

CHALLENGING MATHEMATIC TASK-TYPES FOR STUDENTS FINISHING 7TH GRADE IN BULGARIAN SCHOOLS

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Abstract: The article analyzes the measurement properties of the multiple-choice test items from the National External Assessment for 7th grade, conducted in 2020, 2021, 2022 and 2023. Test Items that did not perform well in terms of difficulty or discrimination coefficients were searched for. Variants of the items are proposed in order to improve their measurement properties. Conclusions were drawn on the differences found in the achievements of students in the districts of Sofia-city, Plovdiv, Pleven and Razgrad.

Keywords: multiple-choice test items, difficulty, discrimination (discriminating power)

1. Introduction

Each stage of the Bulgarian education system ends with a National External Assessment (NEA): after 4th grade, after 7th grade, after 10th grade and after 12th grade. This paper addresses the results in mathematics from NEAs after 7th grade. The exam is composed of 2 modules:

- The first module, lasting 60 minutes, contains:

- 18 test multiple choice questions with a choice of answer A, B, C, D, which are scored with 2 or 3 points;
- 2 test short-answer questions, which are evaluated with worth 5 points each.

- The second module, lasting 90 minutes, contains 3 mathematical open-ended problems. Students should write detailed solutions. They worth a total of 40 points.

The maximum score of the two modules is 100.

The report summarizes the results of the first 18 multiple-choice test items using the raw score. It is obtained by giving 1 point

for a correct answer and 0 points for a wrong answer.

Bulgaria is divided into 28 administrative districts. For the present study, 4 of them were selected – Sofia-city, Plovdiv, Pleven and Razgrad. These are the first two largest districts in Bulgaria, a medium district and one small area district. The ratio of students in these regions is 12:6:2:1 (in 2023 year).

The number of students studied is

- 18836 for 2020
- 19303 for 2021
- 20028 for 2022
- 21060 for 2023

This is about 1/3 of all students who finish 7th grade every year in Bulgaria.

Using the methods of Classical Test Theory (CTT) [1], the coefficients of difficulty and discriminative power were obtained for each multiple-choice task. Summaries are made of the types of tasks that students find difficult and those that are easy for them.

Table 1 Task frequency by difficulty factor

Difficulty		Frequency in			
from	that	2020	2021	2022	2023
0.90	1.00				
0.80	0.89				9
0.70	0.79	5, 6	10, 18, 12	9, 16	13, 15
0.60	0.69	3, 1, 14, 10, 17	4, 15, 16, 9, 2	13, 10, 14, 4, 12, 5, 3	1, 8, 5, 17
0.50	0.59	15, 16, 8, 13, 12, 9, 11	8, 17, 14, 13, 5, 11	6, 17, 18, 1, 8, 2	7, 3, 14, 16
0.40	0.49	2, 4	3, 1, 7	15, 11, 7	12, 18, 4, 11, 10
0.30	0.39	18	6		6, 2
0.20	0.29				
0.10	0.19	7			
0.00	0.09				

It should be noted that the NEA in Bulgaria is used both to assess the level of students' knowledge acquisition and to rank them when applying to high schools after the 7th grade. The conditions of the tasks can be downloaded in Bulgarian from the website of the Ministry of Education and Science [5].

In [3], the measurement qualities of multiple-choice test tasks were analyzed from NAE by using Item responsibility theory (IRT) for the year 2021. In [4], the method for comparing the results of two tests using common (anchor) tasks for 2020, 2021 and 2022 was applied.

2. Difficulty and Discriminative Power

According to CTT, the difficulty of the task is derived from the ratio of those who indicated the correct response to all who answered the task. This value is in the interval $[0;1]$. The higher this number, the easier the task. The discriminative power of a given task is obtained from the difference $D = r_A - r_B$, where r_A is the fraction of correct responses of the high achieved group of students, and r_B is the fraction of correct responses of the lower achieved group of students. This value is in the interval $[-1;1]$. The higher it is, the better the task has discriminative power [2].

7. In November, the price of a laptop was BGN 1,049. For Christmas, its price was reduced by 10%, and in January it was increased by 20% of the new price. The price of the laptop (in BGN) after the two changes is calculated using the expression:

- A) $1049 - 0.10$ B) $1049 + 0.20 * 1049$
 C) $1049 + (0.20 - 0.10) * 1049$ D) $1.20 * (0.90 * 1049)$

Figure 1: Task 7 of 2020

18. Point A is given in a Cartesian coordinate system. Using the drawing find the coordinates of point B, which is symmetric to point A about the abscissa axis.

- A) $B(-3; -4)$
 B) $B(3; -4)$
 C) $B(-3; 4)$
 D) $B(3; 0)$

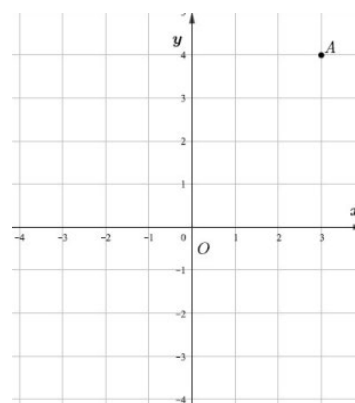


Figure 2: Task 18 of 2020

Table 1 shows the frequency distribution of the tasks according to the difficulty factor. The interval [0;1] in which this coefficient changes is divided into 10 equal parts. As can be seen from the table, for each year the

tasks are similarly distributed. More tasks have an optimal difficulty in the interval (0.40; 0.70). Obviously, there is a lack of tasks that are very easy and tasks that are very difficult.

6. The expression $(y - x)^2 - y + x$ is equivalent to:	
A) $(x - y)(x - y - 1)$	B) $(y - x)(y - x + 1)$
C) $(y - x)(x - y)$	D) $(x - y)(x - y + 1)$

Figure 3: Task 6 of 2021

2. How many of the numbers 1, 3, 9, 11, 15, 18, 21, 23, 27, 29, 31 are prime?			
A) 4	B) 5	C) 6	D) 7

Figure 4: Task 2 of 2023

6. The actual distance between two cities is 40 kilometers. If on a geographical map the distance between these cities is 8 cm, then the scale of the map is:			
A) 1 : 5	B) 1 : 5 000	C) 1 : 500 000	D) 1 : 5 000 000

Figure 5: Task 6 of 2023

3. Examples of Difficult Tasks in NEA

The most difficult task in the four years studied turns out to be task 7 from 2020. It is from the content area “Percentage” that is studied in the 5th grade. In the condition (shown in Figure 1), we are asked to construct an expression for the price of a laptop after two consecutive changes. There are not many such examples in Bulgarian schools. An increase of 20% is represented by addition, not multiplication. The left side of the equation is used, not the right side: $a + 20\%.a = 1,20.a$. The low value of the discriminating power of the task (0.33) shows that even better students fail to indicate the correct answer D). It was chosen by only 17% of all students. Such a task is not particularly suitable when conducting normative tests.

Students encounter difficulty in task 18 of 2020 shown in Figure 2. Distractors A) and C) are indicated almost as often as the correct answer B). The discrimination coefficient is very low (0.22). All this shows that the students are looking for a point of symmetry, but incorrectly. One explanation is that the concepts of abscissa

and ordinate axis, as well as central and axial symmetry, are mistaken. The performance of this task would probably improve if the notation of the axes is given: “...the abscissa axis Ox”.

A serious problem for students is the problems of factoring a polynomial shown in Figure 3. To get the correct answer D) it is necessary to perform two actions:

$$(y - x)^2 = (x - y)^2$$

and

$$-y + x = +(x - y).$$

This task would probably get better results if at least one of these actions was spared. For example, if the expression was given in the form: $(x - y)^2 - y + x$. However, discrimination is very good (0.45), so such a task is relevant for the NEA.

The task that is given to Figure 4 is a typical example of a “prime” concept problem. It is a basic concept in 5th grade, relatively rare in 6th grade, and almost never used in 7th grade. The task has extremely low discrimination (0.14) and needs to be revised.

Another difficult problem is shown in Figure 5. It is from the Proportions area, but

solving it requires students to change the units of measurement. The relatively low value of discrimination (0.26) shows that such type of tasks is not well mastered by the high achieved group of students. There are similar examples in Geography, but obviously this also does not contribute to increasing the success rate.

4. Summary

The presented 5 tasks, which were the most difficult for students in the corresponding years, are from different areas of mathematics. It is noticeable that they are mainly in the field of Algebra. Considering the tasks with a difficulty coefficient in the interval (0.40; 0.49), it can be summarized that the more difficult tasks for students in the 7th grade studying in Bulgarian schools are in the following content areas:

- numeric value of rational expression – 2 (2020); 1 and 3 (2021); 10 and 11 (2023)

- decomposition of expressions – 6 (2021); 4 (2020)
- expression modeling – 7 (2021)
- modular equations – 4 (2023)
- chart reading, percentage, proportion – 7 (2020); 7 (2022); 6 (2023)
- angles in more complex structures – 11 (2022); 11 and 12 (2023)
- elements of cone – 18 (2023)

The content areas in which more than 60% of students were able to indicate the correct answers are:

- event probability at simplest level – 6 (2020); 9 (2021); 9 (2022); 9 (2023)
- angles from the intersection of two lines – 10 (2020); 12 (2021); 12 and 13 (2022)
- right triangle with 30° – 17 (2020); 12 (2021); 16 (2022); 15 (2023)
- angles in a triangle – 14 (2020); 10 (2021); 14 (2022); 13 (2023)

This research can serve as ideas for creating multiple-choice test items.

References List

- [1] Bankov K. *Uvod v testologiyata*. Sofia. Izkustva . 2012 (in Bulgarian).
- [2] Stoimenova E. *Izmeritelni kachestva na testove*. Sofia. NBU. 2000 (in Bulgarian).
- [3] Tsonev P. Choosing an Appropriate Model for Investigating the Tasks of School in Mathematics after Grade VII According to IRT Methods. *Mathematics & Informatics*, LXVI, Volume 6 (2023), 491–505 (in Bulgarian).
- [4] Tsonev P. Comparison of the Results Based on the Multiple-choice Items from the National External Assessment in Mathematics After Grade VII Held in 2020, 2021 and 2022. *Annual International Scientific Conference of VVVU "G. Benkovski", D. Mitropolia*, 2023, 353–363 (in Bulgarian).
- [5] Ministry of Education and Science. Available from: <https://www.mon.bg/obshto-obrazovanie/natsionalno-vanshno-otsenyavane-nvo/nvo-za-vii-klas/testove-i-verni-otgovori-ot-nvo-za-vii-klas-po-godini/>