On regular power fuzzy graphs

T. BHARATHI, S. SHINY PAULIN AND M. JEBA SHERLIN

Abstract

In this paper, we define the notion of regular power fuzzy graph (RPFG), as a combination of regular properties and power fuzzy graphs. We also define totally regular power fuzzy graph as a special case of RPFG. A comparative study between regular and totally regular power fuzzy graphs is investigated. It is also proved that any power fuzzy graph containing a pendant vertex can be neither regular nor totally regular. A necessary condition for a total power fuzzy graph to be perfectly regular is that the vertex and edge membership functions are constants.

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Keywords: Power fuzzy graph, Regular power fuzzy graph, Totally regular power fuzzy graph.

1. INTRODUCTION

The notion of graph theory was introduced by Euler in 1736 and many other properties and concepts were developed later in graph theory. The definition of graph power was developed by [Skiena 1991]. Furthermore, some interesting properties of power graphs can be found in [Ho et al. 2000; Todinca 2003; Chang et al. 2003; An and Wu 2008]. The concept of fuzzy graph was developed by [Rosenfeld 1975] and many other extensions of fuzzy graphs can be found in [Parvathi and Karunambigai 2006; Akram 2011; Akram and Dudek 2011; Zuo et al. 2019]. The new definition of power fuzzy graphs and its properties were discussed [Bharathi et al. 2023]. The regular fuzzy graphs and totally regular fuzzy graphs were introduced by [Gani et al. 2008]. Further regular properties on fuzzy graphs such as bipolar, pseudo, interval valued intuitionistic, picture and many more fuzzy graphs were studied in [Akram and Dudek 2012; Maheswari and Sekar 2016; Mishra and Pal 2017; Cary 2018; Xiao et al. 2020; Ghorai 2021]. Such regular properties were the motivation to extend the concept of regular power fuzzy graph. The objective of this study is to define the vertex regularity and total vertex regularity of power fuzzy graphs and study their properties. The novelty of this study is that the edge membership values according to the vertices and power of the fuzzy graph by considering the edge capacity and communication requirements such that the final o utput a t t he n odes i s e qual. In section 2, basic definitions required for this study are presented. The notions of

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regular and totally regular power fuzzy graph are defined with example in section 3. The properties of regular power fuzzy graphs are given in section 4.

2. PRELIMINARIES

Definition 2.1. [Gani and Ahmed 2003] Let $G_f = (V, E, \sigma, \mu)$ be a fuzzy graph. The degree of a vertex v_1 of G_f is defined as $deg_{G_f}(v_1) = \sum_{v_2 \neq v_1} \mu(v_1, v_2)$ where $v_2 \in V$. The total degree of a vertex v_1 of G_f is defined as $tdeg_{G_f}(v_1) = \sum_{v_2 \neq v_1} \mu(v_1, v_2) + \sigma(v_1)$ where $v_2 \in V$.

Definition 2.2. [Gani and Ahmed 2003] Let $G_f = (V, E, \sigma, \mu)$ be a fuzzy graph. The size of G_f is defined as $S(G_f) = \sum_{(v_1, v_2) \in V \times V} \mu(v_1, v_2)$. The order of G_f is defined by $(O)G_f = \sum_{v \in V} \sigma(v)$.

Definition 2.3. [Gani and Radha 2008] Let $G_f = (V, E, \sigma, \mu)$ be a fuzzy graph on graph $G : (V, E)$. If $deg_{G_f}(v) = k$ for all $v \in V$, that is, if each vertex has the same degree k in G_f, then G_f is said to be a regular fuzzy graph of degree k or a k-regular fuzzy graph. Similarly, if each vertex has the same total degree k in G_f, then G_f is said to be a totally regular fuzzy graph of degree k. Here k is a positive rational number.

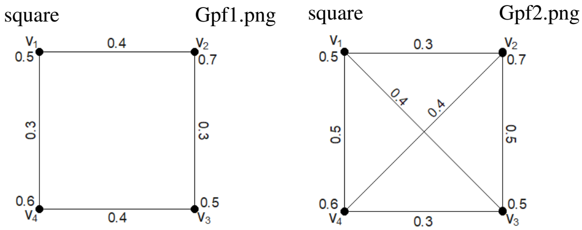
Definition 2.4. [Cary 2018] A perfectly regular fuzzy graph is a fuzzy graph when it is both regular and totally regular.

Definition 2.5. [Bharathi et al. 2024] Let $G_f = (V, E, \sigma, \mu)$ be a fuzzy graph of a graph $G = (V, E)$. The power fuzzy graph (PFG) of G_f is defined as ${G_f}^k = (V, E_k, \sigma, \mu_k)$ where $E_k = E \cup E^*$, for any non-adjacent vertices $u, v \in V$ in G_f there exists $uv \in E^*$ such that $l(u, v) \leq k$ where l is the length from u to v and $k > 1$ provided $min(\sigma(u)^k, \sigma(v)^k) \leq \mu_k(u, v) \leq min(\sigma(u)^{k-1}, \sigma(v)^{k-1})$.

Example 2.6. Consider the fuzzy graph ${G_f}^1$ in Figure 1(a), which satisfies the conditions of power fuzzy graph. ${G_f}^2$ in Figure 1(b), is formed by adding edges v_1v_3 and v_2v_4 to the existing edges v_1v_2, v_2v_3, v_3v_4 and v_1v_4 of ${G_f}^1$, since $l(v_1, v_3) = l(v_2, v_4) = 2$ and $l(v_1, v_2) = l(v_2, v_3) = l(v_3, v_4) = l(v_1, v_4) = 1$.

Definition 2.7. [Bharathi et al. 2024] The power fuzzy graph ${G_f}^k$ is called Total Power Fuzzy Graph ${T_f}^k$ where each vertex is adjacent to all other vertices. The number of edges incident to a vertex is denoted by $e({T_f}^k)$ and the maximum k value referred as diameter is denoted by k_m.

Remark 2.8. [Gani and Ahmed 2003] $\sum_{v \in V} deg(v) = 2 \sum_{(v_1, v_2) \in V \times V} \mu(v_1, v_2)$



(a) $G_f{}^1$ (b) $G_f{}^2$

Fig. 1: Power fuzzy graphs

Edge	**Interval**	**Edge membership value**
(v_1,v_2)	$[0.25,05]$	$\mu_3(v_1,v_2)=0.3$
(v_2,v_3)	$[0.25,0.5]$	$\mu_3(v_2,v_3)=0.5$
(v_3,v_4)	$[0.25,0.5]$	$\mu_3(v_3,v_4)=0.3$
(v_1,v_4)	$[0.25,0.5]$	$\mu_3(v_1,v_4)=0.5$
(v_1,v_3)	$[0.25,0.5]$	$\mu_3(v_1,v_3)=0.4$
(v_2,v_4)	$[0.36,0.6]$	$\mu_3(v_2,v_4)=0.4$

Table I: Edge membership values of $G_f{}^3$

3. REGULAR AND TOTALLY REGULAR POWER FUZZY GRAPHS

Definition 3.1. A power fuzzy graph $G_f{}^k=(V,E_k,\sigma,\mu_k)$ is p-regular power fuzzy graph (RPFG) if for any $p>0$, $deg(u)=p$, $\forall u\in V$.

Example 3.2. Consider the power fuzzy graphs $G_f{}^1$ and $G_f{}^2$ as shown in Figures 1(a) and 1(b) respectively. $G_f{}^1$ is regular since $deg(v_i)=0.3+0.4=0.7$ and $G_f{}^2$ is regular since $deg(v_i)=0.3+0.4+0.5=1.2$ respectively where $i=1,2,3,4$.

Note 3.3. The lowest vertex membership value is denoted as σ_l and the highest vertex membership value is denoted as σ_h. In Figures 1(a) and 1(b), $\sigma_l=0.5$ and $\sigma_h=0.7$.

Definition 3.4. A power fuzzy graph $G_f{}^k=(V,E_k,\sigma,\mu_k)$ is totally regular if for any $q>0, tdeg(u)=deg(u)+\sigma(u)=q, \forall u\in V$.

Example 3.5. Consider the power fuzzy graphs $G_f{}^1$ and $G_f{}^2$ as in Figure 2(a) and Figure 2(b). $G_f{}^1$ is totally regular since

$$tdeg(v_1)=0.1+0.2+0.3=0.6$$

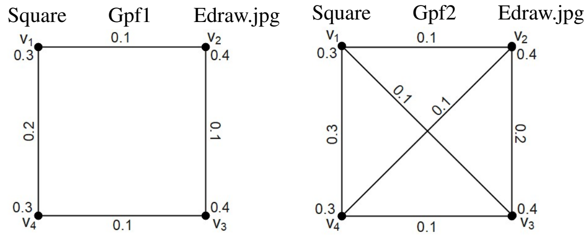
$$tdeg(v_2)=0.1+0.1+0.4=0.6$$

$$tdeg(v_3)=0.1+0.1+0.4=0.6$$

$$tdeg(v_4) = 0.1 + 0.2 + 0.3 = 0.6$$

Similarly, ${G_f}^2$ is totally regular since

$$tdeg(v_1) = 0.1 + 0.1 + 0.3 + 0.3 = 0.8$$
$$tdeg(v_2) = 0.1 + 0.1 + 0.2 + 0.4 = 0.8$$
$$tdeg(v_3) = 0.1 + 0.1 + 0.2 + 0.4 = 0.8$$
$$tdeg(v_4) = 0.1 + 0.1 + 0.3 + 0.3 = 0.8$$



(a) ${G_f}^1$ (b) ${G_f}^2$

Fig. 2: Totally regular power fuzzy graphs

Definition 3.6. A power fuzzy graph ${G_f}^k = (V, E_k, \sigma, \mu_k)$ is called perfectly regular if ${G_f}^k$ is both regular and totally regular.

4. REGULAR PROPERTIES ON VERTICES

LEMMA 4.1. If a power fuzzy graph ${G_f}^k = (V, E_k, \sigma, \mu_k)$ has only two distinct vertex membership values σ_l and σ_h such that multiplicity of σ_h is greater than σ_l then ${G_f}^k$ is neither regular nor totally regular.

Example 4.2. Consider the 4-path power fuzzy graph ${G_f}^2$ as in figure 3., here $\sigma_h = 0.4$ with multiplicity 3 and $\sigma_l = 0.3$ with multiplicity 1. Since $deg(v_1) = 0.26 \neq deg(v_2) = 0.56, {G_f}^2$ is not regular and $tdeg(v_1) = 0.56 \neq tdeg(v_2) = 0.96, {G_f}^2$ is not totally regular.

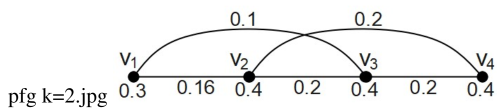


Fig. 3: 4-path power fuzzy graph with $k = 2$

THEOREM 4.3. A power fuzzy graph ${G_f}^k = (V, E_k, \sigma, \mu_k)$ is regular if

$$\sigma_l^{k-1} \in \left[\min\left(\sigma(u)^k, \sigma(v)^k\right), \min\left(\sigma(u)^{k-1}, \sigma(v)^{k-1}\right)\right] \forall u, v \in V.$$

PROOF. Assume that a power fuzzy graph ${G_f}^k = (V, E_k, \sigma, \mu_k)$ is regular and let v be the vertex with σ_l value. Suppose $\sigma_l^{k-1} \notin \left[\min\left(\sigma(u)^k, \sigma(v)^k\right), \min\left(\sigma(u)^{k-1}, \sigma(v)^{k-1}\right)\right]$, then consider a vertex w such that $w < u$ and $w < v$ where $u \neq v$. The edges incident with w has membership values between the boundary

$$\left[\min\left(\sigma(u)^k, \sigma(w)^k\right), \min\left(\sigma(u)^{k-1}, \sigma(w)^{k-1}\right)\right].$$

Then $deg(w) > deg(v)$, which contradicts the assumption.

Example 4.4. Consider 5-path power fuzzy graph with vertex membership values $v_1 = 0.4, v_2 = 0.3, v_3 = 0.2, v_4 = 0.3, v_5 = 0.4$. The power fuzzy graph G_f^4 is not regular since $\sigma_l^{k-1} = \sigma_l^{4-1} = 0.008 \notin [0.0081, 0.027]$.

THEOREM 4.5. The total power fuzzy graph ${T_f}^k$ is regular if $\mu_k(u,v) = m$ such that $m \in \left[\min\left(\sigma(u)^k, \sigma(v)^k\right), \min\left(\sigma(u)^{k-1}, \sigma(v)^{k-1}\right)\right] \forall (u,v) \in E_k$.
PROOF. Consider ${T_f}^k$ be total power fuzzy graph with $\mu_k(u,v) = m$ such that

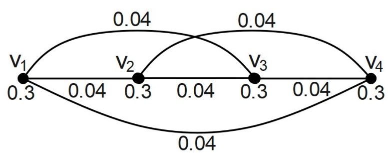
$$m \in \left[\min\left(\sigma(u)^k, \sigma(v)^k\right), \min\left(\sigma(u)^{k-1}, \sigma(v)^{k-1}\right)\right], \forall (u,v) \in E_k \tag{1}$$

Since each vertex is adjacent to all vertices, let e be the number of edges incident to a vertex. (i.e.) $e({T_f}^k) = e$

$$\Rightarrow deg(v) = (m + m + \ldots + m) \text{ e times}, \forall v \in {T_f}^k$$
$$\Rightarrow deg(v) = me, \forall v \in {T_f}^k$$

Hence ${T_f}^k$ is regular.

Example 4.6. Consider the 4-path total power fuzzy graph ${T_f}^k$ as in Figure 4. ${T_f}^3$ is regular since $\deg(v_i) = 0.04 + 0.04 + 0.04 = 0.12$, $\forall v_i \in V$ where $i = 1,2,3,4$, and $\mu_3(u,v) = 0.04$, $\forall (u,v) \in E_3$ such that $0.04 \in [0.027, 0.09]$.



path total pfg .jpg

Fig. 4: 4-path power fuzzy graph ${T_f}^3$

COROLLARY 4.7. The total power fuzzy graph ${T_f}^k$ is perfectly regular if $m \in \left[\min\left(\sigma(u)^k, \sigma(v)^k\right), \min\left(\sigma(u)^{k-1}, \sigma(v)^{k-1}\right)\right], \forall(u,v) \in E_k$ and vertex membership function is constant.

PROOF. By theorem 4.5, the total power fuzzy graph ${T_f}^k$ is regular with $deg(v) = me$, for all vertices in ${T_f}^k$. Assume $\sigma(v) = c_1$, since vertex membership function is constant. We know that

$$tdeg(v) = deg(v) + \sigma(v), \forall v \in {T_f}^k$$
$$\Rightarrow tdeg(v) = me + c_1 = c \text{ (a constant)}, \forall v \in {T_f}^k$$

Therefore, ${T_f}^k$ is totally regular. Hence ${T_f}^k$ is perfectly regular.

Example 4.8. Consider a power fuzzy graph ${G_f}^2 = (V, E_2, \sigma, \mu_2)$ of a 5-cycle graph where $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E_2 = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_1, v_1v_3, v_1v_4, v_2v_5, v_2v_4, v_3v_5\}$ defined by

Vertices	**Membership values**
v_1	$\sigma(v_1) = 0.6$
v_2	$\sigma(v_2) = 0.5$
v_3	$\sigma(v_3) = 0.5$
v_4	$\sigma(v_4) = 0.6$
v_5	$\sigma(v_5) = 0.7$

Table II: Vertex membership values of ${G_f}^2$ of Example 4.8

Edge	**Interval**	**Edge membership value**
(v_1, v_2)	$[0.25, 0.5]$	$\mu_2(v_1, v_2) = 0.4$
(v_2, v_3)	$[0.25, 0.5]$	$\mu_2(v_2, v_3) = 0.4$
(v_3, v_4)	$[0.25, 0.5]$	$\mu_2(v_3, v_4) = 0.4$
(v_4, v_5)	$[0.36, 0.6]$	$\mu_2(v_4, v_5) = 0.4$
(v_5, v_1)	$[0.36, 0.6]$	$\mu_2(v_5, v_1) = 0.4$
(v_1, v_3)	$[0.25, 0.5]$	$\mu_2(v_1, v_3) = 0.5$
(v_1, v_4)	$[0.36, 0.6]$	$\mu_2(v_1, v_4) = 0.5$
(v_2, v_5)	$[0.25, 0.5]$	$\mu_2(v_2, v_5) = 0.5$
(v_2, v_4)	$[0.25, 0.5]$	$\mu_2(v_2, v_4) = 0.5$
(v_3, v_5)	$[0.25, 0.5]$	$\mu_2(v_3, v_5) = 0.5$

Table III: Edge membership values of ${G_f}^2$ of Example 4.8

Then $deg(v_i) = 1.8$ for $i = 1$ to 5 . Hence ${G_f}^2$ is a regular power fuzzy graph but not a totally regular power fuzzy graph, *since* $tdeg(v_1) = 2.4 \neq tdeg(v_2) = 2.3$.

Example 4.9. Consider a power fuzzy graph ${G_f}^2 = (V, E_2, \sigma, \mu_2)$ of a 5-cycle graph where $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E_2 = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_1, v_1v_3, v_1v_4, v_2v_5, v_2v_4, v_3v_5\}$ defined by

Vertices	**Membership values**
v_1	$\sigma(v_1) = 0.3$
v_2	$\sigma(v_2) = 0.2$
v_3	$\sigma(v_3) = 0.3$
v_4	$\sigma(v_4) = 0.3$
v_5	$\sigma(v_5) = 0.2$

Table IV: Vertex membership values of ${G_f}^2$ of Example 4.9

Edge	**Interval**	**Edge membership value**
(v_1, v_2)	$[0.04, 0.2]$	$\mu_2(v_1, v_2) = 0.4$
(v_2, v_3)	$[0.04, 0.2]$	$\mu_2(v_2, v_3) = 0.4$
(v_3, v_4)	$[0.09, 0.3]$	$\mu_2(v_3, v_4) = 0.4$
(v_4, v_5)	$[0.04, 0.2]$	$\mu_2(v_4, v_5) = 0.4$
(v_5, v_1)	$[0.04, 0.2]$	$\mu_2(v_5, v_1) = 0.4$
(v_1, v_3)	$[0.09, 0.3]$	$\mu_2(v_1, v_3) = 0.5$
(v_1, v_4)	$[0.09, 0.3]$	$\mu_2(v_1, v_4) = 0.5$
(v_2, v_5)	$[0.04, 0.2]$	$\mu_2(v_2, v_5) = 0.5$
(v_2, v_4)	$[0.04, 0.2]$	$\mu_2(v_2, v_4) = 0.5$
(v_3, v_5)	$[0.04, 0.2]$	$\mu_2(v_3, v_5) = 0.5$

Table V: Edge membership values of ${G_f}^2$ of Example 4.9

Then $tdeg(v_i) = 0.7$ for $i = 1$ to 5 . Hence ${G_f}^2$ is a totally regular power fuzzy graph but not a regular power fuzzy graph, since $deg(v_1) = 0.4 \neq tdeg(v_2) = 0.5$.

Example 4.10. Consider a power fuzzy graph ${G_f}^2 = (V, E_2, \sigma, \mu_2)$ of a 5-cycle graph where $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E_2 = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_1, v_1v_3, v_1v_4, v_2v_5, v_2v_4, v_3v_5\}$ defined by

Vertices	**Membership values**
v_1	$\sigma(v_1) = 0.6$
v_2	$\sigma(v_2) = 0.6$
v_3	$\sigma(v_3) = 0.6$
v_4	$\sigma(v_4) = 0.6$
v_5	$\sigma(v_5) = 0.6$

Table VI: Vertex membership values of ${G_f}^2$ of Example 4.10

Edge	**Interval**	**Edge membership value**
(v_1,v_2)	$[0.25,0.5]$	$\mu_2(v_1,v_2)=0.4$
(v_2,v_3)	$[0.25,0.5]$	$\mu_2(v_2,v_3)=0.4$
(v_3,v_4)	$[0.25,0.5]$	$\mu_2(v_3,v_4)=0.4$
(v_4,v_5)	$[0.36,0.6]$	$\mu_2(v_4,v_5)=0.4$
(v_5,v_1)	$[0.36,0.6]$	$\mu_2(v_5,v_1)=0.4$
(v_1,v_3)	$[0.25,0.5]$	$\mu_2(v_1,v_3)=0.4$
(v_1,v_4)	$[0.36,0.6]$	$\mu_2(v_1,v_4)=0.4$
(v_2,v_5)	$[0.25,0.5]$	$\mu_2(v_2,v_5)=0.4$
(v_2,v_4)	$[0.25,0.5]$	$\mu_2(v_2,v_4)=0.4$
(v_3,v_5)	$[0.25,0.5]$	$\mu_2(v_3,v_5)=0.4$

Table VII: Edge membership values of ${G_f}^2$ of Example 4.10

Then $deg(v_i)=1.6$ and $tdeg(v_i)=2.2$ for $i=1$ to 5 . Hence ${G_f}^2$ is perfectly regular (both regular and totally regular) power fuzzy graph.

Example 4.11. Consider a power fuzzy graph ${G_f}^2=(V,E_2,\sigma,\mu_2)$ of a 5-cycle graph where $V=\{v_1,v_2,v_3,v_4,v_5\}$ and $E_2=\{v_1v_2,v_2v_3,v_3v_4,v_4v_5,v_5v_1,v_1v_3,v_1v_4,v_2v_5,v_2v_4,$ $v_3v_5\}$ defined by

Vertices	**Membership values**
v_1	$\sigma(v_1)=0.3$
v_2	$\sigma(v_2)=0.2$
v_3	$\sigma(v_3)=0.3$
v_4	$\sigma(v_4)=0.3$
v_5	$\sigma(v_5)=0.4$

Table VIII: Vertex membership values of ${G_f}^2$ of Example 4.11

Edge	**Interval**	**Edge membership value**
(v_1,v_2)	$[0.04,0.2]$	$\mu_2(v_1,v_2)=0.1$
(v_2,v_3)	$[0.04,0.2]$	$\mu_2(v_2,v_3)=0.05$
(v_3,v_4)	$[0.09,0.3]$	$\mu_2(v_3,v_4)=0.2$
(v_4,v_5)	$[0.09,0.3]$	$\mu_2(v_4,v_5)=0.3$
(v_5,v_1)	$[0.09,0.3]$	$\mu_2(v_5,v_1)=0.15$
(v_1,v_3)	$[0.09,0.3]$	$\mu_2(v_1,v_3)=0.15$
(v_1,v_4)	$[0.09,0.3]$	$\mu_2(v_1,v_4)=0.15$
(v_2,v_5)	$[0.04,0.2]$	$\mu_2(v_2,v_5)=0.2$
(v_2,v_4)	$[0.04,0.2]$	$\mu_2(v_2,v_4)=0.1$
(v_3,v_5)	$[0.09,0.3]$	$\mu_2(v_3,v_5)=0.25$

Table IX: Edge membership values of ${G_f}^2$ of Example 4.11

Here ${G_f}^2$ is neither a regular nor a totally regular power fuzzy graph, since $deg(v_1) = 0.55 \neq deg(v_2) = 0.45$ and $tdeg(v_1) = 2.4 \neq tdeg(v_2) = 2.3$.

Remark 4.12. From the examples 4.8, 4.9, 4.10, 4.11 it is clear that there is no relation between regular power fuzzy graph and totally regular power fuzzy graph.

THEOREM 4.13. If a power fuzzy graph ${G_f}^k$ is both regular and totally regular, then vertex membership function is a constant function.

PROOF. Let ${G_f}^k = (V, E_k, \sigma, \mu_k)$ be a power fuzzy graph which is regular and totally regular, where σ is the vertex membership function. Let $u, v \in V$ be any two vertices. Since ${G_f}^k$ is totally regular,

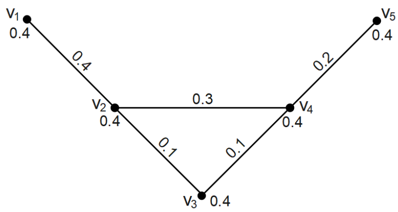
$$tdeg(u) = tdeg(v)$$
$$\Rightarrow deg(u) + \sigma(u) = deg(v) + \sigma(v)$$
$$\Rightarrow \sigma(u) = \sigma(v) (\because deg(u) = deg(v))$$

Therefore, σ is a constant function.

Remark 4.14. The converse of the theorem 4.13 need not be true.

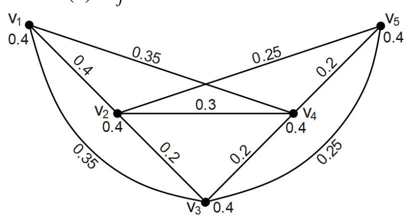
Example 4.15. Consider the power fuzzy graphs in figure 5(a) and 5(b), such that the vertex membership values are constant, (i.e) $\sigma(v_i) = 0.4$ for $i = 1$ to 5 but they are neither regular nor totally regular.

graph Gf1.png



(a) ${G_f}^1$

graph Gf2.png



(b) ${G_f}^2$

Fig. 5: Power fuzzy graphs of bullgraphs

Remark 4.16. Let $G_f{}^p = (V, E_p, \sigma, \mu_p)$ such that $1 \leq p \leq k_m$ be a regular power fuzzy graph. Then $G_f{}^k$ for any $k \neq p$ need not be a regular power fuzzy graph.

COROLLARY 4.17. Any graph with a pendant vertex can be neither a regular power fuzzy graph nor totally regular power fuzzy graph.

PROOF. Let $G_f{}^1 = (V, E_1, \sigma, \mu_1)$ be a power fuzzy graph with more than two vertices. Consider two adjacent vertices v_1 and v_2 such that v_1 is the pendant vertex then the degree of v_1 is given by $deg(v_1) = \mu_1(v_1v_2)$. Suppose v_3 is another vertex adjacent with v_2 since v_2 is not a, then $deg(v_2) = \mu_1(v_1v_2) + \mu_1(v_2v_3)$. Hence $deg(v_1) < deg(v_2)$

Example 4.18. The power fuzzy graph $G_f{}^1$ in figure 6, has $deg(v_5) = 0.3 \neq deg(v_1) = 0.8$ and $tdeg(v_5) = 0.9 \neq tdeg(v_1) = 1.3$. Hence $G_f{}^1$, is neither regular power fuzzy graph nor totally regular power fuzzy graph.

graph Pendant vertex.jpg

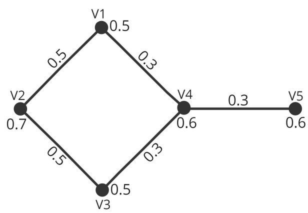


Fig. 6: Power fuzzy graph $G_f{}^1$ of Tadpole graph $T(4,1)$

PROPOSITION 4.19. A path power fuzzy graph, $G_f{}^1$ is neither regular nor totally regular.

PROOF. Since path power fuzzy graph $G_f{}^1$ contains pendant vertices by corollary 4.17 $G_f{}^1$ is neither regular nor totally regular.

THEOREM 4.20. The size of a p-regular power fuzzy graph, $G_f{}^k = (V, E_k, \sigma, \mu_k)$ is $\frac{np}{2}$ where n is the number of vertices. PROOF. The size of $G_f{}^k$ is $S(G_f{}^k) = \sum_{(v_1,v_2)\in V\times V} \mu(v_1, v_2)$.

From Remark 2.8, $\sum_{v\in V} deg(v) = 2 \sum_{(v_1,v_2)\in V\times V} \mu(v_1, v_2)$

$$\Rightarrow 2S(G_f{}^k) = \sum_{v\in V} deg(v) \Rightarrow 2S(G_f{}^k) = \sum_{v\in V} p = np \Rightarrow S(G_f{}^k) = \frac{np}{2}.$$

Example 4.21. Consider the power fuzzy graphs in Figure 7 and 8 where ${G_f}^1$ and ${G_f}^2$ are regular. In ${G_f}^1$, $n = 7$, $p = \sum_{v_i \in V} deg(v_i) = 0.15$, where $1 \leq i \leq 7$ hence $\frac{np}{2} = \frac{7 \times 0.15}{2} = 0.525$. Also, $S({G_f}^k) = \sum_{(v_i, v_j) \in V \times V} \mu(v_i, v_j) = 0.525$, where $i \neq j$ and $1 \leq i, j \leq 7$. Therefore, $S({G_f}^k) = \frac{np}{2}$.

Vertices	**Membership values**
v_1	$\sigma(v_1) = 0.4$
v_2	$\sigma(v_2) = 0.2$
v_3	$\sigma(v_3) = 0.4$
v_4	$\sigma(v_4) = 0.2$
v_5	$\sigma(v_5) = 0.4$
v_6	$\sigma(v_6) = 0.2$
v_7	$\sigma(v_7) = 0.3$

Table X: Vertex membership values of ${G_f}^1$ and ${G_f}^2$ in Figure 7 and 8

graph Gf1 regular.png

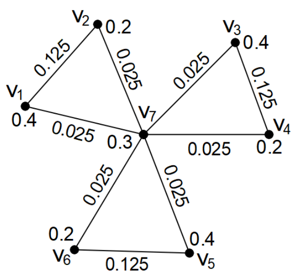
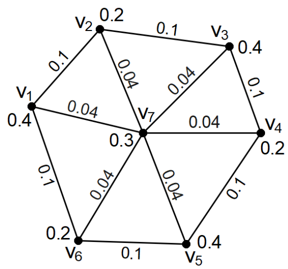


Fig. 7: Power fuzzy graph ${G_f}^1$ of a fan graph

Edge	**Interval**	**Edge membership value**
(v_1, v_2)	$[0, 0.2]$	$\mu_2(v_1, v_2) = 0.125$
(v_3, v_4)	$[0, 0.2]$	$\mu_2(v_3, v_4) = 0.125$
(v_5, v_6)	$[0, 0.2]$	$\mu_2(v_5, v_6) = 0.125$
(v_1, v_7)	$[0, 0.3]$	$\mu_2(v_1, v_7) = 0.025$
(v_2, v_7)	$[0, 0.2]$	$\mu_2(v_2, v_7) = 0.025$
(v_3, v_7)	$[0, 0.3]$	$\mu_2(v_3, v_7) = 0.025$
(v_4, v_7)	$[0, 0.2]$	$\mu_2(v_4, v_7) = 0.025$
(v_5, v_7)	$[0, 0.3]$	$\mu_2(v_5, v_7) = 0.025$
(v_6, v_7)	$[0, 0.2]$	$\mu_2(v_6, v_7) = 0.025$

Table XI: Edge membership values of ${G_f}^1$ in Figure 7



graph Gf2 regular.png

Fig. 8: Power fuzzy graph ${G_f}^2$ of fan graph

Similarly, in ${G_f}^2$, $n = 7$, $p = \sum_{v_i \in V} deg(v_i) = 0.24$, where $1 \leq i \leq 7$ hence $\frac{np}{2} = \frac{7 \times 0.24}{2} = 0.84$. Also, $S({G_f}^k) = \sum_{(v_i, v_j) \in V \times V} \mu\,(v_i, v_j) = 0.84$, where $i \neq j$ and $1 \leq i,\ j \leq 7$. Therefore, $S({G_f}^k) = \frac{np}{2}$.

Edge	Interval	Edge membership value
(v_1, v_2)	$[0.04, 0.2]$	$\mu_2(v_1, v_2) = 0.1$
(v_2, v_3)	$[0.04, 0.2]$	$\mu_2(v_2, v_3) = 0.1$
(v_3, v_4)	$[0.04, 0.2]$	$\mu_2(v_3, v_4) = 0.1$
(v_4, v_5)	$[0.04, 0.2]$	$\mu_2(v_4, v_5) = 0.1$
(v_5, v_6)	$[0.04, 0.2]$	$\mu_2(v_5, v_6) = 0.1$
(v_6, v_1)	$[0.04, 0.2]$	$\mu_2(v_6, v_1) = 0.1$
(v_1, v_7)	$[0.09, 0.3]$	$\mu_2(v_1, v_7) = 0.04$
(v_2, v_7)	$[0.04, 0.2]$	$\mu_2(v_2, v_7) = 0.04$
(v_3, v_7)	$[0.09, 0.3]$	$\mu_2(v_3, v_7) = 0.04$
(v_4, v_7)	$[0.04, 0.2]$	$\mu_2(v_4, v_7) = 0.04$
(v_5, v_7)	$[0.09, 0.3]$	$\mu_2(v_5, v_7) = 0.04$
(v_6, v_7)	$[0.04, 0.2]$	$\mu_2(v_6, v_7) = 0.04$

Table XII: Edge membership values of ${G_f}^2$ in Figure 8

5. APPLICATION

The regular power fuzzy graph model can be applied to achieve a balanced distribution among the vertices in cases like distributing signals or resources to ensure the equitable allocation. This helps in fair allocation of signals and facilitates the mechanism by regulating the frequencies, time slots, and power levels based on fuzzy membership values between nodes. Also, by giving the loads or works in a balanced way ensures efficient performance and prevents overloading. This helps in smooth data flow.

6. CONCLUSION

A new concept of regular power fuzzy graph has been introduced. A general formula for the size of a regular power fuzzy graph has been derived in terms of number of vertices. It is found that there is no connection between regular and totally regular power fuzzy graph. Any network of nodes and edges such that the connected edge carries the network bandwidth or signal strength among these nodes by equal distribution can be viewed as an application of regular power fuzzy graph model. This study can be extended further by analyzing edge regularity, total edge regularity and irregularity properties.

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T. Bharathi
Department of Mathematics,
Loyola College, University of Madras,
Chennai-600034, India.
Email: drtbharathi2024@gmail.com (Corresponding Author)

S. Shiny Paulin
Department of Mathematics,
Loyola College, University of Madras,
Chennai-600034, India.
Email: paulinshiny@gmail.com

M. Jeba Sherlin
Department of Mathematics,
Loyola College, University of Madras,
Chennai-600034, India.
Email: m.sherlinmohan@gmail.com