Revista Minelor – Mining Revue
ISSN-L 1220-2053 / ISSN 2247-8590
vol. 31, issue 4 / 2025, pp. 1-23

ANALYSIS OF ROCK MASS STABILITY UNDER DIFFICULT GEOLOGICAL CONDITIONS: AN INTEGRATED APPROACH FOR LIGNITE DEPOSITS

Mihaela TODERAȘ[1*]

[1]University of Petrosani, Petrosani, Romania, toderasmihaela@yahoo.com

DOI: 10.2478/minrv-2025-0039
received – 08.09.2025 / revised – 15.09.2025
accepted – 02.12.2025 / published – 22.12.2025

Abstract: *This study addresses the complex issue of rock mass stability in mining operations, with a particular focus on lignite deposits in challenging geological conditions. The primary objective was to develop a detailed and integrated geomechanical mathematical model capable of assessing and managing associated instability risks. The study began with an in-depth characterization of the geomechanical properties of the specific rock types found in these environments—marls, clays, aquifer sands, and lignite. This analysis highlighted the lithological variability and rheological behaviour of these materials, including creep phenomena. A crucial component involved analysing the influence of hydrogeological conditions, identifying the complex role of aquifer and aquiclude horizons in groundwater dynamics and infiltration risk. For risk assessment, an integrated cause-and-effect analysis was employed, utilizing visual tools and a risk matrix. This approach revealed that hydrogeological and geological factors represent critical risks, exhibiting the highest impact and probability of occurrence. Mining activities were classified as high-risk, while geomechanical and external factors presented moderate and low risks, respectively. Based on this prioritization, a detailed action plan was structured, including measures for monitoring, controlled drainage, reinforcement, and operational optimization. The central objective of this work, the development of the geomechanical mathematical model, was achieved by describing its theoretical foundations. This model incorporates equilibrium and compatibility equations, advanced constitutive models (elastic-plastic Mohr-Coulomb, rheological Bürgers for creep, and linear elastic), and, critically, hydro-mechanical coupling based on Terzaghi's effective stress principle and Darcy's law. The complexity of this system of nonlinear, coupled partial differential equations justified the necessity of employing numerical methods for its solution. This study provides a robust analytical basis and a dynamic tool for understanding and managing rock mass stability in challenging geological environments. Its contribution lies in providing an integrated framework that enables the safe and efficient design of mining operations, thereby contributing to risk minimization and the sustainability of lignite deposit exploitation*.

Keywords: *rock mass stability, lignite deposit, running ground, cause-and-effect analysis, risk assessment, geomechanical mathematical model, hydro-mechanical coupling*

1. Introduction

Mining activities, while fundamental to economic and social development through the extraction of energy and raw materials, inherently involve a substantial intervention in the pre-existing geological and geomechanical equilibrium [1]. This disturbance introduces new influential factors, generating significant challenges for rock mass stability [2, 3]. Geotechnical instability is a major concern in any mining operation, whether underground or surface, with potential severe consequences for personnel safety, infrastructure integrity, long-term operational viability, and environmental impact [4, 5, 6]. Incidents such as gallery collapses, massive settlements, or landslides can lead to considerable human, material, and financial losses, underscoring the need for a profound understanding and proactive management of these risks [7, 8]. The situation is particularly critical for lignite deposits, which are often found in especially difficult geological

[*] Corresponding author: Toderaș Mihaela, prof. Ph.D. eng., University of Petrosani, Petrosani, Romania, Contact details: University of Petrosani, 20 University Street, toderasmihaela@yahoo.com

vol. 31, issue 4 / 2025
pp. 1-23

conditions. These include the presence of rocks with inferior geomechanical properties, such as marls, clays with pronounced rheological behavior, and unconsolidated sands (referred to as running ground) [9, 10]. The vulnerability of lignite to compression, the low shear strength of clays and running ground, and the presence of major geological discontinuities (faults, fractures) and pressurized groundwater [11] exponentially amplify instability risks. This necessitates a rigorous and integrated approach to geomechanical problems [12, 13].

A bibliographic analysis of existing studies reveals a constant concern within the scientific and engineering community regarding rock mass stability in various mining contexts. While the theoretical foundations are solidly established by classic works in rock and soil mechanics [2, 14], practical applications require continuous adaptations to the unique geological and hydrogeological conditions of each site.

An essential component in understanding rock mass behavior is the hydrogeological regime. Works by authors like Hitchman (1999) [15] and others dedicated to groundwater dynamics demonstrate the fundamental impact of pore water pressure on effective stresses and, consequently, on the shear strength of rocks [14]. Variations in groundwater levels, the existence of pressurized aquifer horizons, or the chemical aggressiveness of water can trigger or accelerate instability phenomena, from internal erosion to liquefaction [5, 16]. The controlled management of groundwater, through drainage and waterproofing systems, is vital for maintaining stability [17]. Beyond hydrogeological factors, the intrinsic geomechanical properties of rocks and their interactions play a decisive role. Rock mass classifications, developed by authors such as Bieniawski (1989) [16] or Hoek and Brown (1997) [12], offer valuable tools for estimating mass strength. However, for rocks with non-elastic or time-dependent behaviour—such as clays and marls—the simple application of instantaneous failure criteria is insufficient. The phenomenon of creep (time-dependent deformation under constant load) requires rheological constitutive models, such as the Bürgers model, to simulate the long-term behavior of the rock mass [9]. Furthermore, the interface between rocks of different lithology and the presence of geological discontinuities further complicate analyses [18].

Risk analysis methodologies have evolved significantly, from qualitative to quantitative and probabilistic approaches. Standards such as ISO 31000:2018 [19, 20] provide comprehensive guidelines for risk management, and tools like Ishikawa diagrams ("fishbone" diagrams) are essential for identifying the root causes of problems [20, 21]. Risk matrices, by quantifying probability and impact, allow for efficient risk prioritization and judicious resource allocation [22, 23, 24, 25]. This systematic approach is vital for transforming identified risks into concrete mitigation and monitoring actions. Advances in numerical modeling have revolutionized geomechanical analysis, enabling the simulation of complex rock mass behavior under dynamic and nonlinear conditions. The Finite Element Method (FEM) and Finite Difference Method (FDM), implemented in advanced software [17, 26, 27], offer the possibility of integrating hydro-mechanical coupling, complex constitutive behaviors (elastic-plasticity, creep), and various construction phases. These numerical tools are indispensable for simulating various scenarios, from normal operating conditions to failure scenarios, such as drainage system failure or seismic overloads [7, 8]. In the specific context of underground mining works, the design and behavior of support systems are also critical. Adequate sizing of concrete linings, anchors, or other support elements requires a precise understanding of rock-support interaction and the distribution of stresses and deformations around the excavation [18, 28]. Current regulations, such as [29, 30] and national legislation [31, 32], provide an essential reference framework for ensuring geotechnical safety.

Given the complexity and criticality of stability problems in lignite deposits situated in difficult geological conditions, this article aims to contribute through the following specific objectives:

a) Characterization of geomechanical and rheological properties: To conduct a comprehensive analysis of the physical and mechanical characteristics of rocks specific to lignite deposits (marls, clays, aquifer sands, lignite), with a particular emphasis on identifying and quantifying the relevant rheological parameters for creep simulation.

b) Evaluation of hydrogeological conditions' influence: To analyze the role of groundwater, pore pressures, and rock permeability in rock mass stability dynamics. This objective also includes identifying optimal drainage and waterproofing solutions to minimize infiltration and liquefaction risks.

c) Integrated analysis and prioritization of instability risks: To apply modern risk analysis methodologies to identify the fundamental causes of rock mass instability and prioritize risk factors (geological, hydrogeological, geomechanical, operational, and external). The goal is to develop an efficient action plan based on this prioritization.

d) Development of a coupled and comprehensive geomechanical mathematical model: To establish the theoretical foundations of a geomechanical model capable of simulating rock mass behavior under the simultaneous influence of mechanical and hydraulic stresses. This model will incorporate nonlinear

vol. 31, issue 4 / 2025
pp. 1-23

constitutive behaviors and the interaction with support systems, thereby justifying the use of advanced numerical methods for its solution.

e) Formulation of strategic recommendations for stability management: Based on the results from the mathematical model and risk analysis, to propose optimal design solutions, supplementary constructive measures, and continuous monitoring strategies. These recommendations are aimed at ensuring the long-term safety and sustainability of mining operations in difficult geological conditions.

By achieving these objectives, this study aims to provide a robust tool and a solid foundation for the proactive understanding and management of instability risks in underground lignite extraction, thereby contributing to operational optimization and enhanced safety in the field.

2. Engineering background

Given the complexity and criticality of stability problems in lignite deposits situated in difficult geological conditions, this article aims to contribute through the following specific objectives:

Rock mass stability represents a major engineering challenge in mining operations, particularly in lignite deposits where geological and hydrogeological conditions are often complex and unfavorable. These conditions, characterized by the presence of rocks with reduced geomechanical properties and pressurized aquifer horizons, can generate instability phenomena with severe consequences. This article proposes an integrated approach to rock mass stability analysis, utilizing specific data from the Vărzari-Nord mining perimeter (part of the Voievozi coal basin, Bihor). The focus will be on how detailed information regarding geology, hydrogeology, geomechanical and rheological properties of rocks, as well as the results of cause-and-effect and risk analyses, converge towards the foundation and application of a comprehensive geomechanical mathematical model, designed to evaluate and predict the rock mass behaviour under operational conditions.

A rigorous analysis of rock mass stability requires, first and foremost, an in-depth understanding of the site's specific geological and hydrogeological context. Popești-Voievozi mining perimeter, located in the north of Bihor County, at the contact between the Apuseni Mountains, the Someș Plateau, and the Pannonian Depression, provides an eloquent example of a complex geological environment with direct implications for the stability of mining works.

2.1. Geological and stratigraphic context of the study area

Geologically, the perimeter is situated within the Șimleu Silvaniei depression, where Pliocene (Pannonian and Pontian) and Quaternary formations rest on a degraded crystalline basement. The Pontian deposits, with thicknesses ranging from 30 to 170 meters, are particularly significant. They are subdivided into a sandy sub-horizon with coal and a marly-argillaceous sub-horizon with sands, the latter containing the majority of the lignite seams. An argillaceous-sandy horizon, with thicknesses between 50 and 400 meters, overlies these deposits.

The lignite deposit itself is part of the productive horizon, which includes sands, marly sands (grey-greenish), alternating with marls and clays, as well as intercalations of carbonaceous clays, argillaceous coals, and pure coals. The economically viable coal seams, Seam II and Seam III, are of paramount importance. Seam II, the primary target for exploitation in Popești-Voievozi, consists of two coal packages (Lower II and Upper II) which, in the southern part of the region, merge into a single seam with a thickness of 2 to 5 meters. The roof of Seam II is composed of an approximately 12-meter-thick aquifer layer, followed by a protective screen of 15 to 20 meters, consisting of marls and clays with high to very high plasticity. Seam III, which is stratigraphically superior to Seam II, has a thickness of 1.2 to 3.44 meters and is separated from Seam II by a 20 to 40-meter-thick interburden of marls, sands, and clays. The structure of the deposit is generally simple, located within a broad syncline with flank dips ranging from 2° to 10°. However, the presence of faults with throws exceeding 10 meters introduces a structural complexity that must be considered in stability analyses.

2.2. Qualitative characterization of coals

The lignite from the Popești-Voievozi perimeter is light to dark brown, with a dull luster and obvious stratification. Its mechanical properties are particularly relevant for stability analyses: the flexural strength ranges from 11 to 15 daN/cm², while the compressive strength varies between 55 and 91 daN/cm² (perpendicular to stratification) and 40 to 60 daN/cm² (parallel to stratification). These values indicate a relatively low strength, classifying the lignite as a soft rock.

vol. 31, issue 4 / 2025
pp. 1-23

2.3. Geomechanical and rheological data of the rock mass

The geomechanical characterization of the rocks within the perimeter is fundamental for any stability analysis. Data obtained from geological boreholes have allowed for the determination of key parameters, which are summarized in Table 1. These parameters include specific gravity, porosity, natural moisture content, compressive and tensile strength, cohesion, internal friction angle, static and dynamic elastic moduli, as well as the coefficients of plasticity and permanent bulking.

Table 1. Physical and Mechanical Characteristics of Rocks in Popești-Voievozi Perimeter

No.	**Characteristic/Rock type**	**Marls**	**Clays**	**Aquifer sand** (Seams II-III)	**Lignite**
1.	Specific gravity, t/m³	2.71	2.65	2.68	1.68
2.	Bulk density, t/m³	2.05	1.88	2.28	1.2
3.	Porosity, n%	38.8	29	30-45	32
4.	Compactness, G %	57.3	71	-	68
5.	Natural moisture content, Wn %	16.92	11.21	34.4	8.34
6.	Uniaxial compressive strength (UCS), MPa	1.9	2.33	-	6.84
7.	Tensile strength, MPa	0.15	0.39	-	0.436
8.	Cohesion, C, MPa	0.227	0.48	0	0.725
9.	Internal friction angle, degrees	19	26	-	33
10.	Static modulus of elasticity, E, MPa	316.2	234.8	-	241.6
11.	P-wave velocity, V, m/s	1295	1555	-	1148
12.	Dynamic modulus of elasticity, Ed, MPa	3246	5035	-	1441.2
13.	Coefficient of plasticity	-	1.7	-	-
14.	Modulus of compactness, MPa	-	40-160	-	-
15.	Coefficient of residual bulking	1.11	1.04	1.0	1.05
16.	Filling coefficient	1-0.9	1-1.3	1	1.35-1.5

According to standard rock classification systems, these materials fall into the categories of soft rocks and semi-hard rocks, which indicates an inherent vulnerability to mechanical stresses. The rheological parameters, presented in Table 2, are particularly critical for the clays and marls, as they reveal a viscous-plastic (creep) behaviour that significantly influences long-term stability.

Table 2. Rheological parameters of intersected rock formations

No.	**Rheological Parameter**	**Rock Type - Clay**
1.	Modulus of Elasticity, 104 MPa (EH, EK)	0.060
2.	Kelvin Relaxation Time, TK (min.)	1.5
3.	Maximum Deformation Value, mm	3
4.	Maximum Stress Value, MPa	0.97
5.	Exponent Value, n	2.7

2.4. Permeability and hydrogeological regime analysis

The hydrogeological regime within Vărzari-Nord perimeter is complex and has a major impact on stability. The phreatic water is confined within Quaternary deposits, while deep groundwater is intercepted in the Pannonian formations, particularly in the aquifer horizons located between the productive coal seams. The primary source of recharge is atmospheric precipitation, which infiltrates through outcrop zones and porous rocks. The drainage of a large volume of water is challenging and occurs through the depressurization or elastic detention of the aquifer horizons.

The aquifer horizon between coal seams II and III is the most significant, with a thickness of approximately 6.5 m (according to borehole data) and a total interburden thickness of approximately 40 m. This horizon exhibits discontinuities and a lenticular character. The initial piezometric pressure in this aquifer was 64.4 m of water column, which decreased to approximately 10 m of water column in the roof and 25 m of water column in the floor after drainage. The filtration coefficients (k) range from 1.5 to 1.9 m/day. The grain size distribution of the aquifer sands in Popești-Voievozi is 40–90% sand, 6–37% silt, and 2–19% clay. The water flow direction is from south to north.

Other aquifer horizons (between seams III–V, V–VII, VII–X) are present, but their influence on mining operations in this perimeter is negligible due to their small thicknesses and discontinuities.

Revista Minelor – Mining Revue
ISSN-L 1220-2053 / ISSN 2247-8590 vol. 31, issue 4 / 2025
pp. 1-23

Figure 1 provides a conceptual representation of the geological succession of layers within the lignite deposit perimeter, along with the identification of critically important zones for rock mass stability. This figure's fundamental role is to illustrate the stratification concept and to highlight the specific geotechnical characteristics of each layer within the context of a geomechanical analysis, visually conveying the rock types.

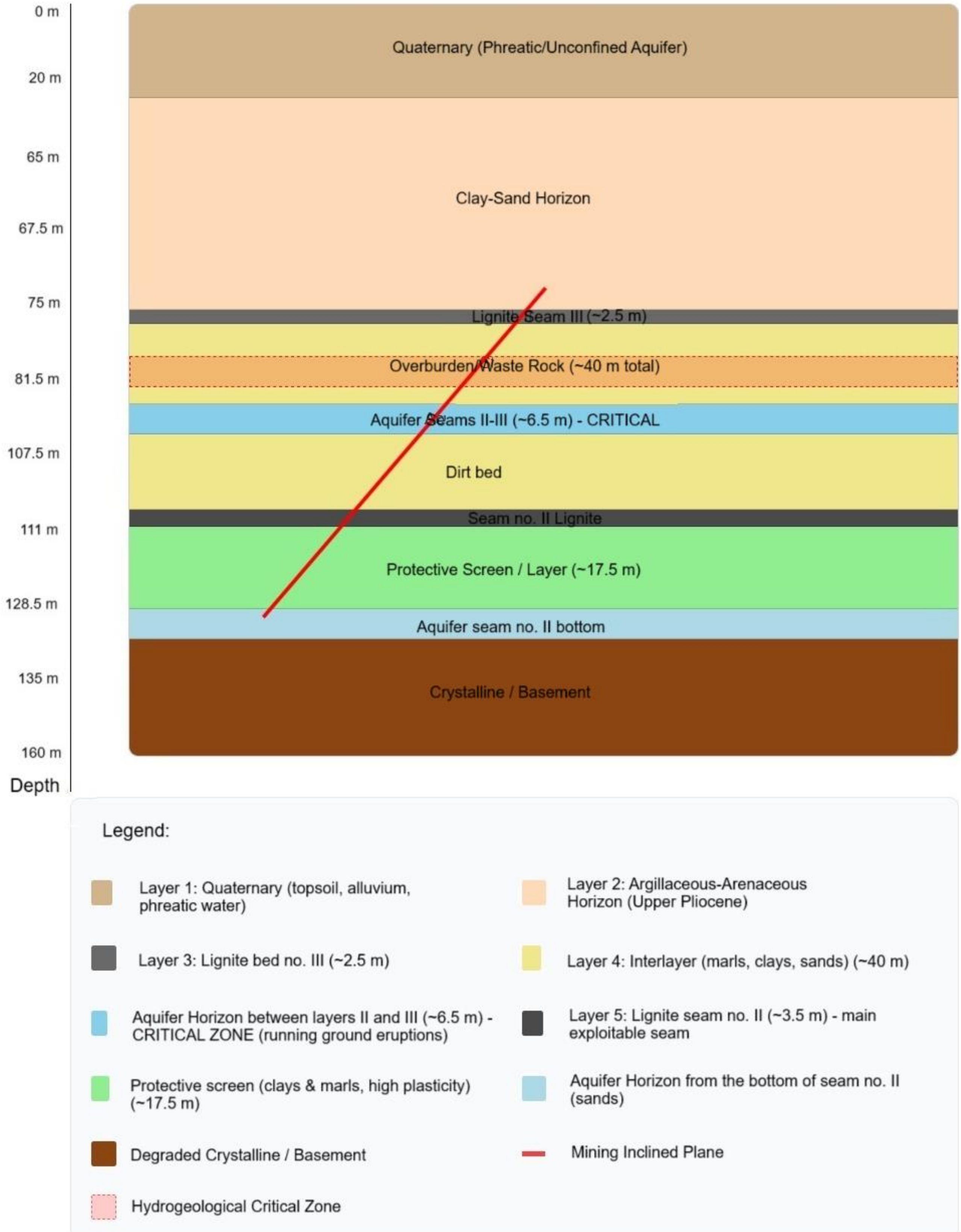

Fig.1. Simplified stratigraphic cross-section of the lignite deposit

 vol. 31, issue 4 / 2025
pp. 1-23

Layer 1: Quaternary is a combination suggesting a heterogeneous, often granular material with a loose structure, characteristic of topsoil and alluvium, which are composed of rock fragments, sand, silt, and organic matter. This layer contains the regional phreatic water. The estimated thickness from boreholes is 20 m.

Layer 2: Argillaceous-sandy horizon (Upper Pliocene) where the clays are characterized as fine, stratified deposits with higher cohesion than sands but with pronounced flexibility and plastic behavior. This also indicates a marly character, which is a sedimentary rock formed from a mixture of clay and calcium carbonate. The estimated thickness from boreholes is 50 m.

Layer 3: Lignite seam III, characterized as a homogeneous, often compact material but with relatively low mechanical strength. It is a stratigraphically superior lignite seam to Seam II, with a thickness of approximately 1.2–3.44 m. The thickness derived from the boreholes is 2.5 m.

Layer 4: Interburden (marls, clays, sands) between coal seams II and III. The total thickness of this layer is approximately 40 m. Within this interval lies the Aquifer Horizon between Seams II and III, an aquifer composed of sands with a thickness of approximately 6.5 m (from 85 m to 91.5 m depth). This is a critical hydrogeological zone where running ground eruptions have been reported and where the piezometric pressure dropped to approximately 10–25 m of water column after drainage. The estimated thickness of these sterile interbeds is 40 m (from 72.5 m to 112.5 m depth).

Layer 5: Lignite seam II (main exploitable seam). This is the primary exploitable lignite seam, with a thickness of 2–5 m in the southern part of the basin (Popești-Voievozi). The roof of this seam is at approximately 95–100 m depth. This is a zone of major interest for stability. The estimated thickness was 3.5 m** (from 107.5 m to 111 m depth).

Layer 6: Protective screen (clays and marls) in the floor of seam II. This layer is composed of clays and marls with high to very high plasticity, which are reactive to water and exhibit significant free swelling. It acts as a protective screen beneath the main lignite seam. The estimated thickness is 17.5 m (from 111 m to 128.5 m depth).

Layer 7: Aquifer Horizon in the floor of Seam II (Sands). This horizon is a widespread sand complex located below the protective screen of Seam II. The thickness is 7 m (from 128.5 m to 135 m depth).

Layer 8: Degraded Crystalline Basement or foundation (consolidated marls). This represents the geological basement, formed of altered and degraded crystalline formations, in the form of medium-to-coarse sands or consolidated marls. It was intercepted by boreholes from a depth of 135 m downwards (at least 25 m represented in the figure, down to 160 m depth and below).

The figure also depicts an inclined opening plane for the coal seams, which represents a subterranean mining work whose stability was analyzed in this study using the developed mathematical model. It is important to note that this work may intersect the Aquifer Horizon between Seams II and III, a critical zone where instability phenomena, such as running ground eruptions, have been reported.

3. Integrated cause-and-effect analysis of rock mass stability in the studied mining perimeter

The integrated cause-and-effect analysis of rock mass stability within Popești-Voivozi mining perimeter represents a complex approach that utilizes a combination of graphical and analytical tools to identify, evaluate, and manage geological and hydrogeological risks. This analysis is based on three primary diagrams: the Ishikawa Diagram, the Flowchart (Process Diagram), and the Mind Map, each playing a specific role in the evaluation process.

3.1. Methodology of cause-and-effect diagram analysis

For this study, we established a cause-and-effect chain, starting from a primary cause and leading to the final effect.

Primary cause: Complex Hydrogeology of the perimeter. Popești-Voievozi mining perimeter faces a complex hydrogeological regime, characterized by significant variations in groundwater levels and high hydrostatic pressures. This complexity is compounded by the presence of diverse geological formations such as clays, marls, and running ground, which influence how water moves through the subsurface.

Intermediate link 1: Variations in groundwater level and hydrostatic pressure. Variations in groundwater level and elevated hydrostatic pressure lead to changes in the pore pressure of the rocks. An increase in pore pressure reduces the shear strength of the rocks, making them more susceptible to deformation and failure.

Intermediate link 2: Influence on the geomechanical properties of rocks. The reduction in shear strength, combined with the influence of water on certain rock types (e.g., swelling clays), leads to changes

vol. 31, issue 4 / 2025
pp. 1-23

in the geomechanical properties of the rock mass. These changes include a decrease in cohesion and internal friction angle, which destabilizes the rock mass.

Intermediate link 3: Instability of mining structures. The destabilized rock mass directly affects the stability of mining structures, such as the network of underground works and surface excavations (e.g., slopes). The increased risk of landslides, collapses, and excessive deformations jeopardizes the safety of mining operations.

Final effect: Increased risks to safety and efficiency of mining operations. Consequently, the complex hydrogeology, through its modification of the geomechanical properties of the rocks, leads to heightened risks to the safety of workers, equipment, and mining infrastructure. Furthermore, the efficiency of mining is compromised, as additional stabilization and monitoring measures are required.

This cause-and-effect relationship highlights the importance of managing hydrogeology in Popești-Voievozi mining perimeter. By carefully monitoring groundwater levels, performing chemical analyses of the groundwater, and implementing efficient drainage systems, the risks associated with rock mass instability can be minimized.

3.2. Identification of primary causes and their relevance for modeling

Ishikawa diagram

The Ishikawa diagram, also known as the "fishbone diagram," is used to identify all possible causes contributing to rock mass instability. This diagram organizes causes into main categories, such as geology, hydrogeology, geomechanics, mining activities, and external factors, and then details them into specific sub-causes. By visualizing all factors influencing rock mass stability, the Ishikawa Diagram allows for a comprehensive understanding of the problem's complexity and facilitates the identification of critical areas for intervention.

In the specific context of this study, we analysed the Ishikawa diagram, explaining each factor that influences rock mass stability within the Popești-Voievozi mining perimeter:

1. Geology: Popești-Voievozi perimeter is characterized by a complex stratigraphic succession with alternations of lignite, marls, clays, and running ground. Each rock type has distinct geomechanical properties, which affect the stability of the rock mass differently. For instance, clays can be moisture-sensitive, causing swelling and landslides, while unconsolidated running ground can be prone to liquefaction. The variable grain size composition of the running ground (sand, gravel, silt, clay) determines variations in permeability and cohesion, affecting stability. The presence of faults, fractures, and discontinuities creates structural planes of weakness in the rock mass [33, 34]. These structures can facilitate groundwater flow and reduce the shear strength of the rocks, promoting landslides and collapses. Variations in the thickness and continuity of the rock layers can create zones of instability, especially during mining operations. Certain areas in the perimeter may be affected by geological phenomena such as karstification or rock dissolution, which reduce the stability of the rock mass.

2. Hydrogeology: Fluctuations in the groundwater level can affect pore pressure and ground stability. An increase in the water level can reduce the shear strength of rocks and cause landslides. The water pressure in the rock pores can reduce the strength of the rock mass, favoring deformations and collapses. The direction and velocity of groundwater flow influence erosion and the transport of material from the rock mass. Water flow through discontinuities can destabilize slopes and underground mining works. Variations in rock permeability determine how water moves through the rock mass. Areas with high permeability may be more prone to infiltration and instability. Groundwater can dissolve certain minerals from the rocks, reducing their strength and promoting corrosion of mining structures.

3. Geomechanics: The compressive strength, shear strength, and deformability of the rocks are critical factors for the stability of the rock mass. Rocks with low strength are more prone to failure and deformation. Natural stresses in the rock mass, combined with stresses induced by mining activities, can cause deformations and failures. Fissures and joints reduce the strength of the rock mass and facilitate the relative displacement of rock blocks. The internal friction angle and cohesion of the rocks are properties that determine their shear strength. Variations in these parameters can affect the stability of slopes and mine tunnels [35, 36].

4. Mining activities: The mining method (underground or surface) influences how stresses are redistributed in the rock mass. Underground mining can cause subsidence and collapses, while surface mining can destabilize slopes [37, 38]. The shape and dimensions of the mining works influence the stability of the rock mass. Galleries with large spans are more prone to collapse. The effectiveness of the support

vol. 31, issue 4 / 2025
pp. 1-23

system is crucial for maintaining the stability of underground structures. Blasting and other mining activities can generate vibrations and shocks that destabilize the rock mass. Uncontrolled drainage can alter the groundwater regime and cause subsidence and landslides.

5. External factors: Earthquakes can cause fractures and displacements in the rock mass, destabilizing slopes and underground mining works. Heavy rainfall can increase groundwater pressure and cause erosion. Temperature variations can induce freeze-thaw cycles, which weaken the rocks. Natural or mining-induced erosion can destabilize slopes. Construction and infrastructure can alter the groundwater regime and induce additional stresses in the rock mass [39, 40].

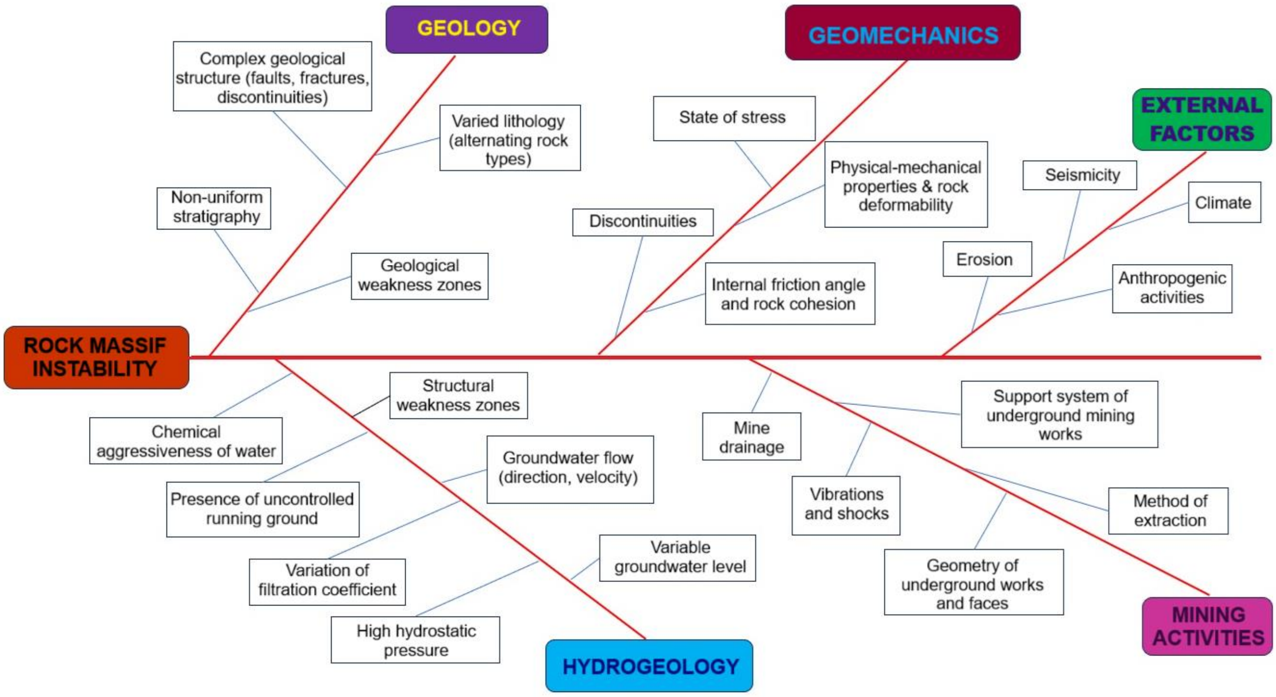

Fig.2. Ishikawa diagram for rock massif instability analysis in Popești-Voievozi mining perimeter

By conducting a detailed analysis of these factors, effective strategies can be developed to ensure rock mass stability and the safety of mining operations in Popești-Voievozi perimeter. The Ishikawa Diagram provides valuable insights and clear directions for managing rock mass stability within the mining perimeter. We present some of the key outcomes and how they assist us:

a) Identification of primary causes and sub-causes. The diagram allows for a clear visualization of all factors contributing to rock mass instability and helps differentiate between primary causes (geology, hydrogeology, etc.) and specific sub-causes (varied lithology, fluctuating groundwater levels, etc.). This differentiation is important for understanding the problem's complexity and for prioritizing actions.

b) Prioritization of causes. By analyzing the diagram, one can identify the causes that have the greatest impact on rock mass stability. The diagram enables the efficient allocation of resources and the concentration of efforts on the most critical aspects. For example, if it is determined that hydrogeology plays a major role, studies and control measures for groundwater can be intensified.

c) Development of prevention and control strategies. The diagram serves as a basis for developing prevention and control strategies tailored to the specific conditions of the mining perimeter. For each identified cause, specific measures can be proposed, such as: improving mine tunnel support systems; implementing efficient drainage systems; continuous monitoring of groundwater levels and ground deformations; and conducting detailed geophysical studies to identify zones of geological weakness. This proactive approach can significantly reduce instability risks and ensure the safety of mining operations [41, 42].

d) Optimization of mining works design. The information obtained from the analysis can be used to optimize the design of mining works, such as the geometry of tunnels and working faces. Specific geological and hydrogeological conditions can be considered to minimize instability risks. For instance, areas with major faults or discontinuities can be avoided, or more robust support systems can be adopted in high-risk zones [22, 23, 24].

vol. 31, issue 4 / 2025
pp. 1-23

e) Improvement of mining safety. By identifying and managing the causes of instability, mining safety can be significantly improved. This reduces the risk of accidents and material losses, protecting the lives of workers and mining equipment [43].

f) Reduction of environmental impact. Proper management of rock mass stability can contribute to reducing the environmental impact. For example, controlling erosion and landslides can prevent land degradation and water pollution.

g) Basis for further research. The diagram can highlight areas where additional research is needed to better understand the behavior of the rock mass. This can guide the development of complex numerical models for simulating ground stability and evaluating different mining scenarios [27].

In conclusion, the Ishikawa diagram is a valuable tool for rock mass stability analysis, providing essential information for decision-making and for ensuring the safety and efficiency of mining operations.

Flowchart (or process diagram)

A flowchart is utilized to illustrate the main causal chain that leads to rock mass instability. This diagram highlights the temporal sequence of events and the direct cause-and-effect relationships between different factors. For instance, it can show how complex hydrogeology, through variations in groundwater levels and high hydrostatic pressures, leads to changes in rock pore pressure, which in turn reduces shear strength and destabilizes the rock massif [14, 16]. By identifying critical intervention points in the causal chain, the Flowchart helps in developing effective prevention and control strategies.

A flowchart illustrating the cause-and-effect relationship for the rock mass stability study in Popești-Voievozi mining perimeter is presented in Figure 3.

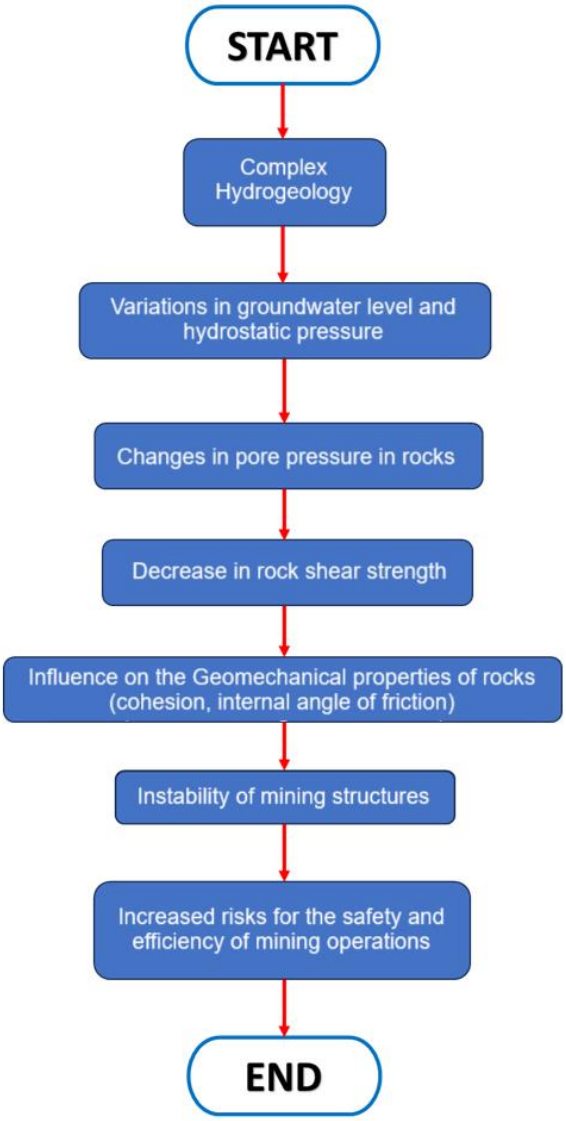

Fig.3. Flowchart (Process Diagram): complex hydrogeology and rock mass instability

 vol. 31, issue 4 / 2025
pp. 1-23

Complex hydrogeology: This represents the primary cause, characterized by the significant fluctuations in groundwater levels and high hydrostatic pressures specific to Popești-Voievozi perimeter.

Variations in groundwater level and hydrostatic pressure: This stage describes the direct effect of the complex hydrogeology, leading to changes in the pore water pressure of the rocks.

Changes in rock pore pressure: An increase in pore pressure reduces the shear strength of the rocks, making them more susceptible to deformation and failure.

Reduction of rock shear strength: This stage highlights the direct impact of changes in pore pressure on the geomechanical properties of the rocks.

Influence on rock geomechanical properties (cohesion, internal friction angle): Changes in geomechanical properties, such as a decrease in cohesion and internal friction angle, destabilize the rock mass.

Instability of mining structures (galleries, inclined planes, shafts, surface mining works): The unstable rock mass directly affects the stability of mining structures.

Increased risks to safety and efficiency of mining operations: This is the final stage, representing the cumulative effect of all previous stages, leading to heightened risks to the safety of workers and equipment, as well as reduced mining efficiency.

This diagram can be used to clearly communicate the causal chain from complex hydrogeology to the risks associated with rock mass instability. Furthermore, the diagram can serve as a basis for developing prevention and control strategies by focusing on the critical stages of the process. Through this diagram, intervention points can be identified, such as monitoring groundwater levels and implementing efficient drainage systems. This simple and clear diagram provides a concise overview of the cause-and-effect relationship, facilitating risk understanding and management in Popești-Voievozi mining perimeter.

Mind Map

A Mind Map is used to generate ideas for control strategies and to visualize the relationships between different factors. This flexible diagram allows for the free organization of information and highlights connections between causes and effects. For example, it can illustrate how hydrogeological monitoring, geological and geomechanical investigations, stabilization measures, optimization of mining activities, safety and continuous monitoring, as well as collaboration and communication can all contribute to ensuring rock mass stability. By stimulating creativity and generating ideas, the Mind Map facilitates the development of innovative and effective solutions. In this study, a Mind Map was conceptualized and used to illustrate the cause-and-effect relationship between complex hydrogeology and rock mass instability in Popești-Voievozi mining perimeter (Figure 4).

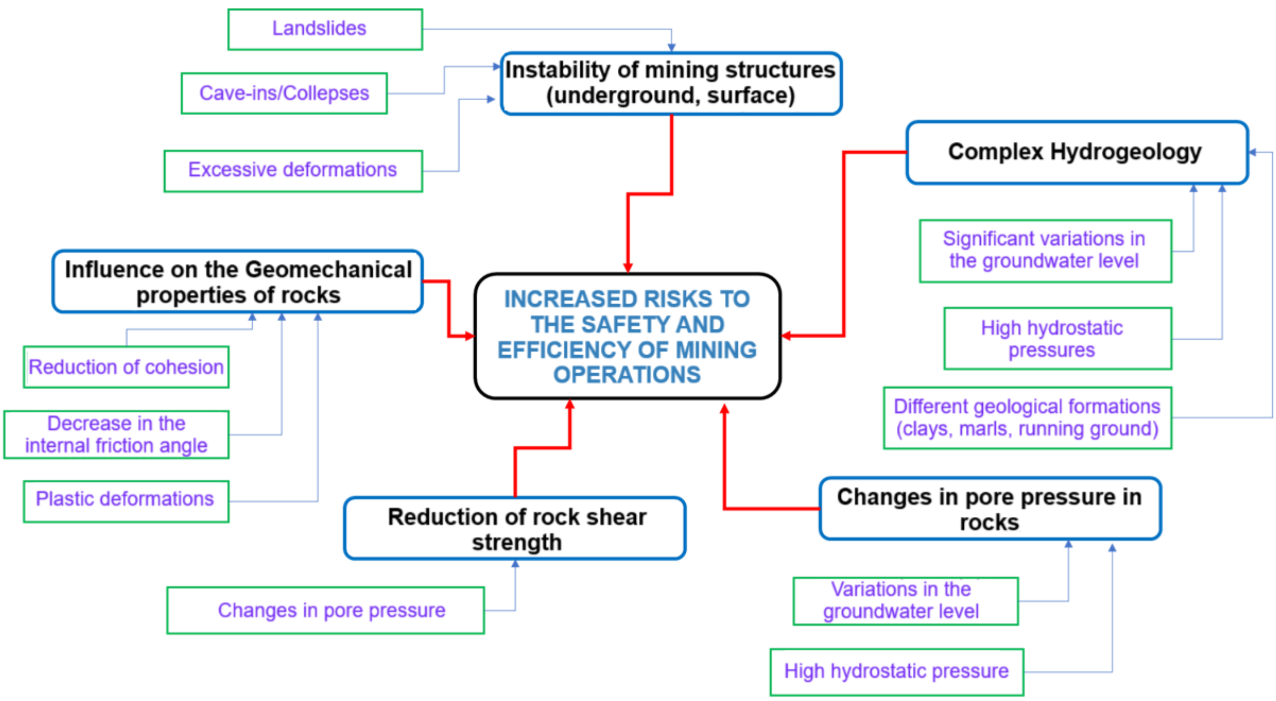

Fig.4. Mind Map: complex hydrogeology and rock mass instability

vol. 31, issue 4 / 2025
pp. 1-23

The branch "Increased Risks to Safety and Efficiency of Mining Operations" is placed at the center of the map, representing the final effect.

The branch "Instability of Mining Structures (underground works, slopes)" describes the direct effects on mining structures, with sub-branches for specific types of instability.

The branch "Influence on Rock Geomechanical Properties" highlights how hydrogeology affects rock properties, with sub-branches for the specific changes.

"Reduction of Rock Shear Strength" is the branch that details the impact on rock strength, with sub-branches for the mechanisms involved.

The branch "Changes in Rock Pore Pressure" describes how hydrogeological variations affect pore pressure.

"Complex Hydrogeology" is the branch that represents the primary cause, with sub-branches for the specific factors contributing to the hydrogeological complexity.

3.3. Prioritization of rock mass instability causes

To prioritize the causes of rock mass instability in Popești-Voievozi mining perimeter, we must evaluate them based on their potential impact and probability of occurrence. Based on the available information, we can establish the following hierarchy:

A) Hydrogeology: High Impact, High Probability. The variable groundwater level and high hydrostatic pressure are critical factors. Fluctuations in the water level can significantly alter pore pressure, reduce rock strength, and promote landslides. The direction and velocity of groundwater flow can erode and destabilize the rock mass, especially in zones with discontinuities. Variations in the filtration coefficient determine how water moves through the mass, with major implications for stability. The chemical aggressivity of water can lead to the dissolution of certain minerals, weakening the rock structure. In the context of Popești-Voievozi perimeter, with its varied formations (lignite, marls, clays, running ground), the behavior of groundwater is complex. Continuous monitoring of water levels and pressure, as well as chemical analysis of the water, is essential to assess corrosion and dissolution risks.

B) Geology: High Impact, High Probability. The complex geological structure, with faults, fractures, and discontinuities, creates zones of weakness in the rock mass. The varied lithology, with alternations of rocks with different geomechanical properties, significantly influences stability. The non-uniform stratigraphy, with variations in thickness and continuity, can create zones of instability. In Popești-Voievozi perimeter, the presence of unconsolidated running ground with a variable grain size composition represents a major risk. These deposits are prone to liquefaction and subsidence. Faults and fractures can facilitate groundwater flow and reduce the shear strength of the rocks.

C) Mining Activities: Moderate to High Impact, Moderate Probability. The mining method, as well as the geometry of tunnels and working faces, influences the redistribution of stresses in the rock mass. The tunnel support system is essential for maintaining stability. Vibrations and shocks generated by blasting can destabilize the rock mass. Uncontrolled mine drainage can alter the groundwater regime. Depending on the mining method used in Popești-Voievozi (underground or surface), the risks may vary. It is important to design mining works by considering local geological and hydrogeological conditions.

D) Geomechanics: Moderate Impact, Moderate Probability. The physical and mechanical properties of the rocks (strength, deformability) determine the behavior of the rock mass under stress. The state of stress in the rock mass and the presence of discontinuities (fissures, joints) influence stability. The internal friction angle and rock cohesion are important parameters for evaluating shear strength. Detailed geomechanical tests are necessary to determine the properties of the rocks in Popești-Voievozi perimeter and to assess the risks of deformation and failure.

E) External Factors: Low to Moderate Impact, Variable Probability. Seismicity, climate (precipitation, temperature variations), and erosion can influence the stability of the rock mass. Anthropogenic activities (construction, infrastructure) can alter the groundwater regime. Although these factors can have a significant impact, their probability of occurrence is lower compared to geological and hydrogeological factors. Nevertheless, it is important to consider them in the design of mining works.

Through this prioritization, an effective action plan can be developed for managing rock mass stability in Popești-Voievozi perimeter. The action plan for managing rock mass stability in Popești-Voievozi mining perimeter must be comprehensive and adapted to the specific conditions of the area. It should include preventive, monitoring, and intervention measures, addressing all relevant aspects identified in the Ishikawa diagram analysis.

Consequently, for the effective management of rock mass stability in mining perimeters where very difficult conditions for the execution and operation of underground works are encountered (as is the case in the mining perimeter considered in this study), an integrated action plan focuses on detailed hydrogeological and geological monitoring of the ground, including in-situ and laboratory investigations [33, 36]. It provides for the design and implementation of robust stabilization measures, such as adapted support systems and controlled drainage. Concurrently, mining activities can be optimized through appropriate mining methods and the control of vibrations resulting from blasting. All these efforts must be supplemented by continuous safety and deformation monitoring measures, supported by interdisciplinary collaboration and effective communication, with the aim of minimizing risks and ensuring operational safety.

By using a combination of these three diagrams, the integrated cause-and-effect analysis of rock mass stability in Popești-Voievozi mining perimeter ensures a comprehensive and efficient approach to managing geological and hydrogeological risks. The Ishikawa Diagram provides a complete picture of all possible causes, the Flowchart illustrates the main causal chain and identifies critical intervention points, and the Mind Map generates ideas for control strategies and visualizes the relationships between different factors. This integrated approach allows for a deep understanding of the problem's complexity and facilitates the development of sustainable solutions for ensuring rock mass stability and the safety of mining operations.

The integrated cause-and-effect analysis, using a combination of the Ishikawa Diagram, Flowchart, and Mind Map, represents a strategic and profound approach to understanding and managing geological and hydrogeological risks in Popești-Voievozi mining perimeter. This method is not just a theoretical exercise but a practical and dynamic tool, adaptable to new information and changes in the field.

First, interactivity and adaptability are essential. These diagrams are not static; they evolve with the project. As new data are obtained, such as results from geomechanical tests or piezometric monitoring, the diagrams are updated. This flexibility allows for a continuous risk assessment and an adjustment of control strategies. Moreover, they stimulate communication among specialists from various fields, facilitating an interdisciplinary and collaborative approach. Second, these diagrams integrate harmoniously with other technical tools. Numerical models, for example, can simulate groundwater flow or the behaviour of the rock mass, and the results are integrated into the diagrams to refine the analysis. Real-time monitoring systems, which track ground deformations or groundwater levels, can provide early warnings, allowing for rapid interventions. Third, the focus is on prevention. By identifying potential causes of instability early, preventive measures can be implemented, thus avoiding high costs for repairs or production interruptions. This proactive approach not only reduces the risk of accidents but also contributes to a safer and more sustainable mining operation. We cannot, however, ignore the economic and environmental implications. Effective management of rock mass stability minimizes the costs associated with instability, ensuring continuous and efficient production. It also protects the environment by preventing landslides and water pollution, which can have serious long-term consequences.

Thus, the integrated cause-and-effect analysis, using these three diagrams, represents a holistic and strategic approach to risk management in mining projects where difficult geological conditions are encountered. It allows for a deep understanding of the complex interaction between geological and hydrogeological factors, facilitating the development of sustainable solutions and ensuring the safety of mining operations.

4. Risk Analysis of rock mass instability

The risk analysis presented herein focuses on the complex evaluation of risks associated with rock mass instability in Popești-Voievozi mining perimeter. Mining activities, by their very nature, intervene in a pre-existing geological and geomechanical equilibrium, modifying natural conditions and introducing new influencing factors. Rock mass instability represents a major concern with potentially serious consequences for personnel safety, the integrity of mining infrastructure, and long-term operational viability [42, 43].

The purpose of this analysis is to identify, evaluate, and prioritize the fundamental causes that contribute to the occurrence of instability phenomena. By using visual tools such as the Ishikawa diagram for cause identification and the risk matrix for their prioritization, we aim to provide a clear picture of the critical factors and their interactions. A deep understanding of these risks is essential for developing effective mitigation and monitoring strategies, thereby ensuring proactive management of rock mass stability and minimizing hazards in mining perimeters situated under similar conditions. This systematic approach will allow for the judicious allocation of resources and the implementation of targeted preventive and corrective measures, contributing to the safety and sustainability of mining operations.

vol. 31, issue 4 / 2025
pp. 1-23

4.1. The risk matrix

To perform this analysis, we used a 3×3 scale for simplicity and clarity, which can be expanded to a 5×5 scale for greater granularity if necessary. The scales are presented in Table 3.

Table 3. Scales used in the analysis

Impact Scale (Severity of Consequences)	Probability Scale (Frequency of Occurrence)
1 (Low): Minor consequences, easily managed, no major impact on operations or safety.	**1 (Low):** Very unlikely to happen, a rare or exceptional event.
2 (Moderate): Significant consequences, requires considerable resources to manage, can affect short-term operations, poses safety risks.	**2 (Moderate):** Likely to happen occasionally, not a common event.
3 (High): Critical consequences, major losses (material, human, financial), long-term interruption of operations, risk of serious accidents or catastrophes.	**3 (High):** Very likely to happen frequently or almost certain, an expected event.

The Risk Level (RL) was calculated as the product of impact and probability. Subsequently, the risk matrix was constructed by assigning corresponding values to the main categories (table 4).

Table 4. Construction of the risk matrix

Main Category	Impact (I)	Probability (P)	Risk Level (I x P)
1. Hydrogeology	High (3)	High (3)	9 (Critical)
2. Geology	High (3)	High (3)	9 (Critical)
3. Mining Activities	Moderate to High (2.5 ≥ 3)	Moderate (2)	5-6 (High)
4. Geomechanics	Moderate (2)	Moderate (2)	4 (Moderate)
5. External Factors	Low to Moderate (1.5 ≥ 2)	Variable (1.5 ≥ 2)	2-4 (Low to Moderate)
Note: We used intermediate values (2.5, 1.5) to reflect "moderate to high" or "low to moderate." However, for a 3×3 matrix, we must round to the nearest level or consider the most pessimistic scenario for prioritization.			

For the visual matrix, we used discrete values: for mining activities, we considered the upper end of the range for prioritization; for external factors, we considered the lower end of the range because the probability is "variable" and the impact is "low to moderate" (Figure 5).

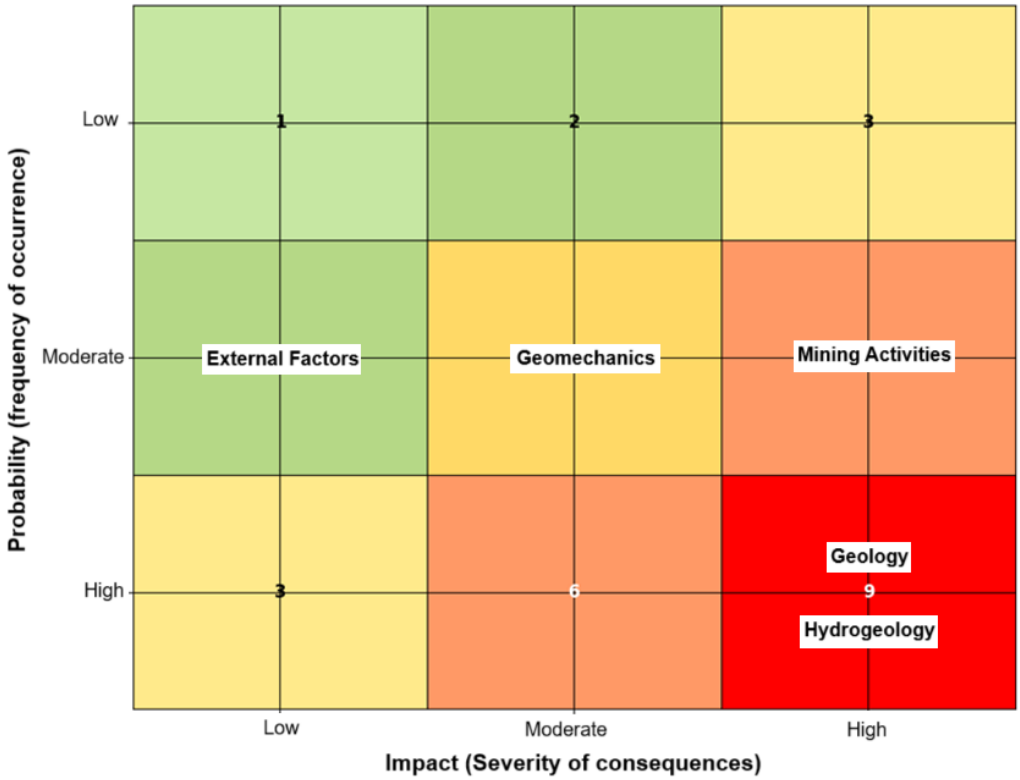

Fig.5. Risk matrix for rock massif instability

 vol. 31, issue 4 / 2025
pp. 1-23

The risk matrix serves as an essential tool for prioritizing and understanding the potential causes of rock mass instability, evaluating each primary factor based on its Impact (severity of consequences) and Probability (frequency of occurrence). By strategically placing the causes within the matrix's nine quadrants, appropriately color-coded according to risk level, a clear visualization of action priorities is obtained.

The risk matrix classifies zones by risk level (Impact × Probability) into distinct colors: red (value 9) indicates critical/very high risk, necessitating immediate actions and major investments; orange (value 6) represents high risk, requiring careful planning, substantial corrective measures, and intensive monitoring; yellow (value 3 or 4) signals moderate risk, mandating active monitoring and contingency plans; light green (value 2) denotes low risk, requiring routine monitoring; dark green (value 1) indicates very low risk, with minimal intervention.

4.2. Analysis of primary cause positioning

Based on this matrix (Figure 5), the hierarchy of instability causes within this mining perimeter becomes evident:

Hydrogeology and Geology (Critical Risk - Red Zone): Both factors are positioned in the maximum risk zone. They are considered the most dangerous and imminent. Due to the intrinsic nature of the rock mass (geology) and its complex interaction with water (hydrogeology), the risk of destabilization is permanent and carries potentially catastrophic consequences. Any stability management plan must prioritize the monitoring, in-depth understanding, and control of these aspects. Mitigation measures here will be the most costly and critical (e.g., advanced drainage systems, structural consolidations).

Mining Activities (High Risk - Orange Zone): This factor is located in the middle-right quadrant (High Impact, Moderate Probability). Although the probability of mining activities triggering instability is considered moderate, the potential impact is as high as that of geological/hydrogeological factors. This implies that if an operational error or a deficiency in mining design occurs, the consequences can be severe. The rigorous application of operational standards, adequate support systems, and vibration control are crucial. Continuous optimization of work methods and adherence to safety procedures are essential to maintain probability at a moderate level and prevent risk escalation.

Geomechanics (Moderate Risk - Yellow Zone): The Geomechanics factor is placed in the center of the matrix (Moderate Impact, Moderate Probability). This is a risk that requires active management. While not as critical as the first two, ignoring rock properties and stress states can lead to the progressive deterioration of the rock mass. It is necessary to monitor deformations, conduct periodic geomechanical tests, and update stability analyses to ensure the rock mass behaves as expected and that no new weakness zones emerge.

External Factors (Low Risk - Light Green Zone): This factor is in the bottom-middle quadrant (Low Impact, Moderate Probability). This category presents the lowest level of risk in the general context of rock mass instability compared to the other factors. Although events such as earthquakes, torrential rains, or erosion can have a localized impact, they are considered either less probable as primary causes of major instability or their direct impact is lower. Routine monitoring of climatic conditions and seismic activity is necessary, and this data should be integrated into overall risk assessments, but without allocating disproportionate resources compared to the critical risks.

The risk matrix clearly confirms and visualizes that Hydrogeology and Geology represent the fundamental drivers of rock mass instability at Popești-Voievozi and require a strategic approach and prioritized resources. Mining Activities, though controllable, demand constant vigilance. Geomechanics and External Factors require monitoring and attention, but not as intensely as the top-tier risks. This prioritization is essential for the efficient allocation of resources and the development of a robust and proactive action plan for stability management within the mining perimeter. The priorities from the risk matrix are clearly defined: Hydrogeology and Geology represent the maximum priority (critical risk), having the highest impact and probability. Mining activities are a high priority, with significant impact. Geomechanics falls under moderate priority, requiring active monitoring. Finally, external factors have a low priority, being the least critical.

4.3. General action plan

For each risk category, including specific causes, we have detailed the proposed management (or at least mitigation) measures based on the aforementioned "action plan" (table 4). The prioritization of the causes of rock mass instability in Popești-Voievozi mining perimeter emphasizes that Hydrogeology and Geology are the most critical aspects and must be at the center of any action plan. Major investments and efforts should be directed towards the monitoring, understanding, and control of these factors [44].

Table 4. Management measures for risk category

Risk Level	Main Category (Cause)	Specific Sub-Causes	Risk Description and Implications	Recommended Mitigation & Monitoring Measures
CRITICAL	**Hydrogeology** I: High (3) P: High (3)	Variable groundwater level and high hydrostatic pressure Groundwater flow (direction, velocity) Variations in the coefficient of filtration Chemical aggressiveness of the water	Fluctuations in pore pressure, erosion, and mineral dissolution can lead to a loss of stability. The complex behaviour of groundwater in the varied formations of Popești-Voivozi indicates a persistent and active influence on the site's stability.	Continuous Monitoring: Piezometers, water level, and pressure monitoring systems. Regular Chemical Analyses: Assess water's chemical aggressiveness and its impact on minerals Controlled Drainage Systems: Implement and optimize drainage to reduce hydrostatic pressure. Hydrogeological Modeling: Create predictive models. Sealing Works: Prevent water infiltration into vulnerable zones.
CRITICAL	**Geology** I: High (3) P: High (3)	Complex geological structure (faults, fractures, discontinuities Varied lithology (alternating rock types) Non-uniform stratigraphy Presence of unconsolidated rock fragments	The complex structure, faults, fractures, and varied lithology create intrinsic and permanent zones of weakness. The unconsolidated rock fragments pose a major risk of liquefaction and subsidence.	Detailed Geological Mapping: Identify and characterize discontinuities and weak zones. In-depth Geotechnical Study: Analyze geomechanical properties, especially of unconsolidated rock fragments. Mass Consolidation: Apply consolidation measures (grouting, rock bolts) in critical areas. Mining Project Adjustment: Design workings to account for the orientation of discontinuities.
HIGH	**Mining Activities** I: High (3) P: Moderate (2)	Mining method, geometry of galleries/working faces Gallery support systems Vibrations and shocks (blasting) Uncontrolled mine drainage	The risk is manageable through best practices, but the potential impact is high if procedures are not followed. An operational error could have severe consequences.	Optimization of Mining Method: Adapt the method to geological and hydrogeological conditions. Rigorous Support Design & Installation: Ensure adequate support and monitor its integrity. Vibration Control: Optimize blasting schemes. Controlled Drainage Management: Ensure drainage does not destabilize the site. Safety Procedures: Implement and strictly adhere to Standard Operating Procedures (SOPs).
MODERATE	**Geomechanics** I: Moderate (2) P: Moderate (2)	Physical-mechanical rock properties (strength, deformability) Stress state and presence of discontinuities Internal friction angle and rock cohesion	Rock properties determine strength, while stresses can activate failures. Although not critical, neglecting these factors can lead to progressive deterioration.	Active Monitoring: Detailed geomechanical testing campaigns (lab and in-situ). Deformation Monitoring: Install instruments (extensometers, inclinometers). Stability Analyses: Use numerical models to evaluate stability. Recalculation of Design Parameters: Adjust parameters based on updated geomechanical data.

vol. 31, issue 4 / 2025
pp. 1-23

LOW	**External Factors** I: Low (1) P: Moderate (2)	Seismicity Climate (precipitation, temperature variations) Erosion Anthropogenic activities (construction, infrastructure)	This category presents the lowest risk compared to other factors. Events are either less likely to be primary causes or their impact is more limited.	Seismic Monitoring: If the area is seismically active. Surface Water Management: Control erosion through superficial drainage and slope stabilization. Anthropogenic Impact Assessment: Analyse nearby projects. Emergency Plans: Elaborate plans for extreme events (e.g., torrential rains, earthquakes)
Note: Impact (I) / Probability (P)				

An efficient action plan must be integrated and iterative, implying:

a) Continuous evaluation, through the periodic re-evaluation of risks based on monitoring data.
b) Multidisciplinary monitoring, through the integration of hydrogeological, geological, geomechanical, and operational data.
c) Preventive measures, through the implementation of mitigation solutions before problems become critical.
d) Rapid response, through the elaboration of emergency and intervention plans for instability scenarios.
e) Education and training, by ensuring that personnel are aware of risks and trained in safety procedures [23, 42, 43].

This risk analysis offers a solid basis for making informed decisions regarding the management of rock mass stability.

5. The Mathematical model conceived for the analysis of the stability of the massif

On the basis of the detailed characterization of Popești-Voievozi perimeter and the prioritization of the risks, a comprehensive geomechanical mathematical model was conceived, meant to analyze the stability of the rock mass. This model is capable of integrating the geological complexity, the behavior of the materials and the critical influence of the hydrogeological regime.

5.1. The Fundamental equations and constitutive models

The model is constructed on the basis of the fundamental equations of the mechanics of continuous media and uses specific constitutive models to describe the behavior of the rocks from the deposit:

a) The Equations of Equilibrium: These ensure that the system is in static equilibrium (or pseudo-static, if reduced inertial forces are also considered). They express the conservation of momentum. In the absence of inertial forces, the sum of the forces in an elementary volume must be zero. Mathematically, in 3D, these equations are expressed in terms of partial derivatives of stresses and volumetric forces (Fx, Fy, Fz):

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + F_x = 0$$
$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + F_y = 0$$
$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + F_z = 0$$

where σ represents normal stresses and τ represents shear stresses.

b) Compatibility (Strain-Displacement) Relations: These equations ensure that strains are compatible with displacements (meaning the material does not fracture or overlap). They define the relationship between the displacement field (u, v, w) and the components of the strain tensor (ε). For small strains:

$$\varepsilon_x = \frac{\partial u}{\partial x}\ ; \qquad \varepsilon_y = \frac{\partial v}{\partial y}\ ; \qquad \varepsilon_z = \frac{\partial w}{\partial z}$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\ ; \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\ ; \qquad \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

where ε are the normal strains and γ are the shear strains.

vol. 31, issue 4 / 2025
pp. 1-23

c) Constitutive Models: These define the stress-strain response of each rock type using the geomechanical parameters from Tables 1 and 2:

- Linear Elastic Model: Used for reversible behavior, incorporating Young's modulus (E) and Poisson's Ratio (μ).
- Mohr-Coulomb Elastic-Plastic Model: Used to simulate shear failure, using Cohesion (C) and Internal Friction Angle (φ). This model is essential for identifying areas where the rock mass loses its bearing capacity.
- Rheological Model (Creep): The Bürgers model is used for marls and clays. This is particularly important for high-plasticity and swelling clays and marls. This viscous-elastic-plastic model allows for the simulation of creep—the time-dependent deformation under constant load—which is necessary for long-term stability evaluation.

5.2. Hydro-mechanical coupling: an essential component

Given the critical importance of groundwater (identified as a major risk), the model includes a bidirectional hydro-mechanical coupling. This coupling is based on:

- **Terzaghi's Principle of Effective Stress:** $\sigma' = \sigma - u$. This principle is fundamental because an increase in pore water pressure (u) reduces the effective stresses (σ′) and, consequently, the shear strength of the rocks. The model simulates how fluctuations in the piezometric level (e.g., those from the Aquifer Horizon between seams no. II and III) influence stability.
- **Darcy's Law**: This describes the flow of water through porous media, using the filtration coefficients (k) determined from boreholes. The model calculates the distribution of pore pressures and water flows, which allows for the evaluation of the impact of infiltration and drainage systems on stability.

Integrating these principles allows the model to simulate complex phenomena, such as the liquefaction of saturated sands (unconsolidated rock fragments) under pressure, which is a major problem in this mining perimeter.

5.3. Problem formulation and stability criteria

The mathematical model must account for the time evolution of the construction process and rheological phenomena.

A) Initial (Geostatic) Equilibrium: The initial stresses are calculated based on the specific weight of the layers (γ) and the coefficient of lateral earth pressure at rest (K_0).

$$\sigma_v = \sum(\gamma_i \times h_i) \quad ; \quad \sigma_h = K_0 \times \sigma_v \tag{1}$$

where: $K_0 = \frac{\vartheta}{1-\vartheta}$ for rocks.

The initial pore water pressures are established based on the piezometric level and the specific weight of water:

$$u = \gamma_w \times \left(h_{piezometric} - z\right) \tag{2}$$

B) Incremental model construction process:

Element deactivation/activation: Mathematically, this involves modifying the global stiffness matrix of the finite/discrete element network by removing the contribution of "deactivated" elements or adding the contribution of "activated" elements. Each increment of excavation/support represents a new state of equilibrium to which the system converges.

C) Time-dependent analysis (creep and consolidation): The rheological (Bürgers) model introduces the time dependency of deformations, calculating the creep deformation rates $\dot{\varepsilon}_{fluaj}$. Consolidation processes (the dissipation of excess pore water pressures) are also time-dependent and governed by coupled equations, including the term for the variation of pore pressure over time $\left(\frac{\partial u}{\partial t}\right)$.

5.4. Problem formulation and stability criteria

Based on the calculated stresses and deformations, engineering criteria are applied.

A) Rock mass safety factor (SRF - Strength Reduction Factor): This is a method for evaluating overall stability by iteratively reducing the strength parameters (C and tgϕ) until the model fails. The safety factor (F) is the reduction factor at which the model becomes unstable:

$$F = \frac{\tau_f}{\tau_{mob}} = \frac{C + \sigma' tg\emptyset}{\tau_{mob}} \tag{3}$$

where τ_f is the shear strength and τ_{mob} is the mobilized shear stress. In the SRF method:

$$C_{trial} = \frac{C}{SRF} \quad ; \quad \emptyset_{trial} = arctg\left(tg\frac{\emptyset}{SRF}\right) \tag{4}$$

B) Verification of Support Bearing Capacity: This involves comparing the calculated stresses in the concrete ($\sigma_{concrete}$) with the specified characteristic compressive strength (f_{ck}=30 MPa).

$$\sigma_{concrete} \leq f_{ck} \tag{5}$$

C) Acceptable deformation thresholds: Comparison of calculated deformations, which for radial convergences (U_r) with the limits allowed in design, is: $U_r \leq U_{admis}$.

5.5. The system of governing equations for the coupled geomechanical model

The geomechanical model presented for 3D numerical geomechanical modeling is a complex system of coupled, non-linear, partial differential equations that govern [17], [26], [27]: mechanical equilibrium (three equations for each X, Y, Z direction); constitutive behavior (non-linear Mohr-Coulomb stress-strain relations and creep); fluid flow (a continuity equation based on Darcy's law); and fluid-solid interaction (the principle of effective stresses. These equations are locally valid at every point in the medium and evolve over time (due to creep, consolidation, and construction phases). Material properties (E, ν, C, ϕ, K, etc.) vary spatially (from layer to layer) and, for some materials, depend on the state of stresses/strains or loading history. The solution to this system of equations is strongly influenced by the boundary conditions (prescribed displacements, applied forces, pore pressures at the boundaries). The failure criteria (Mohr-Coulomb) and plastic behavior introduce a pronounced nonlinearity, which means that simple superposition cannot be applied and the solution requires iterative methods. To solve this system of equations, software such as **FLAC3D** or **ABAQUS** uses numerical methods (Finite Differences or Finite Elements) to discretize these complex equations on a mesh of elements. Thus, the system of partial differential equations is transformed into a massive system of algebraic equations (matrices) with thousands or millions of unknowns (displacements, stresses, pore pressures at nodes or element centers), which is solved iteratively. The geomechanical model is described by a system of coupled, non-linear, partial differential equations that express mechanical equilibrium, material behavior, and the conservation of fluid mass in a porous medium.

1) Mechanical equilibrium equation (in terms of effective stresses and pore pressure): Starting from the total stress equilibrium equations and applying the principle of effective stresses ($\sigma = \sigma' + u\delta_{ij}$, where δ_{ij} is the Kronecker delta), the equilibrium equations can be written in terms of effective stresses $\left(\sigma'_{ij}\right)$ and pore water pressure (u):

$$\frac{\partial \sigma'_{ij}}{\partial x_j} + \frac{\partial u}{\partial x_i} + \rho g_i = 0 \tag{6}$$

where: σ'_{ij} represents the components of the effective stress tensor; u is the pore water pressure; g_i is the gravitational acceleration in the i direction; ρ is the total density of the medium.

This equation is coupled because the pore water pressure directly influences the mechanical equilibrium through its gradient term.

2) Constitutive relations (stress-strain): These relations define the material behavior and are generally non-linear. They link the effective stress rate $\left(\dot{\sigma}'_{ij}\right)$ to the total strain rate$\left(\dot{\varepsilon}_{ij}\right)$ and the plastic/viscous strain rate $\left(\dot{\epsilon}_{ij}^{p/v}\right)$:

$$\left(\dot{\sigma}'_{ij}\right) = D_{ijkl}\left(\dot{\varepsilon}_{kl} - \dot{\epsilon}_{kl}^{p/v}\right) \tag{7}$$

where: s the material's elastic stiffness tensor; ϵ˙kl is the total strain rate (derived from the displacement rates); $\dot{\epsilon}_{kl}^{p/v}$ is the plastic or viscous strain rate, governed by:

- **Mohr-Coulomb failure criterion:** $F\left(\sigma'_{ij}, C, \emptyset\right) = 0$F (where F is the yield function).
- **Plastic flow rule:** $\dot{\varepsilon}_{ij}^{p} = \lambda \frac{\dot{\partial G}}{\partial \sigma'_{ij}}$ (where λ is a plastic multiplier and G is the plastic potential).
- **Creep law (for example, for the Bürgers model):** $\dot{\varepsilon}_{ij}^{v} = f\left(\sigma'_{ij}, t, viscous\ parameters\right)$ (a function dependent on stresses and material parameters, which evolves over time).

3) Fluid flow equation (coupled hydromechanical consolidation): This equation expresses the conservation of fluid mass and is coupled with mechanical deformations through the variation of pore

vol. 31, issue 4 / 2025
pp. 1-23

volume. For a saturated medium and an incompressible fluid, the continuity equation combined with Darcy's law and the relationship between pore volume and volumetric strain ϵv is given by:

$$\frac{K}{\gamma_w} \times \frac{\partial^2 u}{\partial x_i x_i} = \frac{1}{M} \times \frac{\partial u}{\partial t} + \frac{\partial \varepsilon_v}{\partial t} \tag{8}$$

where: K is the permeability; γw is the specific weight of water; M is the Biot modulus (a material constant related to the compressibility of the solid skeleton and pores); ε_v is the volumetric strain $\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$

This equation is coupled because the rate of change of pore water pressure (u) is related to the volumetric strain rate $\left(\frac{\partial \varepsilon_v}{\partial t}\right)$, which in turn depends on the material's mechanical response.

The geomechanical model is thus governed by the system formed by equations (6), (7), and (8), along with the compatibility relations (strain-displacement), the specific properties of each material, and the spatial and temporal boundary conditions. This system of equations does not have a simple analytical solution for complex geometries and non-linear behaviors. Therefore, numerical methods (FEM or FDM) are used, which discretize space and time to approximately solve these equations through massive, iterative matrix systems. Essentially, the geomechanical mathematical model for the considered inclined plane is a complete, coherent, and quantifiable description of the geological and engineering system, which is necessary before unstable any simulation. It is, in practice, the foundation upon which any subsequent numerical analysis is built and without which the modeling process would be meaningless. This is the main reason why we chose the elaboration of the geomechanical mathematical model as the main objective of this study, a model that can be used for any type of underground work carried out under similar conditions. Figure 6 describes a sequential and interconnected flow of the components of the integrated geomechanical mathematical model.

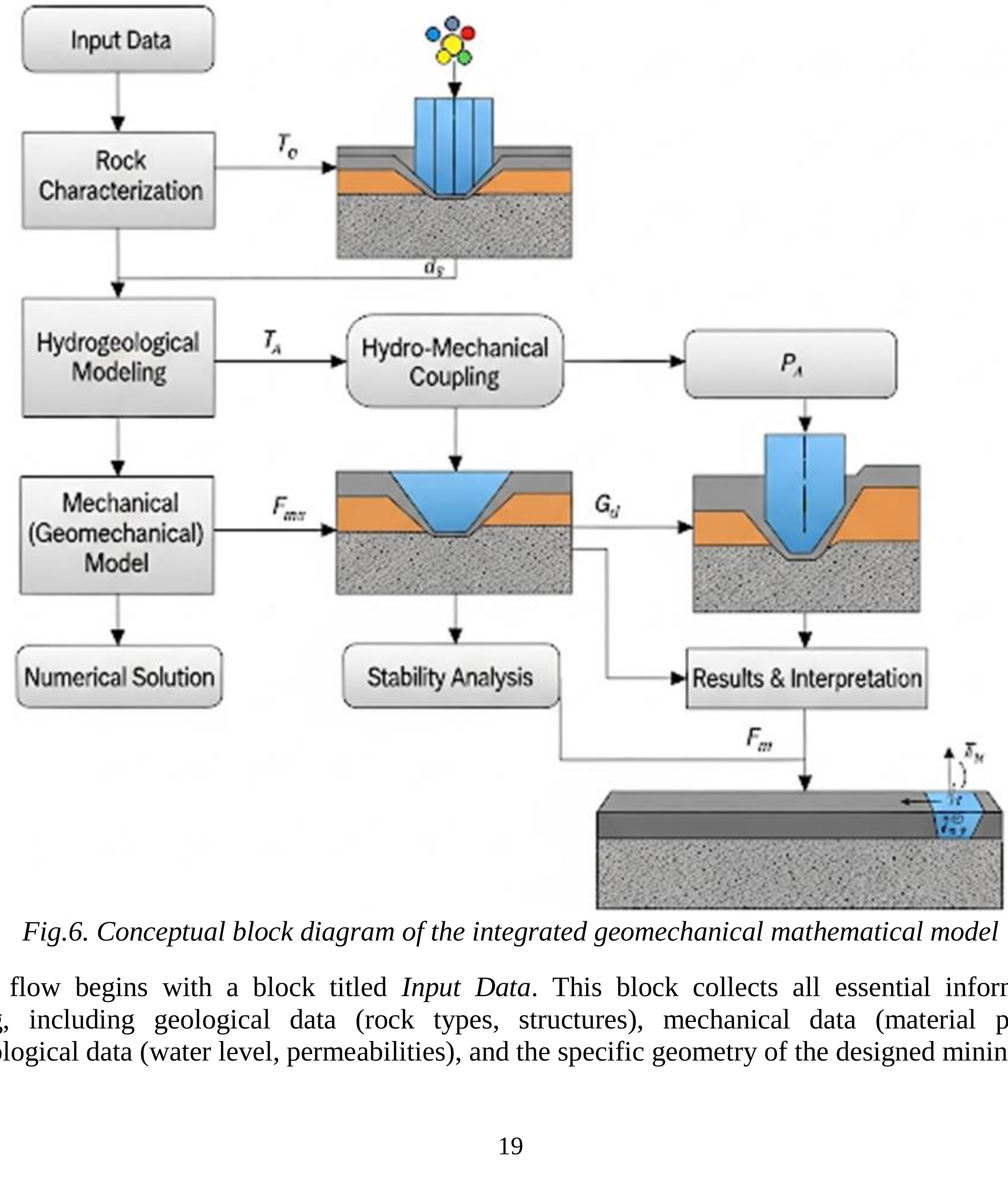

Fig.6. Conceptual block diagram of the integrated geomechanical mathematical model

The flow begins with a block titled *Input Data*. This block collects all essential information for modeling, including geological data (rock types, structures), mechanical data (material properties), hydrogeological data (water level, permeabilities), and the specific geometry of the designed mining works.

From the Input Data block, the information proceeds to *Rock Characterization*. Here, the intrinsic properties of the constitutive materials are determined and defined: physical properties (density, porosity), mechanical properties (strengths, elastic moduli), and rheological properties (creep behavior).

The characterized properties, especially those related to permeability, then feed into *Hydrogeological Modeling*. This block is based on Darcy's law for water flow, considers specific hydraulic boundary conditions (e.g., surface water level, presence of isolated aquifers), and allows for the calculation of the piezometric level distribution within the rock mass.

A central element of the model is the *Hydro-Mechanical Coupling*, which receives information from the Rock Characterization block (via poroelastic parameters) and the Hydrogeological Modeling block (pore water pressures). This block represents the fundamental interaction between the state of mechanical stresses and the hydraulic regime, based on Terzaghi's principle of effective stresses. This coupling ensures that changes in water pressure affect the strength and deformability of the rock, and vice versa.

The results of the coupling feed the *Mechanical (Geomechanical) Model*. This block includes the equilibrium equations (which ensure force conservation), compatibility relations (which link strains to displacements), and various constitutive models for rocks: the linear elastic model (for elastic behaviour), the Mohr-Coulomb model (for shear failure conditions), and the Bürgers model (for simulating creep and time-dependent behavior).

Next is the *Numerical Solution* block. Given the complexity of the system of equations, this stage is essential. Here, advanced numerical methods, such as the Finite Element Method (FEM) or the Finite Difference Method (FDM), are applied. This involves the space-time discretization of the study domain and the use of iterative solution algorithms to obtain approximate solutions to the equations.

The results obtained from the numerical solution are processed in the *Stability Analysis* block. This stage involves the calculation of the Safety Factor (SRF), verification of the bearing capacity of the designed support systems, and comparison of the calculated deformations with the allowable deformation thresholds. These criteria provide a quantitative assessment of safety.

Finally, the *Results & Interpretation* block presents the model's outputs in a format useful for engineers. These include distributions of deformations (displacements), stresses within the rock mass, identification of failure or plastic yielding zones, and water flows. All this information is important for making engineering decisions regarding the safety and optimization of the mining works.

6. Discussions and applications of the mathematical model

The conceived geomechanical mathematical model, which integrates geological, geomechanical, and hydrogeological data, provides a powerful tool for the stability analysis of the rock mass in Popești-Voievozi perimeter. It allows not only for a static evaluation of stability but also for a simulation of time-dependent behaviour and complex interactions with groundwater.

By applying this model, the following important analyses can be performed:

- **Prediction of deformations and stresses**: The model can calculate the distribution of stresses and deformations around mine excavations, identifying zones of stress concentration and plastic yielding. This is essential for preventing collapses and ensuring the integrity of the workings.
- **Stability evaluation in the presence of water**: The coupled hydro-mechanical module allows for the simulation of the impact of pore pressures on stability. Scenarios of water influx (similar to mud and water inrushes) can be analyzed, evaluating the efficiency of drainage systems in reducing pressures and maintaining the Safety Factor. The model can quantify the risk of liquefaction of pressurized aquifer sands.
- **Optimization of support systems**: Through modeling, different configurations and materials for supports can be tested, evaluating their capacity to accommodate the deformations induced by the rock mass and to ensure long-term stability, including under creep conditions. Practical observations regarding support displacements can be used for model calibration.
- **Time-dependent behavior analysis**: Using the Bürgers rheological model, the progressive deformations of clayey and marly rocks can be simulated over time. This is crucial for designing supports with an adequate service life and for anticipating the need for maintenance interventions.
- **Risk scenario evaluation**: The model allows for the simulation of the impact of external factors (e.g., extreme rainfall, seismic events) or failure scenarios (e.g., drainage interruption), providing a basis for emergency and risk mitigation plans.

vol. 31, issue 4 / 2025
pp. 1-23

In the specific context of this perimeter, where severe instability phenomena related to groundwater and rock properties have been recorded, the application of this mathematical model is indispensable. It allows for a transition from a reactive approach (responding to incidents) to a proactive one (prediction and prevention), optimizing the design of mining works and increasing operational safety.

7. Conclusions

The stability analysis of the rock masses in the considered mining perimeter demonstrates the inherent complexity of lignite extraction under difficult geological and hydrogeological conditions. The integrated approach, which included a detailed characterization of the geo-mining context, the application of cause-and-effect and risk analysis methodologies, and the establishment of a coupled geomechanical mathematical model, was essential for understanding and managing the challenges encountered.

The main instability factors, identified through the Ishikawa diagram and prioritized as critical by the risk matrix, are of a hydrogeological nature (pressurized aquifer horizons, running ground phenomena) and a geological nature (unfavorable lithology, discontinuities). The conceived mathematical model is capable of integrating these aspects, simulating the behavior of materials, their interaction with groundwater, and the response of the rock mass to the stresses induced by the works.

By applying this model, the safety factor, the bearing capacity of the supports, and deformations can be evaluated, providing a solid scientific basis for the safe and efficient design of underground mining works. The study underscores that the success of mining operations in such environments depends on a deep understanding of the complex interactions among geological, hydraulic, and mechanical factors, as well as on a constant capacity for adaptation and monitoring.

References

[1] **Brady, B. H. G., & Brown, E. T.,** 2006.
Rock Mechanics for Underground Mining (third ed.), Springer, Kluwer Academic Publishers, Netherlands (**2006**). [Google Scholar]

[2] **Goodman, R. E.,** 1989
Introduction to Rock Mechanics (2nd ed.). 2nd Edition, John Wiley & Sons Ltd., New York. ISBN: 978-0-471-81200-5.

[3] **Hudson, J. A., & Harrison, J. P.** 1997
Engineering Rock Mechanics: An Introduction to the Principles. Pergamon.

[4] **Que, C.T., Nevskaya, M., Marinina, O**., 2021
Coal Mines in Vietnam: Geological Conditions and Their Influence on Production Sustainability Indicators. Sustainability 2021, 13, 11800. https://doi.org/10.3390/su132111800

[5] **Toderas, M.,** 2014
Mecanica rocilor, pământurilor şi construcţii subterane. Editura Universitas, Petroşani, Vol.I-II, ISBN 978-973-741-381-9, 1167 pg.

[6] **Toderas, M.,** 2021
Constructii miniere subterane. Vol. I – II. Editura Universitas, Petroşani, 1266 pg., ISBN 978-973-741-806-7

[7] **Wang, J., & Lee, H. S**., 2007
Risk analysis in tunneling. Journal of Tunnelling and Underground Space Technology, 22(4), 437-448.

[8] **Xiao, H., & Liu, R.,** 2008
Risk Assessment of Rock Burst in Deep Underground Mining. Safety Science, 46(8), 1294-1300.

[9] **Cristescu, N. D., & Hunsche, U.,** 1998
Rock Mechanics: From Deformation to Fracture. Kluwer Academic Publishers.

[10] **Hammah, R.; Yacoub, T.; Corkum, B.,** 2005
T*he Shear Strength Reduction Method for the Generalized Hoek-Brown Criterion*. American Rock Mechanics Association (ARMA): Anchorage, AK, USA. [Google Scholar]

[11] **Jaeger, J.A.,** 1988.
Excess Shear Stress in the Assessment of Geological Hazardous Situations. Journal of the South African Institute of Mining and Metallurgy, vol. 88. pp. 27–39.

vol. 31, issue 4 / 2025
pp. 1-23

[12] **Hoek, E., & Brown, E. T.,** 1997
Practical Estimates of Rock Mass Strength. International Journal of Rock Mechanics and Mining Sciences, 34(8), 1165-1186.

[13] **Read, S. E., & Stacey, T. R.**, 2009
A Discussion on the Engineering Geological Assessment of Rock Mass Strength for Underground Excavations. International Journal of Rock Mechanics and Mining Sciences, 46(1), 1-13.

[14] **Terzaghi, K., Peck, R. B., & Mesri, G.**, 1996
Soil Mechanics in Engineering Practice (3rd ed.). John Wiley & Sons.

[15] **Hitchman, S.,** 1999
Groundwater and Geotechnics. Blackwell Science.

[16] **Bieniawski, Z. T.**, 1989
Engineering Rock Mass Classifications: A Practical Guide for Engineers in Mining, Civil, and Petroleum Engineering. John Wiley & Sons.

[17] *** * ***
Software Plaxis 2D/3D [accessed May 2025]

[18] **Stacey, T. R.,** 2009
The Engineering Rock Mass Classification Systems: Recent Developments and Comparisons. Journal of the Southern African Institute of Mining and Metallurgy, 109(5), 263-274.

[19] **ISO 31000:2018**., 2018
Risk Management – Guidelines. International Organization for Standardization. Second edition 2018-02. Published in Switzerland. www.iso.org

[20] **ASQ (American Society for Quality)** ASQ/ANSI E4:2014 (R2019)
Quality Management Systems for Environmental Information and Technology Programs— Requirements with Guidance for Use

[21] **U.S. Geological Survey**, 2021
Mineral Commodity Summaries 2021: U.S. Geological Survey, 200 p., https://doi.org/10.3133/mcs2021

[22] **Chapman, R. J**., 2019
Exploring the Value of Risk Management for Projects: Improving Capability Through the Deployment of a Maturity Model. IEEE Engineering Management Review, Vol.47, No. 1., First Quarter March 2019, IEEE DOI: 10.1109/EMR.2019.2891494.

[23] **Cooper, D., Grey, S., Raymond, G., & Walker, P**., 2005
Project Risk Management Guidelines: Managing Risk in Large Projects and Complex Procurements. John Wiley & Sons.

[24] **Fantoni, R., & Monteleone, A.**, 2011
Risk Management for Mining Projects. Society for Mining, Metallurgy, and Exploration (SME).

[25] **Raiffa, H**., 1968
Decision Analysis: Introductory Lectures on Choices Under Uncertainty. Addison-Wesley.

[26] **Itasca Consulting Group**, 2023
FLAC3D 9.0: Advanced Modeling Solutions Applied to Challenging Geomechanical Problems for Three-Dimensional Analyses of Soil, Rock, Concrete, Structural Ground Support, and Groundwater Flow, Minneapolis, Minnesota.

[27] **Zienkiewicz, O. C., & Taylor, R. L.**, 2000
The Finite Element Method, Vol. 1: The Basis (5th ed.). Butterworth-Heinemann.

[28] **Potvin, Y., & Hadjigeorgiou, J.**, 2008
Rock Mass Classification for Open Pit Mines. Journal of the Southern African Institute of Mining and Metallurgy, 108(12), 769-778.

[29] **SR EN 1997-1:2004**
Eurocode 7 - Design of Geotechnical Structures - Part 1: General rules. [accessed July 2025]

[30] **Project Management Institute (PMI)**, 2021
The Standard for Project Management and a Guide to the Project Management Body of Knowledge (7th ed.). PMBOK Guide, Project Management Institute (PMI).

vol. 31, issue 4 / 2025
pp. 1-23

[31] **Legea Minelor nr. 85 din 18 martie 2003**
Legea Minelor 85/2003. Monitorul Oficial, Partea I nr. 197 din 27 martie 2003 [accessed July 2025]

[32] **ANRM**
Autoritatea Națională de Reglementare în Domeniul Minier, Petrolier și al Stocării Geologice a Dioxidului de Carbon https://www.namr.ro/home/ [accesed july 2025]

[33] **Lingampally, S.V., Bhattacharjee, R.M., Ghosh, N.**, 2023
Underground Coal Mining Methods and Their Impact on Safety. Natural Hazards - New Insights. IntechOpen. doi:10.5772/intechopen.109083.

[34] **Petlovanyi, M., Malashkevych, D., Sai, K., Zubko, S.**, 2020
Research into Balance of Rocks and Underground Cavities Formation in the Coal Mine Flowsheet when Mining Thin Seams. Min. miner. depos. **2020**, 14(4):66-81. https://doi.org/10.33271/mining14.04.066

[35] **Shen, B., Duan, Y., Luo, X., van de Werken, M., Dlamini, B., Chen, L., Vardar, O., Canbulat, I.**, 2020
Monitoring and Modelling Stress State near Major Geological Structures in an Underground Coal Mine for Coal Burst Assessment, International Journal of Rock Mechanics and Mining Sciences, Volume 129, **2020**, 104294, ISSN 1365-1609, https://doi.org/10.1016/j.ijrmms.2020.104294

[36] **Klimov. V.V.**, 2019
Geomechanical Feasibility of Underground Coal Mining Technology Using Control Systems of Electro-Hydraulic Shield Supports for Longwall Mining IOP Conf. Ser.: Mater. Sci. Eng. 560 012067

[37] **Kolapo, P., Oniyide, G.O., Said, K.O., Lawal, A.I., Onifade, M., Munemo, P.**, 2022
An Overview of Slope Failure in Mining Operations. Mining 2022, *2*, 350-384. https://doi.org/10.3390/mining2020019

[38] **Gundewar, C.**, 2014
Application of Rock Mechanics In surface and Underground Mining. Indian Bureau of Mines: Nagpur, Maharashtra, India, pp. 62–66. [Google Scholar]

[39] **Strzałkowski, P., Ścigała, R.**, 2024
Unusual Conditions for the Formation of Linear Discontinuous Deformations Triggered by Underground Mining. Environ Earth Sci 83, 611. https://doi.org/10.1007/s12665-024-11910-w

[40] **Trzeciak, M., Jonek-Kowalska, I.**, 2021
Monitoring and Control in Program Management as Effectiveness Drivers in Polish Energy Sector. Diagnosis and Directions of Improvement. Energies 2021, 14, 4661. [Google Scholar]

[41] **Kazanin, O., Sidorenko, A., Drebenstedt, C.**, 2020
Intensive Underground Mining Technologies: Challenges and Prospects for the Coal Mines in Russia. Acta Montanistica Slovaca, Volume 26 (1), 60-69. https://doi.org/10.46544/AMS.v26i1.05

[42] **Nikulin, A., Ikonnikov, D., Dolzhikov, I.**, 2019
Increasing Labour Safety on Coal Mines. Int. J. Emerg. Trends Eng. Res. 2019, 7, 842–848. [Google Scholar]

[43] **Smith, N.J., Merna, T., Joblin, P.**, 2006
Managing Risk: In Construction Projects. Second Edition. Blackwell Publishing, ISBN-13: 978-1-4051-3012-7; ISBN-10: 1-4051-3012-1

[44] **Matayev, A., Uakhitova, B., Kaumetova, D., Imangazin, M., Sarkulova, Z., Issengaliyeva, G., Orazbekova, R.**, 2024
Substantiation and Selection of Parameters for Supporting Mine Workings at Deep Levels. Min. miner. depos. **2024**, 18(4):125-138. https://doi.org/10.33271/mining18.04.125