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MODELLING THE HEAT AND MASS TRANSFER IN DEVICES WITH DENSE-LAYER DISPERSED SYSTEMS

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Abstract:
The work is devoted to the processes of heat and mass transfer in moving and stationary dense layers of dispersed materials. One and two-component models of heat and mass transfer in a layer with internal heat sources caused by chemical and phase transformations in the presence of submerged heat exchange processes are given. A review of the literature showed that for a layer containing heat sources, not only information on these parameters is missing, but also methods for their determination. This paper describes the theoretical basis that forms the analytical dependencies of such methods. Satisfactory qualitative and quantitative agreement between experimental and calculated data indicates that the models accurately describe the main patterns of heat transfer in a blown layer with submerged heat transfer surfaces. The research results showed that when calculating temperature fields, reliable information is needed on the heat transfer coefficients of the layer and its components.

Key words: minerals, chemical and phase transformations, immersed heat transfer processes, mathematical model, dispersed layer, heat and mass transfer, air mixture supply method

INTRODUCTION

Devices with dense layer dispersed systems are used in power engineering, metallurgy, chemical, food and other industries. These are, for example, installations for energy-technological processing of solid fuels, catalytic heat generators, apparatus for methane conversion, enrichment of non-ferrous metal ores, processing of various wastes, catalytic chemical reactors, and dryers for various bulk materials. In this regard, extensive literature is devoted to the study of heat and mass transfer processes in such systems. The work [1] analysed and summarised data on the structural and aerodynamic characteristics of a stationary blown layer, intercomponent thermal and mass transfer. Considerable attention is paid to the mathematical description of heat and mass transfer processes in layers filtered by gas or liquid.

In the general case, such a description cannot be performed based on concepts of continuum mechanics due to the macroscopic heterogeneity of dispersed systems. The continuum approximation is valid only in cases where the structural size of the elements of a dispersed medium is significantly smaller than the linear scale of changes in average temperatures and concentrations [2, 3]. This condition is satisfied in many problems of heat and mass transfer in dispersed media that are interesting for practice. In this regard, the need arises to formulate equations of conservation of heat and mass for continua that model the components of the system or the system as a whole.

Various methods are used to derive transport equations [4]:

a) phenomenological [5],
b) method of spatial or temporal averaging of microscopic local equations [6],
c) statistical method of averaging local equations over an

ensemble of possible configurations of a group of particles [7].

Conservation equations are postulated using the phenomenological approach, and transport coefficients are determined experimentally. During spatial averaging, variable averages for volumes and differently oriented areas are introduced into the equations. With this approach, it is necessary to make assumptions about the nature of the relationship between different average quantities and assumptions that allow one to reduce integrals over the volume (surface) of an ensemble of particles to integrals for one particle. The most rigorous is the statistical method, which makes it possible to obtain conservation equations and relations that close the system of equations based on the same model concepts and does not require the formulation of hypotheses about the type of connection between volumetric and surface averaging.

Phenomenological single-component homogeneous models (for example, [4] and others) have become widespread, allowing analytical solutions to be obtained for various heat transfer problems. In such models, equality of the temperatures of the components is postulated. Meanwhile, under certain conditions (during reactions with a large thermal effect in the layer, short-term nonstationary processes, low intensity of intercomponent heat exchange, high frame thermal conductivity of the layer), the temperatures of the components can differ significantly. In such cases, two-component homogeneous models are used, in which the layer is considered as a system consisting of two quasi-solid components, between which intercomponent heat and mass transfer processes occur (for example, [8].

In [3, 6], the system of heat transfer equations for two-component models was reduced to a certain equivalent equation. A comparison of one- and two-component models showed that the use of one-component models in short-term processes, with intense heat release and high thermal resistance of intercomponent heat transfer, leads to significant errors in predicting temperatures [9]. The models of heat transfer known in the literature in the presence of internal heat sources are valid for a free layer; there are no models for a layer with submerged surfaces. This study is devoted to processes for which information in the literature is absent or extremely limited. Two- and one-component mathematical models are used to describe heat and mass transfer in a layer with internal sources (sinks) of heat caused by chemical and phase transformations, in the presence and absence of submerged heat exchange surfaces.

RESEARCH METHODOLOGY

Let us consider the interconnected processes of heat and mass transfer in a fixed catalyst bed through which gas (reacting mixture) is blown. An exothermic or endothermic reaction occurs on the surface and in the volume of particles. The optimal temperature regime for a specific reaction in the bed is organised with the help of heat exchange surfaces immersed in it (for example, a tube bundle) [10]. The transfer processes in such a system can be described based on a two-component homogeneous model, according to which the bed consists of two quasi-homogeneous components - gas and solid. Conductive heat transfer in components is characterized by effective thermal conductivity coefficients λ_g^*, λ_m^*, diffusion transfer in gas is characterized by the effective diffusion coefficient D_g^*, intercomponent heat and mass transfer is characterized by heat and mass transfer coefficients a_M, β_M, heat transfer of each component with submerged surfaces is characterized by heat transfer coefficients a_{cm}^g, a_{cm}^T.

To simplify the model, the internal thermal and diffusion resistances of the particles are assumed to be negligibly small compared to the external ones, which allowed us to exclude the equations of heat conductivity and diffusion in them. However, if necessary, these resistances can be taken into account by making appropriate corrections to the values of the intercomponent exchange coefficients [7, 9]. Heat transfer in the gas component due to mass transfer is also not taken into account.

With a uniform distribution over the cross section of the layer roughness, gas velocity, and the area of immersed surfaces, the transverse temperature and concentration gradients are negligibly small compared to the longitudinal ones, as a result of which the problem will be considered one-dimensional.

The mathematical two-component model of non-stationary heat and mass transfer in a layer with immersed surfaces during a chemical reaction in the external diffusion area, obtained based on the laws of conservation and transfer of energy and mass, includes [11]:

– heat transfer equation in the gas component [12]:

$$\rho_g \cdot c_{pg} \cdot \varepsilon \cdot (1-\beta_2) \cdot \frac{\partial t_g}{\partial \tau} + \rho_g \cdot c_{pg} \cdot (1-\beta_1) \cdot w_g \cdot \frac{\partial t_g}{\partial x} = (1-\beta_1) \cdot \frac{\partial}{\partial x} \cdot \left(\lambda_g^* \cdot \frac{\partial t_g}{\partial x}\right) + \tag{1}$$

– heat transfer equation in the solid component [13]:

$$\rho_m \cdot c_m \cdot (1-\varepsilon) \cdot (1-\beta_2) \cdot \frac{\partial t_m}{\partial \tau} = (1-\beta_1) \cdot \frac{\partial}{\partial x}\left(\lambda_m^* \cdot \frac{\partial t_m}{\partial x}\right) - -a_M \cdot a \cdot (1-\beta_2) \cdot \left(t_m - t_g\right) - a_{cm}^m \cdot F_{cm} \cdot \left(t_m - t_{cm}\right) + r_P \cdot \beta_M \cdot a \cdot (1-\beta_2) \cdot \left(m_g - m_m\right) \tag{2}$$

– mass transfer equation in the gas component [14]:

$$\rho_g \cdot \varepsilon \cdot (1-\beta_2) \cdot \frac{\partial m_g}{\partial \tau} + \rho_g \cdot (1-\beta_1) \cdot w_g \cdot \frac{\partial m_g}{\partial x} = (1-\beta_1) \cdot \frac{\partial}{\partial x} \cdot \left(D_g^* \cdot \rho_g \cdot \frac{\partial m_g}{\partial x}\right) - -\beta_M \cdot a \cdot (1-\beta_2) \cdot \left(m_g - m_m\right) \tag{3}$$

Initial conditions:

at $\tau = 0$

$$t_m = t_{m0}, m_g = m_{g0}, t_g = t_{g0}. \tag{4}$$

Boundary conditions:

at $x = 0$

$$t_m = t'_m, m_g = m'_g, t_g = t'_g; \quad \frac{\partial t_m}{\partial x} = \frac{\partial t_g}{\partial x} = \frac{\partial m_g}{\partial x} = 0. \tag{5}$$

here:

D_g^* is effective diffusion coefficient of the gas component, m^2/s;

F_{cm} is specific area of submerged surfaces, m^2/m^3;

m_g, m_m are dimensionless mass concentrations of the reagent in the gas component and on the surface of the particles.

λ_g^*, λ_m^* are effective axial thermal conductivity coefficients of the gas and solid components, W/(m·K) [15].

Solving equations (1-3) with the corresponding conditions of unambiguity makes it possible to find the distribution over the layer height of the temperatures of the components, the concentrations of the reagent, the degree of conversion, and the amount of heat transferred to the immersed surfaces. The area of immersed surfaces required to achieve a given component temperature (i.e. transfer a given heat flow) and organise an optimal temperature profile for a specific reaction can also be determined. The presented mathematical model is suitable for design and verification calculations of layer apparatuses. If a chemical reaction occurs in the kinetic region, the mass transfer equation is excluded, and the heat of reaction is taken into account in the heat transfer equation as an internal heat source.

The mathematical model of such processes takes the following form:

– heat transfer equation in the gas component (see equation 1);

– heat transfer equation in the solid component.

$$\rho_m \cdot c_m \cdot (1-\varepsilon) \cdot (1-\beta_2) \cdot \frac{\partial t_m}{\partial \tau} = (1-\beta_1) \cdot \frac{\partial}{\partial x}\left(\lambda_m^* \cdot \frac{\partial t_m}{\partial x}\right) - -a_M \cdot a \cdot (1-\beta_2) \cdot \left(t_m - t_g\right) - a_{cm}^m \cdot F_{cm} \cdot (t_m - t_{cm}) + q_v \cdot (1-\beta_2) \quad (6)$$

Initial conditions:

at $\tau = 0$

$$t_m = t_{m0},\ t_g = t_{g0}. \quad (7)$$

Boundary conditions:

at $x = 0$

$$t_m = t_m',\ \frac{\partial t_g}{\partial x} = \frac{\partial t_m}{\partial x} = 0, t_g = t_g'; \quad (8)$$

Equation (6) includes the productivity of the internal heat source q_v determined by the heat and rate of the chemical reaction, which depends on the temperature and concentration of the reagent:

$$q_v = \rho_g \cdot \frac{\partial m_g}{\partial \tau} \cdot r_P \text{ , Вт/м}^2 \quad (9)$$

Mathematical models in special cases can be simplified. If we accept the assumption of a negligibly small contact area of the solid component with the immersed surfaces, we can ignore the heat exchange between them, i.e. in equations (2), (7) we can assume that $a_{cm}^m = 0$. Under certain conditions, following from a comparative assessment of the order of the terms of the equations, we can neglect conductive heat transfer and diffusion mass transfer:

$$(1-\beta_1) \cdot \frac{\partial}{\partial x} \cdot \left(\lambda_g^* \cdot \frac{\partial t_g}{\partial x}\right) = 0,\ (1-\beta_1) \cdot \frac{\partial}{\partial x} \cdot \left(\lambda_m^* \cdot \frac{\partial t_m}{\partial x}\right) = 0,\ (1-\beta_1) \cdot \frac{\partial}{\partial x} \cdot \left(D_g^* \cdot \rho_g \cdot \frac{\partial m_g}{\partial x}\right) = 0$$

With a slight change in the effective coefficients of thermal conductivity and diffusion, they can be taken as constant: $\lambda_m^* = const, \lambda_g^* = const, D_g^* = const$. In several cases, the productivity of internal heat sources can be considered constant ($q_v = const$), or dependent on the coordinate: $q_v = f(x)$.

In a certain range of parameter changes, when the temperatures of the components are practically the same ($t_g = t_m = t$), a single-component model is applicable for the blown layer. The heat transfer equation in the layer has the form:

$$\rho_l \cdot c_l \cdot (1-\beta_2) \cdot \frac{\partial t}{\partial \tau} + \rho_g \cdot c_g \cdot (1-\beta_1) \cdot w_g \cdot \frac{\partial t}{\partial x} = (1-\beta_1) \cdot \frac{\partial}{\partial x} \cdot \left(\lambda^* \cdot \frac{\partial t}{\partial x}\right) + +a \cdot F_{cm}(\mathrm{t} - t_{cm}) + q_v(1-\beta_2) \quad (10)$$

Here ρ_l is the density, c_l is the heat capacity, λ^* is the effective coefficient of axial thermal conductivity of the layer; a is the coefficient of heat exchange of the layer with the submerged surfaces [16].

Initial and boundary conditions:

$\tau = 0$

$$t = t_0, \quad (11)$$

x = *0*

$$t = t'. \quad (12)$$

The one-component model is applicable under the following conditions:

$$\gamma = \frac{a_{cm}^m \cdot F_{cm}}{a_M \cdot a \cdot (1-\beta_2)} \ll 1;\ \varepsilon_M = \frac{\lambda_m^* \cdot (1-\beta_1)}{a_M \cdot a \cdot (1-\beta_2) \cdot d_m^2} \ll 1$$

That is when the intercomponent heat exchange in the layer significantly exceeds the heat transfer due to thermal conductivity in the solid component and the heat exchange of this component with the immersed surfaces.

Below are analytical solutions of the system of equations (1) – (6) and equation (10), which allow us to perform approximate calculations for qualitative and quantitative analysis of the influence of regime and geometric factors on the temperature distribution in the layer [17]. They can also be the theoretical basis for experimental methods of determining heat exchange coefficients of layer components with submerged surfaces.

Temperature field in a stationary blown layer with immersed surfaces. Stationary heat transfer in a layer without internal heat sources

In a stationary mode, $\mathrm{q_v} = 0$ and constant transfer coefficients, the system of equations (1) – (6) takes the form [18]:

$$\mathrm{G_g \cdot c_{pg} \cdot (1-\beta_1) \cdot \frac{d\upsilon_g}{dx} = \lambda_g^* \cdot (1-\beta_1) \cdot \frac{d^2\upsilon_g}{dx^2} + a_M \cdot a \cdot (1-\beta_2) \cdot (\upsilon_m - \upsilon_g) - a_{cm}^g \cdot F_{cm} \cdot \upsilon_g} \quad (13)$$

$$\lambda_m^* \cdot (1-\beta_1) \cdot \frac{d^2\upsilon_m}{dx^2} - a_M \cdot a \cdot (1-\beta_2) \cdot (\upsilon_m - \upsilon_g) - a_{cm}^g \cdot F_{cm} \cdot \upsilon_g = 0 \quad (14)$$

Boundary conditions:

for $\mathrm{x} = 0$: $\upsilon_g = \upsilon_{g0},\ \upsilon_m = \upsilon_{m0}$,

for $\mathrm{x} \to \infty$: $\upsilon_g \to 0,\ \upsilon_m \to 0$.

here:

$G_g = \rho_g \cdot w_g$ is mass velocity of gas, kg/(m²s);

$\upsilon_g = t_g - t_{cm};\ \upsilon_m = t_m - t_{cm}$ are excess temperature of components, K.

The solution was performed using the asymptotic perturbation method for a small parameter – the value of $\lambda_m^*/(a_M a)$ with an accuracy of up to $(\varepsilon_M)^2$. The following

formulas were obtained for the temperature distribution in the gas and solid components of the equations [19]:

$$\upsilon_g = \upsilon_{g0} \cdot exp(A \cdot X) \quad (15)$$

$$\upsilon_m = \upsilon_{m0} \cdot \left(exp(-BX) - \frac{exp(-BX)-exp(A\cdot X)}{1+\gamma-\varepsilon_M \cdot A^2}\right) \quad (16)$$

here:

$$\gamma = \frac{a_{cm}^m \cdot F_{cm}}{a_M \cdot a \cdot (1-\beta_2)}; \quad \varepsilon_M = \frac{\lambda_m^* (1-\beta_1)}{a_M \cdot a \cdot (1-\beta_2) \cdot d_m^2}; \quad X = \frac{x}{d_m} \quad (17)$$

$$A = \frac{Pe_M^*}{2} - \sqrt{\left(\frac{Pe_M^*}{2}\right)^2 + \frac{d_m^2 \cdot (1+\gamma) \cdot (a_{cm}^g \cdot F_{cm} \cdot (1+\gamma) + a_{cm}^m \cdot F_{cm})}{[\lambda_g^* \cdot (1+\gamma)^2 + \lambda_m^*] \cdot (1-\beta_1)}} \quad (18)$$

$$Pe_M^* = \frac{G_g \cdot c_{pg} \cdot (1+\gamma)^2 \cdot d_m}{[\lambda_g^* \cdot (1+\gamma)^2 + \lambda_m^*] \cdot (1-\beta_1)}, B = \sqrt{\frac{1+\gamma}{\varepsilon_M}} \quad (19)$$

The complexes γ, ε_M, Pe_M^* characterise the relationships of heat flows caused by different transfer mechanisms. The system of equations (13), (14) and solutions (15), (19) take into account conductive transfer and heat exchange with the wall for each component.

Under certain conditions, some mechanisms can be neglected and simplified modifications of the model can be considered [20].

a) The heat exchange of the solid component with the submerged surfaces is negligible.

In equations (14) $a_{cm}^m = 0$. The solution to the problem for this case has the form:

$$\upsilon_g = \upsilon_{g0} \cdot exp(R \cdot X) \quad (20)$$

$$\upsilon_m = \upsilon_{m0} \cdot \left(exp(-BX) - \frac{exp(-BX)-exp(RX)}{1-\varepsilon_M \cdot R^2}\right) \quad (21)$$

here:

$$R = \frac{Pe^*}{2} - \sqrt{\left(\frac{Pe^*}{2}\right)^2 + \frac{a_m^g \cdot F_{cm} \cdot d_m^2}{[\lambda_g^* + \lambda_m^*] \cdot (1-\beta_1)}}, B = \sqrt{\frac{1}{\varepsilon_M}} \quad (22)$$

$$Pe^* = \frac{G_g \cdot c_{pg} \cdot d_m}{[\lambda_g^* + \lambda_m^*] \cdot (1-\beta_1)} \quad (23)$$

If in (20)-(22) instead of a_{cm}^g we use the value a_cm, equal to the sum of the heat transfer coefficients of the components included in equations (13), (14), i.e. $a_{cm}^{g*} = a_{cm}^g + a_{cm}^m$, then the temperatures calculated using formulas (15), (16), and (20), (21) practically coincide.

As indicated above, for $\varepsilon_M \ll 1$ and $\gamma \ll 1$ the two-component model can be replaced by a one-component one.

b) Conductive heat transfer in both components is negligibly small.

With this assumption, i.e. when in (13) $\lambda_g^* \cdot (1-\beta_1) \cdot \frac{d^2 \upsilon_g}{dx^2} = 0$ and in (14) $\lambda_g^* \cdot (1-\beta_1) \cdot \frac{d^2 \upsilon_m}{dx^2} = 0$, the temperature field is described by the following formulas obtained by the integral Laplace transform method:

$$\upsilon_g = \upsilon_{g0} \cdot exp(P \cdot X); \; \upsilon_m = \frac{\upsilon_g}{1+E_2} \quad (24)$$

$$P = -\frac{B_1}{1+E_2} - B_1 - E_1 \quad (25)$$

$$B_1 = \frac{a_M \cdot a \cdot (1-\beta_2)}{G_g \cdot c_{pg}}; \; E_1 = \frac{a_{cm}^g \cdot F_{cm}}{G_g \cdot c_{pg}}; \; E_2 = \frac{a_{cm}^m \cdot F_{cm}}{a_M \cdot a \cdot (1-\beta_2)} \quad (26)$$

When $E_2 \ll 1$ we obtain $\upsilon_m \cong \upsilon_g$. Under such conditions, a single-component model is applicable, according to which the formula determines the temperature of the equation:

$$\upsilon = \upsilon_0 \cdot exp(-P' \cdot x), \quad (27)$$

where:

$P' = \frac{a_l \cdot F_{cm}}{G_g \cdot c_{pg}}$, a_l is the heat exchange coefficient of the layer with immersed surfaces.

Stationary heat transfers in a layer with internal heat sources

At constant transfer coefficients, system of equations (5), (6) has the form:

$$\begin{gathered}\rho_g \cdot c_{pg} \cdot (1-\beta_1) \cdot \frac{d\upsilon_g}{dx} = \lambda_g^* \cdot (1-\beta_1) \cdot \frac{d^2 \upsilon_g}{dx^2} + a_M \cdot a \cdot \\ (1-\beta_2) \cdot (\upsilon_m - \upsilon_g) - a_{cm}^g \cdot F_{cm} \cdot \upsilon_g, \\ \lambda_m^* \cdot (1-\beta_1) \cdot \frac{\mathrm{d}^2 \upsilon_\mathrm{m}}{\mathrm{dx}^2} - \mathrm{a_M} \cdot \mathrm{a} \cdot (1-\beta_2) \cdot (\upsilon_\mathrm{m} - \upsilon_\mathrm{g}) - \\ \mathrm{a_{cm}^m} \cdot \mathrm{F_{cm}} \cdot \upsilon_\mathrm{m} - \mathrm{q_{v0}} \cdot (1 \pm \mathrm{b} \cdot \mathrm{x}) \cdot (1-\beta_2),\end{gathered} \quad (28)$$

when $x = 0$ $\upsilon_g = \upsilon_{g0,}$

$$\upsilon_m = \upsilon_{m0} \; \frac{d\upsilon_g}{dx} = \frac{d\upsilon_m}{dx} = 0. \quad (29)$$

Here, q_{v0} is the productivity of internal heat sources in the input section, W/m^2.

With negligibly small conductive heat transfer in the solid component (i.e. $\lambda_m^* \cdot (1-\beta_1) \cdot \frac{d^2 \upsilon_g}{dx^2} = 0$), the temperature field of the components is described by the following equations.

$$\begin{gathered}\upsilon_g = \frac{\upsilon_{g0}}{P_1 - P_2} \cdot \left[\left(P_1 + \frac{1}{A_1}\right) \cdot exp(P_1 \cdot x) - \left(P_2 + \frac{1}{A_2}\right) \cdot \right. \\ \left. exp(P_2 \cdot x)\right] - \frac{B_1 \cdot H}{A_1 \cdot (B_2 + E_2)} \cdot \left[\frac{exp(P_1 \cdot x)}{P_1 \cdot (P_1 - P_2)} - \frac{exp(P_2 \cdot x)}{P_2 \cdot (P_1 - P_2)} + \right. \\ \left. \frac{1}{P_1 \cdot P_2}\right] + \frac{B_1 \cdot H \cdot b}{A_1 \cdot (B_2 + E_2)} \cdot \left[\frac{exp(P_1 \cdot x)}{P_1^2 \cdot (P_1 - P_2)} - \frac{exp(P_2 \cdot x)}{P_2^2 \cdot (P_1 - P_2)} + \frac{P_1 + P_2}{P_1^2 \cdot P_2^2} + \right. \\ \left. \frac{x}{P_1 \cdot P_2}\right];\end{gathered} \quad (30)$$

$$\upsilon_m = \frac{\upsilon_g \cdot B_2 + H \cdot (1 \pm b \cdot x)}{B_2 + E_2} \quad (31)$$

$$\begin{gathered}A_1 = \frac{\lambda_g^* \cdot (1-\beta_1)}{G_g \cdot c_{pg}}; \; B_1 = \frac{\mathrm{a_M} \cdot \mathrm{a} \cdot (1-\beta_1)}{G_g \cdot c_{pg}}; \; B_2 = \mathrm{a_M} \cdot \mathrm{a} \cdot (1-\beta_2). \\ E_1 = \frac{a_{cm}^g \cdot F_{cm}}{G_g \cdot c_{pg}}; \; E_2 = a_{cm}^m \cdot F_{cm}; H = q_{v0} \cdot (1-\beta_2)\end{gathered} \quad (32)$$

$$P_{1,2} = -\frac{1}{2 \cdot A_1} \pm \sqrt{\frac{1}{4 \cdot A_1^2} - \frac{1}{A_1} \cdot (B_1 + E_1) - \frac{B_1 \cdot B_2}{B_2 + E_2}}. \quad (33)$$

From (30), (31) equations can be obtained for cases when the power of the sources is constant (i.e. $b = 0, q_v = q_{v0} = const$), when there are no sources (i.e. $q_v = 0, H = 0$).

For the values of the complexes $\frac{E_2}{B_2} = \frac{a_{cm}^m \cdot F_{cm}}{a_M \cdot a \cdot (1-\beta_2)} \ll 1$ (when the thermal resistance of intercomponent heat exchange is significantly less than the resistance of heat exchange of the solid component with the immersed surfaces), and $\frac{\mathrm{H}}{B_2} = \frac{q_{v0} \cdot (1-\beta_2)}{a_\mathrm{M} \cdot a \cdot (1-\beta_2)} \ll 1$ when the heat flux caused by the action of sources is significantly less than that transferred in the process of intercomponent heat exchange), the one-component model (10) can be used. Under such conditions, at ∂t/∂τ = 0, the following formula is valid for the temperature distribution in the equations:

$$\begin{gathered}\upsilon_g = \frac{\upsilon_0}{P_1 - P_2} \cdot \left[\left(P_1 + \frac{1}{A_1}\right) \cdot exp(P_1 \cdot x) - \left(P_2 + \frac{1}{A_2}\right) \cdot \right. \\ \left. exp(P_2 \cdot x)\right] - \frac{H}{A_1} \cdot \left[\frac{exp(P_1 \cdot x)}{P_1 \cdot (P_1 - P_2)} - \frac{exp(P_2 \cdot x)}{P_2 \cdot (P_1 - P_2)} + \frac{1}{P_1 \cdot P_2}\right] - \frac{H \cdot b}{A_1} \cdot \\ \left[\frac{exp(P_1 \cdot x)}{P_1^2 \cdot (P_1 - P_2)} - \frac{exp(P_2 \cdot x)}{P_2^2 \cdot (P_1 - P_2)} + \frac{P_1 + P_2}{P_1^2 \cdot P_2^2} + \frac{x}{P_1 \cdot P_2}\right],\end{gathered} \quad (34)$$

$$A_1 = \frac{\lambda^* \cdot (1-\beta_1)}{G_g \cdot c_{pg}}; \; E_1 = \frac{\mathrm{a_l} \cdot \mathrm{F_{cm}}}{G_g \cdot c_{pg}}; \; H = \frac{q_{v0} \cdot (1-\beta_2)}{G_g \cdot c_{pg}}, \quad (35)$$

$$P_1 = \frac{1}{2\cdot A_1} \pm \sqrt{\frac{1}{4\cdot A_1^2} - \frac{E_1}{A_1}}. \quad (36)$$

If the power of the sources (34) remains constant, $b = 0$ should be set, and if they are absent, $H = 0$.

For values of the Peclet number $Pe^* = \frac{\rho_g \cdot c_{pg} \cdot w_g \cdot D}{\lambda_g^*} \geq 50$ in the two-component model, conductive heat transfer in the gas component can be neglected, i.e., $\lambda_g^* \cdot (1-\beta_1) \cdot \frac{d^2 \upsilon_g}{dx^2} = 0$ can be taken into account in (28).

In this case, the distribution of temperatures of the components at a constant power of internal heat sources ($q_v = const$) is described by the equations:

$$\upsilon_g = \upsilon_{g0} \cdot \left[exp(P \cdot x) + \frac{B_1 \cdot H}{P \cdot (E_2+1) \cdot \upsilon_{g0}} \cdot \left(1 - exp(P \cdot x)\right)\right] \quad (37)$$

$$\upsilon_m = \frac{\upsilon_g + H}{1+E_2} \quad (38)$$

$$B_1 = \frac{a_M \cdot a \cdot (1-\beta_2)}{G_g \cdot c_{pg}}. \quad (39)$$

$$E_1 = \frac{a_{cm}^g \cdot F_{cm}}{G_g \cdot c_{pg}};\ E_2 = \frac{a_{cm}^m \cdot F_{cm}}{a_M \cdot a \cdot (1-\beta_g)};\ H = \frac{q_v}{a_M \cdot a}. \quad (40)$$

$$P = \frac{B_1}{1+E_2} - B_1 - E_1 \quad (41)$$

In the absence of sources ($q_v = 0$), formulas (37), (38) turn into (24), (27).

In the one-component model (10), conductive heat transfer in the equation can be neglected at Peclet number values $Pe^* \geq 0$. In such cases, the temperature field in the layer at ($q_v = const$) is calculated according to the equations:

$$\upsilon = \upsilon_0 \cdot \left[exp(P \cdot x) - \frac{H}{P \cdot \upsilon_{g0}} \cdot \left(1 - exp(P \cdot x)\right)\right] \quad (42)$$

$$P = \frac{a_{cm} \cdot F_{cm}}{G_g \cdot c_{pg}};\ H = \frac{q_v \cdot (1-\beta_2)}{G_g \cdot c_{pg}}. \quad (43)$$

When ($q_v = 0$) formula (42) turns into (27), unsteady heat transfer in a layer without internal heat sources. The task of unsteady heat transfer, described by the two-component model (5), was solved without taking into account conductive heat transfer:

$$(\lambda_g^* \cdot (1-\beta_1) \cdot \frac{\partial^2 t_g}{\partial x^2} = 0 \text{ and } \lambda_m^* \cdot (1-\beta_1) \cdot \frac{\partial^2 t_g}{\partial x^2} = 0),$$

heat exchange of the solid component with immersed surfaces ($a_{cm}^m \cdot F_{cm} \cdot (t_m - t_{cm}) = 0$), in the absence of internal heat sources ($q_v = 0$).

The solution was obtained by the operator expansion method, which allows one to reduce the system of nonstationary equations of heat transfer of components to an equivalent equation.

Temperature distribution under boundary conditions: $\tau = 0\ t_m = t_{m0},\ x = 0,\ t_g = t_g = t_{g0}$ is determined by the equations:

$$\begin{gathered}\upsilon_g = \upsilon_{g0} \cdot exp(-(1+h) \cdot X) \cdot \int_0^T \exp(-T) \cdot J_0 \cdot \\ \left(2 \cdot \sqrt{T \cdot X}\right) \cdot dT + \upsilon_{g0} \cdot \exp(-(1+h) \cdot X) \cdot \exp(-T) \cdot \\ J_0 \cdot \left(2 \cdot \sqrt{T \cdot X}\right) + \upsilon_{m0} \cdot exp(-T) \cdot \int_0^X exp(-(1+h) \cdot \\ X) \cdot J_0 \cdot \left(2 \cdot \sqrt{T \cdot X}\right) \cdot dx,\end{gathered} \quad (44)$$

$$\begin{gathered}\upsilon_m = \upsilon_{g0} \cdot exp(-(1+h) \cdot X) \cdot \int_0^T \exp(-T) \cdot J_0 \cdot \\ \left(2 \cdot \sqrt{T \cdot X}\right) \cdot dT + \upsilon_{m0} \cdot \exp(-(1+h) \cdot X) \cdot \exp(-T) \cdot \\ J_0 \cdot \left(2 \cdot \sqrt{T \cdot X}\right) + \upsilon_{m0} \cdot (1+h) \cdot \\ exp(-T) \cdot \int_0^X exp(-(1+h) \cdot X) \cdot J_0 \cdot \left(2 \cdot \sqrt{T \cdot X}\right) \cdot dx,\end{gathered} \quad (45)$$

here:

$$T = \frac{a_m \cdot a}{\rho_m \cdot c_m \cdot (1-\varepsilon)} \cdot \tau;\ X = \frac{a_M \cdot a \cdot (1-\beta_2)}{G_g \cdot c_{pg}} \cdot x;\ h = \frac{a_{cm}^g \cdot F_{cm}}{a_M \cdot a \cdot (1-\beta_g)}.$$

The temperature distribution in the layer according to the one-component model taking into account conductive heat transfer is described by the expression:

$$\begin{gathered}\upsilon = \frac{(\upsilon_0 - \upsilon_m) \cdot exp(-\mu \cdot \tau)}{2} \cdot \left[erfc\left(\frac{\omega \cdot x}{2 \cdot \sqrt{\tau}} - \chi \cdot \sqrt{\tau}\right) + exp\left(\frac{x}{v}\right) \cdot \right. \\ \left. erfc\left(\frac{\omega \cdot x}{2 \cdot \sqrt{\tau}} + \chi \cdot \sqrt{\tau}\right)\right] + \frac{\mu \cdot \upsilon_0}{2} \cdot \left[\int_0^\tau exp(-\mu \cdot \tau) \cdot \right. \\ \left. erfc\left(\frac{\omega \cdot x}{2 \cdot \sqrt{\tau}} - \chi \cdot \sqrt{\tau}\right) \cdot d\tau\right] + exp\left(\frac{x}{v}\right) \cdot \int_0^\tau exp(-\mu \cdot \tau) \cdot \\ erfc\left(\frac{\omega \cdot x}{2 \cdot \sqrt{\tau}} + \chi \cdot \sqrt{\tau}\right) \cdot d\tau + \upsilon_m \cdot exp(-\mu \cdot \tau)\end{gathered} \quad (46)$$

In the equation (46):

$$\begin{gathered}\mu = \frac{a_l \cdot a}{\rho_m \cdot c_m \cdot (1-\varepsilon) \cdot (1-\beta_2)};\ \omega = \sqrt{\frac{\rho_m \cdot c_m \cdot (1-\varepsilon) \cdot (1-\beta_2)}{\lambda^* \cdot (1-\beta_1)}} \\ \chi = \frac{G_g \cdot c_{pg}}{2 \cdot \sqrt{\rho_m \cdot c_m \cdot (1-\varepsilon) \cdot (1-\beta_1) \cdot (1-\beta_2) \cdot \lambda^*}};\ v = \frac{\lambda^* \cdot (1-\beta_2)}{G_g \cdot c_{pg}}\end{gathered} \quad (47)$$

The equations given for $\tau \to \infty$ are consistent with the formulas for the steady state. In [21, 22], an approximate analytical method for calculating the heat transfer coefficients of a layer and its components with submerged cylindrical surfaces is given based on a two-component model.

RESULTS

Based on the above analytical dependencies, calculations of stationary temperature fields were performed for various regimes and geometric characteristics. The heat transfer coefficients of the components of the layer with submerged surfaces required in the calculations were determined according to the recommendations of [8], and the coefficients of intercomponent heat transfer were determined according to [2, 23]. The calculations were performed using two modifications of the two-component model, differing in the method of taking into account heat exchange with submerged surfaces:

- in modification 1, the heat exchange of each component is taken into account using the corresponding heat transfer coefficients a_{cm}^g, a_{cm}^m;
- in modification 2, the heat exchange of the solid component is considered negligible, and only the heat transfer of the gas component is taken into account using the value $a_{cm}^{g*} = a_{cm}^g + a_{cm}^m$.

A quantitative comparison of the calculation results for both modifications was carried out; to assess the discrepancy, the value of the relative difference between the calculated temperatures of the gas and solid components $\delta_m = \frac{t_{m1} - t_{m2}}{t_{m1}}$ was used; $\delta_g = \frac{t_{g1} - t_{g2}}{t_{g1}}$ was used.

The calculations were carried out for different gas velocities (0.05-0.5) m/s, particle sizes (0.002-0.01) m, their thermal conductivity (0.75-11) W/(m K), and Reynolds numbers (10-1000) at $q_v = 0$.

Figure 1 shows the temperature distributions along the layer height according to modification 1 for two modes. Modification 2 gives qualitatively the same results with certain quantitative discrepancies.

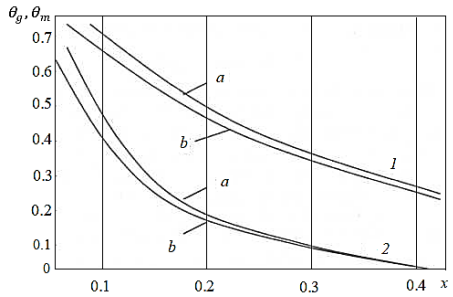

Fig. 1 Temperature fields of the gas (a) and solid (b) components of the layer, calculated according to modification 1 at d_m = 9.3 mm, λ_m = 11 W/(m·K). 1 – Re = 500, 2 – Re = 50

The analysis shows that the values of δ_m, δ_g are significantly affected by the Reynolds number. Thus, at $\mathrm{d_m} = 9.3$ mm, $\lambda_m = 111$ W/(m·K), $\mathrm{x} = 0.5$ m, at $\mathrm{Re} = 50 - \delta_g = 5.7\%$, $\delta_m = 14.3\%$, and at $Re = 350 - \delta_g = 1\%$; $\delta_m = 5.8\,\%$. As the particle diameter increases, the discrepancy between the models grows. Thus, at $\mathrm{Re} = 100$, $\mathrm{x} = 0.17$ m, a change in diameter from 2.95 mm to 9.3 mm leads to an increase in δ_g from 1.4% to 3.3%, δ_m – from 3.2% to 10.5%.

With an increase in the thermal conductivity of the particle material, the discrepancy also increases. Thus, at $\mathrm{Re} = 100$, $\mathrm{x} = 0.17$ m, $\mathrm{d_m} = 2.95$ m, a change in λ_m from 2 to 11 W/(m K) leads to an increase in δ_g from 0.5 to 1.4%, δ_m is from 2.2 to 3.3%.

Thus, for a fixed geometry of the immersed surfaces, the difference in temperatures calculated using both modifications of the model depend on the Reynolds number, the properties of the components, and the longitudinal coordinate.

The convergence criterion can be the complex $\gamma = \frac{a_{cm}^{m} \cdot F_{cm}}{a_M \cdot a \cdot (1-\beta_2)}$ (1/γ), which takes these factors into account.

Fig. 2 shows the dependence of the relative temperature differences on 1/γ. The values of δ_g, δ_m in the region of small values of the $1/\gamma$ complex decrease noticeably with its increase; at $1/\gamma$ >12 the rate of dependence decreases, and at $1/\gamma$ > 30 practical self-similarity is observed.

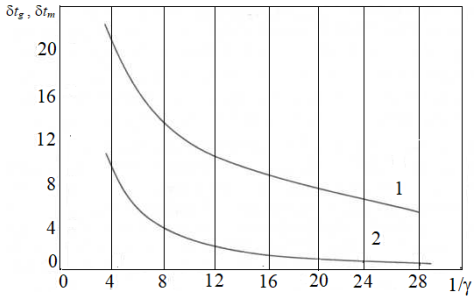

Fig. 2 The influence of complex 1/γ on the difference in temperatures calculated using modifications 1 and 2:
1 – solid component, 2 – gas component λ = 11 W/(m·K), Re = 50, d_m= 9.3 mm, x = 0.17 m

In the considered range of change of 1/γ, the values of δ_m are $(4 - 20)\%$, δ_g $(1 - 10)\%$, at 1/γ = *idem*, $\delta_m > \delta_g$.

The condition 1/γ >30 (or γ < 0.03) is satisfied at high Reynolds numbers, small λ_m, d_m, x, when the thermal resistance of intercomponent heat exchange is small compared to the resistance of heat transfer from the solid component to the wall. Fulfilment of this condition ensures satisfactory agreement between the temperatures calculated using both modifications of the models.

Fig. 3 compares the experimental temperature distributions of the components in the layer are compared with the calculated ones (for the experimental conditions, both modifications of the model give almost identical results). Satisfactory qualitative and quantitative agreement between experimental and calculated data indicates that both modifications of the model correctly describe the main patterns of heat transfer in a blown layer with immersed heat transfer surfaces [24, 25, 26].

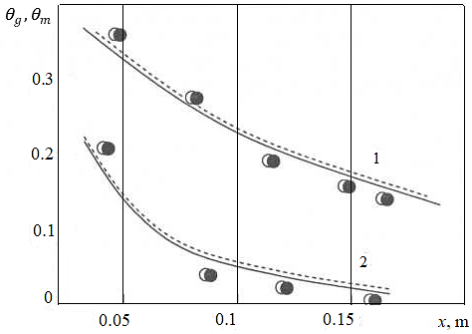

Fig. 3 Comparison of experimental and calculated temperature fields in the layer

$$D = 0.0335\ \mathrm{m},\ \frac{S_1}{D} = 2,\ \frac{S_2}{D} = 2,\ d_m = 3\ \mathrm{mm},$$
$$\lambda_m = 0.756\ \mathrm{W/(m \cdot K)}.$$
$$1 - Re = 290, 2 - Re = 29.$$

Solid and dotted curves are the calculated temperatures of gas and particles, respectively; dots are experimental temperatures of gas and particles

Based on these models, analytical dependences were obtained for the temperature distribution of the components in a stationary blown layer with heat sources and submerged surfaces, and in a moving layer during conductive-convective and microwave-convective drying.

CONCLUSIONS

The sensitivity of the models to errors in determining the heat transfer coefficients of the components a_{cm}^{m}, a_{cm}^{g}, a_{cm}^{g*} was analyzed. It was found that this error significantly affects the values of the calculated temperatures for both modifications of the model.

For example, with $Re = 25$, $\lambda_m = 0.75$ W/((m·K)), $d_m = 2.95$ mm, $x = 0.17$ m, when a_{cm}^{g} and a_{cm}^{m} change by 10% and 20%, the temperatures of the components change by 14% and 31%, respectively. Thus, inaccuracy in determining the heat transfer coefficients leads to significant errors in the calculation of temperature fields. These data make it possible to formulate requirements for the accuracy of experimental data. Experiments are significantly simplified and give less error in determining the heat transfer coefficient a_{cm}^{g*} than a_{cm}^{g} and a_{cm}^{m}. Therefore, if the condition $\gamma \ll 0.03$ is met, it is more

convenient to use the second modification of the calculation model, determining a_{cm}^{g*} from experimental experiments. If this condition is not met, the first modification of the model should be applied using experimental values a_{cm}^{g}, a_{cm}^{m} for each of the components.

Further research will be aimed at developing original methods and stands for determining the heat transfer coefficients of layer components, moisture and heat transfer coefficients in dispersed materials during drying. Experimental data will be obtained on the structure, aerodynamic resistance and heat exchange of the blown layer with bundles of smooth and finned pipes, the heat transfer coefficients of the layer components with the surface in the presence and absence of internal sources, on the change in the heat and moisture transfer coefficients of a layer of a number of materials during the drying process, as well as formulas for calculating these characteristics.

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