. Finally, a fictitious edge beam as shown in Figure 7 was included in all simulations to quantify its influence on the results.

#### EXPERIMENTAL TESTS AND LOADCASES

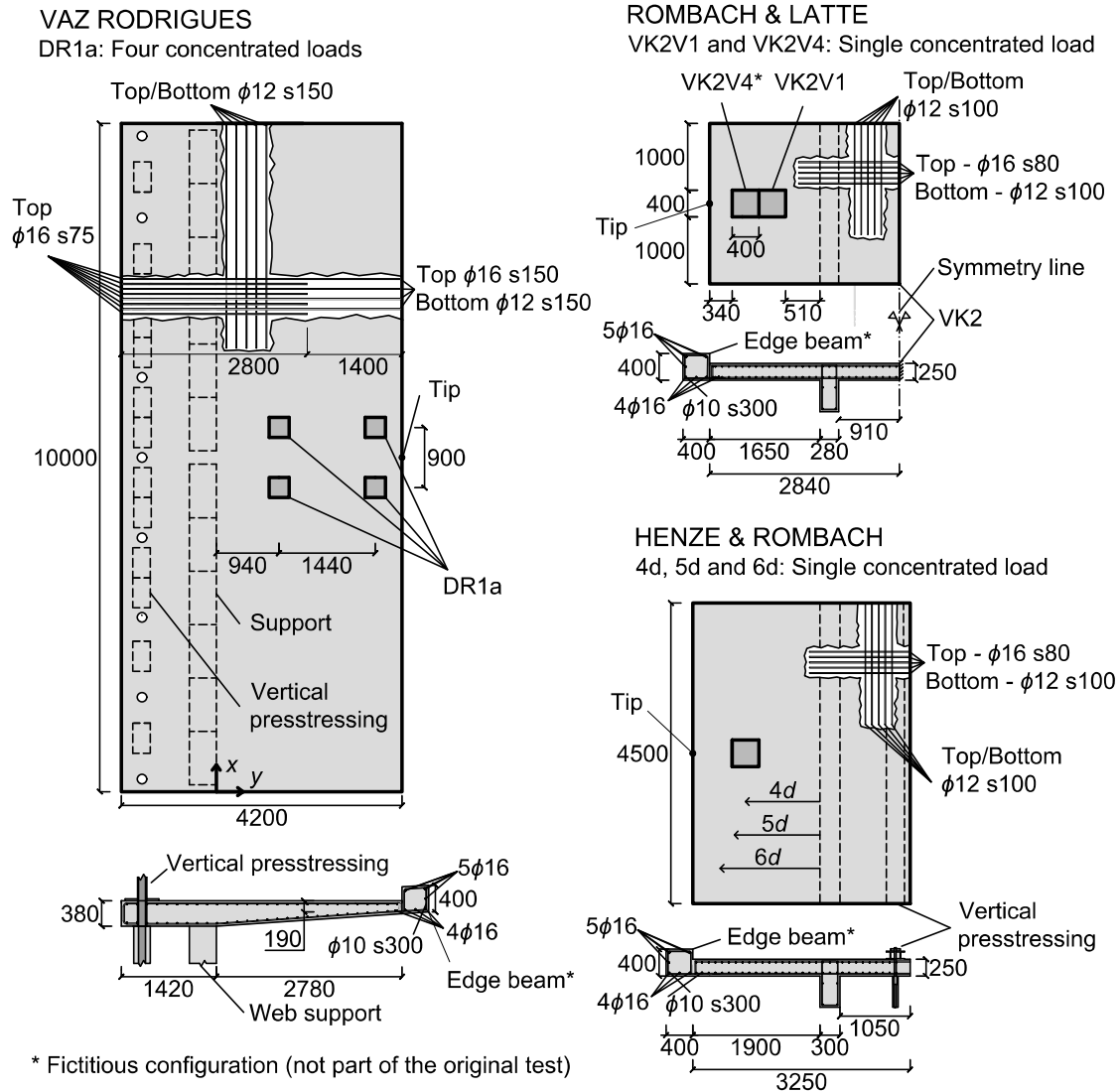


Figure 7 – Experimental tests DR1a, VK2V1 (including fictitious case VK2V4), and 4d, 5d and 6d. Only half of VK2 is shown (but the whole slab and the two webs were modelled in the non-linear FE-analysis). The fictitious edge beam is shown in the cross-section. Units in mm.

Table 4 –  $b/s$  ratios and corresponding range of simulated  $b$  in m for tests 4d, 5d, 6d, VK2V1, VK2V4 and DR1a ( $b$  of the original test is underlined)

| $b/s$      | 6.8  | 6.1  | 5.3  | 4.5  | 4.0  | 3.6         | 3.3 | 3.0 | 2.7 | 2.4        | 1.8 | 1.5        | 1.0 |
|------------|------|------|------|------|------|-------------|-----|-----|-----|------------|-----|------------|-----|
| 4d, 5d, 6d | 13.0 | 11.6 | 10.0 | 8.5  | 7.6  | 6.9         | 6.3 | 5.7 | 5.1 | <u>4.5</u> | 3.5 | 2.8        | 1.9 |
| VK2 V1, V4 | 11.2 | 10.0 | 8.7  | 7.4  | 6.6  | 6.0         | 5.5 | 5.0 | 4.5 | 4.0        | 3.0 | <u>2.4</u> | 1.7 |
| DR1a       | 19.0 | 16.9 | 14.7 | 12.5 | 11.1 | <u>10.0</u> | 9.2 | 8.3 | 7.5 | 6.7        | 5.0 | 4.1        | 2.8 |

*Step 2: Determination of the minimal width  $b_{\min}$  and the threshold width  $b_{\text{th}}$* 

The minimal width  $b_{\min}$  and the threshold width  $w_{\text{th}}$  were derived from the  $Q_u - b$  curves as depicted in Figure 2.  $Q_u$  referred to the load capacity of each specimen obtained from the non-linear simulations and denoted as  $Q_{\text{Rd,nlFE}}$ .  $b_{\min}$  of a certain specimen is defined as the minimum width whose  $Q_{\text{Rd,nlFE}}$  does not differ more than 4 % of the  $Q_{\text{Rd,nlFE}}$  obtained with the widest specimen from the non-linear FE simulations.

The threshold width  $w_{\text{th}}$  was calculated graphically by the intersection between two straight lines. One of them referred to the linear growth of  $Q_{\text{Rd,nlFE}}$  with  $b$  and was represented by joining the origin with  $Q_{\text{Rd,nlFE}}$  of the narrowest slab. The other one characterized  $Q_{\text{Rd,nlFE}}$  corresponding to the previously calculated  $b_{\min}$  of the test and was represented by a horizontal line from that value of  $Q_{\text{Rd,nlFE}}$ .

*Step 3: Assessment of the effective widths  $b_{\text{eff}}$* 

The assessment of  $b_{\text{eff}}$  was performed from a load-bearing capacity perspective, that is, by comparing the calculated load capacity  $Q_{\text{Rd}}$  to the one obtained from the non-linear FE-analyses  $Q_{\text{Rd,nlFE}}$ .  $Q_{\text{Rd}}$  was derived according to Equation (10) with  $V_G = 0$  and with  $b_{\text{eff}}$  according to Equations (3-6) and (8) as well as  $b_{\text{th}}$  defined above. No safety coefficients were used. The value of  $Q_{\text{Rd,nlFE}}$  corresponded in this case to the widest specimen to ensure the full capacity of the RC overhang slab. Eurocode 2 [10] is used to calculate  $v_{\text{Rd}}$ .

### 3.1 Description of the FE-model

*General assumptions and limitations*

Bond slip between concrete and reinforcement will influence the crack spacing. However, since no anchorage failure was reported in the tests and the aim of the numerical model was to predict the failure load, bond slip was not considered. The fluctuation of the load magnitude applied onto each load plate during the experiment was not considered. For DR1a, the same load on each loading plate on the overhang was assumed. The web was assumed to be monolithically connected to the slab. The initial load cycling in tests DR1a, 4d, 5d and 6d was not simulated.

*Geometry and reinforcement*

The geometry of the models followed the experiments (Table 3 and Figure 7). For DR1a, the longitudinal reinforcement consisted of two layers (top and bottom) of  $\varnothing 12$  mm bars with a spacing 150 mm. The transversal reinforcement was composed of  $\varnothing 16$  mm bars with a spacing 75 mm, except for the last 1.4 m of the overhang span where the spacing was 150 mm.

For VK2V1, 4d, 5d and 6d, the longitudinal reinforcement consisted of two layers (top and bottom) of  $\varnothing 12$  mm bars with a spacing 100 mm. The transversal reinforcement was composed of  $\varnothing 16$  mm bars with a spacing 80 mm at the top and  $\varnothing 12$  mm bars with a spacing 100 mm at the bottom. An edge beam of  $400 \times 400 \text{ mm}^2$  was added at the free edge of the slab according to Figure 7. The longitudinal reinforcement consisted of 10 bars of  $\varnothing 16$  mm, and the transversal one was  $\varnothing 10$  mm with a spacing 300 mm.

*Material properties and models*

The mechanical properties of concrete and steel reinforcement used were based on the tests (Table 5). The material model for concrete was ‘Concrete Damaged Plasticity’ [22, 23] with default parameter values under multiaxial stress state. The tensile constitutive model consisted of a linear-

elastic branch until the tensile strength, and a post-peak behaviour in terms of stress versus crack opening [24]. The fracture energy was calculated according to Model Code 2010 [12].

The compressive constitutive model was adapted to the provisions of Eurocode 2 [10] and based on the available information from the tests. A linear-elastic behaviour was assumed up to 40 % of the compressive strength followed by a parabolic approximation for the compression hardening and softening until crushing, following the recommendations by Malm [25]. The material model for the reinforcement was ‘Plastic’, which allowed plastic isotropic hardening after yielding.

*Table 5 – Mechanical properties of concrete and steel reinforcement (Rombach & Henze [20]; Rombach & Latte [19]; Vaz Rodrigues et al. [16]).*

|                                        | 4d    | 5d    | 6d     | VK2V1 | VK2V4 | DR1a  |
|----------------------------------------|-------|-------|--------|-------|-------|-------|
| <b>Concrete</b>                        |       |       |        |       |       |       |
| Young modulus [GPa]                    | 29.7  | 27.7  | 25.7   | 33.9  | 33.9  | 36.0  |
| Tensile strength [MPa]                 | 2.5   | 2.5   | 2.8    | 3.4   | 3.4   | 2.9   |
| Compressive strength [MPa]             | 40.0  | 46.4  | 43.0   | 46.0  | 46.0  | 39.1  |
| Dilation angle [degrees]               | 30    | 35    | 37     | 35    | 35    | 35    |
| <b>Steel reinforcement</b>             |       |       |        |       |       |       |
| Young modulus [GPa]                    | 198.0 | 198.0 | 195.3  | 195.0 | 195.0 | 200.0 |
| Yield strength, ø10 and 12 mm [MPa]    | 632.0 | 632.0 | 386.0* | 550.0 | 550.0 | 499.0 |
| Ultimate strength, ø10 and 12 mm [MPa] | 693.0 | 693.0 | 567.0  | 607.0 | 607.0 | 600.0 |
| Yield strength, ø16 mm [MPa]           | 544.0 | 544.0 | 591.0  | 554.0 | 554.0 | 541.0 |
| Ultimate strength, ø16 mm [MPa]        | 625.0 | 625.0 | 661.0  | 646.0 | 646.0 | 629.0 |

\* 0.2 % offset yield strength

#### *Mesh size and non-linear analysis procedure*

A mesh of 8-noded solid elements with a characteristic element size of 25 mm was used for the concrete in all the simulations. This enabled at least ten elements across the thickness, which is suitable to describe non-linear shear behaviour. This mesh size is normally used in the literature for these tests [4, 19, 20]. A sensitivity analysis by the authors also showed its suitability [26]. The reinforcement was modelled using truss elements with the same mesh size. An explicit procedure to avoid convergence issues was followed. The loading step was smoothed using a predefined function to avoid inertial forces affecting the results. Since the problem is quasi-static, the total and kinetic energies of the model during the simulation were monitored to ensure the accuracy of the analysis.

#### *Loading, boundary conditions and constraints*

The load was applied using a statically determinate structure built of beams and trusses connected to the loading plates, following the guidelines in [27]. A prescribed displacement-control was assigned to a node at the top of the structure and the reaction force was then determined. The web support was modelled by restraining all the degrees of freedom of the bottom surface. For the pre-stressing for tests DR1a, 4d, 5d and 6d, all the degrees of freedom at the two contact surfaces of the slab were restrained. Fully bonded reinforcement was assumed by constraining the nodes of the truss elements to follow the displacement degrees of freedom of the surrounding concrete.

### **3.2 Model validation**

A validation for experimental tests DR1a and VK2V1 was presented in previous work [3, 11]. Concerning tests 4d, 5d and 6d, Figures 8 and 9 show a comparison of the load-deflection curves

$Q - u$ , a ratio of load capacities and failure mechanisms from the experimental tests and the non-linear FE-analyses. The displacement in the diagrams corresponds to the central point on the slab under the load application. The overhang slabs failed in shear failure in agreement with the tests. Figure 10 represents the steel strains of the central top reinforcement bar from the original test (device DS4) and the non-linear FE-analyses. The authors consider the results accurate enough to validate the FE-models and thus proceed with the campaign of overhang slabs with varying  $b$ , with and without an edge beam.

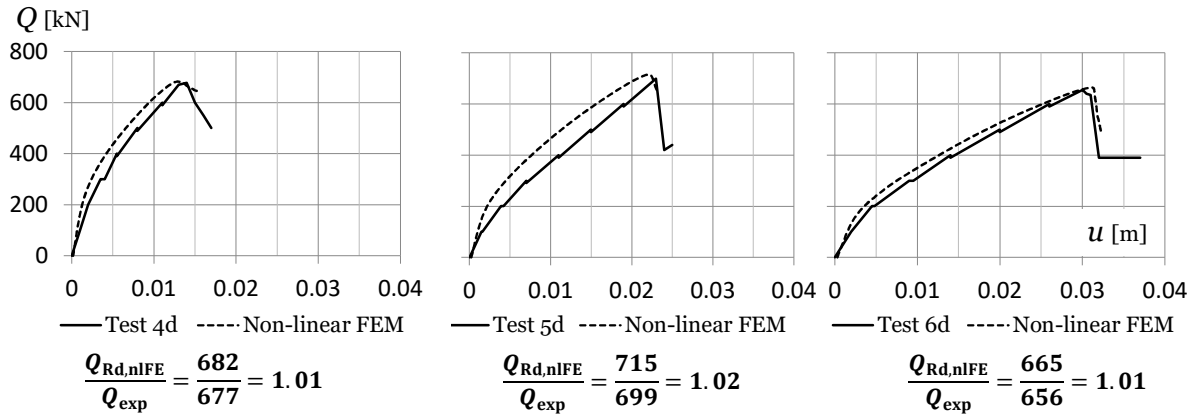


Figure 8 – Load-displacement  $Q - u$  curves for experimental tests 4d, 5d and 6d and non-linear FE-simulations.

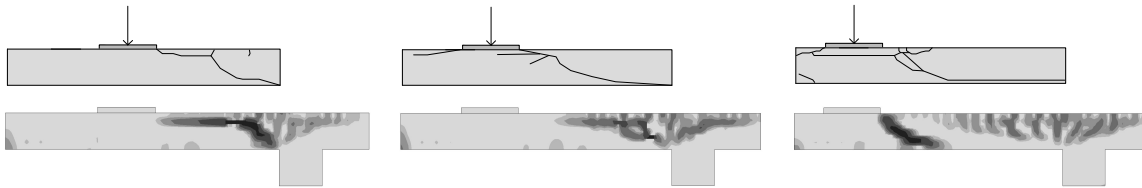


Figure 9 – Shear cracks at failure of tests 4d, 5d and 6d. Top: Illustration of the tests by Rombach and Henze [20]. Bottom: Plastic strains of the non-linear FE-simulations.

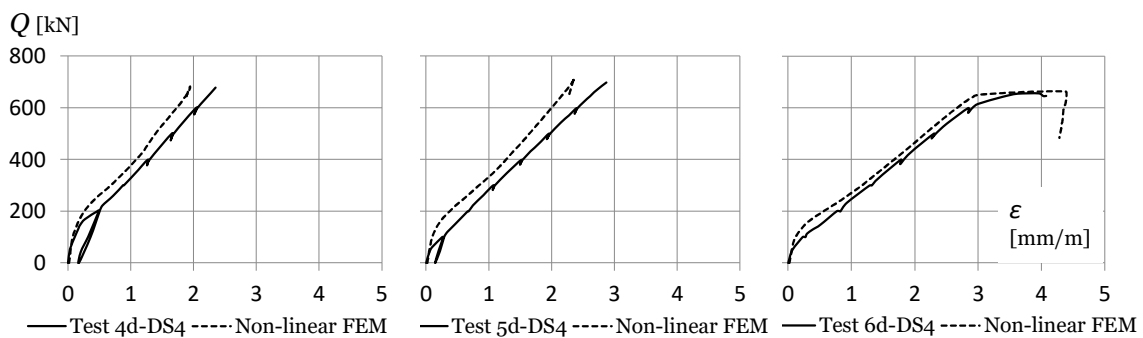


Figure 10 – Load - steel strains  $Q - \epsilon$  curves from experimental tests 4d, 5d and 6d and non-linear FE-simulations.

## 4. RESULTS AND DISCUSSION

### 4.1 Campaign of non-linear FE-simulations

Tables 6 and 7 show the results of  $Q_{Rd,nlFE}$  for different  $b$  of the studied specimens and their corresponding ratio  $b/s$ . The ratio  $r$  represents the variation of  $Q_{Rd,nlFE}$  with  $b$  from the original

test specimen (underlined italics row). From these tables, curves  $Q_{Rd,nlFE} - b$  were drawn (Figure 11). The shape of the curves had a good agreement with the one hypothesized in Figure 2. The load increase for tests 4d, 5d and 6d from the original tests was 23-24 %. For test VK2V1, it went up to 87 %. For the fictitious case VK2V4, using the same reference, it was up to 75 %. In test DR1a such increase was not relevant (5 %), showing that the slab was wide enough. The edge beam implied even higher increases due to its contributing effect coupled to wider slabs.

Table 6 –  $Q_{Rd,nlFE}$  of tests 4d, 5d and 6d of different  $b$ , and variation ratio  $r$  from the original test. The results obtained for the  $b$  of the original tests are underlined. EB: edge beam

| $b/s$             | $b$               | $Q_{Rd,nlFE}$     |                   |                   | $r$                |                    |                    | $Q_{Rd,nlFE}$     |                   |                   | $r$                |                    |                    |
|-------------------|-------------------|-------------------|-------------------|-------------------|--------------------|--------------------|--------------------|-------------------|-------------------|-------------------|--------------------|--------------------|--------------------|
|                   |                   | 4d                | 5d                | 6d                | 4d                 | 5d                 | 6d                 | 4dEB              | 5dEB              | 6dEB              | 4dEB               | 5dEB               | 6dEB               |
| <b>6.8</b>        | <b>13.0</b>       | 846               | 877               | 826               | 1.24               | 1.23               | 1.24               | 902               | 950               | 959               | 1.30               | 1.34               | 1.34               |
| <b>6.1</b>        | <b>11.6</b>       | 836               | 885               | 835               | 1.23               | 1.24               | 1.26               | 895               | 934               | 952               | 1.29               | 1.32               | 1.33               |
| <b>5.3</b>        | <b>10.0</b>       | 856               | 872               | 827               | 1.25               | 1.22               | 1.24               | 878               | 908               | 935               | 1.26               | 1.28               | 1.31               |
| <b>4.5</b>        | <b>8.5</b>        | 828               | 849               | 791               | 1.21               | 1.19               | 1.19               | 848               | 865               | 892               | 1.22               | 1.22               | 1.25               |
| <b>4.0</b>        | <b>7.6</b>        | 813               | 828               | 793               | 1.19               | 1.16               | 1.19               | 824               | 829               | 864               | 1.18               | 1.17               | 1.21               |
| <b>3.6</b>        | <b>6.9</b>        | 789               | 817               | 779               | 1.16               | 1.14               | 1.17               | 811               | 819               | 844               | 1.17               | 1.16               | 1.18               |
| <b>3.3</b>        | <b>6.3</b>        | 764               | 787               | 715               | 1.12               | 1.10               | 1.08               | 771               | 795               | 820               | 1.11               | 1.12               | 1.15               |
| <b>3.0</b>        | <b>5.7</b>        | 745               | 761               | 716               | 1.09               | 1.06               | 1.08               | 744               | 780               | 779               | 1.07               | 1.10               | 1.09               |
| <b>2.7</b>        | <b>5.1</b>        | 714               | 736               | 701               | 1.05               | 1.03               | 1.06               | 724               | 756               | 760               | 1.04               | 1.07               | 1.06               |
| <b><u>2.4</u></b> | <b><u>4.5</u></b> | <b><u>682</u></b> | <b><u>715</u></b> | <b><u>665</u></b> | <b><u>1.00</u></b> | <b><u>1.00</u></b> | <b><u>1.00</u></b> | <b><u>695</u></b> | <b><u>708</u></b> | <b><u>715</u></b> | <b><u>1.00</u></b> | <b><u>1.00</u></b> | <b><u>1.00</u></b> |
| <b>1.8</b>        | <b>3.5</b>        | 618               | 634               | 629               | 0.91               | 0.89               | 0.95               | 622               | 621               | 630               | 0.89               | 0.88               | 0.88               |
| <b>1.5</b>        | <b>2.8</b>        | 545               | 543               | 502               | 0.80               | 0.76               | 0.76               | 526               | 536               | 515               | 0.76               | 0.76               | 0.72               |
| <b>1.0</b>        | <b>1.9</b>        | 391               | 397               | 361               | 0.57               | 0.56               | 0.54               | 369               | 379               | 325               | 0.53               | 0.54               | 0.45               |

Table 7 –  $Q_{Rd,nlFE}$  of tests V1, V4 and DR1a of different  $b$ , and variation ratio  $r$  from the original test. The results obtained for the  $b$  of the original tests are underlined. EB: edge beam

| $b/s$             | $b$               | $Q_{Rd,nlFE}$     |                   | $r$                |                    | $Q_{Rd,nlFE}$     |                   | $r$                |                    | $Q_{Rd,nlFE}$      |                     | $r$                |                     |
|-------------------|-------------------|-------------------|-------------------|--------------------|--------------------|-------------------|-------------------|--------------------|--------------------|--------------------|---------------------|--------------------|---------------------|
|                   |                   | V1                | V4                | V1                 | V4                 | V1EB              | V4EB              | V1EB               | V4EB               | $b$                | DR1a                | DR1aEB             |                     |
| <b>6.8</b>        | <b>11.2</b>       | 1,393             | 1,087             | 1.87               | 1.75               | 1,590             | 1,289             | 2.03               | 2.12               | <b>19.0</b>        | 1,387               | 1.05               | 1,651               |
| <b>6.1</b>        | <b>10.0</b>       | 1,388             | 1,078             | 1.86               | 1.74               | 1,575             | 1,268             | 2.01               | 2.08               | <b>16.9</b>        | 1,381               | 1.04               | 1,628               |
| <b>5.3</b>        | <b>8.7</b>        | 1,388             | 1,059             | 1.86               | 1.70               | 1,562             | 1,219             | 2.00               | 2.00               | <b>14.7</b>        | 1,369               | 1.03               | 1,686               |
| <b>4.5</b>        | <b>7.4</b>        | 1,347             | 1,023             | 1.81               | 1.65               | 1,529             | 1,162             | 1.95               | 1.91               | <b>12.5</b>        | 1,380               | 1.04               | 1,641               |
| <b>4.0</b>        | <b>6.6</b>        | 1,336             | 982               | 1.79               | 1.58               | 1,472             | 1,103             | 1.88               | 1.81               | <b>11.1</b>        | 1,334               | 1.01               | 1,614               |
| <b>3.6</b>        | <b>6.0</b>        | 1,295             | 956               | 1.74               | 1.54               | 1,384             | 1,067             | 1.77               | 1.75               | <b><u>10.0</u></b> | <b><u>1,325</u></b> | <b><u>1.00</u></b> | <b><u>1,593</u></b> |
| <b>3.3</b>        | <b>5.5</b>        | 1,255             | 942               | 1.68               | 1.52               | 1,349             | 1,021             | 1.72               | 1.68               | <b>9.2</b>         | 1,320               | 1.00               | 1,575               |
| <b>3.0</b>        | <b>5.0</b>        | 1,267             | 919               | 1.70               | 1.48               | 1,277             | 962               | 1.63               | 1.58               | <b>8.3</b>         | 1,315               | 0.99               | 1,548               |
| <b>2.7</b>        | <b>4.5</b>        | 1,176             | 890               | 1.58               | 1.43               | 1,233             | 955               | 1.58               | 1.57               | <b>7.5</b>         | 1,309               | 0.99               | 1,486               |
| <b>2.4</b>        | <b>4.0</b>        | 1,135             | 848               | 1.52               | 1.36               | 1,138             | 884               | 1.45               | 1.45               | <b>6.7</b>         | 1,254               | 0.95               | 1,493               |
| <b>1.8</b>        | <b>3.0</b>        | 922               | 728               | 1.24               | 1.17               | 950               | 748               | 1.21               | 1.23               | <b>5.0</b>         | 1,187               | 0.90               | 1,290               |
| <b><u>1.5</u></b> | <b><u>2.4</u></b> | <b><u>745</u></b> | <b><u>621</u></b> | <b><u>1.00</u></b> | <b><u>1.00</u></b> | <b><u>783</u></b> | <b><u>608</u></b> | <b><u>1.00</u></b> | <b><u>1.00</u></b> | <b>4.2</b>         | 1,085               | 0.82               | 1,103               |
| <b>1.0</b>        | <b>1.65</b>       | 563               | 433               | 0.76               | 0.70               | 544               | 420               | 0.69               | 0.69               | <b>2.8</b>         | 731                 | 0.55               | 732                 |

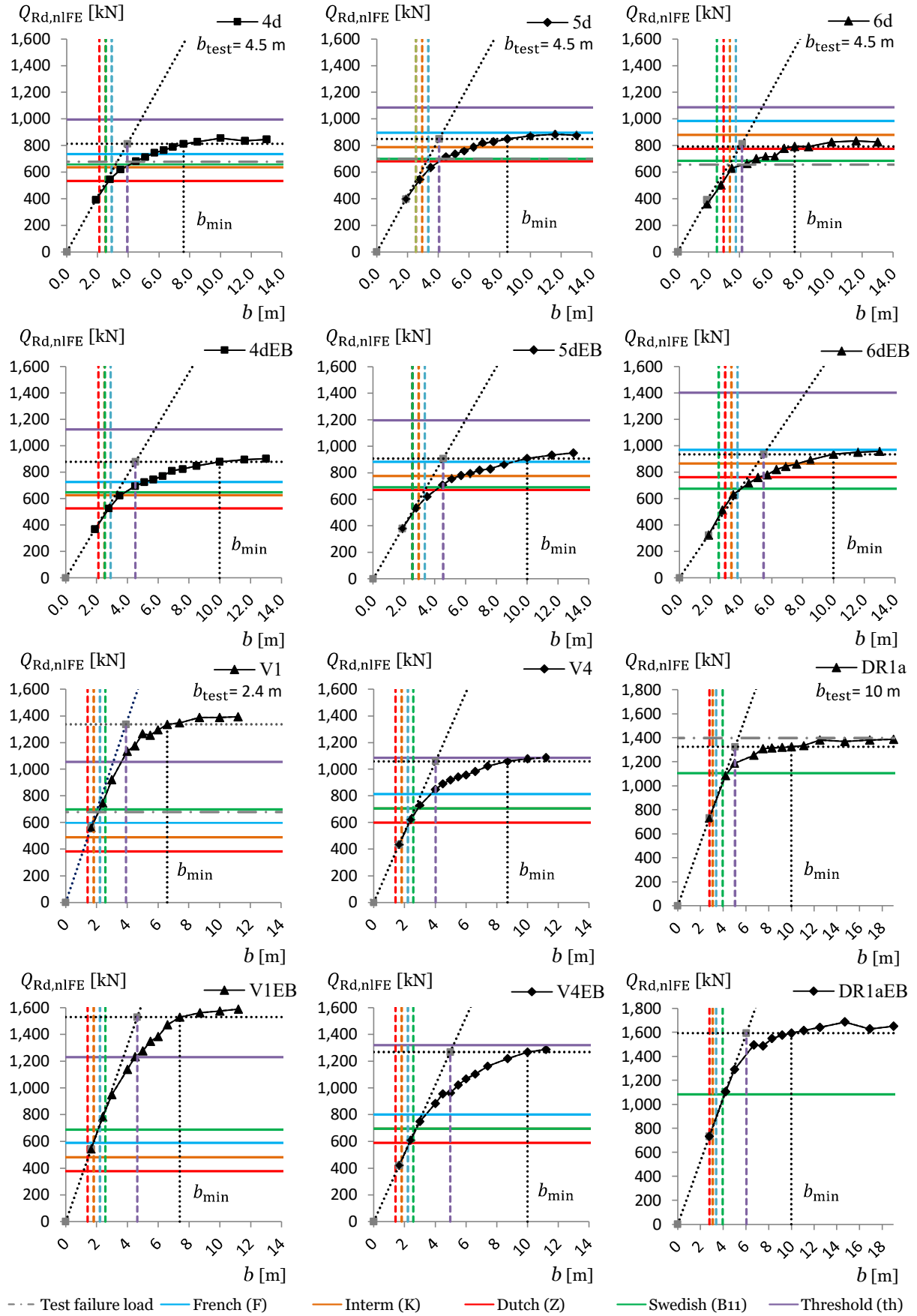


Figure 11 – Curves  $Q_{Rd,nlFE} - b$  for tests 4d, 5d, 6d, V1, V4 and DR1a. EB: edge beam.

## 4.2 Determination of minimal and threshold widths

Table 8 served to determine the value of  $b_{\min}$  for each test according to the definition proposed. Values below 4 % are underlined. In simulations without the edge beam, for tests 4d and 6d the value for  $b_{\min}$  was 7.6 m ( $b/s=4.0$ ), and for 5d it was 8.5 ( $b/s=4.5$ ). A reason for the difference in 5d may be a higher compressive strength. For tests V1 and V4, the determined values for  $b_{\min}$  were 6.6 m ( $b/s=4.0$ ) and 8.7 m ( $b/s=5.3$ ), respectively. Given in this case the same material properties, it may be said that the load position influenced  $b_{\min}$ .

For test DR1a a value of 10.0 m ( $b/s=3.6$ ) was obtained which comparatively gave a lower ratio  $b/s$  than the other tests. DR1a failed in shear locally, which could explain a reduced relevance of  $b$ . Therefore, the failure mode also affected  $b_{\min}$ . This was in agreement with the conclusions from previous work [11]. Another factor that might explain this result is that the span  $s$  was considerably larger in DR1a than the other tests.

Concerning simulations with an edge beam, for all tests 4d, 5d and 6d the value of  $b_{\min}$  was 10 m ( $b/s=5.3$ ). For V1 and V4, the obtained values of  $b_{\min}$  were 7.4 m ( $b/s=4.5$ ) and 10 m ( $b/s=6.1$ ), respectively. For DR1a,  $b_{\min}$  was 10 m ( $b/s=3.6$ ). The edge beam had then a general tendency to increase the value of  $b_{\min}$ , which is understandable since the edge beam has a more accentuated contribution for wider specimens. All values of  $b_{\min}$  are represented in Figure 11.

Table 8 – Determination of  $b_{\min}$  for the tests without/with an edge beam (values below 4 % are underlined). EB: edge beam

| $b/s$ | $b$  | 4d          | 5d          | 6d          | 4dEB        | 5dEB        | 6dEB        | $b$  | V1          | V4          | V1EB        | V4EB        | $b$  | DR1a        | DR1aEB      |
|-------|------|-------------|-------------|-------------|-------------|-------------|-------------|------|-------------|-------------|-------------|-------------|------|-------------|-------------|
| 6.8   | 13.0 | <u>1.00</u> | <u>1.00</u> | <u>1.00</u> | <u>1.00</u> | <u>1.00</u> | <u>1.00</u> | 11.2 | <u>1.00</u> | <u>1.00</u> | <u>1.00</u> | <u>1.00</u> | 19.0 | <u>1.00</u> | <u>1.00</u> |
| 6.1   | 11.6 | <u>0.99</u> | <u>1.01</u> | <u>1.01</u> | <u>0.99</u> | <u>0.98</u> | <u>0.99</u> | 10.0 | <u>1.00</u> | <u>0.99</u> | <u>0.99</u> | <u>0.98</u> | 16.9 | <u>1.00</u> | <u>0.99</u> |
| 5.3   | 10.0 | <u>1.01</u> | <u>1.00</u> | <u>1.00</u> | <u>0.97</u> | <u>0.96</u> | <u>0.98</u> | 8.7  | <u>1.00</u> | <u>0.97</u> | <u>0.98</u> | 0.95        | 14.7 | <u>0.99</u> | <u>1.02</u> |
| 4.5   | 8.5  | <u>0.98</u> | <u>0.97</u> | <u>0.96</u> | 0.94        | 0.91        | 0.94        | 7.4  | <u>0.97</u> | 0.94        | <u>0.96</u> | 0.90        | 12.5 | <u>1.00</u> | <u>0.99</u> |
| 4.0   | 7.6  | <u>0.96</u> | 0.94        | <u>0.96</u> | 0.94        | 0.91        | 0.94        | 6.6  | <u>0.96</u> | 0.90        | 0.93        | 0.86        | 11.1 | <u>0.96</u> | <u>0.98</u> |
| 3.6   | 6.9  | 0.93        | 0.93        | 0.94        | 0.90        | 0.86        | 0.88        | 6.0  | 0.93        | 0.88        | 0.87        | 0.83        | 10.0 | <u>0.96</u> | <u>0.96</u> |
| 3.3   | 6.3  | 0.90        | 0.90        | 0.87        | 0.85        | 0.83        | 0.86        | 5.5  | 0.90        | 0.87        | 0.85        | 0.79        | 9.2  | 0.95        | 0.95        |
| 3.0   | 5.7  | 0.88        | 0.87        | 0.87        | 0.83        | 0.82        | 0.81        | 5.0  | 0.91        | 0.85        | 0.80        | 0.75        | 8.3  | 0.95        | 0.94        |
| 2.7   | 5.1  | 0.84        | 0.84        | 0.85        | 0.81        | 0.80        | 0.80        | 4.5  | 0.86        | 0.82        | 0.78        | 0.74        | 7.5  | 0.94        | 0.90        |
| 2.4   | 4.5  | 0.81        | 0.82        | 0.80        | 0.79        | 0.75        | 0.79        | 4.0  | 0.81        | 0.78        | 0.72        | 0.69        | 6.7  | 0.90        | 0.90        |
| 1.8   | 3.5  | 0.73        | 0.72        | 0.76        | 0.69        | 0.65        | 0.66        | 3.0  | 0.66        | 0.67        | 0.60        | 0.58        | 5.0  | 0.86        | 0.78        |
| 1.5   | 2.8  | 0.64        | 0.62        | 0.61        | 0.58        | 0.56        | 0.49        | 2.4  | 0.53        | 0.57        | 0.49        | 0.47        | 4.2  | 0.78        | 0.67        |
| 1.0   | 1.9  | 0.46        | 0.45        | 0.44        | 0.41        | 0.40        | 0.34        | 1.65 | 0.40        | 0.40        | 0.34        | 0.33        | 2.78 | 0.53        | 0.44        |

## 4.3 Assessment of effective widths

The threshold width  $b_{th}$  was calculated graphically from Figure 11 from the line intersection as described in the methodology. Table 9 lists the values of  $b_{eff}$  considered for this study.  $b_F$ ,  $b_K$  and  $b_Z$  were evaluated at the root S0. For DR1a, the superposition principle for a group of loads applied. Table 10 compares the resultant values of the calculated  $Q_{Rd}$  against  $Q_{Rd,nlFE}$  of the widest specimen as a reference. The values of some  $b_{eff}$  and the corresponding  $Q_{Rd}$  for each test

were also represented in Figure 11 by means of vertical and horizontal lines, respectively. Note that in DR1a some of the calculated  $Q_{Rd}$  went beyond the axis scale.

Table 9 – Derived values of  $b_{eff}$  for the studied experimental tests using different approaches.

| <i>EB: edge beam</i> |      |      |      |      |      |      |      |      |      |      |      |        |
|----------------------|------|------|------|------|------|------|------|------|------|------|------|--------|
| $b_{eff}$            | 4d   | 4dEB | 5d   | 5dEB | 6d   | 6dEB | V1   | V1EB | V4   | V4EB | DR1a | DR1aEB |
| $b_F$                | 2.92 |      | 3.35 |      | 3.78 |      | 2.22 |      | 3.02 |      |      | 6.26   |
| $b_K$                | 2.52 |      | 2.95 |      | 3.38 |      | 1.82 |      | 2.62 |      |      | 5.96   |
| $b_Z$                | 2.12 |      | 2.55 |      | 2.98 |      | 1.42 |      | 2.22 |      |      | 5.66   |
| $b_{MC}$             | 2.49 |      | 2.92 |      | 3.35 |      | 1.79 |      | 2.59 |      |      | 5.62   |
| $b_{B11}$            | 2.55 |      | 2.55 |      | 2.55 |      | 2.57 |      | 2.57 |      |      | 2.92   |
| $b_{th}$             | 3.95 | 4.53 | 4.06 | 4.55 | 4.18 | 5.47 | 3.92 | 4.64 | 4.03 | 4.98 | 5.04 | 6.04   |

The threshold width  $b_{th}$  resulted in clear unsafe estimates. The expressions for  $b_{eff}$  based on the geometry were diffuse: from safe to unsafe predictions and with a good agreement in certain cases. The Swedish formulation remained somewhat stable in the results and was always safe. Conservative estimates for tests VK2V1 were derived. This may be associated with the presence of an arch action due to a load close to the support, which should instead be addressed in the calculation of  $v_{Rd}$ .

Table 10 –  $Q_{Rd,nlFE}$  and comparison through ratios  $Q_{Rd,nlFE}/Q_{Rd}$  for different  $b_{eff}$  (see sub index). Unsafe values are underlined. EB: edge beam

| $Q_{Rd}$        | 4d          | 4dEB        | 5d          | 5dEB        | 6d          | 6dEB        | V1    | V1EB  | V4    | V4EB        | DR1a        | DR1aEB      |
|-----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------|-------|-------|-------------|-------------|-------------|
| $Q_{Rd,nlFE}$   | 846         | 902         | 877         | 950         | 826         | 959         | 1,393 | 1,590 | 1,087 | 1,289       | 1,387       | 1,651       |
| $/ Q_{Rd,nlFE}$ | 1.00        | 1.00        | 1.00        | 1.00        | 1.00        | 1.00        | 1.00  | 1.00  | 1.00  | 1.00        | 1.00        | 1.00        |
| $/ Q_{Rd,F}$    | 0.87        | 0.80        | <u>1.02</u> | 0.93        | <u>1.19</u> | <u>1.01</u> | 0.43  | 0.37  | 0.75  | 0.62        | 0.78        | 0.65        |
| $/ Q_{Rd,K}$    | 0.75        | 0.69        | 0.90        | 0.82        | <u>1.06</u> | 0.90        | 0.35  | 0.30  | 0.65  | 0.54        | 0.71        | 0.59        |
| $/ Q_{Rd,Z}$    | 0.63        | 0.58        | 0.78        | 0.71        | 0.94        | 0.80        | 0.27  | 0.24  | 0.55  | 0.46        | 0.64        | 0.53        |
| $/ Q_{Rd,MC}$   | 0.74        | 0.68        | 0.89        | 0.81        | <u>1.06</u> | 0.90        | 0.35  | 0.30  | 0.64  | 0.53        | 0.63        | 0.53        |
| $/ Q_{Rd,B11}$  | 0.78        | 0.72        | 0.80        | 0.73        | 0.83        | 0.70        | 0.50  | 0.43  | 0.65  | 0.54        | 0.80        | 0.66        |
| $/ Q_{Rd,th}$   | <u>1.18</u> | <u>1.24</u> | <u>1.24</u> | <u>1.26</u> | <u>1.32</u> | <u>1.46</u> | 0.76  | 0.77  | 1.00  | <u>1.02</u> | <u>1.16</u> | <u>1.16</u> |

#### 4.4 Analysis of results

##### Minimal width $b_{min}$

The minimal width  $b_{min}$  depends on the load position. Therefore, the purpose of the tests may partly influence the value of  $b_{min}$ . This can be the case of loads close to the root if the compression strut (arch effect) needs to be studied. Loads closer to the longitudinal free edge may influence the failure mode, which makes it a relevant factor as well. In this context, the thickness of the overhang plays a role since tapered slabs will likely fail locally in the surroundings of the load, as was the case of DR1a. The number of loads across the longitudinal direction should also be taken into consideration for  $b_{min}$ . However, non-conclusive results were reached in this study because of the local failure mode of the only analysed test with several loads, DR1a.

Given these considerations, some preliminary recommendations could be provided from the results. The slab definition according to the Eurocode 2 and the value of  $b_{\text{eff}}$  – including  $b_{\text{th}}$  – should not be used as a reference for  $b_{\text{min}}$ . Figure 12 illustrates the concept of  $b_{\text{min}}$  against  $b_{\text{eff}}$  for a clearer distinction. The authors' proposal is presented in relative terms considering the  $b/s$  ratio instead of absolute values of  $b_{\text{min}}$ . Based on this study, a recommended value for  $b/s$  that can give safe dimensions to reach the full capacity of RC overhang slabs is between 4.0 and 4.5. The edge beam implies an increase of  $b/s$  to values between 5.3 and 6.1 due to its associated effect with  $b$ .

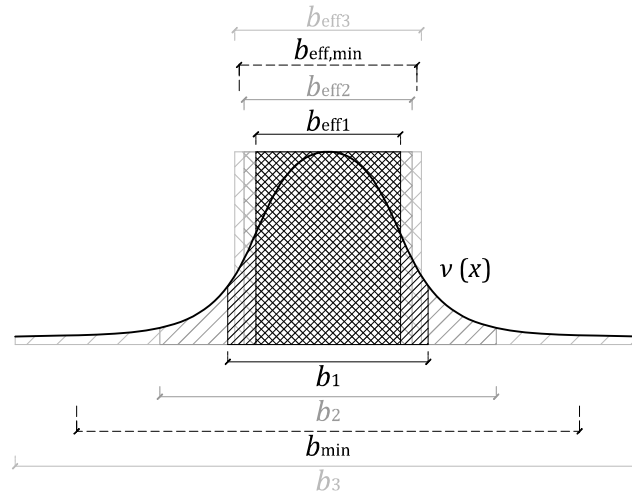


Figure 12 – Illustration of the concept of  $b_{\text{min}}$  and distinction from  $b_{\text{eff}}$  for increasing  $b$ .

This paper outlines more in detail the influence of  $b$  on  $Q_u$  and the failure mode. According to the authors' opinion, it is of paramount importance to perform experimental tests on specimens wide enough to remove any uncertainty from the interpretation of the results and thus be able to draw adequate conclusions. The following discussion presumes RC overhang slabs at its full capacity without such uncertainty.

#### Effective width $b_{\text{eff}}$

In the context of RC overhang slabs, the effective width  $b_{\text{eff}}$  is a tool used in design to calculate the maximum value of the shear force distribution  $v_Q$  straight from the concentrated load(s). The expressions for  $b_{\text{eff}}$  based on geometry propose an attractive and intuitive approach for the designer since they illustrate the force distribution from the load to the support. They give a perception of dissociation between the shear concrete strength  $v_{Rd}$  and the geometry of the slab.

However, such formulae have no theoretical connection to the real value of  $b_{\text{eff}}$ , which is implicitly linked to the material properties. The fit or mismatch of the load bearing capacities shown in this and previous research is merely a pure coincidence. Thus, the use of  $b_{\text{eff}}$  for the design of RC overhang slabs by engineers should be done with caution. The hypothesis of the threshold width  $b_{\text{th}}$  to match the real  $b_{\text{eff}}$  considers geometry and material properties but it does not provide reliable results either, at least following the proposed definition and based on non-linear FE-analyses.

Linear-elastic FE-analyses can instead provide a direct value of  $v_Q$  without the need of  $b_{\text{eff}}$ . They also consider the boundary conditions and the load arrangement, and to a certain extent the positive effect of the edge beam [3]. The authors therefore suggest using this approach for the

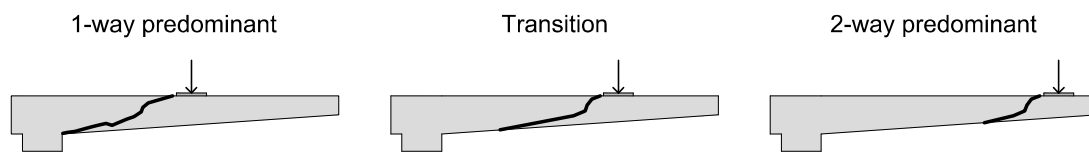
engineering practice, in agreement with previous references in the literature [4]. A distribution width for RC overhang slabs as proposed in Pacoste et al. [28] is recommended for the post-processing of the results and leads to adequate predictions [3].

#### Load position

In the design practice, the position of the loads will be according to the control sections that give the most unfavourable result. For tapered overhang slabs, one of the control sections will be furthest from the root to account for the minimum thickness. In case of flat overhang slabs, the position of the loads should be closest to the root, considering the maximum value of the self-weight. Note that this location should be outside the area that activates the direct compression strut (arching effect). The linear-elastic FE-analysis will not reflect this mechanism. Instead, it will give an even higher value  $v_Q$  instead, which will be conservative and result in an opposite effect.

In this context, the first location of the load which activates the compression strut according to Eurocode ( $2d$ ) is questionable. Previous research suggests increasing this value to  $2.5d$ ,  $2.75d$  or  $3.3d$  as proposed in the literature in Netherlands, Switzerland and Germany, respectively [1, 4, 17]. The highest value is recommended since this mechanism was observed in test VK2V1 with  $a/d = 3.3$  in previous work [11]. This effect can motivate the low ratios obtained in Table 10 for this test.

The design approaches described above may not be fully consistent with the results of experimental tests. Experimental test 6d failed locally in shear similarly to DR1a. Since this was a flat slab, the root would have been chosen as a control section. In spite of different material properties, tests 4d and 5d did not give considerable higher load capacities but the shear crack developed from the load towards the root, in contrast to 6d. Based on the failure mode and the crack development, the authors believe that there is a transition area connected to the distance of the load to the root (Figure 13). In this area, the direct load transfer strut starts to activate smoothly until it has an impact on the load capacity. If the load is not close enough, a local failure resembling punching would take place.



*Figure 13 – Shear failure for different load positions: Possible transition from one- to two-way.*

In this context, the effect of the edge beam is relevant since it affected the failure mode of test 6d. The shear crack developed towards the root in contrast to the specimen without the edge beam (Figure 14). This evidences that the edge beam accentuates the one-way shear force distribution not only towards the root but also towards the edge beam. This results in a more efficient distribution of the shear forces that increase the load capacity and even the failure section may change if the overhang slab has an edge beam, in agreement with previous research [3, 11].

An aspect not explicitly considered in the Eurocode 2 is the shear and flexural interaction for the calculation of  $v_{Rd}$ , in contrast to ACI [29] or Model Code [12]. A load close to the free edge will

lead to a higher value of the bending moment at the root. This might imply that the root would become the control section and affect the load position for design.



Figure 14 – Shear crack at the central transversal cross section of tests 4d, 5d and 6d with a  $b$  of 13 m.

#### Control section

Considering a load at any position in the overhang slab, the reasonability of the choice of the root as one of the control cross sections has been criticized [4]. As explained previously, there is no theoretical connection between the formulations of  $b_{\text{eff}}$  with load spread considering the root as the failure section. However, to discard this control section may not be appropriate. In fact, one-way shear failure caused by line loads will typically occur at the root.

For RC overhang slabs under concentrated loads, a definite shear failure is a hybrid between one and two-way [2]. In both cases, the control sections should ideally match the shear crack initiation. Experiments contribute to identify this complex mechanism, but the exact position and nature of the failure might be diffuse to outline. A precise definition of the control section is therefore complicated.

Nevertheless, from a design perspective, an engineer might question the practicality of such detailed information. For hand calculations, the verification at the root will consider the total self-weight, whereas the one at the section close to the load will account for the reduced thickness. For linear-elastic FE-analysis, the total shear force from the concentrated load(s) integrated from the distribution should be the same across different longitudinal cross sections. The difference will be its shape, which will affect the peak value of  $v_Q$  for design as higher values are obtained for control sections closer to the load. The location usually used for evaluation is at a distance of  $d/2$  which is in the authors' opinion reasonable.

#### Other considerations

In the context of non-linear FE-simulations, the influence of the material properties is remarkable. Quantitatively, a considerable difference of magnitude of  $Q_{\text{Rd,nlFE}}$  between tests VK2 and 4-5-6d was observed. This could be associated with the values of the Young modulus, the compressive and tensile strength of concrete. The equation to calculate the shear concrete strength according to Eurocode 2 only accounts for the compressive strength by means of a cubic root.

Another factor to be highlighted is the total span of the overhang slab  $s$ , which was also different in these tests. Given the current design equations, it only contributes to additional self-weight. However, for a fixed load distance, the fact of having different values of  $s$  might influence the bending moment and thus the structural behaviour.

## 5. CONCLUSIONS

A campaign of non-linear FE-simulations was performed to investigate the structural behaviour of RC overhang slabs without shear reinforcement. These were based on validated experimental tests available in the literature. A range of slab widths  $b$  was simulated to see the impact on the load capacity and assess effective widths  $b_{\text{eff}}$  in the current design practice. The conclusions of this work are:

- Increase of load capacity  $Q_u$  with the slab width  $b$ . The results from the campaign confirm the increase of  $Q_u$  with  $b$  until a transition area is reached, in agreement with previous research. The edge beam plays both a quantitative and a qualitative role as it leads to an increase of  $Q_u$  and affects the failure mode. Some experimental tests on overhang slabs have not been wide enough to reach their full shear capacity. This may imply misleading evaluation of results and hence inaccurate conclusions.
- Minimal width  $b_{\text{min}}$ . The results suggest designing experimental tests using  $b_{\text{min}}$  based on a width-span ratio  $b/s$  of 4.0-4.5 to ensure that full capacity of the specimens is reached. In case of an edge beam, this value should be increased to 5.3-6.1. The slab definition according to the Eurocode or the expressions for  $b_{\text{eff}}$  should not be used as a reference for  $b$ .
- Effective width  $b_{\text{eff}}$ . The use of  $b_{\text{eff}}$  based on geometric expressions do not provide reliable results to calculate the load capacity  $Q_{\text{Rd}}$ . In fact, there is no theoretical connection between the definition of  $b_{\text{eff}}$  and such expressions. To dissociate between the shear concrete strength and the slab geometry by means of  $b_{\text{eff}}$  is not possible because the material properties are implicitly latent.
- Threshold width  $b_{\text{th}}$ . The hypothesis of the threshold width  $b_{\text{th}}$  as a way to calculate  $b_{\text{eff}}$  considering both geometry and material properties of the slab does not provide reliable results either, at least following the definition of this paper and based on non-linear FE-analyses. Therefore, this approach should be discarded for future evaluations of  $b_{\text{eff}}$ .
- Design practice. The authors suggest using linear-elastic FE-analyses that consider the load arrangement and boundary conditions with a traditional control section at a distance of  $d/2$  for the evaluation of  $v_Q$ .
- Experimental tests. The authors recommend that decision-making for the design of the experimental test including such parameters as dimensions, reinforcement ratios, load position, etc. should be based on preliminary non-linear FE-analyses. This should serve to identify the influence of these parameters on the main purpose of the test. The application in real context should be considered as well. In this context, bridge overhang slabs are usually wide enough and may have an integrated edge beam system.

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