International Conference KNOWLEDGE-BASED ORGANIZATION
Vol. XXIX No 3 2023

MATHEMATICAL MODELS OF THE UAV OPERATORS

Róbert SZABOLCSI

Óbuda University, Budapest, Hungary
szabolcsi.robert@bgk.uni-obuda.hu

Abstract: *The operators maintaining different kinds of activities might have different mathematical models regarding their physical and mental status, their skills and experiences, their level of training they own. Recent days more and more attention is paid to wide spread of different UAVs applied both in military and in non-military governmental applications. The UAV production industry is able to manufacture UAVs for different and diverging purposes. However, the effective maintenance of these UAVs requires highly trained and qualified operators. When massive UAV applications support combating ground forces in the war theatre the training of the UAV operators becomes to be one of the crucial criteria. Diversity of the UAV types often fabricated and gunned for military purposes shows difficulties of the training program created for UAV operators. This paper addresses mathematical models of the UAV operators applied both in open and closed loop control systems of the UAVs. Additionally, findings of this paper might support and improve selection process of the UAV operators to reach maximum efficiency of both military reconnaissance and combat applications.*

Keywords: operator, UAV operator, mathematical model, open loop control, closed loop control

1. Introduction

In military UAV applications it is crucial to find the most skilled operators able to conduct different missions with highest efficiency.
The training of UAVs' military operators is properly regulated and organized best way in many countries with different learning outcomes of the alumni of the training organizations.
In spite of this, when large amount of operators is needed, or when large variety of different types of the UAVs is applied in the war theatre, to meet all criteria of the formal regulations might bump into massive walls of the reality reigning on the battlefield.
The variety of the UAVs flown today in different flight missions starts from nano UAVs of few grams and ranges to the category of the HALE UAVs of several tons. Easy to agree that to train universal UAV operators able to handle all categories of the UAVs is a challenge solved never before.
The mathematical modelling of the human operators is in the focus of attention of many scholars since the early 1950's, when it was needed and required by different fields of aeronautics and astronautics [1], [2].
The operators are central elements and important parts in the closed loop control systems [3], [4].
Thus, evaluation of their role, their physical and mental status, their level of training gained in stability analysis or in evaluation of closed loop dynamic performances is very important [6], [7], [8].
The human operators' mathematical models were identified leaning on different

DOI: 10.2478/kbo-2023-0081

laboratory tests and are thoroughly analysed [5].

2. Mathematical Models of the UAV Human Operators

The mathematical model of the UAV human operators acting in any control systems represents and consists of reaction time, different gains, different time constants, nonlinearity, deterministic feature, and sampled in time.

This way, the UAV pilot model can be derived as [1], [2], [3], [4], [5]:

$$Y(s) = \frac{x_{out}(s)}{x_{in}(s)} = K\frac{1+sT_L}{(1+sT_I)(1+sT_N)(s^2+2\xi\omega s+\omega^2)}e^{-s\tau}, \quad (1)$$

where:

- $x_{in}(s)$ is the Laplace-transform of the test input signal;
- $x_{out}(s)$ is the Laplace-transform of the output signal of the operator;
- K is the operator's gain;
- τ is the reaction (dead) time;
- $\frac{1}{(1+sT_N)(s^2+2\xi\omega s+\omega^2)}$ is the model of the neuro-muscular system [2], [3];
- $(1+sT_L)$ is a real derivative lead term describing prediction ability of the operator;
- $\frac{1}{(1+sT_I)}$ is a lag term describing data transmission properties of the human operator.

In the dynamic model given by eq (1) the second order term is often modelled with unit gain, and (1) is often simplified to the form of given below [1], [2], [3], [4], [5]:

$$Y(s) = \frac{x_{out}(s)}{x_{in}(s)} = K\frac{1+sT_L}{(1+sT_I)(1+sT_N)}e^{-s\tau}. \quad (2)$$

The mathematical model (2) involves data given by [1], [2], [3], [4], [5], [6], [7] and the parameters shall lie in the ranges given below:

$$K = 2 \div 60; \quad T_L = (3 \div 15)s; \; T_I = (0{,}08 \div 0{,}7)s; \; T_N = (0{,}05 \div 0{,}15)s, \quad (3)$$

The time constant of T_N describes the dynamics of the neuro-muscular system of the UAV operator. The less this time constant is the faster reaction of the UAV operator could be achieved to accomplish flight missions with highest level of efficiency.

The time constant T_L describes ability of the UAV operator to predict different processes in control of UAV. This ability might help operators to ensure proper level of the flight safety, and additionally, to extend flight envelope of the UAV in many situations for example flight time, flight range, active flight radius, optimal management of use of the energy stared aboard of the UAV etc.

It mostly depends on experiences gained in the past by the UAV operator. The gain K represents ability of the UAV operator to magnify signals during his signal tracking processes [5], [6], [7], [8].

The full process of the signal tracking has dead time τ describing times need for data management both in human vision and brain system.

Regarding parameter ranges given above, the best case is when the high skilled, well-trained, experienced UAV operator with his best fitness for flights operates the UAV.

On the contrary, the worst case might happen when a low-skilled, low trained UAV operator with low level of his fitness for flights operates the UAV.

3. Computer Simulation of the Mathematical Models of the UAV Human Operators

Computer simulation allows to model UAV operators in virtual space not threatening the loss of the UAVs in real flights.

Moreover, it supports to make difference between behaviour of the UAV pilots with different dynamic model data.

The first data subjected to evaluation is the time delay (dead time) of the UAV operator.

It is well-known that '$e^{-s\tau}$' type on nonlinearity can be linearized using Padé-series expansion [8]:

$$e^{-s\tau} \cong P_d(s) = \frac{N_d(s)}{D_d(s)} = \frac{\sum_{k=0}^{n}(-1)^k c_k \tau^k s^k}{\sum_{k=0}^{n} c_k \tau^k s^k}, \quad (4)$$

where:

$$c_k = \frac{(2n-k)!\cdot n!}{2n!\cdot k!\cdot (n-k)!};\ n = 1,2,3,\cdots;\ k = 0,1,2,3,\cdots,n \quad (5)$$

To reach 2% of the accuracy in time delay model (4) linearization, it is supposed by the author to apply 5th order approximation, hence $n = 5$.

This kind of linearization will lead to the following linear model of the dead time:

$$e^{-\tau s} \cong \frac{1-\frac{\tau}{2}s+\frac{\tau^2}{12}s^2-\frac{\tau^3}{120}s^3+\frac{\tau^4}{1680}s^4-\frac{\tau^5}{30240}s^5}{1+\frac{\tau}{2}s+\frac{\tau^2}{12}s^2+\frac{\tau^3}{120}s^3+\frac{\tau^4}{1680}s^4+\frac{\tau^5}{30240}s^5}. \quad (6)$$

Figure 1 shows how the proposed approximated linear model of (6) matches with the real nonlinearity of '$e^{-s\tau}$'. Computer-aided simulation was supported by MATLAB 2022b.

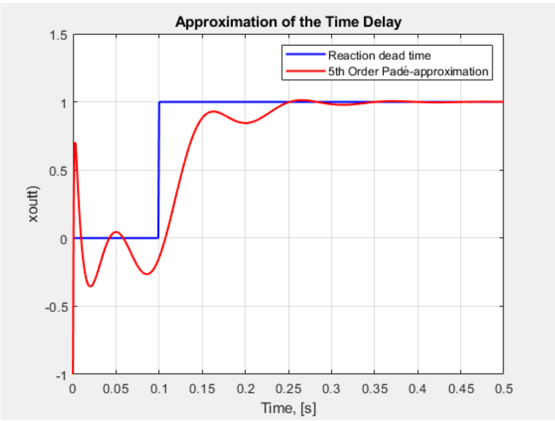

Figure 1: Reaction time approximation.

For further computer-aided simulations of the UAV operators let us select his parameters as they are listed below with various gain K:

$$K_1 = 30;\ K_2 = 40;\ K_3 = 52{,}5;\ K_4 = 60;\ T_L = 0{,}1026s;\ ;\ T_I = 0.103s;\ T_N = 0{,}05s \quad (7)$$

Results of the computer simulation for unit step function of $x_{in}(t) = 1(t)$ can be seen in Figure 2.

From Figure 2 it is easy to see that any increase in gain K will increase distortion of the dead zone approximation, whilst steady-state gains will increase to its steady state values via very few oscillations.

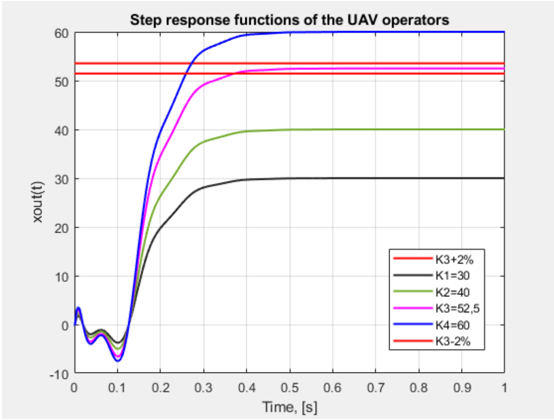

Figure 2: UAV operator step responses.

Frequency domain responses (Figure 3) of the UAV operators show that changes in the operator's gain K will lead to change of amplitudes keeping phase angles unchanged.

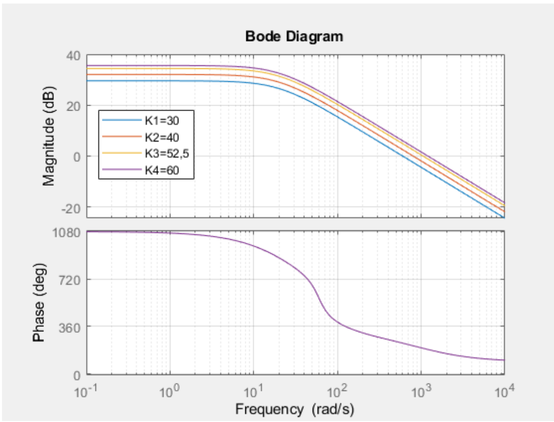

Figure 3: UAV operator frequency domain responses.

The time constant of T_L describing ability of the UAV operator to foresee events and to predict UAV behaviour is very important.

Hence, the UAV model was tested for different prediction times of T_L. Results of the computer simulation can be seen in Figure 4.

It is easy to state that as prediction time constant of T_L increases the transient time lessens.

For this, 2% of the steady-state gain of K has been set as a tolerance field.

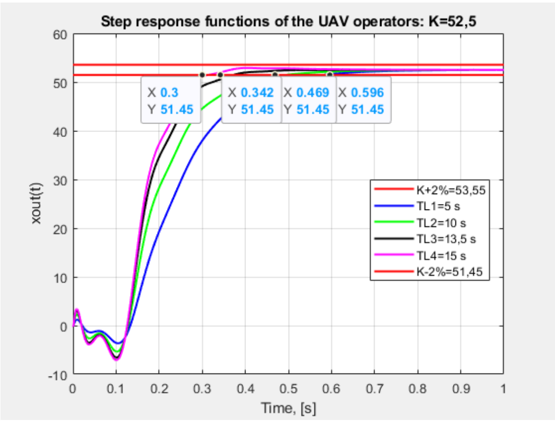

Figure 4: UAV operator step responses.

The frequency domain behaviour of the human operator when to control harmonic input signals can be seen in Figure 5.

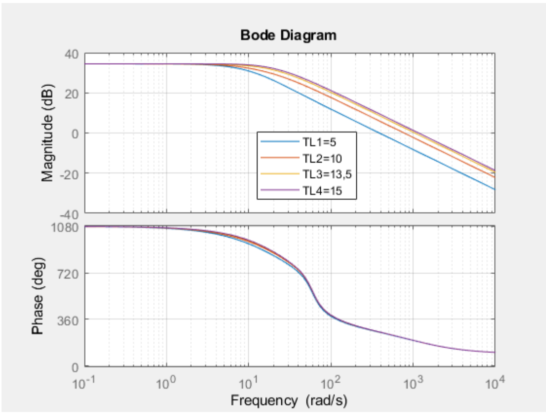

Figure 5: UAV operator frequency domain responses.

From Figure 5 it can be seen that in the vicinity of the crossover frequency increase of the prediction time of T_L leads to increase of the operator's gain with no sufficient changes of the phase shift.

The time constant of T_N describes the dynamics of the neuro-muscular system of the human operator. For different values of T_N the model (2) of the human operator was tested both in time and frequency domain. Results of the computer simulation can be seen in Figure 6 and Figure 7.

From Figure 6 it can be depicted that any increase of T_N leads to increase of the transient time. It is easy to see that for the full range of T_N the transient time is doubled for 2% of the tolerance field set for the steady state gain.

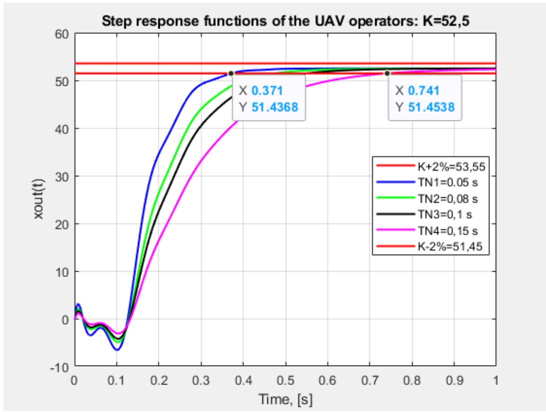

Figure 6: UAV operator step responses.

Figure 7 shows Bode diagrams of the human operator describing transmission of harmonic sinusoidal signals of different frequencies.

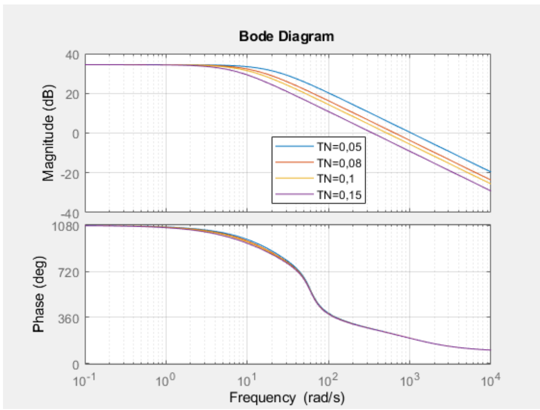

Figure 7: UAV operator frequency domain responses.

From Figure 7 it can be seen that any increase of the time constant T_N leads to decrease of the human operator's gain in the vicinity of the crossover frequency. However, in low frequency domain there is meaningful impact on pilot gain.

The time constant of T_I describes time derived by integral feature of the pilot behaviour and his activity. The human operator model was tested for different time constants of T_I. Results of the computer simulation can be seen in Figure 8 and Figure 9. From Figure 8 it can be easily depicted that any decrease in time constant T_I will lead to decrease in overall transient time.

However, oscillations might appear in step responses changing exponential pattern of the step response found for lesser time constants of T_I.

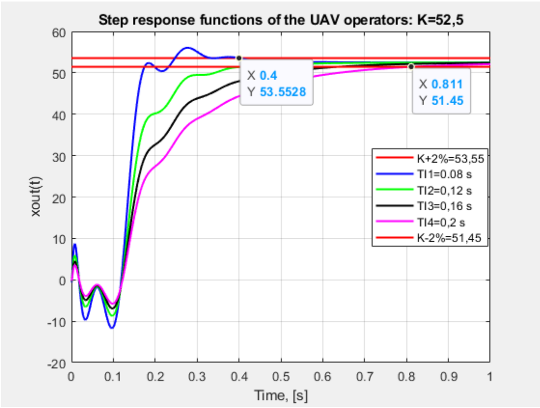

Figure 8: UAV operator step responses.

Figure 9 shows Bode diagrams of the human operators.

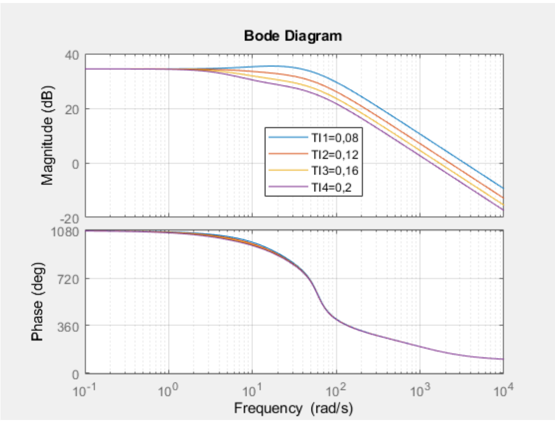

Figure 9: UAV operator frequency domain responses.

As it can be seen from Figure 9 increase of the time constant will decrease the pilot gain, starting also from the low frequency ranges with no sufficient changes in phase angles.

4. Highly Trained and Experienced vs Less-trained and Non-experienced UAV Operators' Behaviour

Obviously, UAV operators' responses shall differ both in tracking and in holding control missions.

Moreover, as it was stated before, the UAV operator models shall vary with level of their training, expertise and professionalism.

The highly trained and experienced UAV operator transfer function is as follows [1], [2], [3], [4], [5], [6]:

$$Y_{pro}(s) = \frac{x_{out}(s)}{x_{in}(s)} = K\frac{1+sT_L}{(1+sT_I)(1+sT_N)}e^{-s\tau} = 52{,}5\frac{1+0{,}0795s}{(1+0{,}08s)(1+0{,}05s)}e^{-0{,}15s} \quad (8)$$

On the contrary, the less-trained and less-skilled UAV operator has transfer function as it given below [1], [2], [3], [4], [5], [6]:

$$Y_{unpro}(s) = \frac{x_{out}(s)}{x_{in}(s)} = K\frac{1+sT_L}{(1+sT_I)(1+sT_N)}e^{-\tau s} = 52{,}5\frac{1+0{,}16s}{(1+0{,}16s)(1+0{,}15s)}e^{-0{,}4s} \quad (9)$$

Step responses of these models are depicted in Figure 10.

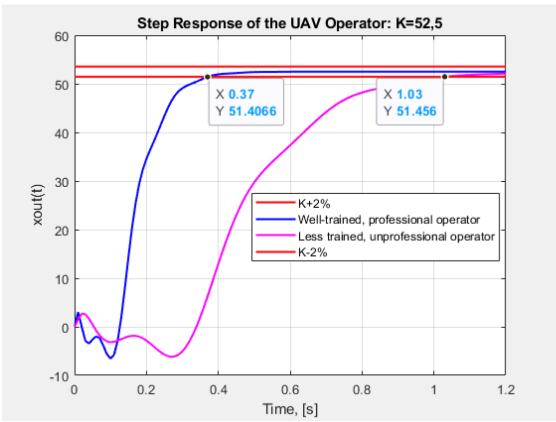

Figure 10: UAV operator step responses.

From Figure 10 it is obvious that the professional pilot acts very fast and very intensively close to some aggressive intent trying to follow the unit input subjected.

The unprofessional pilot gives time for

himself to act and to follow the unit input signal. His behaviour is very slow, careful and safe. The increase of the transient time reaches one second, which still can be considered for fast reaction.

There are several test input signals which play important role in UAV control, like squared signals, exponential signals, different sigmoid functions etc.

Figure 11 depicts time domain responses of both well-trained and less-trained UAV operators to a single square test input.

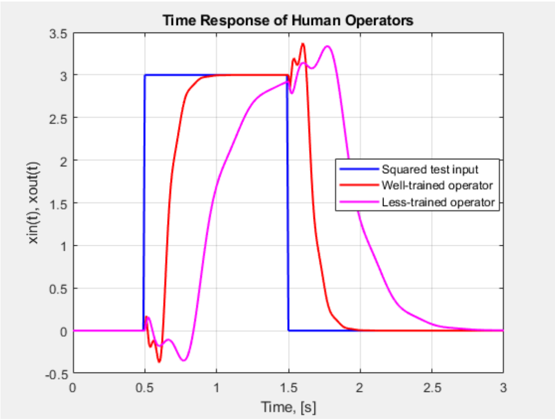

Figure 11: UAV operator time domain responses.

It is evident that the well-trained pilot has faster response to the test input, and he is able to maintain his initial zero status rapidly. The less-trained pilot has larger dead time, and slower reaction to the test input.

Moreover, he returns to its initial status very carefully. In the practice the sign changing squared test input signals are widely used in UAV manual control.

It is well-known that mostly in highly stressed control situations like UAV flights conducted in operational theatre operators might over actuate the manoeuvre requesting further corrections in the control with actuations in opposite meaning. Results of the computer simulation when asymmetric, sign changing square signal has been used to test abilities of the operators can be seen in Figure 12. Figure 12 depicts reactions of those two pilot models we put in the focus of attention earlier.

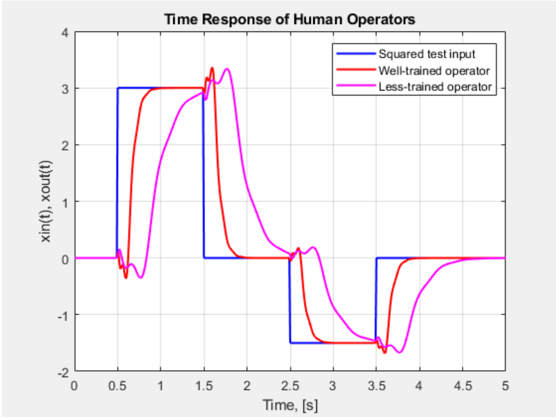

Figure 12: UAV operator time domain responses.

From Figure 12 it is easy to see that the more trained operator is fast in tracking the test input signal, having few dead times in his reactions.

The less-trained operator requires more time to reach the test input maximums, and is very careful in setting initial reference values.

Both from Figure 11 and Figure 12 it is evident that the sudden appearance, the sudden disappearance and the sign changes of the test signals subjected to the UAV operators mean difficulties also for the well-trained pilots to follow.

In many cases UAV's test inputs might vary with very smooth dynamics, like the

$$x_{in}(t) = (1 - e^{-1,5t}). \qquad (10)$$

Exponential function.

In this case, the magnitude of the reference input changes relatively slowly. For this case, results of the computer simulation can be seen in Figure 13.

From Figure 13 it is evident that there is no large difference between behaviour of the two pilot models examined in this article.

The well-trained pilot is a bit faster than the less-trained operator.

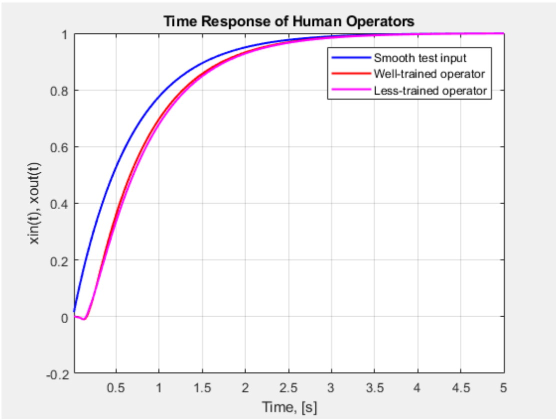

Figure 13: UAV operator time domain responses.

It is important to have another smooth test input like the logistical sigmoid function of the form

$$x_{in}(t) = \frac{5}{1+e^{2*(2,8-t)}}. \qquad (11)$$

The UAV pilot models were subjected to this sigmoid function. Results of the computer simulation are depicted in Figure 14.

From Figure 14 it is easy to see that both pilots perform well when the smooth test signal is to be tracked by them.

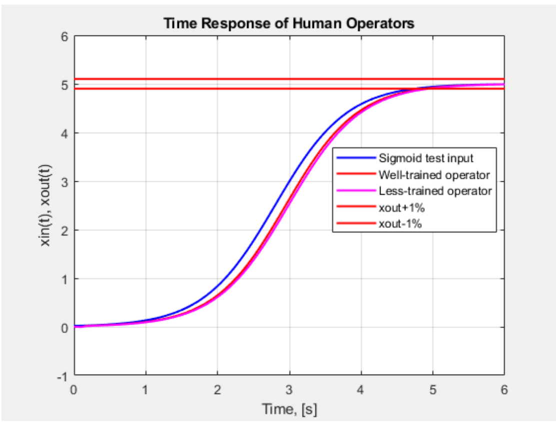

Figure 14: UAV operator time domain responses.

Also has importance to discuss a case when '(1-cos)'-type test input signal is applied with the following equation:

$$x_{in}(t) = 5 * (1 - \cos(0{,}6283 * t)). \qquad (12)$$

Results of the computer-aided simulation can be seen in Figure 15. From Figure 15 it is evident that both pilots are able to follow the test input signal very accurately.

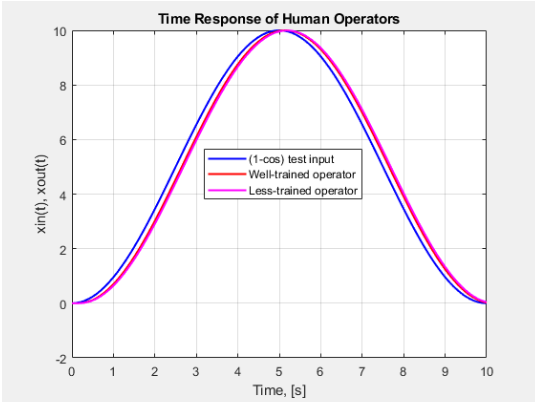

Figure 15: UAV operator time domain responses.

Regarding typical control inputs used and applied by UAV operators it can be stated that computer-aided simulation is able to support development and improvement of basic skills of the UAV operators.

This way, the training programs established for UAV operators shall be enriched with knowledge about their behaviour expected.

5. Discussions

The human operator model used widely in recent days is known since long time. The identified mathematical models help system designers to get idea about human behaviour.

The model identified has parameters differing for different human operators. Regarding parameters of the operator model it can be stated that its parameters are time depending.

If a UAV operator improves his knowledge and skills in UAV control parameters of his mathematical model would be improved, his activity would be safer than ever before.

If a UAV operator, due to any reason, has degraded level of his fitness and readiness to accomplish UAV flights, his model parameters shall worsen and also might be degraded to the ranges of his unfitness.

The operator gain K is a crucial element of his dynamic model. The larger is the gain it is the best from the point of view of signal transmission.

Nevertheless, if this gain is too large, in

Pilot-in-the-Loop (PiL) cases it might tend the closed loop flight control system to its stability margins, or as the worse, the large gain might destabilize the closed loop control systems at all.
The time constant of the prediction ability of the operator T_N describes ability of the well-trained highly skilled UAV operator to foresee for instance flight path manoeuvres of the UAV, and might help to avoid underactuation of the vehicle.

6. Conclusions and Outlook

Recent days UAVs represent promising applications both in military and in non-military purposes. The modern world is very busy with challenges in climate change, in modern digitalized precise agriculture, and also in safety and security matters.In dirty-dull-dangerous missions UAVs can serve the best purpose not to threaten the operators.
Worth to mention that unlimited accessibility to a modern and powerful UAV might lead to cases when the role of the UAV operator gains extremely high level of importance.
Any operator training program will focus on training minimums ensuring those basic skills needed for the safe operations in UAV spatial motion control. However, the emergency flight situations are handled rarely in basic training programs.
Hence, research work introduced in this paper shall be continued with discussing both the open and the closed loop control systems when the pilot accomplishes his mission as part of the control system.
In this framework, promising results might be predicted in finding critical parameters of the UAV operator leading the whole closed loop control system to loss of their stability margins.

References List

[1] McRuer D.T. & Krendel E.S. Dynamic Response of Human Operators, WADC Technical Report, 56-524, 1957.
[2] McRuer D.T. & Krendel E.S. The Human Operator as a Servo System Element. Journal of the Franklin Institute, Vol267, Issue5, pp (381-403), 1959.
[3] McRuer D.T., Graham G., Krendel E. & Reisener W. Human Pilot Dynamics in Compensatory Systems. US Government Report, AFFDL-TR-65-15, 1965.
[4] Bekey G.A. The Human Operator in Control Systems. University of Southern California, USCEE Report 359, 1969.
[5] McRuer D.T. & Krendel E.S. Mathematical Models of Human Pilot Behavior. NATO AGARD-AG-188, 1974.
[6] Aslanyan A.E. Aircraft Automatic Flight Control Systems, Part I. Kiev Military Aviation Technical Academy, Kiev, Ukraine, 1984.
[7] McRuer D.T., Clement W.F., Thompson P.M. & Magdaleno R.E. Minimum Flying Qualities, Volume II, Pilot Modeling for Flying Qualities Applications. Technical Report WRDC-TR-89-3125, 1990.
[8] Stevens B.L., Lewis F.L. & Johnson E.N. Aircraft Control and Simulation: Dynamics, Controls, Design, and Autonomous Systems. 3rd Edition, ISBN 978-1-118-87098-3, Wiley-Blackwell, 2016.