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CONTINUOUS BEAM STRUCTURE SUBJECTED TO SYNCHRONOUS VERSUS ASYNCHRONOUS DISPLACEMENTS SOLVED USING THE THREE MOMENTS EQUATION METHOD

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Abstract: In the following report are presented the hand calculation of a continuous 2 bay beam solved using The Method of Three Moments considering the imposed displacements due to earthquake. The ground motions can be uniform applied to the structure whereas in the mean time they can be considered as asynchronous imposed displacements. The dynamic equations can be adapted for imposed displacements that can take into account the spatial variability of seismic ground motions.

Keywords: displacement, accelerogram, asynchronous displacements, rigid supports, synchronous displacements, modal analysis

1. Introduction

Bridges represent art works which show the ability of the structural engineer to avoid various obstacles due to nature. Structural systems used in the design of bridges are many and variate, and can be continuous beams, truss beams etc.

For buried pipelines and tunnels, the spatial variability in the excitation constitutes, essentially, the seismic load for the structures. For pipelines and tunnels, that are buried in uniform site conditions and subjected to uniform excitations, the response is a stress-free rigid body motion. In-between these two extreme cases, lie the response patterns of dams and bridges. The pattern that emerges in the comparison of the response of these structures to uniform and spatially variable excitations is complex. Spatially variable excitations induce pseudo-static internal forces in the structures, which the uniform excitations do not, because they result in a pseudo-static, rigid-body deformation. For the dynamic response of lifeline systems, all studies conclude that spatially variable seismic ground motions excite the dynamic modes of the structures differently than uniform excitations.[12]

In the majority of structures ground motion recordings, accelerograms are applied uniformly at the base of the structure. What has been studied over the years is the following problem: If structures with support conditions at comparable distances (e.g. Bridge piers, rafts for equipment in a nuclear powerplant) are there any significant differences if the ground motions are applied as asynchronous movement.

In the present report is presented a way of solving by hand a structure of the type of a continuous beam with the help of the Method of the Three Moments considering support displacement. At the same time, it will be presented the hand calculations for seismic loading of the supports with synchronous and asynchronous ground motions.

In current applications supports are considered that the ground motions for them are uniform and synchronous.

The effect of the spatial variation of seismic ground motions on long-span bridges has been recognized since the middle 1960's, when pioneering research work was conducted by Bogdanoff.[12]

2. The equation of movement for the continuous beam structure

This chapter presents the equations of movement corresponding to multi-support excitation and to the phenomenon of spatial variable ground motions. The types of excitations are for the supports of a specific portfolio of structures above mentioned.

In order to analyze these types of structures is necessary to include in the equations of motion the movement of the supports from the interface between the structure and the ground.

The dynamic displacement vector has two components $\{u^t\}$ that includes N degrees of dynamic freedom, corresponding to the structure itself, where the t index refers to the total displacement; whereas $\{u_g\}$ corresponds to the N_g components of displacement of the ground supports.

The equation of instantaneous dynamic equilibrium for all degrees of dynamic freedom is partitioned as follows:

$$\begin{bmatrix} [m] & [m_g] \\ [m_g]^T & [m_{gg}] \end{bmatrix} \begin{Bmatrix} \ddot{u}^t \\ \ddot{u_g} \end{Bmatrix} + \begin{bmatrix} [c] & [c_g] \\ [c_g]^T & [c_{gg}] \end{bmatrix} \begin{Bmatrix} \dot{u}^t \\ \dot{u_g} \end{Bmatrix} + \begin{bmatrix} [k] & [k_g] \\ [k_g]^T & [k_{gg}] \end{bmatrix} \begin{Bmatrix} u^t \\ u_g \end{Bmatrix} = \begin{Bmatrix} 0 \\ p_g(t) \end{Bmatrix} \tag{1}$$

In equation (1) the following terms have the meanings presented here:

The mass matrix contains the following matrices:

$[m]$ represents the mass matrix of the superstructure.

$[m_g]$ represents the mass matrix associated with the ground motions.

$[m_g]^T$ is the transpose matrix associated with the ground motions.

$[m_{gg}]$ represents the lumped masses in the supports.

The damping matrix contains the following matrices:

$[c]$ represents the damping matrix of the supertructure

$[c_g]$ represents the damping matrix associated with the ground motion of the supports.

$[c_g]^T$ represents the transpose damping matrix associated with the ground motion of the supports

$[c_{gg}]$ represents the matrix of damping associated with the ground motions.

The stiffness matrix contains the following matrices:

$[k]$ represents the stiffness matrix of the supertructure

$[k_g]$ represents the stiffness matrix associated with the ground motion of the supports.

$[k_g]^T$ represents the transpose stiffness matrix associated with the ground motion of the supports

$[k_{gg}]$ represents the stiffness matrix associated with the ground motions.

The first vector in equation (1) contains the two accelerations regarding the superstructure' degrees of freedom ($\ddot{u}^t$)-the total acceleration and the ground motion degrees of freedom ($\ddot{u_g}$).

The second vector in equation (1) contains the two velocities regarding the superstructure' degrees of freedom ($\ddot{u}^t$)-the total velocities and the ground motion degrees of freedom ($\ddot{u_g}$).

The third vector in equation (1) contains the two displacements regarding the superstructure' degrees of freedom ($\ddot{u}^t$)-the total displacements. and the ground motion degrees of freedom ($\ddot{u_g}$).

All accelerations, velocities, and displacements marked with the superscript "t" presented here in equation (1) are imposed to the structure by the ground movement. In fact, the matrix multiplied by a vector represents a force. Hence each term represents the inertial force of a part of the system over which act the ground motions.

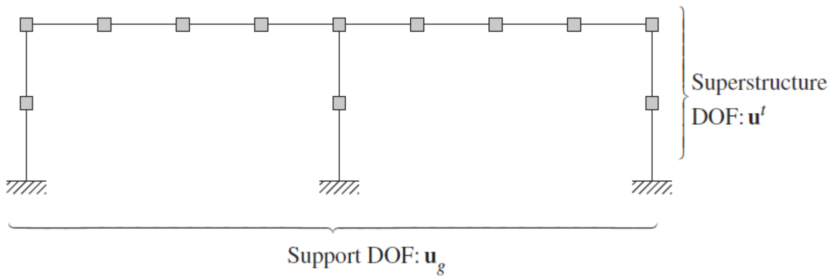


Fig. 1 - Specifying the degrees of freedom for the superstructure and for the support.[2]

In equation (1) mass, damping and rigidity characteristics are determined using known procedures from dynamics of structures whereas the support's displacement $\{\ddot{u}_g(t)\}, \{\dot{u}_g(t)\}$ $și$ $\{u_g(t)\}$ need to be specified.

The next step determines the vector of total displacements $\{u^t(t)\}$,associated with the degrees of dynamic freedom of the superstructure and the forces in the supports.

The displacements vector is separated in two vector components

$$\begin{Bmatrix} \{u^t\} \\ \{u_g\} \end{Bmatrix} = \begin{Bmatrix} \{u^s\} \\ \{u_g\} \end{Bmatrix} + \begin{Bmatrix} \{u\} \\ \{0\} \end{Bmatrix} \tag{2}$$

In this equation $\{u^s\}$ are the structural displacements due to statical action of the imposed ground displacements at each moment of time.

The two types of displacements are governed by the following matrix equation.

$$\begin{bmatrix} [k] & [k_g] \\ [k_g]^T & [k_{gg}] \end{bmatrix} \begin{Bmatrix} \{u^s\} \\ \{u_g\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{{p_g}^s\} \end{Bmatrix} \tag{3}$$

Where the term ${p_g}^s$ represents the necessary forces to impose the u_g displacements, that vary with time. The u^s vector is called quasi-static displacement vector.

It can be observed that if ${p_g}^s = 0$ the structure has movements of a solid body and has zero redundancy. The rigid body movement of the structure is when all imposed displacements to the support are synchronous.

The other structural displacements denoted by u(t)are dynamic displacements, due to the fact that there is a dynamic analysis required to obtain them.

Now that the equation for the total displacement has been divided in the quasi-static and the dynamic component, we revisit the initial equation.

$$[m]\{\ddot{u}^t\} + [m_g]\{\ddot{u_g}\} + [c]\{\dot{u}^t\} + [c_g]\{\dot{u_g}\} + [k]\{u^t\} + [k_g]\{u_g\} = 0 \tag{4}$$

$$[m]\{\ddot{u}^t\} + [m_g]\{\ddot{u_g}\} + [c]\{\dot{u}^t\} + [c_g]\{\dot{u_g}\} + [k]\{u^t\} + [k_g]\{u_g\} = 0 \tag{5}$$

$$[m]\{\ddot{u}^t\} + [c]\{\dot{u}^t\} + [k]\{u^t\} = \{p_{eff}(t)\} \tag{6}$$

$\{p_{eff}(t)\}$ represents the forces that act on the structure due to the imposed ground movement.

$$\{p_{eff}(t)\} = -([m]\{\ddot{u}^s\} + [m_g]\{\ddot{u_g}\}) + -\left([c]\{\dot{u}^s\} + [c_g]\{\dot{u_g}\}\right) + -([k]\{u^s\} + [k_g]\{u_g\}) \tag{7}$$

$$[k]\{u^s\} + [k_g]\{u_g\} = 0. \tag{8}$$

We denoted the influence matrix as $[l] = -[k]^{-1}[k_g]$.This matrix describes the influence of the support displacements over the structure's displacements.

$$\{p_{eff}(t)\} = -([m]\{l\} + [m_g])\{\ddot{u_g}(\mathrm{t})\} + -([c]\{l\} + [c_g])\{\dot{u}_g(t)\} \tag{9}$$

If the accelerations and velocities $\ddot{u_g}(\mathrm{t})$ și $\dot{u}_g(t)$ are prescribed then $p_{eff}(t)$is known so the formulation of the problem is complete.

Simplifying the term $p_{eff}(t)$.

For many practical applications the next simplification of the force vector is possible due to two reasons: the first reason is that the term regarding the damping is zero if the damping matrix is proportional to the rigidity matrix($c = a_1k$ și $c_g = a_1k_g$) due to the equation

$$[k]\{u^s\} + [k_g]\{u_g\} = 0.$$

Whereas when the term regarding the damping in the equation of $p_{eff}(t)$ is not equal to zero for arbitrary forms of damping, the contribution is relatively small regardless to the inertial term, so it could be negligible.

The second reason is that lumped as concentrated mass models in the dynamic degrees of freedom the mass matrix is diagonal and m_g is a null matrix.

Hence with these observations $\{p_{eff}(t)\} = -[m]\{l\}\{\ddot{u_g}(\mathrm{t})\}$.

The interpretation of $\{p_{eff}(t)\}$.

In the following equation we will analyze$\{u^s(t)\}$

$$\{u^s(t)\} = \sum_{l=1}^{N_g} \{l_l\}\, u_{gl}(t) \tag{10}$$

Where $\{l_l\}$ is the ”lth”–column of the influence matrix and it is associated with the displacements in the supports $u_{gl}(t)$.

The lth column represents the forces regarding the supports due to acceleration at the l h degree of freedom associated to the support which has the same form as structures with one support (this idea is used at structures which move synchronous)

2.1. The structure’s stiffness matrix obtained through the three moments equation considering imposed displacements

There will be analyzed a structure which is represented as a continuous beam. This continuous beam describes the structure of a bridge with the following geometric characteristics: EI is the flexural stiffness and the mass of each part of the structure between the two supports are lumped as concentrated mass. [2]

The numerical values used in the application

L =10.000m-the length between two consecutive supports

E=2.1x10^5 MPa-modulus of elasticity

I=14.40*1x10^6 MPa-moment of inertia

The method comes from structural analysis and is known as the three moments equation. The idea of the method is that between two portions of a continuous beam the relative rotation is equal to zero.

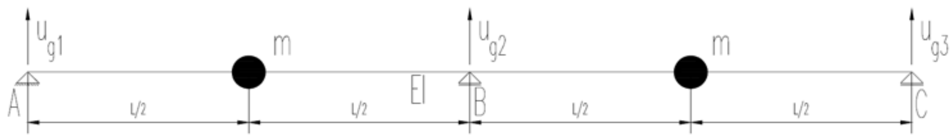


Fig. 2 -The Geometry of the structure

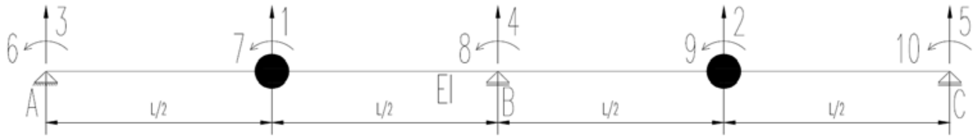


Fig. 3 - The degrees of freedom for each characteristic point in the structure.

In the following steps we will determine the stiffness matrix.

For the imposed displacement at support, j

$$I_c = I - \text{conventional moment of inertia}$$

There will be determined the transformed lengths for each part of the structure

$$\lambda_{ij} = \frac{L}{2} * \frac{I_c}{I}\ ; \lambda_{jk} = \frac{L}{2} * \frac{I_c}{I};\ \lambda_{kl} = \frac{L}{2} * \frac{I_c}{I};\ \lambda_{lm} = \frac{L}{2} * \frac{I_c}{I} \tag{11}$$

Now we will impose the contour conditions regarding the bending moment:

$$M_i = 0;\ M_m = 0;$$

Due to the fact that there is not any loading in the vertical direction the bending moments due to them are equal to zero.

$m_{j.left.} = 0;$ represents the bending moment for the "j" node of the structure, on the left side of "j"

$m_{j.right.} = 0;$ represents the bending moment for the "j" node of the structure on the right side of "j"

Hence the rotation and signs of the rotation are established here:

$$\psi_{ij} = \frac{2}{L};\ \psi_{jk} = -\frac{2}{L}$$

The „ i-j-k" Region of the structure.

$$\lambda_{ij}M_i + 2(\lambda_{ij} + \lambda_{jk})M_j + \lambda_{jk}M_k + \lambda_{ij}m_{j.stg.} + \lambda_{jk}m_{j.drp.} + 6EI_c(\psi_{jk} - \psi_{ij}) = 0 \tag{12}$$

Due to the fact that there is not any loading in the vertical direction the bending moments due to them are equal to zero.

$m_{k.left.} = 0$; represents the bending moment for the "k" node of the structure, on the left side of "k"

$m_{k.right.} = 0$; represents the bending moment for the "k" node of the structure, on the right side of "k"

Hence the rotation and signs of the rotation are established here:

$$\psi_{kl} = 0$$

The „ j-k-l" Region of the structure.

$$\lambda_{jk}M_j + 2(\lambda_{jk} + \lambda_{kl})M_k + \lambda_{kl}M_l + \lambda_{jk}m_{k.stg.} + \lambda_{kl}m_{k.drp.} + 6EI_c(\psi_{kl} - \psi_{jk}) = 0 \tag{13}$$

Due to the fact that there is not any loading in the vertical direction the bending moments due to them are equal to zero.

$m_{l.left.} = 0$; represents the bending moment for the "l" node of the structure, on the left side of "l".

$m_{l.right.} = 0$; represents the bending moment for the "l" node of the structure, on the right side of "l".

Hence the rotation and signs of the rotation are established here:

$$\psi_{lm} = 0$$

The „k-l-m". region of the structure.

$$\lambda_{kl}M_k + 2(\lambda_{kl} + \lambda_{lm})M_l + \lambda_{lm}M_m + \lambda_{kl}m_{l.stg.} + \lambda_{lm}m_{l.drp.} + 6EI_c(\psi_{lm} - \psi_{kl}) = 0 \tag{14}$$

Hence the stiffness term of the matrix corresponding to the first column using the following principle from mechanics of structures: Each part of the structure is loaded with the end moments and the shear forces represent the stiffnesses needed.

Below we obtain the stiffness matrix

$$[\hat{k}] = \frac{EI}{L^3}\begin{bmatrix} 78.86 & 30.86 & -29.14 & -75.43 & -5.14 \\ 30.86 & 78.86 & -5.14 & -75.43 & -29.14 \\ -29.14 & -5.14 & 12.86 & 20.57 & 0.86 \\ -75.43 & -75.43 & 20.57 & 109.71 & 20.57 \\ -5.14 & -29.14 & 0.86 & 20.57 & 12.86 \end{bmatrix}$$

Next step is to partition the displacements vector and the stiffness matrix

$$\{u\} = \{u_1 \quad u_2\}^T$$

$$\{u_g\} = \{u_3 \quad u_4 \quad u_5\}^T$$

$$[\hat{k}] = \begin{bmatrix} [k] & [k_g] \\ [k_g^T] & [k_{gg}] \end{bmatrix}$$

$$[k] = \frac{EI}{L^3}\begin{bmatrix} 78.86 & 30.86 \\ 30.86 & 78.86 \end{bmatrix}$$

$$[k_g] = \frac{EI}{L^3}\begin{bmatrix} -29.14 & -75.43 & -5.14 \\ -5.14 & -75.43 & -29.14 \end{bmatrix}$$

$$[k_{gg}] = \frac{EI}{L^3}\begin{bmatrix} 12.86 & 20.57 & 0.86 \\ 20.57 & 109.71 & 20.57 \\ 0.86 & 20.57 & 12.86 \end{bmatrix}$$

The mass matrix for the two degrees of freedom.

$$[\mathrm{M}] = m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Next the influence matrix is determined.

$$[l] = -[k]^{-1}[k_g]$$

$$[l] = \begin{bmatrix} +0{,}40625 & +0{,}68750 & -0{,}09375 \\ -0{,}09375 & +0{,}68750 & +0{,}40625 \end{bmatrix}$$

The influence vectors are as follows:

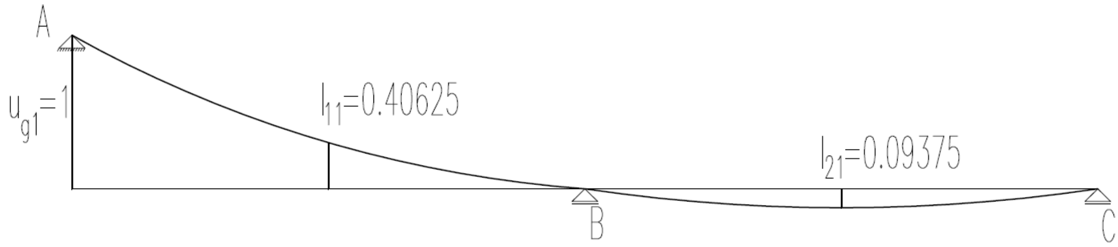
$$\{l_1\} = [+0{,}40625 \quad -0{,}09375]^T$$

$$\{l_2\} = [+0{,}68750 \quad +0{,}68750]^T$$

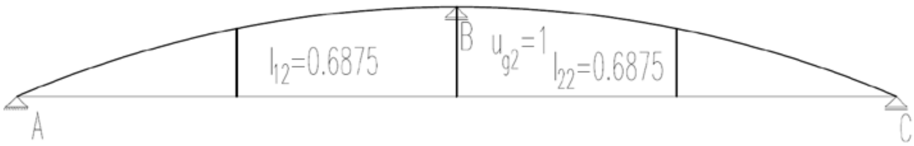
$$\{l_3\} = [-0{,}09375 \quad +0{,}40625]^T$$

The displacement diagrams are presented below:

$$\{l_1\} = [+0{,}40625 \quad -0{,}09375]^T$$



$$\{l_2\} = [+0{,}68750 \quad +0{,}68750]^T$$



$$\{l_3\} = [-0{,}09375 \quad +0{,}40625]^T$$

We conclude this part of the application with the equations of movement.

$$[\mathrm{M}]\ddot{u} + [k]u = p_{eff}(t)$$

3. Seismic analysis for multi-support excitation

In this part of the analysis the theory is developed for systems with multiple degrees of freedom imposed at supports $\ddot{u}_{gl}(t)$ $(l = \overline{1, N_g})$.

The modal equation has the following expression:

$$\ddot{q_n} + 2\zeta_n\omega_n\dot{q_n} + \omega_n q_n = -\sum_{l=1}^{N_g} \Gamma_{nl}\,\ddot{u}_{gl}(t) \tag{15}$$

$$\Gamma_{nl} = \frac{L_{nl}}{M_n} \tag{16}$$

$$L_{nl} = \{\Phi_n\}^T[m]\{l_l\} \tag{17}$$

$$M_n = \{\Phi_n\}^T[m]\{\Phi_n\} \tag{18}$$

The solution of the above equation can be summarized as follows:

$$q_n(t) = \sum_{l=1}^{N_g} \Gamma_{nl} D_{nl}(t) \tag{19}$$

The term $D_{nl}(t)$ represents the response in terms of displacement of the nth mode of vibration for the system with one degree of dynamic freedom at the imposed displacement $\ddot{u}_{gl}(t)$.

The response of the structure in terms of displacement contains the two components:

The dynamic displacements have the following form:

$$u(t) = \sum_{l=1}^{N_g}\sum_{n=1}^{N} \Gamma_{nl}\{\Phi_n\}D_{nl}(t) \tag{20}$$

The quasi-static displacements.
Combining the two components we obtained the total response.

$$\{u^t(t)\} = \sum_{l=1}^{N_g}\{l_l\}u_{gl}(t) + \sum_{l=1}^{N_g}\sum_{n=1}^{N} \Gamma_{nl}\{\Phi_n\}D_{nl}(t) \tag{21}$$

The forces in the structural elements can be obtained through the total displacements and the prescribed displacements without dynamic analysis. There exist two methods of obtaining the forces in the structure's elements.

In the first method the forces in the structure's elements are calculated starting from the displacements of the nodes and the stiffness properties. This method is used at computer programs in case of multi support-imposed displacements.

For the purpose of our study, it is instructive to generalize the second method based on statical equivalent forces. The statical forces in case of the degree of freedom are given by the following equation.

$$\{f_S\} = [k]\{u^t\} + [k_g]\{u_g\} \tag{22}$$

$$\{u^t\} = \{u^s\} + \{u\}$$

$$[k]\{u^s\} + [k_g]\{u_g\} = 0.$$

Hence

$$\{f_S\} = [k]\{u(t)\} \tag{23}$$

These forces depend only on the dynamic displacements

$$\{f_S(t)\} = \sum_{l=1}^{N_g}\sum_{n=1}^{N}\Gamma_{nl}k\{\Phi_n\}D_{nl}(t) \tag{24}$$

Which can be expressed in term of the mass matrix as follows:

$$\{f_S(t)\} = \sum_{l=1}^{N_g}\sum_{n=1}^{N}\Gamma_{nl}[m]\{\Phi_n\}A_{nl}(t) \tag{25}$$

$$A_{nl}(t) = \omega^2 D_{nl}(t)$$

This is a pseudo-acceleration of the n th degree of freedom due to the imposed acceleration in the support.

The equivalent static forces on the direction of the degree of freedom are defined as:

$$\{f_{Sg}\} = [k_g]^T\{u^t\} + [k_{gg}]\{u_g\} \tag{26}$$

$$\{u^t\} = \{u^s\} + \{u\}$$

$$[k_g^T]u^s + k_{gg}u_g = p_g^s(t).$$

$$f_{Sg}(t) = k_g^T u(t) + p_g^s(t) \tag{27}$$

Substituting $\{u^t\}$ in the expression of $\{u^t\} = \{u^s\} + \{u\}$ și $[k_g^T]u^s + k_{gg}u_g = p_g^s(t)$.

$$f_{Sg}(t) = k_g^T u(t) + p_g^s(t) \tag{28}$$

We must observe the fact that the forces $f_{Sg}(t)$ depend on the structural displacements of the degrees of freedom but also of the displacements from the supports, hence the forces can not be obtained through static analysis.

The forces that result in the supports can be expressed as:

$$f_{Sg}(t) = \sum_{l=1}^{N_g}\left(k_g^T l_l + k_{gg}{}^l\right)u_{gl}(t) + \sum_{l=1}^{N_g}\sum_{n=1}^{N}\Gamma_{nl}k_g^T m\Phi_n A_{nl}(t) \tag{29}$$

Hence the forces in the structure are evaluated at each instance of time through static analysis of the structure.

3.1. Seismic response due to asynchronous movement

For the continuous beam analyzed above we will consider that the supports have the same accelerogram but with a delay of 0t, t and 2t.[2]

Now there will be determined the following instantaneous characteristics: The displacements of the two masses, the bending moments at the middle of each part of the beam and the bending moments in the central support.

We will express all results in terms of $D_n(t)$ and $A_n(t)$,respectively the response in displacements and in pseudo-accelerations associated with the nth degree of freedom at the vertical accelerations

The purpose of this report is to compare the results obtained through synchronous asynchronous movement described by the accelerogram $\ddot{u}_g(t)$.

Now there will be evaluated the eigen value and eigen vectors for the structure using the stiffness matrix.

$$|[K]-\omega^2[M]|=0$$

$$[K]=\frac{EI}{L^3}\begin{bmatrix}78,86 & 30,86\\ 30,86 & 78,86\end{bmatrix}$$

$$[M]=m\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\begin{vmatrix}78,86\frac{EI}{L^3}-\omega^2 m & 30,86\frac{EI}{L^3}\\ 30,86\frac{EI}{L^3} & 78,86\frac{EI}{L^3}-\omega^2 m\end{vmatrix}=0$$

$$\left(78,86\frac{EI}{L^3}-\omega^2 m\right)^2=\left(30,86\frac{EI}{L^3}\right)^2$$

$$78,86\frac{EI}{L^3}-\omega^2 m=\pm 30,86\frac{EI}{L^3}$$

$$\omega_{1<}\omega_2$$

$${\omega_1}^2 m=48\frac{EI}{L^3}$$

The eigen values are obtained through known procedures.

$$\omega_1=6,928\sqrt{\frac{EI}{mL^3}}$$

$$\omega_2=10,474\sqrt{\frac{EI}{mL^3}}$$

The eigen vectors are obtained.

$$\Phi_{11}=-1$$

$$\{\Phi_1\}=\begin{Bmatrix}-1\\ 1\end{Bmatrix}$$

$$\{\Phi_2\}=\begin{Bmatrix}1\\ 1\end{Bmatrix}$$

$$\Phi=[\{\Phi_1\}\quad\{\Phi_2\}]$$

The modal participating factors.

$$\Gamma_{nl}=\frac{\mathrm{L}_{nl}}{M_n}$$

$$\mathrm{L}_{nl}=\Phi_n^T m l_l$$

$$n=\overline{1,2}$$

$$l=\overline{1,3}$$

$$\mathrm{L}_{11}=\Phi_1^T m l_1$$

$$\mathrm{L}_{11}=-0,5m$$

$$L=[L_{nl}]=m\begin{bmatrix}-0,5 & 0 & 0,5\\ 0,3125 & 1,3750 & 0,3125\end{bmatrix}\begin{matrix}\leftarrow mode\ 1\\ \leftarrow mode\ 2\end{matrix}$$

$$M_n = \Phi_n^T m \Phi_n, n = \overline{1,2}$$

$$M_1 = 2m$$

$$M_2 = 2m$$

$$\Gamma_{nl} = \begin{bmatrix} -0,25000 & 0 & 0,25000 \\ 0,15625 & 0,6875 & 0,15625 \end{bmatrix} \begin{matrix} \leftarrow mode\ 1 \\ \leftarrow mode\ 2 \end{matrix}$$

It is determined the modal response through the accelerogram.

$$\ddot{u}_{g1}(t) = \ddot{u}_g(t)$$

$$\ddot{u}_{g2}(t) = \ddot{u}_g(t - t')$$

$$\ddot{u}_{g3}(t) = \ddot{u}_g(t - 2t')$$

$$D_{n1}(t) = D_n(t)$$

$$A_{n1}(t) = A_n(t)$$

$$D_{n2}(t) = D_n(t - t')$$

$$A_{n2}(t) = A_n(t - t')$$

$$D_{n3}(t) = D_n(t - 2t')$$

$$A_{n3}(t) = A_n(t - 2t')$$

The total response is evaluated in term of displacements.

$$\begin{Bmatrix} u_1^t(t) \\ u_2^t(t) \end{Bmatrix} = \begin{Bmatrix} 0,40625 \\ -0,09375 \end{Bmatrix} u_g(t) + \begin{Bmatrix} 0,6875 \\ 0,6875 \end{Bmatrix} u_g(t - t') + \begin{Bmatrix} -0,09375 \\ 0,40625 \end{Bmatrix} u_g(t - 2t')$$
$$+ -0,25 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} D_1(t) + 0 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} D_1(t - t') + 0,25 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} D_1(t - 2t')$$
$$+ 0,15625 \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} D_2(t) + 0,6875 \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} D_2(t - t')$$
$$+ 0,15625 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} D_2(t - 2t')$$

The statical forces are evaluated for $N = 2$ $și$ $N_g = 3$ hence they are:

$$f_S(t) = + - 0,25 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} mA_1(t) + 0 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} mA_1(t - t') + 0,25 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} mA_1(t - 2t')$$
$$+ 0,15625 \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} mA_2(t) + 0,6875 \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} mA_2(t - t')$$
$$+ 0,15625 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} mA_2(t - 2t')$$

The statical forces from the supports are evaluated

$$f_{Sg}(t) = \begin{Bmatrix} -0,125 \\ 0 \\ 0,125 \end{Bmatrix} mA_1(t) + \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} mA_1(t - t') + \begin{Bmatrix} 0,125 \\ 0 \\ -0,125 \end{Bmatrix} mA_1(t - 2t')$$
$$+ \begin{Bmatrix} -0,0488 \\ -0,2148 \\ -0,0488 \end{Bmatrix} mA_2(t) + \begin{Bmatrix} -0,2149 \\ 0,9454 \\ 0,2149 \end{Bmatrix} mA_2(t - t')$$
$$+ \begin{Bmatrix} -0,0488 \\ -0,2148 \\ -0,0488 \end{Bmatrix} mA_2(t - 2t') + \begin{Bmatrix} +1,50 \\ -3,00 \\ +1,50 \end{Bmatrix} \frac{EI}{L^3} u_g(t)$$
$$+ \begin{Bmatrix} -3,00 \\ +6,00 \\ -3,00 \end{Bmatrix} \frac{EI}{L^3} u_g(t - t') + \begin{Bmatrix} +1,50 \\ -3,00 \\ +1,50 \end{Bmatrix} \frac{EI}{L^3} u_g(t - 2t')$$

The specified bending moments are evaluated

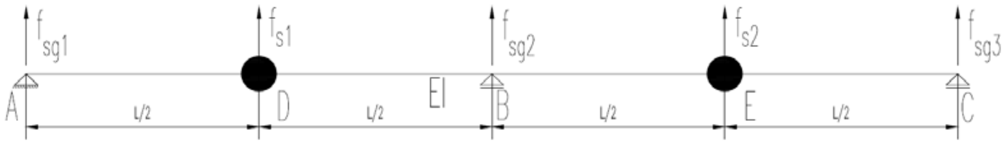


Fig. 4 -The continuous beam structure with the prescribed displacements

$$M_D = mL\big(-0{,}0625A_1(t) + 0A_1(t-t') + -0{,}0625A_1(t-2t')\big) + mL\big(-0{,}0244A_2(t) + -0{,}1074A_2(t-t') + -0244A_2(t-2t')\big) + \frac{EI}{L^2}(0{,}75u_g(t) + -1{,}50u_g(t-t') + 0{,}75u_g(t-2t'))$$

$$M_E = mL\big(+0{,}0625A_1(t) + 0A_1(t-t') + 0{,}0625A_1(t-2t')\big) + mL\big(-0{,}0244A_2(t) + 0{,}1074A_2(t-t') + -0244A_2(t-2t')\big) + \frac{EI}{L^2}(0{,}75u_g(t) + -1{,}50u_g(t-t') + 0{,}75u_g(t-2t')$$

$$M_B = mL\big(-0{,}0293A_2(t) + 0{,}1289A_2(t-t') + 0{,}0293A_2(t-2t')\big) + \frac{EI}{L^2}(1{,}50u_g(t) + -3{,}00u_g(t-t') + 1{,}50u_g(t-2t')$$

For the synchronous displacements $u_g(t)$

$$\Gamma_{nl}, l = 1$$

$$\Gamma_1 = 0$$

$$\Gamma_2 = 1$$

$$u(t) = \Gamma_2\Phi_2D_2(t)$$

It must be observed that only the second eigen mode is used in the analysis due to the fact that it is symmetrical, the first eigen mode being symmetrical.

The statical forces are given by:

$$f_S(t) = \Gamma_2 m\Phi_2A_2(t)$$

$$f_S(t) = m\begin{Bmatrix}1\\1\end{Bmatrix}A_2(t)$$

We obtain $f_{Sg}(t)$ by introducing the $u_g(t) = u_g(t-t') = u_g(t-t')$.

$$f_{Sg}(t) = \begin{Bmatrix}-0{,}3125\\-1{,}3750\\-0{,}3125\end{Bmatrix}mA_2(t)$$

The bending moment in the three points of the structure.

$$M_D = -0{,}15625mLA_2(t)$$

$$M_E = -0{,}15625mLA_2(t)$$

$$M_B = +0{,}18750mLA_2(t)$$

4. Conclusions and remarks

Structures that extend for long distances parallel to the ground, such as pipelines, tunnels, bridges and dams, are subjected to the effect of spatially variable ground motions during an earthquake.[12]

Newmark and Rosenbluth stated in 1971 regarding the seismic resistant design of bridges: “Within their diversity of dimensions, materials, and structural solutions, bridges have certain features in common from the viewpoint of their response to earthquakes. Their salient characteristic lies in the fact that their supports tend to undergo differential motions during earthquakes. This is partly due to distance between supports and partly due to the differences in geologic and topographic features at the supports or surrounding them. Therefore, even for short spans, there is a tendency for the abutments to move differentially.”[12]

If the prescribed displacements are identical, respectively the quasi-static forces are equal to zero and the is not any quasi-static component in the bending moments and all forces at the supports are calculated through elementary statics starting with the degrees of freedom.

In contrast with the synchronous displacements when the displacements are different the evaluation is more complex, in particular the statical forces associated with different displacements of the supports it is mandatory to include the forces from the supports.

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