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Simple Switch between Single Trip Vehicle Routing Problem and Multiple Trip Vehicle Routing Problem

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Abstract: Complexity of the mathematical model development for vehicle routing problems increases with the addition of more variables, constraints, and instances to single trip vehicle routing models. In this study, mathematical model was developed for the single trip vehicle routing problem in the initial phase and then it was converted to a multiple trip vehicle routing model using a simple approach. Novelty is brought to the study through simple three index formulation developed for the multiple trip vehicle routing problem, with a fewer constraint. Both the models were developed with Mixed integer linear programing techniques and were tested with the real-world data set using Cplex optimizer. Output of the experimental analysis showed a clear reduction in distance travel and the number of vehicles used. It implies that the optimization algorithm proposed in the study is applicable to real world cases to enjoy cost benefits and easiness in scheduling and optimizations. This study contributes to the existing knowledge gap through the development of novel and simple mathematical model, and the model's testing and validation serve to the industrial applications as well.

Keywords: Linear programing, mathematical model, multiple trips, single trip, vehicle routing

1. Introduction

Transportation is a major function in the logistic and supply chain management, and it is also considered as the main cost attribute of the logistic management process [1-2]. With the rising complexity of the supply chain, surprisingly, it provides easier approaches to production and business expansion. Adding geographically scattered new partners such as suppliers, manufacturers, and distributors, leads to more frequent transportation. With the emergence of the global supply chain, different modes of transportation such as air, water, land (railways and roads), cable, pipeline etc. were started to be heavily utilized by business organizations. Further, advanced transportations models in logistic such as multimodal transportation and inter model transportation came into arena to address the timely need of the industries like cost benefits, customer satisfaction and time reduction. However, scarcity of energy sources, complexity of the supply chain, uncertainty & supply chain disruption etc. immensely boost the need of transportation planning [3]. Vehicle Routing Problem (VRP) plays a vital role in transportation planning. History of the VRP goes back to 1959, and the truck dispatching problem was formulated and discussed in that study [4]. There was a simple model with few constraints on demand conditions, minimum travel distance and homogeneous fleet of vehicles. Later different variants of the classical VRP emerged. Mathematical models represent the back end of the VRP problems. Mathematical techniques regarding optimizations have been applied, mostly with the intention of cost and time minimization.

Introduction of Multiple Trip Vehicle Routing problem (MTVRP), rises the complexity of mathematical formulations due to the addition of more indices [5], new variables and new constraints. Effort was taken by several researchers [4-5], to develop simple form of mathematical models for MTVRP with three index and two index formulations. However, there has been no significant contribution towards the development of a simple mathematical model for MTVRP, where complexity still remains. This study introduced a novel mathematical model for the MTVRP with three index decision variables, where trip index has been removed. This also reduces the solving complexity of the model. In the initial stage, a mathematical formulation was developed for the single trips and then with some slight changes of a constraint and a variable, it was transformed to a mathematical model for MRVRP. Novelty has been brought to the study by usage of simple switch between two variants of VRP, which has not been discussed in any prior study. Further two models have been solved using exact algorithm in Cplex. Experimental analysis was conducted to validate the developed mathematical models and to compare the results.

2. Evolution of VRP and its Variants

The primary definition of the classical VRP is heavily based on assumptions and most of the relevant past research works contribute to the development of theories. According to survey on MTVRP by basics assumptions of the classical VRP are [5]:

- Demand of the deport or starting node is zero,
- Should visit all the customers,
- Each customer should visit only by one vehicle,
- When the vehicle in the fleet leaves the deport, it should visit at least one customer and return to the deport,
- Demand of the customers is indivisible.

These assumptions act as the foundation for all type of VRP variants [6]. In the beginning, researchers focus on the homogeneous fleet of vehicles, but today concern is on heterogeneous fleets

[7]. Capacitated Vehicle Routing Problem (CVRP) evolves with the constraint related to vehicle capacity. Known demand of the customers and central deport is another common feature of CVRP [8]. In recent past CVRP has aligned with the advanced solving algorithms and novel solving approaches [8-10]. A new variant which is known as Vehicle Routing Problem with Time Window (VRP-TW) was introduced with the objective of minimizing the total transportation cost within given time bound. This was also heavily discussed in past studies, together with the other variants like CVRP and heterogeneous fleets. The condition of serving to a customer or node, just once using a single vehicle has been relaxed in 1989, with the introduction of the concept of split delivery by Dror and Trudeau [11]. Cost saving in term of distance travelled and number of vehicles used are the key features of this variant. Findings showed that spilt delivery have the possibility to reduce cost on logistic activities more than the 50% [12].

In 1990, multiple trip vehicle routing was discussed for the first time in history by Fleischmann [2-3,13]. While following assumptions of the classical VRP, he suggested a new criterion that more than one trip could be allocated to each vehicle without exceeding the given time limit. MTVRP hold the key objective of travel time minimization and try to identify the appropriate set of trips and finally assign vehicles to identifies trips [7].

2.1.1 Mathematial Formulation

Appropriate mathematical modelling is crucial in VRP and its variants. Due to mathematical complexity in formulation and solving, MTVRP is included in the nondeterministic - polynomial hard problem category[14-16]. Increase in number of variables, constraints and instances also leading MTVRP to

In general, bidirected graph allowed by $g = (N, A)$, here nodes represented by $N = \{0,1,2,3. \ldots N\}$ while set of arcs are represented by $A = \{(i,j) | i,j \in N\}$. Further, $(i,j) \in A$ is associated with travel time T_{ij}, Node 0 serves as the depot, where a fleet v of identical vehicles is stationed. Then $r \text{ or } t$ is used as indices to represent the trip. The 4-index formulation is the most common formulation available for the MTVRP, incorporating a 4-index binary decision variable as follows

$$X_{ij}^{vr} = \begin{cases} 1 \text{ if trip r of vehicle v travels through arc (i j)} \in A \\ 0 \quad \text{Otherwise} \end{cases}$$

Scholars [17-19] have successfully developed 4-index formulations, including the aforementioned 4-index binary variable. Similarly, a typical 3-index formulation was generated by removing the trip index (r) from the binary decision variable as below.

$$X_{ij}^{v} = \begin{cases} 1 \text{ if vehicle v travels through arc (i j)} \in A \\ 0 \quad \text{Otherwise} \end{cases}$$

This leads to complexity by adding more decision variables and constraints [20]. The main decision variable of this formulation is more complex than the formulations with separate indices for the transporter, plant, starting node, ending node and the trip corresponding to a particular period. Later three index formulations were introduced by removing trip index and by removing vehicle index as well. In the case of removing.

In general reduction of 4th index of the decision variables leads to complexity by adding more decision variables and constraints [20]. The main decision variable of this formulation is more complex than the formulations h separate indexes for the transporter, plant, starting node, ending node and the trip corresponding to a particular period. Later three index formulations were introduced by removing trip index and by removing vehicle index as well. In the case of removing trip index researchers have extended capacity constraints using Miller-Tucker-Zemlin-like constraints [7]. Meantime for the case of removing vehicle index, new variables have been introduced. Literature survey paper [7], present all the mathematical formulations developed in past studies for three index formulation as a summary.

3. Data and Methods

Different mathematical formulations with the knowledge of Linear Programing (LP) are available for the single trip models. Application of LP problems in real world cases are limited by the assumption of divisibility [21]. Under this assumption non integer values need to be permissible for the decision variables. This is hard to apply in real world cases, as in practical cases, decision variables only make sense if they have integer values. This can be formulated with the derivative of LP which is named Integer programming. Integer programming problems have widely utilized in the areas where problem can be solved with number of interrelated Yes or No decision, and Mixed Integer Linear Programing (MILP) use when there is set of variables and only some decision variables deal with the integer values [22-23].

In this study, the author focused on the MILP technique for mathematical model development. The model development of this study is conducted under two phases. In the first phase a mathematical model was developed for the single trip VRP with heterogeneous fleet of vehicles, capacity constraints, pickup problem and time constraints. In the second phase MTVRP was developed with the same set of conditions.

3.1 Single Trip VRP

This is a 3- index formulation where indices are used for the vehicle(v), leaving node(i) and entering node(j).

Indices

i, j - Indices for the nodes (i, j = 0,1,2, ……. n)
where: i, j = 0; deport
v – Indices for the vehicle (v = 1, 2, 3…. n)

Parameters

L_{ij} *– Distance from node i to j*
w_j *– Weight of the product supply by node j*
C_v *– Capacity of the vehicle v*
t_{ij} *- Time taken to travel from node i to j*

Decision variables

$$X_{ij}^{v} = \begin{cases} 1 & \text{If vehicle } v \text{ visit node } j \text{ after node } i \\ 1 & \text{Otherwise} \end{cases}$$

$$y_{j}^{v} = \begin{cases} 2 & \text{If vehicle } v \text{ visit node } j \\ 0 & \text{Otherwise} \end{cases}$$

$$Z_{v} = \begin{cases} 1 & \text{If vehicle } v \text{ is in use} \\ 0 & \text{Otherwise} \end{cases}$$

$w_j^v -$ Cumulative weight of vehicle *v*, after serving to node *j*
$T_{ij}^v -$ Total time taken by vehicle *v* in a particular trip to visit node *j* through node *i*
$t_i -$ Cumulative time taken to visit node *i* from the deport.
$t_{ij} -$ Time taken to travel from node i to j

Objective function

$$Min\ Z = \sum_{v=1}^{n} \sum_{i=0}^{n} \sum_{j=0}^{n} X_{ij}^{v} L_{ij}\ , \quad [\text{m.s}^{-2}] \tag{1}$$

The objective function (1) of this study has been designed to minimize the total distance travelled by chosen fleet of vehicles in a given day/ time period. Nodes covered by vehicles will be identified through the decision variable X_{ij}^v and distance between the nodes must be identified from the given data set. This decision variable carries three indices and the same decision variable identification is found in several studies including [9,24,25]. Three index formulation comes with the i, j indices to represent the arc and another index to represent the vehicle. This became more realistic with the heterogeneous fleet of vehicles.

Subject to

For all v and j, (j≠0, j=k)

$$\sum_{i=0}^{n} X_{ij}^{v} = y_j^v \quad (2)$$

$$\sum_{i=0}^{n} X_{ji}^{v} = y_j^v \quad (3)$$

For all j, (j≠0)

$$\sum_{v=1}^{n} \sum_{i=0}^{n} X_{ij}^{v} = 1 \quad (4)$$

For all v,

$$\sum_{j=0}^{n} X_{0j}^{v} = \sum_{i=0}^{n} X_{i0}^{v} \quad (5)$$

$$\sum_{j=0}^{15} X_{0j}^{v} = Z_v \quad (6)$$

For all j and v, (j≠0)

$$X_{0j}^{v}(t_{0j}) = X_{0j}^{v}(t_j) \quad (7)$$

$$X_{0j}^{v}(w_j^v) = X_{0j}^{v}(w_j) \quad (8)$$

For all i,j and v, (i, j≠0)

$$X_{ij}^{v}(t_i + t_{ij}) = X_{ij}^{v}(t_j) \quad (9)$$

$$X_{ij}^{v}(w_i^v + w_j) = X_{ij}^{v}(w_j^v) \quad (10)$$

$$X_{i0}^{v}(T_{i0}^{v}) = X_{i0}^{v}(t_i + t_{i0}) \quad (11)$$

$$T_{i0}^{v} \le Tmax \quad (12)$$

For all i,j and v

$$X_{ij}^{v}(C_v - w_j^v) \ge 0 \quad (13)$$

$$w_j^v \ge X_{ij}^{v}(w_j) \quad (14)$$

$$X_{ij}^{v} \in \{0,1\} \quad (15)$$

$$y_j^v \in \{0,1\} \quad (16)$$

$$0 \le w_j^v,\ Z_v,\ t_i, T_{ij}^{v} \quad (17)$$

Constraint (2), ensures that Vehicle v can enter to node j at most once. Decision variable $\boldsymbol{y_j^v}$ is a binary variable, which has just two values one or zero, where one means vehicle v enters to the node j. Since the single entrance is permitted by constraint (2), every node should leave at most once by vehicle v. This requirement is satisfied by the constraint (3). Constraint (4) has been used to represent all the visits made by all the vehicles to a particular node. Since this model does not support split delivery sum of all the visits to a particular node by all the vehicles should be equal to 1. Accordingly, the number of times entered to any node has been restricted by one time entry. This constraint is usually relaxed in the Split delivery VRP, which is another important variant of VRP. Constraint (5) is designed to address the single trip criteria along with constraint (4). Since zero denotes the deport, the number of times a particular vehicle departure from the deport should be equal to the number of times it arrives to the deport. In general, center deports use in vehicle routing problems to start and end the trip, same context has been considered in here. The focus of the initial stage is to develop a single trip vehicle routing model, therefore the number of departures and arrivals to the deport made by each vehicle should be equal to one or zero. Thus, with the support of constraints (4) and (5), it is able to satisfy the single trip condition of the vehicles in the fleet.

Most of the past studies considered the number of vehicles available for the routing plan with a separate index [26-28]. It is rare to find studies with a separate variable to denote the vehicle utilization. Whether a particular vehicle is in use on a specific day can be easily identified by constraint (6). For this $\boldsymbol{Z_v}$ is introduced to the mathematical model as a binary variable, which was not recognized in previous studies. If vehicle v has left the deport, it means that vehicle is performing a trip then $\boldsymbol{Z_v}$ will become 1. Time and distance between two nodes are provided as the input data for the model, but cumulative time taken to visit node j from the deport "$\boldsymbol{t_j}$" is not given and must generate it from the model. It is not a fixed value and depends on the route which has been chosen. Therefore, it is unnecessary to calculate "$\boldsymbol{t_j}$" values for all j. In constraint (7), component $\boldsymbol{X_{0j}^v}$ is used in both right and left side of the constraint to ensure the calculation of cumulative taken to visit j from deport if a particular vehicle has entered to node from the deport. Time taken to travel from deport (node "0") to any other node can be easily retrieved from the data provided. The same logic is used in constraint (8) to calculate the cumulative weight of the vehicles at the first serving node. Cumulative time taken to visit each node (except the first serving node from the deport) is calculated in the constraint (9). Cumulative time for node i (previous node) will be calculated first and then time from node 1 to node j will be taken from the given data set. Like the time calculation, cumulative weight of vehicle v at node j can be calculated using constraint (10).

$\boldsymbol{T_{ij}^v}$ has been inserted to this model to calculate the total time consumption of a trip. Accordingly, constraint (11) calculates the total duration of each trip. $\boldsymbol{X_{i0}^v}$ component is included in this constraint to ensure end of a trip by vehicle v, because it says vehicle v travels from node i to node zero/deport. The calculated cumulative time at node i and available data for the time from node i to j are used to calculate the total time of a trip. Time constraint is considered in the (12). Maximum time allocated for the vehicles to complete its routing is denoted by the "Tmax". Accordingly, the total time taken by every vehicle to complete its trip should be less than the allocated maximum time. This can satisfy the time window constraint. Vehicle capacity is another constraint. A heterogeneous fleet of vehicles has been considered in this model; therefore, the capacity of each vehicle can be given as input data. Capacity requirements will be checked through constraints (13). Vehicle capacity should not exceed after collecting goods from the nodes. These criteria check each node that a vehicle visits by using constraint (13). Once vehicle v visits a particular node, cumulative weight of the vehicle v increases as it collects items from that node. Therefore, it is needed to make sure that the cumulative weight of the vehicle v at node j should be equal or greater than the weight provided by the node j. Constraint (14) is designed to achieve the forementioned requirement. Binary variables are indicated by (15) and (16). Integer variable use in this model is represented by non-negative constraint (17).

3.2 Simple Switch to MTVRP

The model developed for the single trip routine in the above phase was further extended to multiple trip vehicle routing model with few modifications. This paper provides a simple and easy approach for the conversion of single trip vehicle routing model to complex multiple trip vehicle routing model. Accordingly, binary variable Z_v in the single trip model was altered to an integer decision variable while there is no change is occured on indices.

$$Z_v : Number\ of\ \ trips\ assign\ to\ vehicle\ v\ in\ a\ perticular\ day$$

Constraints (6) and (12) in the single trip vehicle route plan have been replaced by the following constraints (18) and (19) to satisfy the multiple trip requirements.

For all v,

$$\sum_{i=0}^{15} X_{i0}^{v} = Z_v \tag{18}$$

$$\sum_{i=0}^{15} T_{i0}^{v} \leq Tmax \tag{19}$$

Constraint (18) is used for the relaxation of single trip model and it indicates the number of trips performed by vehicle v in a particular day is equal to number of times that vehicle v has entered to deport from another location.

Constraint (19) compiles the model with time constraint. Accordingly, the total travelling time of each vehicle on all its trips should not exceed the allocated maximum time per day. This proposed mathematical model is much simpler than the available model in past studies. Furthermore, this model is straightforward to comprehend, as it closely resembles single trip VRP models, with no additional constraints.

3.3 Numerical Result

The solving procedure is specific for MIP, as it uses relation of LP. This is what is identified as integrality constraints in LP. Usually in LP, simplex methods and barrier methods are used to solve this type of problems and able to obtain solutions in a general rational pool. Further, optimization will derive integer solutions by utilizing exact methods or heuristic methods.

To illustrate applicability of the above introduced mathematical models, a case study was conducted from Sri Lankan context (company specific VRP instance) using Cplex optimizer. To illustrate applicability of the above introduced mathematical models, a case study was conducted from Sri Lankan context using Cplex optimizer. Cplex optimizer is widely used to solve LP problems and special variants like MILP. It has state of art algorithms like branches and cuts, which facilitate addressing complex real-world problems. Python language was utilized to convert the mathematical model into a format that can be understood by the solver.

For the numerical experiment input data were taken from a medium scale enterprise in Sri Lanka, which is daily engaged in the tea leaves pickup process. The company currently uses 4 vehicles and visits 15 villages nearby. Capacities of heterogeneous vehicles are given in Table 1.

Table 1 Vehicles information. Source: author

Vehicle Name	Payload (Kg)
Vehicle A	750
Vehicle B	1200
Vehicle C	975
Vehicle D	1125

Distance between each node (village) was gathered and developed to a matrix and time taken to travel between nodes was also calculated and inserted to a matrix (Time and distance data available at public repository [29]). Two matrixes were used as input data in the solving procedure. For the testing and comparison of the two models, 20 instances were selected from the subjected company. Results were obtained from the Cplex solver studio by optimizing the problem with the 0.1 MIP gap and 120 seconds time limit. Both the models were successfully executed with the Cplex solvers within a considerable time, approximately close to 2 minutes. Outputs of both models are included in the following Table 2.

Table 2 Comparison of models' output. Source: author

Instance	Pickup quantity	Original Fleet Size	Used Fleet Size		Z*		Reduction in Distance
			ST	MT	ST	MT	
1	3940	4	4	2(B, D)	102.45	91.05	11.4
2	3988	4	4	3(A, B, D)	91.3	90.55	0.75
3	3990	4	4	2(B, D)	91.3	90.55	0.75
4	3954	4	4	3(A, B, C)	102.35	91.55	10.8
5	3960	4	4	3(B, C, D)	100.25	91.55	8.7
6	3955	4	4	2(B, D)	91.3	89.2	2.1
7	3964	4	4	2(B, D)	101.15	89.2	11.95
8	3962	4	4	2(B, D)	91.3	90.1	1.2
9	3980	4	4	3(B, C, D)	91.3	89.2	2.1
10	3985	4	4	2(B, D)	103.35	90.55	12.8
11	3960	4	4	3(B, C, D)	91.3	90.7	0.6
12	3894	4	4	3(B, C, D)	92.7	89.5	3.2
13	3999	4	4	2(B, D)	91.3	90.55	0.75
14	3968	4	4	2(B, D)	92.85	89.2	3.65
15	3974	4	4	2(B, D)	94.6	89.2	5.4
16	3962	4	4	2(B, D)	91.3	90.1	1.2
17	3973	4	4	3(A, B, D)	91.3	89.2	2.1
18	3965	4	4	3(A, B, D)	98.3	90.1	8.2
19	3989	4	4	2(B, D)	106.5	89.2	17.3
20	3960	4	4	3(A, B, D)	94.6	90.45	4.15

(ST- Single Trip/ MT- Multiple Trip)

In here same quantity, vehicles, distance matrix and time matrix have been used as the input data set for both the models. Accordingly, 20 problem instances which are possible to use for single trip model were utilized in both the models. Hence each single problem instance has a total quantity less than 4000Kg. Optimal distance travelled was denoted by using single trip model and multiple trip model, which is summarized under the column Z*. All the instances have achieved less travelling distance in multiple trip models than the single trip model.

A further five out of 20 results have more than 10 km reduction in the total distance travelled. The highest reduction in the distance travelled was achieved in the 19th problem instance with 17.3 Km. There are three instances that have achieved more than 5Km reduction in total distance travelled. As a summary 16 problem instances out of tested 20 instances have reduced more than 1Km of total distance travelled. This result also represents a significant reduction in the used fleet size.

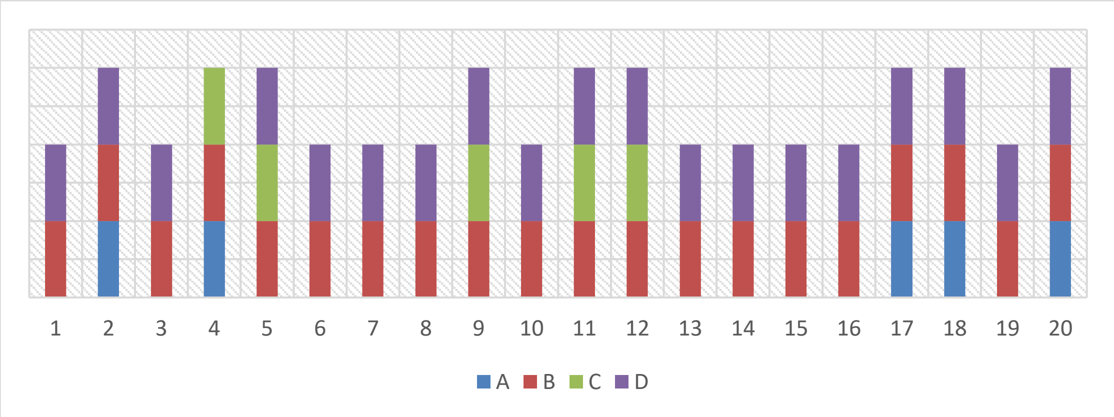

Fig. 1 Vehicle utilization (MTVRP). Source: author

In every problem instance, the single trip model has utilized full fleet size, while no single instance has utilized full fleet in the multiple trip model. Nine instances out of 20 have utilized only three vehicles while the remaining 11 instances have utilized two vehicles for the routing plan in multiple trip model. All the instances that used two vehicles, vehicle B and vehicle D are for the routing plan and when using datasets which are compatible with both single trip and multiple trip models, multiple trip model has generated output using vehicle A than in the above scenario. Resource utilisation or vehicle utilisation during the MTVRP for considered 20 days are visualised in the above Figure 1. In this figure, A, B, C, D denotes the 4 vehicle types with four separate colors. Twenty days are corresponding to the data set numbered in the Horizontal axis. This clearly shows that vehicle A and C have been used just for 5 days while keeping those idle in the remaining 15 days. Model has optimally utilized resources to minimize distance travel, but this shows the issues within the company related to the underutilization of resources.

4. Conclusion

Mathematical models which address several components of VRP such as heterogeneous fleet of vehicles, capacity constraints, time constraints and multiple trip vehicle routing has increased the number of constraints in the model. Hence there is complexity in mathematical modeling as well as solving difficulties. This simple structure of the mathematical model has added novelty to the study. The model has been applied to a real-world instance and showed the ability to reduce the distance travelled and optimization of vehicles utilized. Thus, study fills the knowledge gap through generation of new mathematical model and numerical result assessment to address the real-world applications. Yet, there are few more essentials criteria to address today such as fuel consumption, gas emission to address sustainability and service times, which were not discussed in this study. Future research directions open to extend this model by adding essentials consideration of the present.

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