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STUDY OF STRESSES IN METAL REINFORCEMENTS SUBJECTED TO NON-UNIFORMLY DISTRIBUTED LOADS

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Abstract: *The paper analyses the stresses in metal reinforcements used in underground structures, essential for their support under rock pressure. The construction of these structures alters the initial stress equilibrium of the ground, requiring efficient support solutions. The mathematical modelling of vertical and horizontal load distribution and their impact on bending moments, normal forces, and shear forces is presented. Load determination can be carried out through experimental measurements and mathematical interpolation. The bearing capacity of the support depends on the material and the applied forces. Optimizing support systems increases the safety and stability of constructions. Through proper design, risks can be minimized, and operational efficiency improved. Thus, stress distribution analysis helps adapt the support to specific geotechnical conditions. Optimization methods for reducing costs and enhancing resistance are proposed. Ultimately, the study provides solutions for underground structure stability.*
Keywords: *metal reinforcements, underground structures, distributed loads, bending moment, load-bearing capacity*

1. Introduction

The extensive lengths of underground constructions require efficient and durable support solutions, considering the significant pressures exerted by rock masses and the complex operating conditions. In this context, metal reinforcements made from laminated profiles are widely used due to their high strength, ease of installation, and ability to absorb structural stresses generated by the terrain.

Proper sizing of these metal structures, along with optimized design and carefully monitored operation, represents essential technical and economic challenges for engineers involved in research and design, as well as for those responsible for production and implementation. Effective support must be adapted to the geological specifics of each construction, taking into account the factors influencing its stability and durability.

To ensure the safety and functionality of underground structures, a detailed understanding of the stress distribution in metal support elements is necessary, correlated with the pressure exerted by the rock and the behaviour mechanisms of the support system. This analysis allows for the selection of appropriate technical solutions, contributing to increased efficiency and safety in challenging underground environments.

2. Stress Field Analysis

The construction of an underground mining structure induces a modification in the initial equilibrium of the rock mass, leading to the replacement of the primary stress field with a newly established one. This transition necessitates the implementation of efficient support methods to regulate stress redistribution and mitigate excessive deformations in the surrounding rock formations.

The shift from the initial stress field to the one induced by underground construction does not occur instantaneously but rather through a sequence of intermediate equilibrium phases. These transitional stages require meticulous management throughout the execution process and necessitate the adoption of technical solutions that ensure the preservation of structural stability.

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3. Stress Modelling in Metal Reinforcements

Based on the presented analysis, it can be considered that a metal reinforcement is subjected to a vertically distributed load p_y and a horizontally distributed load p_x=f(x), as illustrated in Figure 1.

Through analytical resolution using the force method, mathematical expressions have been derived for determining the bending moment M, the normal force N, and the shear force TTT at any section of the arch, regardless of the type of applied load [1, 2].

$$M = M_p + V \cdot x - H \cdot y - M_o$$

$$N = N_p + V \cdot cos\,\varphi + H \cdot \sin\varphi$$

$$T = T_p + V \cdot \cos\varphi - H \cdot \cos\varphi$$

Where:

M_p, N_p, and T_p represent the bending moment, normal force, and shear force of a statically determined beam under a given load.

x, y denote the coordinates of the analysed section of the arch.

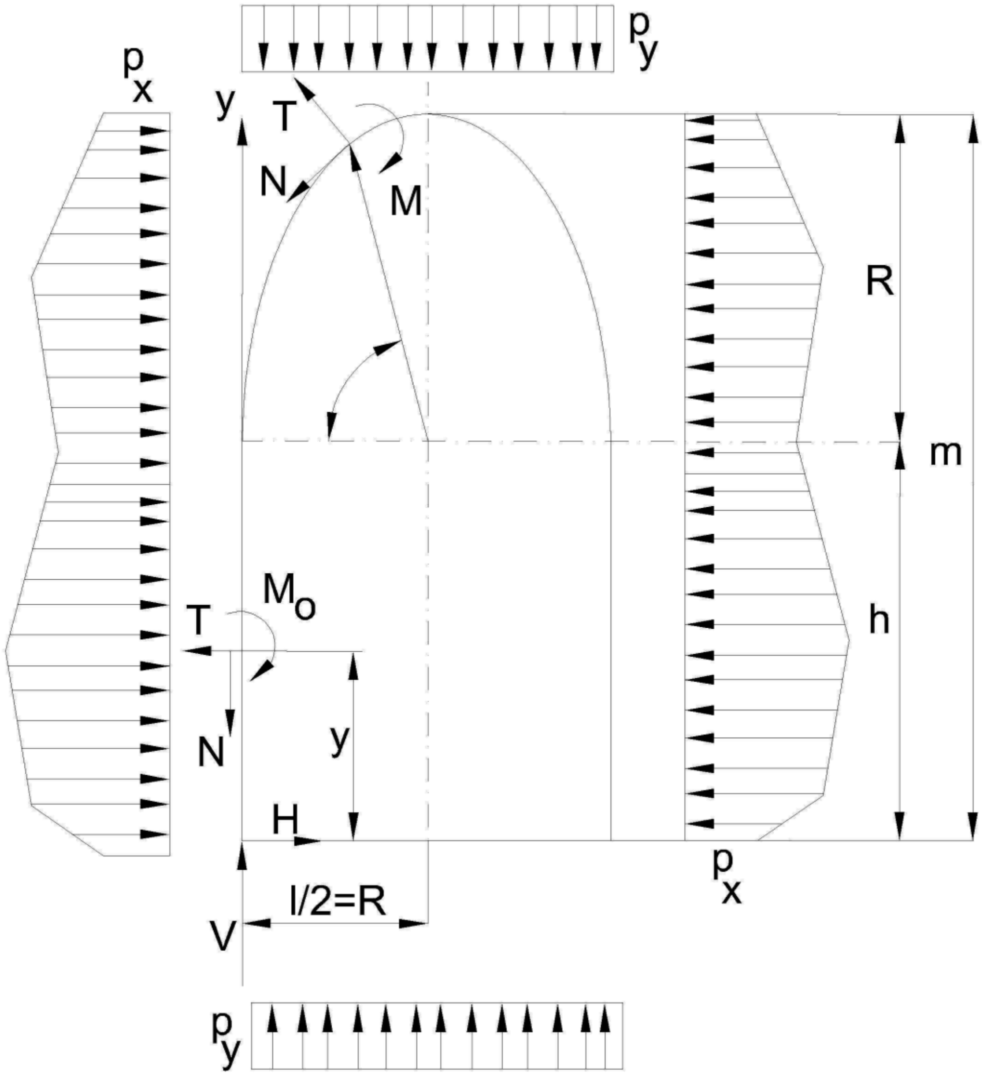

Fig. 1 The tasks apply to the metal support

These coordinates for the arch in figure 1 are as follows:

$$x = \frac{l}{2} - R \cdot \cos\varphi$$

$$y = h + R \cdot \sin\varphi$$

Φ represents the inclination angle of the metallic arch axis at the analysed section, ϕ=0 for the straight portion.

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V, H, M_o are constant values associated with the metallic arch and the type of applied load, where:
V represents the vertical reaction force,
H represents the horizontal reaction force,
M_o represents the bending moment, which is considered zero (M_o=0) if the support columns are assumed to be fixed.

If the load from the mine roof (vertical load) p_y is uniformly distributed and the horizontal load is given by p_x=f(x), then the internal forces M, N, M and T in a given section of the support structure can be determined using the generalized equations: [3, 4, 5]

$$M = V \cdot x - H \cdot y + p_y \cdot \frac{x^2}{2} + \frac{y^2}{2} \int_0^y f(x)\ dx$$

$$T = V \cdot \sin\varphi - H \cdot \cos\varphi + p_y \cdot x \cdot \sin\varphi + y \cdot \cos\varphi \cdot \int_0^y f(x)\ dx$$

$$N = -V \cdot \cos\varphi - H \cdot \sin\varphi + p_y \cdot x \cdot \sin\varphi - y \cdot \cos\varphi \cdot \int_0^y f(x)\ dx$$

As observed, the vertical reaction force is given by:

$$V = \frac{p_y \cdot l}{2}$$

The horizontal reaction force H is determined by considering the deformation of the arch under the influence of the bending moment M and the normal force N. This deformation must be null at the articulation point of the foundation support. Thus, it can be expressed as follows [3, 4, 5]:

$$\int_0^h \frac{M}{E \cdot I_x} \cdot \frac{\partial M}{\partial H} dy + \int_0^h \frac{N}{E \cdot A} \cdot \frac{\partial N}{\partial H} dy + \int_0^{\frac{\pi}{2}} \frac{M}{E \cdot I_x} \cdot \frac{\partial M}{\partial H} ds + \int_0^{\frac{\pi}{2}} \frac{N}{E \cdot A} \cdot \frac{\partial N}{\partial H} ds$$

where:
$E \cdot I_z$ and E·A represent the bending rigidity and axial rigidity (tension-compression) of the metallic reinforcement profile, respectively.
d_S represents an arc element (a differential length element of the metallic arch).

In general:
d_y refers to a differential element in the vertical portion of the structure (support columns).
d_s refers to a differential element in the curved portion of the metallic arch.

In the analysis of the deformability of a metallic arch, the element $\mathbf{d_s}$ is used to describe variations along the length of the arch, considering that the curved portion cannot be represented solely by the y-coordinate.

In practice, ds can be expressed as a function of the angular coordinate ϕ as follows:

$$ds = R \cdot d\varphi$$

where:

R is the radius of the arch,

dϕ is the angular variation corresponding to an infinitesimal arc element.

Thus, for the lateral column domain, where 0≤y≤h, we have: [4]

$$M = -H \cdot y + \frac{y^2}{2} \int_0^y f(x)\ dx$$

$$\frac{\partial M}{\partial H} = -y$$

$$N = -V$$

$$\frac{\partial N}{\partial H} = 0$$

and for the circular arch portion**,** where 0≤φ≤π/2, we have:

$$M = -H \cdot h - \frac{h^2}{2} \cdot \int_0^{R\sin\varphi} f(x) - H \cdot R \cdot \sin\varphi + V \cdot R \cdot (1 - \cos\varphi) - \frac{R^2}{2} \cdot sin^2\varphi \cdot \int_0^{R\sin\varphi} f(x) - p_y \cdot \frac{R^2}{2} \cdot (1 - \cos\varphi)^2$$

$$\frac{\partial M}{\partial H} = -h - R \cdot \sin\varphi$$

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$$N = -V \cdot \cos\varphi - H \cdot sin\,\varphi - h \cdot \sin\varphi \cdot \int_0^{R\sin\varphi} f(x) - R \cdot sin^2\,\varphi \; + p_y \cdot R \cdot (1 - cox\varphi) \cdot cos\varphi$$

$$\frac{\partial N}{\partial H} = -\sin\varphi$$

Solving the above equation, the lateral thrust H is determined in the following form:

$$H = \frac{T_1 + T_2 + T_3}{\frac{h^3}{3 \cdot E \cdot I_x} + \frac{h^2 \cdot \pi}{2 \cdot E \cdot I_x} + \frac{2 \cdot h \cdot R}{E \cdot I_x} + \frac{R^2 \cdot \pi}{4 \cdot E \cdot I_x} + \frac{\pi}{4 \cdot E \cdot A}}$$

where:

$\frac{h^3}{3 \cdot E \cdot I_x}$ It represents the influence of the bending rigidity of the vertical support column.

$\frac{h^2 \cdot \pi}{2 \cdot E \cdot I_x}$ describes the effect of rigidity in the curved section of the arch.

$\frac{R^2 \cdot \pi}{4 \cdot E \cdot I_x}$ represents the influence of rigidity on the arch.

$\frac{\pi}{4 \cdot E \cdot A}$ reflects the effect of the normal force on tension and compression.

$$T_1 = -\frac{1}{E \cdot I_x} \cdot \int_0^h \frac{y^3}{2} \cdot \int_0^y f(x)dxdy$$

$$T_2 = \frac{1}{E \cdot I_x} \cdot \int_0^{\pi/2} \left[\frac{h^2}{2} \cdot \int_0^{Rsin\varphi} f(x)dx - V \cdot R \cdot (1 - \cos\varphi) + \frac{R^2}{2} \cdot sim^2\varphi \cdot \int_0^{Rsin\varphi} f(x)\,dx + p_y \cdot \frac{R^2}{2} \cdot (1 - \cos\varphi)^2\right] \cdot (h + R \cdot \sin\varphi)d\varphi$$

$$T_3 = \frac{1}{E \cdot A} \cdot \int_0^{\frac{\pi}{2}} \left[h \cdot \sin\varphi \cdot \int_0^{R \cdot \sin\varphi} f(x)dx + R \cdot sin^2\varphi - p_y \cdot R \cdot (1 - \cos\varphi) \cdot \cos\varphi \cdot\right] \cdot \sin\varphi\, d\varphi$$

4. Final equations

By substituting the values of V and H into the above equations, the relationships for the loads acting on the metallic support system in mining constructions are obtained.

The function f(x) can be determined experimentally by measuring pressure at specific points along the support columns. Through interpolation and mathematical regression, an explicit expression for this function can be obtained.

5. Conclusions

Analysing the presented relationships, it becomes evident that the loads acting on the support system can be determined at any point within the structure. This capability provides a significant advantage in the design and optimization of support structures, allowing for adjustments in the dimensions and configuration of metallic reinforcements to ensure optimal resistance.

The determination of stress distribution within the support structure, correlated with the pressure exerted by the rock and the deformable behavior of the rock mass, contributes to a more precise adaptation of technical solutions to specific geotechnical conditions. Thus, through careful design and rigorous execution, greater stability of underground constructions can be achieved, minimizing risks and optimizing operational efficiency.

Knowing the load distribution and their values, the load-bearing capacity of the beam can be determined, based on the strength condition of the material used in the support structure. This corresponds to the ultimate load, meaning the maximum load that the beam can withstand before the material reaches the plastic deformation domain in a given section of the arch.

The fundamental equation for evaluating the load-bearing capacity is:

$$\sigma_c = \frac{M}{W} + \frac{N}{F}$$

where:

- σ_c represents the yield strength of the support material,
- W represents the section modulus of the support profile;

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- F represents the cross-sectional area of the arch profile;
- M and N are the torsional moment and the longitudinal force, respectively, acting in the considered section of the beam.

Determining the load-bearing capacity allows for the optimization of the support system, ensuring its adaptation to specific underground conditions. Depending on the rock pressures and the geomechanical properties of the surrounding environment, structural modifications to the support system can be implemented to improve stability and safety in mining operations.

This approach ensures not only technical efficiency but also optimization of material and economic resources involved in the design and execution of underground works.

References

[1] *** * ***, 1998
Optimizarea parametrilor geometrici ai susținerii eliptice cu trei elemente (KEP) – adapted from Șahtnoe stoitelstvo*,* No. 10/1988, pp. 11-14

[2] **Ivan M., et. al.**, 1982
Statica, stabilitatea și dinamica construcțiilor, Ed. Didactică și Pedagogică, București

[3] **Džaparidze P.A.**, 1975
Rasčet metaliceskoi krepi gornîh vîrabotok. Nedra, Moscova

[4] **Elczner G.**, 1987
Elemente de calcul privind eforturile din armăturile metalice ale galeriilor, Revista Mine, Petrol și Gaze, No. 11/1987

[5] **Medves E.R.**, 2018
Calculul susținerii lucrărilor subterane. Teorie și aplicații, Editura A.G.I.R.