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AN ALTERNATIVE METHOD FOR CHECKING REINFORCED CONCRETE ELEMENTS UNDER BIAXIAL BENDING

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Abstract: The article presents an original method for verifying reinforced concrete elements under biaxial bending with axial force, in structures subjected to seismic action. This method was easily implemented in automated calculation programs based on numerical analysis. The nonlinear behavior of the reinforced concrete cross section was considered. More importantly, we propose a new method of presenting the results of numerical analysis. This representation involves a two-dimensional graph that can be printed on paper, and it shows the boundary between the resistance reserve of the analyzed element and the direction of horizontal loading. The described method exhibits a very good convergence, allowing results to be calculated and presented rapidly to the program user. The presented method can be successfully applied to any type of geometry and reinforcement.

Keywords: fiber element; biaxial bending; reinforced concrete; cross section

1. Introduction

As human evolution and technology progressed, construction projects have become increasingly sophisticated, reaching impressive levels of complexity. The development of these projects requires specialists in increasingly diverse fields. Nowadays, it is unimaginable to execute such projects without the assistance of computers, utilizing CAD (computer-aided design) technology, and recently, even the integration of BIM (building integrated model) technology has become essential. Considering the highly complex and numerous calculations performed through computing technology, the need for the development of a representation method of results from numerical modeling becomes stringent. These systems should enable the reader to interpret analysis results and even to perform manual verifications to some degree. Those checks have the potential to identify errors in creating the digital model and even some program errors. The significance of these checks is evident from statements made by software authors who do not take any responsibility for the usage of their programs, or the results provided by them. Additionally, we must not overlook the catastrophic consequences that could arise from potential errors in analysis results. Considering the legislation in Romania, which requires the project to be verified by an independent individual or team separate from the design team, representing results should be in a printable format, allowing interpretation of the results outside the program's usage.

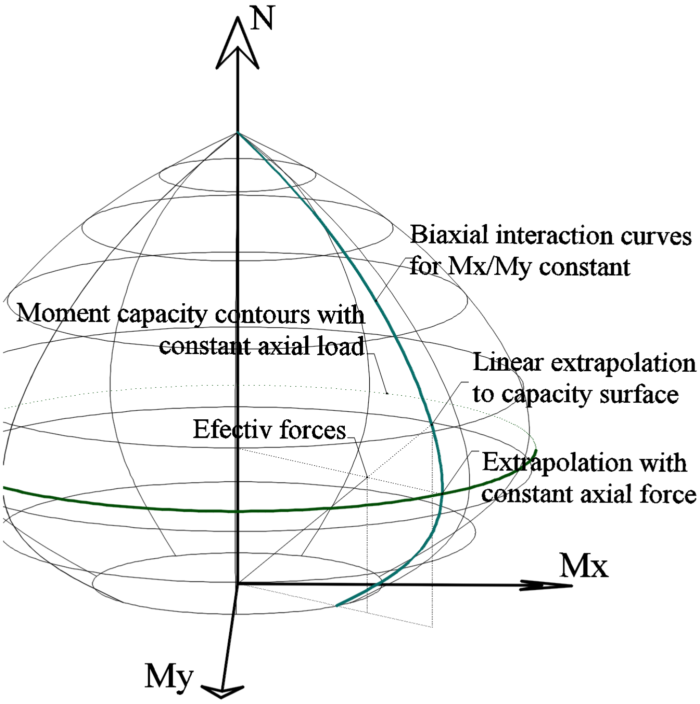


Fig. 1 - Interaction diagram N-Mx-My

An integral part of the results analysis and verification concerning the model of a reinforced concrete structure is to check the vertical elements for biaxial bending with axial force, particularly for small eccentricity compression. Before the widespread use of automated calculation in project development, abacuses were employed to assess the capacity of elements under this type of load. Presently, automated calculation methods with specialized software like ASEP [1], Extract, C.S.I. Col [2], are utilized for these verifications. They provide a three-dimensional interaction diagram in the form of an enclosed surface for each section, on which the effective values of forces resulting from structural calculations are overlaid manually or automatically. That enclosed surface is called "*Interaction diagram N-Mx-My*" Fig. 1 and is represented as a three-dimensional surface in a cartesian system with one of each component of the section force on each axis. To establish if a section of an element has enough strength capacity to carry the effective loads, it becomes necessary to extrapolate a given point on the Interaction diagrams. But as can be seen they are hard to read, especially when more data are shown. The need to simplify the view is obvious, so a two-dimensional plot is going to be used in this article. In order to achieve that the programs usually present some slice of the interaction diagram in the form of intersections of the *Interaction diagram* with various planes, but it is hard to get the right section, which contains the right point. In some cases, for a given section forces composed from the three components some program can give the maximum strain (or stress) in concrete respectively in reinforced steel. On the other hand, those programs can extrapolate the given two out of three components and compute the third component to obtain a point on the *Interaction diagrams.* There are some methods to do that extrapolation, for example in [4] two of those options are presented(Fig. 2). "Option I" is to keep the axial force constant, and a capable moment is determined. The other option is to have all effective internal resultants (both N and M) scaled, and a capable force is obtained. We will call this "option II". For practical reasons, engineers tend to synthesize the verifications by comparing certain quantities.

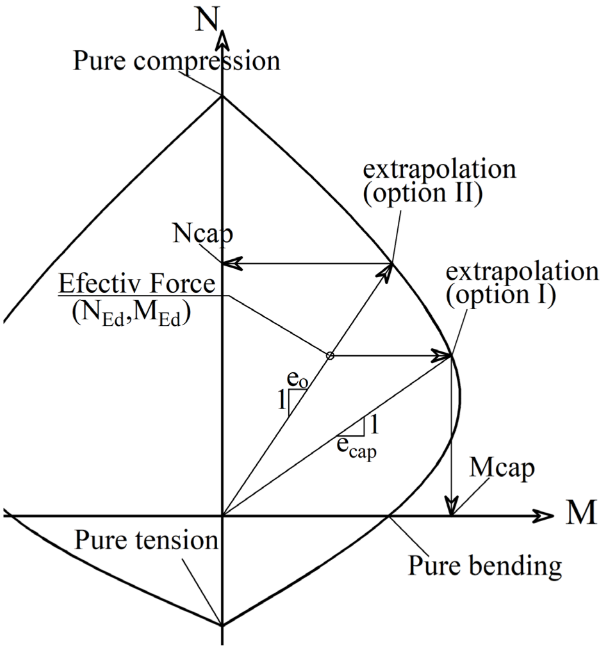


Fig. 2 - Extrapolation option

Considering the arbitrary nature of seismic loading, it would be better to have a graphic diagram which shows the variation of the reserve resistance of the section with a change of the direction of horizontal load. Because of the combination of vertical loads typically generated by gravity forces with the horizontal loads generated by the seismic load the proposed extrapolation does not work properly. "Option I" with axial force kept constant does not work because of the indirect effect of the beam which changes the axial force as a function of the horizontal load. "Option II" with proportionally scaling all the components of section force is not a perfect solution because the effect of gravity loads should be constant. Moreover, if some other element of the structure develops plastic hinges, multiplying the effective force is not a good idea because the effect of those elements should not be multiplied. Comparing maximum strains (or stresses) gives no clue about what changes correspond to a variation of the seismic load.

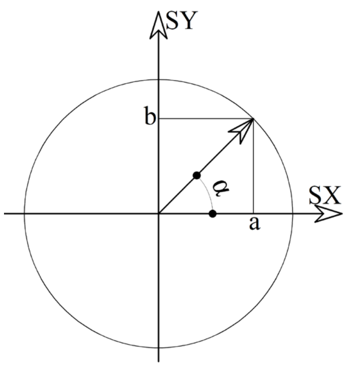


Fig. 3 - Representation system

Hence, we should aim to find a representation of the variation of the resistance reserve to some changes in the value or direction of the seismic load. Usually, the analysis of a structure consists in a three-dimensional model with some cases of vertical load and at least two cases of horizontal load for the seismic loads. If we admit linear superposition of loads, we can stipulate that if the seismic load is applied to the structure at an arbitrary direction at an angle α, that load can be decomposed into our two principal directions. If our cases of horizontal loads are in two normal directions, we will denote them as "SX" and "SY". For any direction of the seismic load there are two parameters (*a, b*) for which the effect of seismic load is identical with effect of SX multiplied with parameter *a* summed with effect of SY multiplied by *b*. Also, we have: $a^2+b^2=1$ which leads to a circle of radius one in the reference system presented in Fig. 3. We will try to represent the capacity of a given section in a plane, cartesian coordinate system which will represent *a* on the ordinate and *b* on the coordinate. Then, for example, when we have represented a point at (1.2;0.7) it means the interaction diagram is reached if we summed the effect of gravity load with the effect of SX multiplied by 1.2 and the effect of Sy multiplied by 0.7. If all the points associated with our section are outside the unit circle it means the section will resist at any direction of horizontal load. If any point is in the circle, let's say (0.5;0.5) it means if the horizontal load will act at 45º, and the limit state of the section will be reached at 0.71 of the given horizontal loads.

The aim of our research was to develop a method for rapidly, even automatically verifying if a certain section can carry certain forces. The first option is to evaluate the *Interaction diagrams*, then store it and then for each combination of forces determine if the corresponding point is inside or outside the *Interaction diagrams*. A faster method was developed in this article to represent the point of the force and then check naturally via the described representation.

2. The presentation of the calculation method.

2.1. Hypothesis

The proposed method assumes we have the cross section of the element and it's reinforced with rebars. The cross section can have any arbitrary geometry and have any type of reinforcement.

We also require 3 sets of sectional forces *(N, Mx, My).* The first set is composed of the effect of those loads which should not be multiplied since they are considered constant. This set will be denoted as GS, and it is composed from axial force N_{GS}; bending moment along axis y M_{GS}^y and bending moment along axis x M_{GS}^x and accounts usually for the gravitational loads which remain constant. The second set will be denoted as SX and represents the section force produced by the horizontal load considered on direction x. This set will be composed of axial forces N_{SX}, and moments M_{SX}^x respectively M_{SX}^y. The third set is composed of the components N_{SY}, M_{SY}^X and M_{SY}^Y, it will represent the effect on our section from the seismic load in y direction. The last two sets are considered variables, and we want to compute how many times we can multiply those forces to reach the interaction diagram of the section.

Since we are interested in biaxial bending with axial forces, we will consider that the eccentricity is quite low, and the effect of axial forces significant. We are interested in compressed elements, but the procedure was tested and works in tension situations also. So, the limit state is considered reached when a limit strain in concrete occurs, or a specific strain is achieved in reinforcement. An arbitrary cartesian system with "z" axes along the element length can be considered, but it will be considered with origin "O" in the center of gravity of the section. The integration of stresses over the section is done by considering fiber method so:

$$N_{Rd} = \int_S \sigma_x * da + \sum_i A_i * \sigma_{xi} \approx \sum_j \sigma_{xj} * \Delta a + \sum_i A_i * \sigma_{xi}; \quad (1)$$

$$M_{Rd}^x = \int_S \sigma_x * y * da + \sum_i A_i * \sigma_{xi} * y_i \approx \sum_j \sigma_{xj} * \Delta a * y_j + \sum_i A_i * \sigma_{xi} * y_i; \quad (2)$$

$$M_{Rd}^y = \int_S \sigma_x * x * da + \sum_i A_i * \sigma_{xi} * x_i \approx \sum_j \sigma_{xj} * \Delta a * x_j + \sum_i A_i * \sigma_{xi} * x_i; \quad (3)$$

In those equation was considered:

N_{Rd} – Axial force corresponding to a given point on the *Interaction diagram*

M_{Rd}^{x} – Bending moment around x axis of that point on the *Interaction diagram*

M_{Rd}^{y} – Bending moment around y axis corresponding to the same point

S – Our section considered as domain

$\sigma_x; \sigma_{xj}$ – the axial stresses in point of the section at coordinate (x,y); respectively on the concrete fiber "j"

σ_{xi} – the axial stresses in reinforcement "i"

Δa – the area of the concrete fiber

A_i – the area of reinforcement "i", respectively the area of the concrete fiber

Even though the fiber method for integration is not exact, it is preferred due to its simplicity. The computer subroutine was made under AutoCAD and the surface definition of the section was considered easier to compute this way. Since the computing time is negligible even for big sections with fiber dimensions at 1x1cm, this method was considered acceptable.

Both the sliding between fibers and the sliding between concrete and reinforcement will be neglected.

A point of the *Interaction diagram* is achieved either when the strain in the compressed concrete fiber or the strain in reinforcement reach a certain value. Since the following examples are columns with significant axial forces, in these particular cases the reinforcement limit strain did not occur.

In the following examples a bilinear behavior without consolidation of the reinforced steel was considered. The first domain is characterized by elasticity modulus *E = 210GPa* and a limit of f_{yd} = *43.5kPa*. The concrete behavior was considered similar to Eurocode 2 [5]. The tension in concrete fibers was neglected, but the method does not require that. The compression behavior of the concrete is calibrated to model the concrete C20/25 according to 3.1.7 of Eurocode 2 [5] where:

$$\sigma_c = \begin{cases} f_{cd}\left[1-\left(1-\frac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right] for\ 0 \le \varepsilon_c \le \varepsilon_{c2} \\ f_{cd}\ for\ \varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu2} \end{cases} \tag{4}$$

In our case $n=2$; $f_{cd} = 16$ *MPa*; $\varepsilon_{c2} = 2.0‰$; $\varepsilon_{cu2} = 3.5‰$.

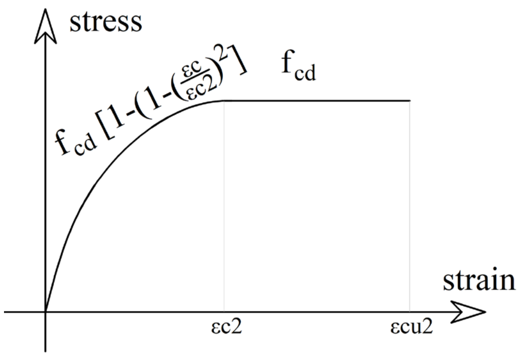


Fig. 4 - Stress-strain relationships for concrete in compression

The strain-softening effect exhibited by the concrete in compression was not considered, not because it will change the proposed procedure but since it is aimed at the final stage of the section behavior this strain-softening does not change the capacity of the cross section. The strain-softening effect becomes important when we try to evaluate moment-curvature diagrams, but if one wants to take strain-softening it easily can be done by changing constitutive relation (4).

2.2. Calculus

In order to evaluate very fast the resistance capacity of a section, at start we considered the position of the neutral axis known which allows us to compute a random point on the *Interaction diagram*. The position of the neutral axis is governed by two parameters: the inclination of the neutral axis and the distance to the assumed center "O" of the section. The angle formed by the neutral axis with the horizontal one will be further denoted as φ, and ρ as the distance from the assumed center of the section to the neutral axis. The assumed center of the section is the point in relation to which the global stresses are calculated.

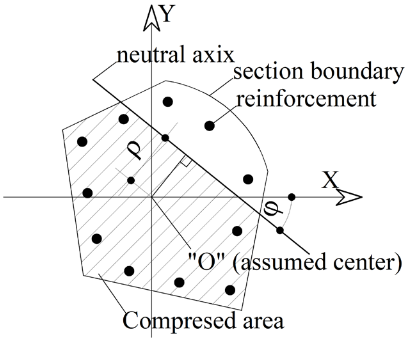


Fig. 5 - Neutral axis position

For this random position of the neutral axis, a point on the *Interaction diagram* is evaluated, but we can't link it to our set of data. However, the angle φ will most certainly change on a variation of the changes in the horizontal load direction. Hence, we will presume that for any angle φ there is a direction for the horizontal load direction for which a state of limit state can be found. Moreover, there probably is a certain kind of unique value for ρ corresponding to a limit state corresponding to a certain direction of the direction of horizontal load. After testing a lot of section geometry with various reinforcement for many sets of forces we can state that there are three cases: first of all, the huge percent of cases conduct to a unique value of ρ; second, there are some cases when two values for ρ are found; thirdly there are some cases when the value of ρ can't be found. However, for plausible sets of loads there is a unique value for ρ.

Therefore, a value for the angle φ can be assumed and the algorithm reaches static equilibrium on the section between the capacity of section and the internal forces computed from overlapping gravity action and horizontal load. That equilibrium consists of three equations and will have three unknown variables: first of them is the value of ρ, which has been discussed before, and other two which must determine the position of horizontal loads. The last two are presented in Fig. 3 denoted with *a,* and *b.*

Hence, we have three equations with three unknown variables:

$$\begin{cases} M_{GS}^{x} + a * M_{SX}^{x} + b * M_{SY}^{x} = M_{Rd}^{x} \\ M_{GS}^{y} + a * M_{SX}^{y} + b * M_{SY}^{y} = M_{Rd}^{y} \\ N_{GS} + a * N_{SX} + b * N_{SY} = N_{Rd} \end{cases} \tag{5}$$

If the system is linear in relation with unknown *a* and *b* the first unknown ρ brings nonlinearity to the system. The simplest solution was to solve the system in an iterative manner. For a chosen value of ρ from the first two equations the unknown values of *a* & *b* are computed and the value of ρ is established. The last equation is assessed, and a variable denoted err is evaluated.

$$err = N_{GS} + a * N_{SX} + b * N_{SY} - N_{Rd} \tag{6}$$

Practically a very big negative value ρ_1 and one positive ρ_2 is considered for ρ and two values of err are computed: *err1* and *err2*. By interpolation a new value for ρ is chosen and a new value of err is computed. The value of ρ_1 or ρ_2 is replaced depending on the sign of the new err value. The process is repeated until an acceptable value for "*err*" is obtained. Finding ρ requires a couple of iterations. Actually, the important values are the values of unknown *a* & *b* which are retained and printed on the graphic diagram in coordinate of SX and SY (see Fig. 3). The meaning of unknown *a* & *b* is that if we subjected the section by the axial force which corresponds to gravity loads overlayed by a horizontal load with projection has at direction *X* equal with a and a projection on *Y* equal with b the limit state of the section is reached.

Since a mandatory condition for section resistance is that it must withstand the full seismic load regardless of the direction it means $a^2+b^2 \geq 1$ hence the point represented in Fig. 3 should be outside the unit circle for which $a^2+b^2 = 1$.

2.3. Logical schema of the procedure

The procedure starts with reading input data. Those input data are the following: cross-section geometry; dimensions and position of the reinforcement on the section; physical parameters of the materials; and the three sets of section forces to which the section is subjected.

The next step is to divide the concrete section into fibers for numerical integration of axial stress over the concrete section.

Next it begins a loop in which the angle of the neutral axis φ takes certain values. For each value of φ a point on the plot representation is computed.

An iteration procedure for determining a specific ρ value follows. The targeted value for ρ is the one leading to a real solution for equation system (5), meaning the value that leads to zero for the variable *err* from (6). Values of ρ range between "-∞" and "+∞," where -∞ corresponds to a pure tension state of stresses and +∞ corresponds to a pure compression state. Between these values, *err* exhibits continuous variation with the variation of ρ.

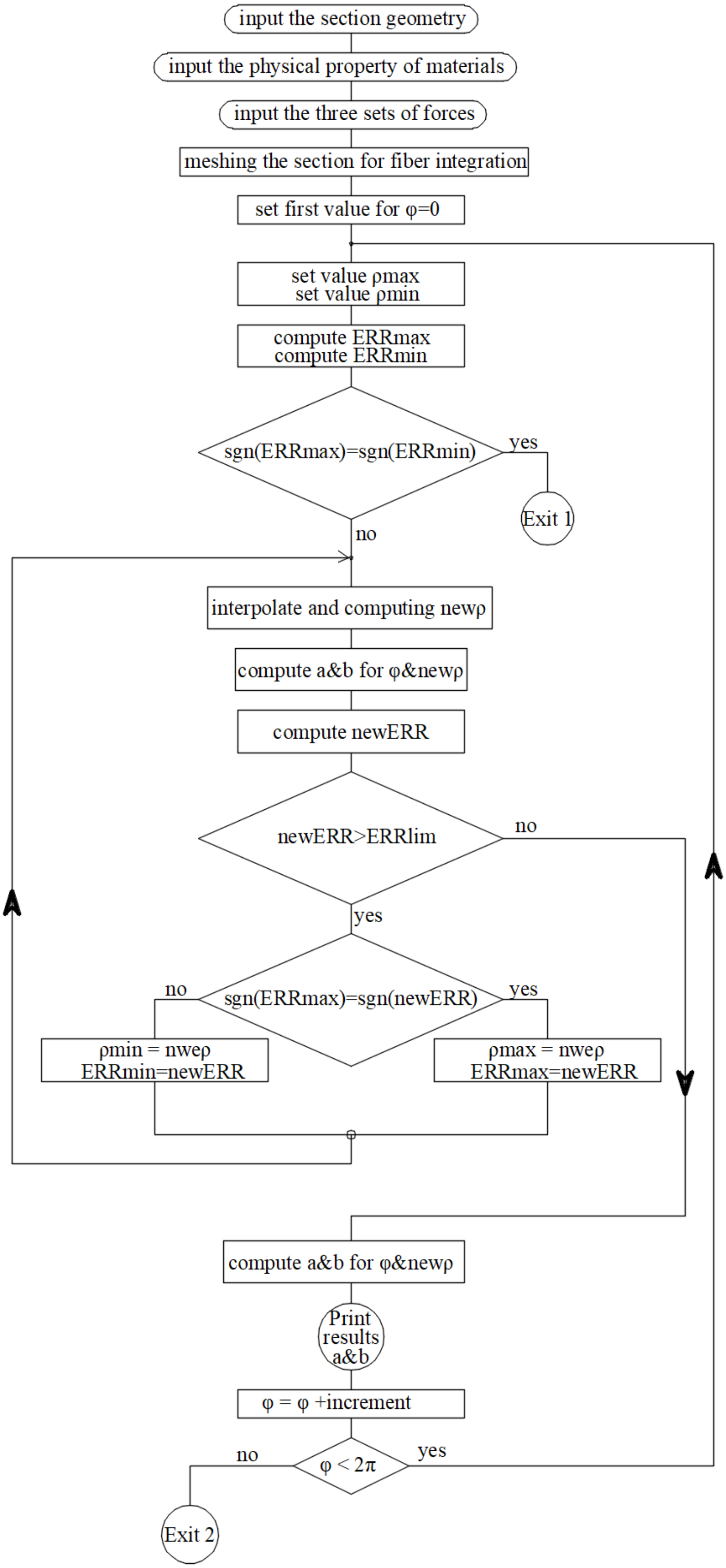
In practical applications, columns are neither in a pure tension state nor in a pure compression state. Additionally, after a sufficiently large value of ρ, changes in the *err* value become negligible. Therefore, the variable ρ can be considered within practical boundaries, denoted as ρmin and ρmax. For these presented values, we find the values of *err* in the variable *ERRmin* corresponding to ρmin and *ERRmax* corresponding to ρmax.

The typical variation of value of err as function of ρ is illustrated in Fig. 7 . However, it is common for the sign of *ERRmin* to differ from the sign of *ERRmax*. This indicates that, as err is a continuous function of ρ, there is a certain value for ρ that leads to *err = 0*. Since *err* as a function of ρ is usually monotonic, this value is determined through a few iterations.

For determining the value of ρ a bisection-based method was employed [7,8].

Once the aimed value of ρ is found and because the value of φ is known all the other parameters can be computed. Those parameters are: $M_{Rd}^{y}, M_{Rd}^{y}, N_{Rd}, a, b,$ maximum strain in reinforcement, level of compression of the section.

The next value of φ is set and the process is repeated until φ has taken all desired values.



If "Exit I" occurred it means that either the first value of ρ is too small and should set bigger ones, or, usually, the set of forces are not genuine.

If "Exit 2" occurred, then the diagram is successfully computed.

3. Numerical examples.

We will see how this procedure works and how the representation looks for a particular section and a pair of forces. In the following numerical evaluations, we will consider the variation of the angle φ to be 4°, resulting in 90 points on the capacity curve. The concrete section will be discretized into square fibers with a side length of 1 cm.

3.1. Rectangular section

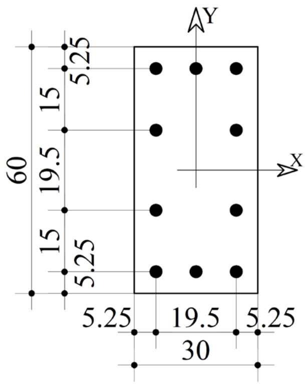


Fig. 6 - Rectangular section

A rectangular concrete section Will be considered with dimensions of 30x60cm reinforced with 10Ø16 see Fig. 6. The sets of forces which will be applied to the section are presented in table 1.

Table 1

Sets of forces on rectangular section

Cases	My	Mx	N
GS	-21.497	17.442	13.82
SX	-59.897	-4.718	7.31
SY	17.53	152.378	-4.38

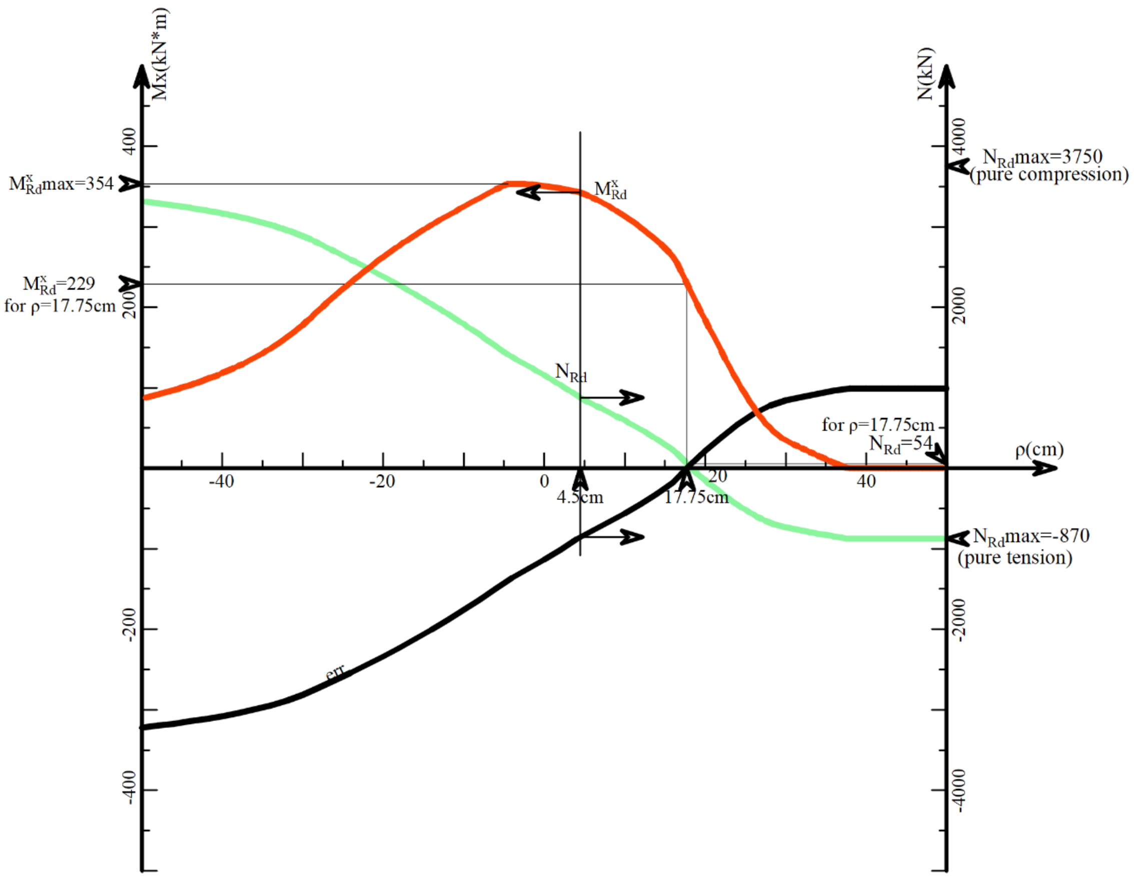


Fig. 7 - Variation of err for rectangular section.

In fig 7 will be presented the evaluation for a single point of the proposed diagram, but the entire diagram will be shown. The point for which computation will be presented corresponds to an angle $\varphi = 0°$ so the neutral axis will be horizontal see Fig. 5. Given the symmetry of the section a horizontally neutral axis leads to $M^y_{Rd} = 0$. The system from (5) become (7):

$$\begin{cases} 17.4 - a * 4.7 + b * 152.4 = M_{Rd}^{x} \\ -21.5 + a * 59.9 + b * 17.5 = M_{Rd}^{y} \\ 13.8 + a * 7.31 - b * 4.38 = N_{Rd} \end{cases} \tag{7}$$

The variation of variable *err* will be presented graphically even if this is not done in the procedure, for a better understanding and to prove the good convergence of the method see Fig. 6. Besides variation of *"err"* the figure also shows the variation of the capable axial force (N_{Rd}) and capable bending moment (M_{Rd}).

On each vertical line there will be three intersections: the intersection with the red graph leads to the capable moment on the left vertical axis; the intersection with the green graph leads to the value of axial forces on the right vertical axis; the intersection with black graph leads to the value of err. Since all values are on the same vertical then we will actually have the point on the *Interaction diagram.* If we want to compute the point on the *Interaction diagram* corresponding to a horizontal neutral axis located at 4.5 cm from the assumed center (in our case gravity center) we will have $M_{Rd}^{y} = 0$; M_{Rd}^{x} on the left axis; N_{Rd} on the right axis. The *err* is represented on the right axis since it is in terms of axial forces coming from the last equation of the system. For graphical purposes the values of ρ are displayed between values of ± 50 cm. Because of that, the position of the neutral axis which is consistent with pure compression does not appear. As it can be seen in Fig. 6 the value of ρ that we are looking for is 17.75 cm. These value leads to $M_{Rd}^{x} = 290kNm$; $M_{Rd}^{y} = 0kNm$ and N_{Rd}=45kN. The right value for ρ can be verified by (7) which leads to *a = -0.32; b = 1.38*.

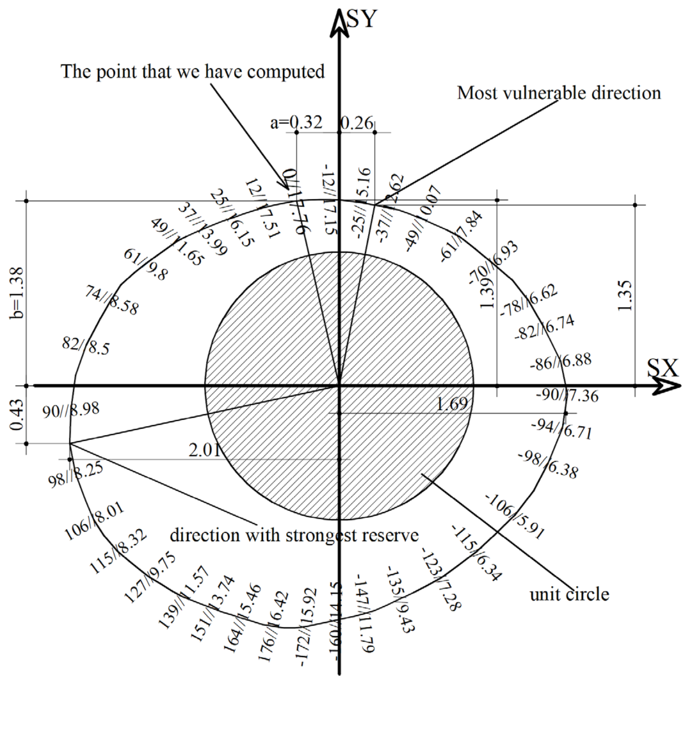


Fig. 8 - Capacity plot for rectangular section

In Fig. 7 it can be seen that the variation of err between the x-balance and the almost tension situation is almost linear, which can explain why the interpolation method for driving ρ is successful.

In Fig. 8 the graphical representation that we are aiming for is presented. All the points which are used to construct the diagram are presented. For each point "$\varphi//\rho$" are displayed which means that each point corresponds to a limit state with neutral axes rotated with angle φ from the horizontal axis and is located at a distance ρ from the center of the section. The section can be considered with sufficient strength if the value of horizontal load capable of the section is greater than the forces used for the analyzed set on each direction of horizontal forces. Particularly, the section must sustain the force from gravity combination overlaid with force from SY case of horizontal load multiplied by a coefficient above a unit value. That coefficient can be found at the intersection of our graph with the vertical axis. If that coefficient is smaller than unit value, it means the limit state occurred at a force smaller than the effective force resulting from superposition of gravity action and the seismic one acting in y direction. The same reasoning can be applied on each direction because the section must resist the seismic load from any direction the $(a^2+b^2)>1$, where *a* & *b* are coefficient multiplying the horizontal load projections on the two normal directions. Hence if all the points computed are outside the unit circle it means the section can withstand the loads for any direction of horizontal load. If any of those points is in the unit circle it means the section is inadequate for that horizontal load applied from that particular direction. For this example, it is shown clearly that the section can successfully withstand any direction of horizontal load. The next example will show a situation when the section is inadequate for a certain direction. To sum up, the square section in fig. 6, subjected to sectional forces resulting from the superposition of values from table 1, with "GS" values kept constant, and "SX" and "SY" values each multiplied by different coefficients, leads to the results in fig. 8, from which some can interpret the following information:

- the limit state will occur when horizontal forces acting in y direction multiplied by 1.39 are combined with gravity forces.
- the limit state will occur when horizontal forces acting in x direction multiply by 1.69 are combined with gravity forces.
- the most vulnerable direction consists with a value of the horizontal load multiplied by $\sqrt{(1.35^2 + 0.26^2)} = 1.37$, acting in a direction as is shown in figure Fig. 8.
- the strongest direction consists of a value of the horizontal load multiplied by $\sqrt{(2.01^2 + 0.34^2)} = 2.04$, acting in a direction as is shown in figure Fig. 8.

The automatic verification can be done by comparing the value of *(a^2+b^2)* with one for every direction computed, but the visual confirmation can be done effortlessly, and give to the operator a clue about how the section will behave or how it should be changed in case of inadequate behavior.

3.2. Section of "L" geometry

Next, we will present the results for an 'L' shaped concrete section with dimensions of 65x40x25cm reinforced with 10Ø16 see Fig. 9. This section will be subjected to forces presented in Tab. 2. The section in this example will be divided into square-shaped fibers with a size of 3cm. The stresses are reduced at the assumed center which is in the center of gravity of the section.

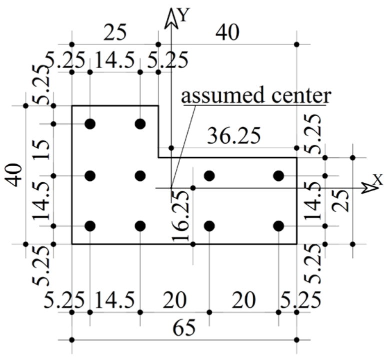


Fig. 9 - "L" shape section

Table 2

Sets of forces on "L" shape section

Cases	My	Mx	N
GS	-30	25	150
SX	-180	-5	-70
SY	2	-80	-50

Following the same procedure used for the rectangular section, we will follow the calculation steps but for a vertical neutral axis which will lead to angle *φ=90°* but, since the vertical axis is not a symmetry axis as previews hence $M_{Rd}^{y} \neq 0$. Instead of determining the value of ρ iteratively as the procedure of analysis was explained, we will present its variation, and this value will be evaluated graphically. In Fig. 10 the variation in the capable stresses and the failure and the variation of variable err is presented, as described in (6).

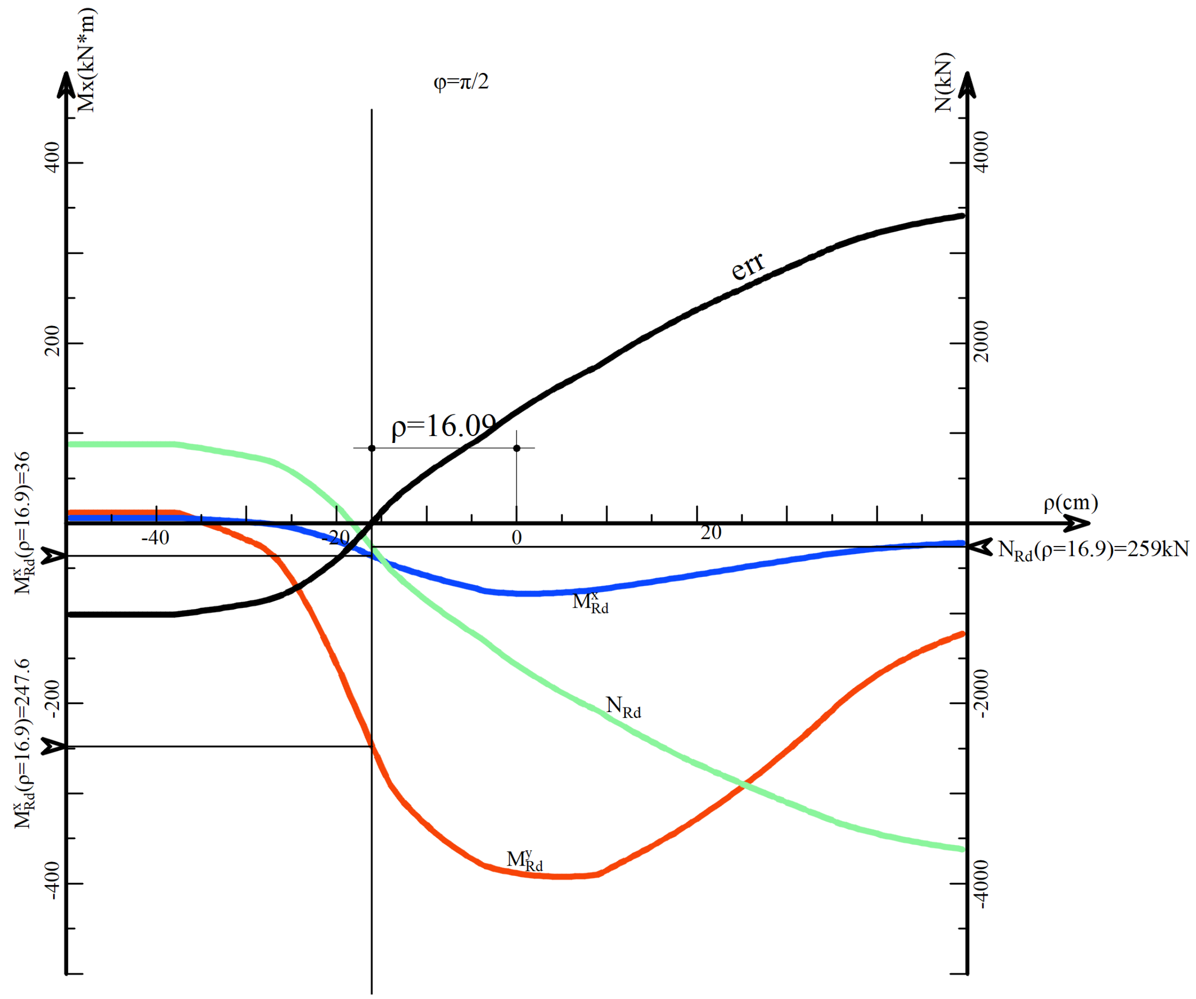


Fig. 10 - Variation of err for “L” shape considering $\varphi=\pi/2$

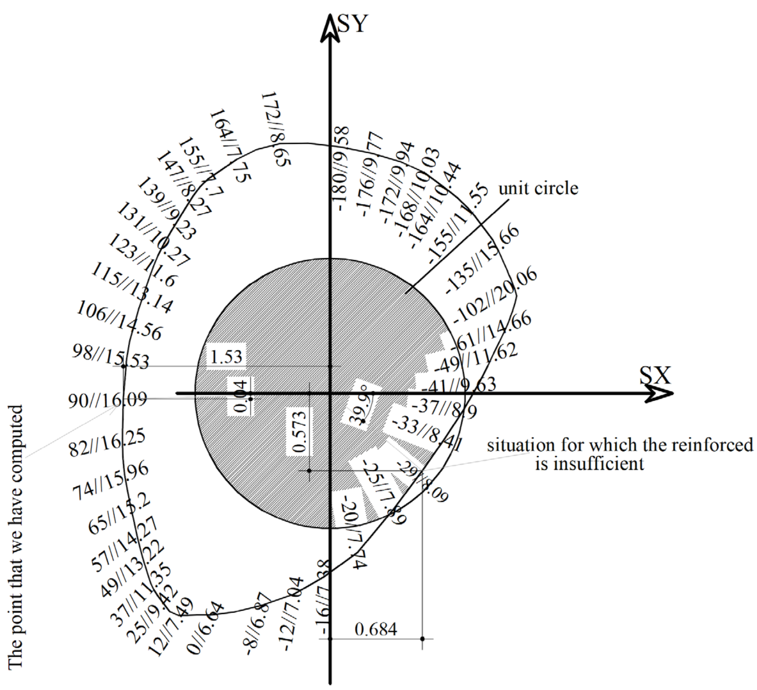


Fig. 11 - Capacity plot for “L” shape

The plotted functions are the same as the ones explained in the previous chapter. As depicted in Fig. 9, the value of the parameter ρ is 16.09cm. At this value of ρ, given the position of the neutral axis the algorithm can easily evaluate the component of section forces which conduct to a limit state. Those values are: $M^x_{Rd} = 36kNm$; $M^y_{Rd} = 247.6kNm$ and N_{Rd}=259kN. The system of equations (5) will become:

$$\begin{cases} -30 - a*180 + b*2 = 247.6 \\ 25 - a*5 - b*80 = 36 \\ 150 - a*70 - b*50 = 259 \end{cases} \tag{8}$$

The solution for the system of equations (8) is $a = -1.532$ and $b = 0.042$. It can be verified that those values validate all three equations. Hence a point of graph is computed, that point states that a limit state is reached if the section is simultaneously subjected to the force of gravity combination and the effect of horizontal load on x direction multiplied by *-1.53* and the horizontal load acting on y direction is multiplied by 0.042. After computing more points, the entire graph can be obtained and plotted see Fig. 10. In this figure it can be seen how the computed graph looks for a section which does not check requirements for the resistance criteria. As can be seen in the graph the section meets the requirements for the combination of gravity action combined with the effect of horizontal load considered on each of the principal directions. But there is a weak spot where the horizontal load results from an inclined direction. This is well known in current structural analysis and the design codes specify verification for such direction, anyway in this graphical representation a situation like that is more correctly explained. It seems that the worst scenario for our section and given set of forces occurs when the horizontal load is applied from a direction inclined at arctg *(0.753/0.684) = 39.9°*. This load leads to a critical state of strain with the neutral axis inclined by φ*=-29°*. Let's see how our capable forces and err from (7) vary as a function of ρ for the given φ.

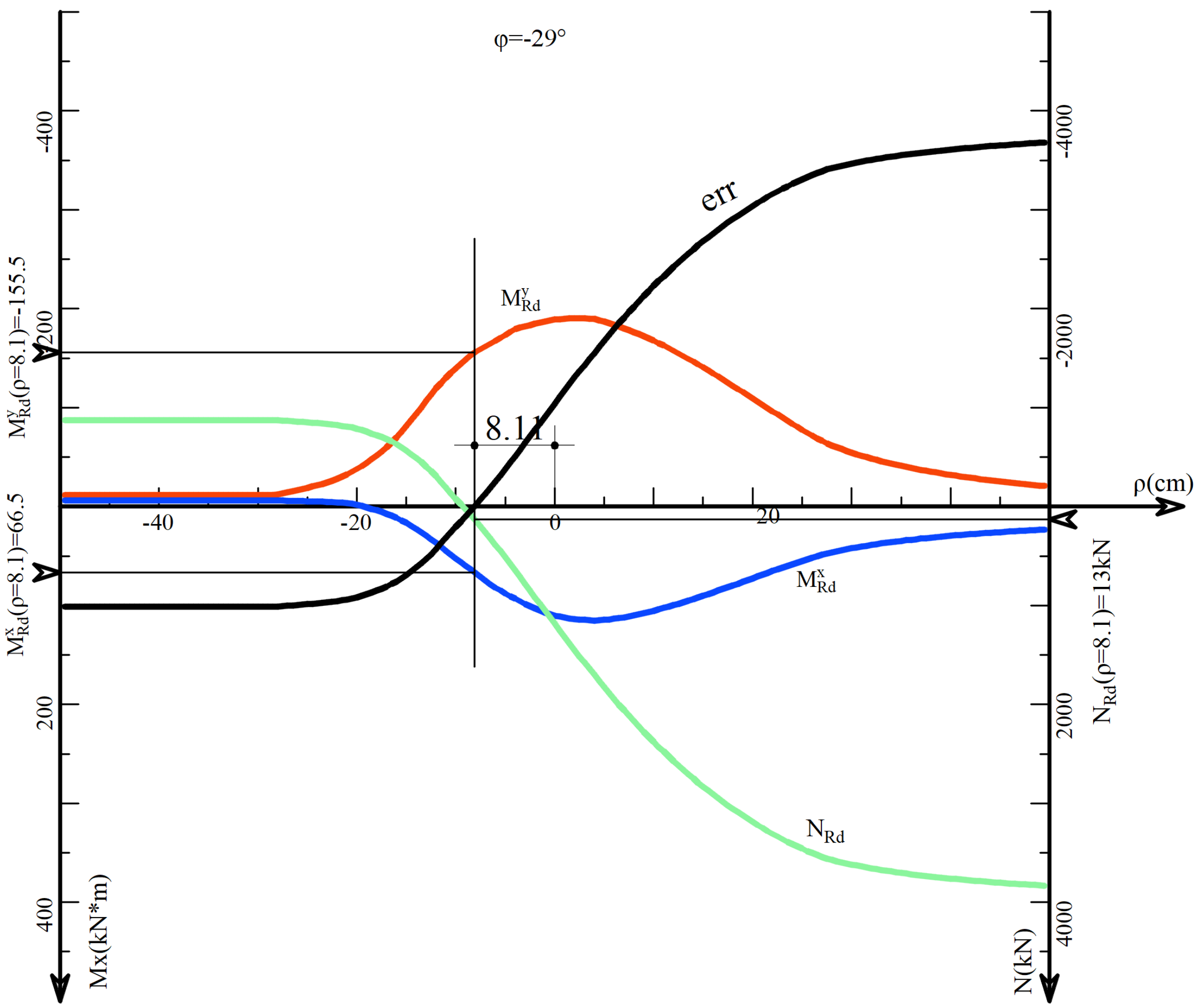


Fig. 12 - Variation of err for "L" shape considering φ*=-29°*

From Fig. 12 the value of the point of the *Interaction diagram* which have the following properties can be extracted: corresponding to a neutral axis inclined with φ*=-29°;* the capable force can be achieved by superposing the gravity actions effects with the effects of the horizontal load acting on a certain direction and multiplied by a scalar factor. The capable forces corresponding to that point are: $M_{Rd}^{x} = 66.5kNm$; $M_{Rd}^{y} = -155.5kNm$ and $N_{Rd}=13kN$. Those values lead to following particularization of the (5):

$$\begin{cases} -30 - a * 180 + b * 2 = -155.5 \\ 25 - a * 5 - b * 80 = 66.5 \\ 150 - a * 70 - b * 50 = 13 \end{cases} \tag{9}$$

The system has a unique solution which consists of $a = 0.69$ and $b = -0.562$. That means if the horizontal force is acting on angle of about -40° from the horizontal axis then at $\sqrt{(0.69^2 + 0.562^2)} = 0.89$ of considered force the section will reach a limit state and can't support the entire desired horizontal force.

4. Conclusion

A variety of methods for evaluating the load capacity of reinforced concrete sections have been developed, and some are presented in references [1-4],[6]. All aim to construct an interaction diagram in the form of a 3D graph within a Cartesian system. Typically, on the vertical axis the axial forces component is represented, while the other two depict the bending moments in each direction. Some papers also include an evaluation of forces-curvature plots [6]. In all cases, the procedures analyze the section's behavior using a single section force (with its three components). If the point corresponding to the given force lies inside the interaction diagram surface, it means the section can carry the load; if it's on the surface, the force corresponds to a limit state, and if it's outside the surface, the section is deemed inadequate for the given force. While this approach aligns with the study of the strength of materials, it overlooks the fact that these forces result from a superposition of different types of loads. In reinforced concrete column analysis, gravity load and seismic load are typically involved. While gravity loads are usually precisely determined, with minimal variation, seismic loads have an unknown direction and intensity. For design purposes, it's crucial to evaluate the strength reserve, examining the behavior of the concrete element beyond the load intensity given by code prescriptions. Engineers prefer not just a yes/no verification but want to understand how much more load the elements can carry. Therefore, in all cases, one or more extrapolations of the section forces are made to anticipate the limit state situation based on that section force. Based on the proposed extrapolation, certain slices of the interaction diagram are used for graphical representation.

Instead of that, a procedure is proposed, which takes into account three section forces describing separately the gravitational effect and the variation of requirements for the section when changes occur in the direction of the seismic load. These three section forces are combined to determine the weakest direction of the seismic load and then verify how many times the seismic load should be multiplied for the section to reach a limit state of stress. The proposed procedure exhibits good convergence. There was no intention to provide moment-curvature diagrams; instead, the focus is on offering a method for checking sections of reinforced concrete elements subjected to biaxial bending with axial force.

The procedure was exemplified in two different cases. In the first case, the section is verified and can be considered to have sufficient strength regardless of the direction of the seismic load. However, in the second example, a direction for the seismic load was identified for which the resistance capacity of the section is exceeded. In both examples, it was explained how a particular point on the plot was computed and how to interpret the plotted results.

A new method for presenting section capacity is proposed, and the graphical representation format of the results offers some advantages. The first of them is that the computation based on iteration is required only for a single variable. Second is that the proposed graph format is highly convenient for designers, as it facilitates easy identification of how the section will be impacted by variations in the horizontal load direction. Third is that the results take into account that some actions are constant, while others are variable; it does not rely on a single section force as it used to. Nevertheless, due to the simplicity of viewing results, multiple sections can be superimposed, allowing designers to discern whether a specific horizontal load direction affects one column or multiple columns. For complex structures and asymmetrical sections, a more profound understanding of how the resistance reserve varies with seismic load fluctuations is essential.

In the practical version of the representation, information displayed at each point does not present details about the position of the neutral axis; instead, it provides information about the level of axial stresses. In the current paper, information about the position of the neutral axis was necessary for explanatory purposes, but in design applications, the focus is primarily on the level of compression. This information is not displayed for all points; it is only shown where axial forces exceed certain thresholds.

The procedure should be improved by employing a better fiber integration method and considering concrete strain-softening. It would be beneficial to establish an exact rule for determining when the system lacks solutions. Additionally, some situations have been encountered where a minor change in the inclination of the neutral axis resulted in a completely different direction of seismic load. This implies that the incremental growth of neutral axis inclination led to a nonuniform distribution of computed points on plotted results, hence an enhanced variation of neutral axis inclination should be developed. The procedure should also be enhanced to account for plastic hinge effects on the beams.

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