# Impact of random variations in the sampling period on the stability of embedded control systems

D. GABRISKA


## Abstract

Control of embedded systems is a key task in many industrial and technological applications, and their performance and stability are critically dependent on the ability to effectively implement control algorithms within limited computational and time resources. One of the main challenges in this area is the need to process data with a random variable that comes to processing randomly and with variable periodicity. This variability in the sampling period can cause a decrease in the quality of the control, which threatens the stability of the entire system. The article examines the limits to which this variability can be tolerated without losing the stability of the controlled system.

The study includes an analysis of systems dynamics with a variable sampling period and determining the necessary period of system discretization. It is also essential to consider the impact of the random component of the sampling period on the system's transient response. The proposal is based on determining the permissible limits of the period change based on analysis and simulation.

The result of this research provides recommendations for the design and implementation of control systems that can maintain high-quality regulation even with unpredictable changes in sampling periodicity. The present article also opens space for further research in the field of control of systems with variable periodicity, especially regarding their practical implementation in real embedded devices.


**Mathematics Subject Classification 2000**: 93C62
**General Terms**: System stability, Simulation
**Additional Key Words and Phrases**: embedded systems, discrete event dynamic system, technological processes, processing time.

## 1. INTRODUCTION

When implementing numerical control for various types of embedded systems through a single central processor, complications arise. The complications are related to the synchronization and scheduling of data processing. These complications are especially evident when the system has to respond to events with diverse processing time requirements. These events occur at random or unpredictable moments [Ogata, 2014]. As a result, there is variability in the sampling period, which is a critical parameter crucial for the stability and performance of the control system.

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In real time, this change in the sampling period can fundamentally affect the dynamic behavior of the system [Chaves et al., 2009; Yassa et al., 2019; Luo et al., 2020]. The stability, quality of regulation, and overall performance of the control algorithm can be compromised [Chen, Ricles 2009; Hetel et al., 2017]. This can lead to deterioration in the reliability and safety of the entire controlled process. Random changes during the sampling period can therefore be understood as disturbances in the system, which must be adequately analyzed and resolved.

However, the analytical evaluation of the impact of such disturbances is complex because it depends on many factors, such as the nonlinear characteristics of the controlled processes, the complexity of the control algorithms, and the diversity of types of embedded systems [Yao et al., 2008]. Therefore, other approaches can be used to assess the impact of random variability in the sampling period on the system behaviour.

One of the effective methods for solving this problem is the modelling and simulation method, which allows to experimentally investigate the dynamics of the system under various scenarios of changing the sampling period. Through modelling, it is possible to determine the limits of the permissible variability of the sampling period at which the system maintains stability and the required quality of regulation. In this way, it is possible to identify optimal strategies for designing robust control algorithms that will be resistant to random timing deviations in timing [Weinmann, 2012].

The main emphasis will be placed on the use of the modelling method as a tool for understanding system dynamics and determining appropriate measures to ensure the reliable operation of control systems in real time [Kulkarni et al., 2014].

Random changes during the sampling period are unlikeof interest in many studies. Such disturbances are specific because they can significantly affect the stability and performance of control systems. For example, in the literature [Lis et al., 2022], the focus was on determining the control parameters and the influence of the noise signal on the modelled system. The control was focused on an induction furnace, the characteristics of which were the basis for the proposed model, while [Zhang et al., 2015] focused on a variable sampling period. The implementation is carried out using a scheduler that refers to different sampling periods. In the article [Tasoujian et al.,2019], stability and performance were investigated from the perspective of a closed loop with variable parameters. The influence of measurement noise and disturbances is also considered in the tracking. However, in their study work, the small gain theory is used. Based on their theory, robust stability conditions are investigated. In the paper [M. Aboud, 2023] analyzed H-infinity algorithms and robust control techniques are analyzed. However, they are less suitable for embedded systems with low computational resources. Compared to the papers [Raji et al., 2022], which used dynamic task scheduling to compensate for time deviations and the papers [G. Wang et al.,2024] which implement machine learning for adaptive control, this paper is oriented towards simplicity and practical feasibility. Simulations allow for efficient system modelling and identification of the effect impact of random variations on the system behaviour. Unlike previous studies, this paper investigates the issue of random changes in the sampling period using simulation methods. The goal is to determine the limits of random changes at which the system can maintain stability and the desired quality of control, while focusing on applications with limited resources where advanced algorithms may be inappropriate.

## 2. THE INFLUENCE OF CHANGES IN THE SAMPLING PERIOD ON REGULATION

When changing the sampling period, it is essential to ensure that the modified model preserves the key properties of the original system, such as stability and desired dynamic characteristics.

Changing the sampling period requires changing the mathematical description of the numerical control system [Gabriska, Gese, 2013]. Let us assume that the control system is described by a transfer function of the form:

$$a\,G(s) = \frac{Y(s)}{U(s)} = \frac{\prod_{\beta=1}^{m}\left(1 + T_\beta s\right)}{(1 + 2DT_s + t^2t^2)\prod_{\alpha=1}^{m-2}(1 + T_\alpha s)} \cdot e^{-T_t s}$$
$$= \frac{b_0 + b_1 s + \cdots + b_m s^m}{1 + a_1 s + \cdots + a_m s^m} \cdot e^{-T_t s} \qquad (1)$$

Equations (1) and (2) represent simplified models of control objects in separate discrete systems. Equation (1) assumes that the dynamic properties of the object remain constant. These properties do not change with the presence or absence of feedback [Isermann, 2013]. This approximation is commonly used in the analysis of linear systems. The feedback effect, which primarily influences the stability of the system, is considered, while the dynamics of the object itself remains unchanged.

The dynamic properties of the control system can remain practically constant if the balance condition of the total sum of the time constants is satisfied [Isermann, 2013]:

$$T_\Sigma = 2DT + \sum_{\propto=1}^{m-2} T_\alpha - \sum_{\beta=1}^{m-2} T_\beta + T_t = (a_i - b_i) + T_t \qquad (2)$$

From this result, we conclude that a simplified model of the system should be created so that the total amount of energy of the transient process does not change or changes only slightly. This approach is important for maintaining model accuracy, especially in the context of dynamic systems [Elhassan et al., 2021], while the energy properties are fundamental for understanding and predicting the behaviour of the system. Therefore, the rule can be formalized in the form of mathematical relations that ensure the consistency of the energy characteristics. It is logical to assume that this rule should be applied to a discrete model of the object as follows:

$$T_t = \sum_{i=1}^{t} T_i \qquad (3)$$

## 2. DETERMINING THE REQUIRED SAMPLING PERIOD OF THE SYSTEM

Typically, for dynamic systems, the sampling time constant $\boldsymbol{T_0}$ is determined and includes only the time $\boldsymbol{T_R}$ spent on calculating the required control signals and ensuring their stability. In contrast, for combined control systems, the sampling period is variable because of the inclusion of the time $\boldsymbol{Tp}$ required to process random events in the system. To determine the time $\boldsymbol{Tp}$, we statistically carry out a statistical assessment of events and the most probable time $\boldsymbol{Tp}$, using the normal probability distribution law. With the normal probability distribution, a uniform distribution of events is assumed relative to the calculated time according to the equation $\boldsymbol{T_a = T_c / N}$, where $\boldsymbol{T_c}$ is the time required to process N interrupts, where N is the number of interrupts for processing events occurring in each cycle.

Then, the standard deviation of the sampling period from the average $\boldsymbol{Ta}$ is determined by the relation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n-1}}\ , \tag{4}$$

Where μ=Ta is the average arithmetic time required to process new events.

Using the root mean square deviation of the sampling period from the specified one, we can establish criteria to process incoming random requests within embedded system events. In this scenario, the sampling period corresponds to the average value μ of the processing time. The probability of an event occurring within the time interval $\boldsymbol{T_P}$ is 99.7%, where $\boldsymbol{T_P = T_a \pm 3\sigma}$ . This implies that the time $\boldsymbol{T_a}$ represents the duration required to process all events within the embedded system. Possible options for the ratio of processing periods:

1. $\boldsymbol{T_a} < \boldsymbol{T_0}$ – the interrupt processing time is less than the sampling period T0, and these options are possible:
   a. $\boldsymbol{T_R} >> \boldsymbol{T_a}$– – the sampling period of real-time clock systems $\boldsymbol{T_R}$ is much greater than the time required to process events from

embedded systems $\boldsymbol{T_a}$. In this case, the influence of time $\boldsymbol{T_a}$ on the whole can be neglected,

b. $\boldsymbol{T_R} \approx \boldsymbol{T_a}$ is the sampling period of real-time systems $\boldsymbol{T_R}$ approximately equal to the service period of all events $\boldsymbol{T_a}$.

c. $\boldsymbol{T_R} \ll \boldsymbol{T_a}$ is the service period of all events $\boldsymbol{T_a}$ much greater than the sampling period of real-time systems $\boldsymbol{T_R}$.

2. $\boldsymbol{T_R} > \boldsymbol{T_0}$ is the service period of real-time systems $\boldsymbol{T_R}$ greater than the sampling period of the entire system, then the minimum number of processing steps ***Kmin*** required for servicing is determined by the ratio ***Kmin*** $\geq \boldsymbol{T_a/T_p}$. With this information processing method, the time required to implement all interrupts for servicing events is divided into steps ***Kmin***.

## 3. THE IMPACT OF THE RANDOM VARIATION IN THE SAMPLING PERIOD ON THE SYSTEM'S TRANSIENT RESPONSE

The objective was to assess the influence of the random component of the sampling period using the numerical built-in real-time control subsystem with a sampling period T0 = 0.16 sec. Setting the regulators [Weinmann, 2012] to the "symmetric optimum" ensures stable operation of the subsystem with a cutoff frequency ω_c=1 rad/sec.

The transfer function of the subsystem

$$D_2(z) = \frac{0.004168z^4 + 0.03156z^3 - 0.002851z^2 - 0.02262z - 0.001879}{z^5 - 3.384z^4 + 4.77z^3 - 3.675z^2 + 1.58z - 0.2894} \quad (5)$$

The proposed model in the Simulink Matlab environment allows us to analyze the influence of the random component of the sampling period on the stability and performance of the control system. The model includes several key blocks: distinct Discrete Transfer Function, Variable Fractional Delay, and Band-Limited White Noise, which simulate the subsystem dynamics of the subsystem, variable time delay, and random noise interference. The distinct Discrete Transfer Function block implements the discrete transfer function of the subsystem, which is based on the transfer function equation (5). This block models the dynamics of the subsystem at distinct times with a set sampling period.

The Variable Fractional Delay block is introduced into the schedule, where the value is variable. With a fixed setting, the delay is a single sample, but the variability of the delay affects the input signal from the Band-Limited White Noise block. This mechanism simulates various changes in the sampling period, which can be factors such as variable interrupt processing in a real system. The Band-Limited White Noise block generates a random white-noise disturbance signal in a limited frequency range. This signal represents the variable sampling periods and acts as an input to the Variable Fractional Delay block. The signal then causes a simulation of a variable time delay.

The model is designed to simulate the dynamics of a control system with a controlled delay and random disturbance. The input signal passes through a distinct Transfer Function block, which models the subsystem dynamic of the subsystem. It is then affected by a random delay in a variable fractional delay. The output signal can be analyzed for stability and control quality. This provides important insights for the design of robust control algorithms capable of handling potential real-time impacts. The model allows for a systematic study of the behavior of control systems under random conditions of the sampling period and provides tools for evaluating the stability and performance of control under various conditions. The block diagram of the model is shown in Figure 1.

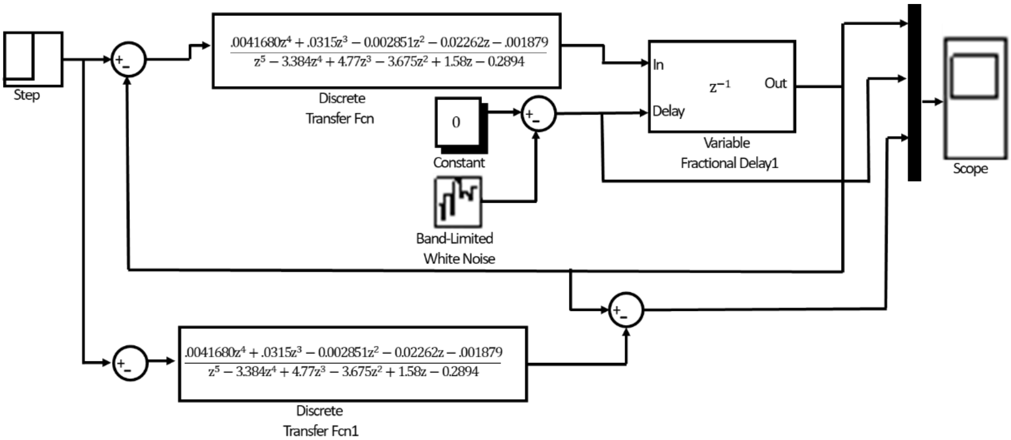


Figure 1. Block diagram of the real-time subsystem model

Figures 2a and 2c show the transient characteristics of the system without changing the sampling period during the transient process (Figure 2a) and with a change (Figure 2c) of the white noise type with a power of 0.1 and with a frequency band limitation with an average value of changes equal to zero. As follows from the transient processes, this type of disturbance has virtually no significant effect on the quality of regulation. The high frequency of changes in the sampling period relative to the characteristics of the object is well filtered by the control object under a centered law of changes (the average value of the period changes. $\Delta\bar{T}_0 = 0$).

Another group of experiments was carried out with a constant increase in the sampling period relative to the calculated period. Figure 2c shows the transient characteristic of the system with a twofold increase in the period. The system is on the stability boundary. In Figure 2d, the sampling period changed during the transient process with a frequency of f=0.2 Hz and with an amplitude of $\pm T_0$ and an average value of $T_0$. The system has a similar transient process with a constant increase in the sampling period, as shown in Figure 2c. It follows from these experiments that when choosing a common sampling period for a group of subsystems with time and demand control, a safety margin should be provided by increasing the ratio between the system's operating bandwidth and the sampling frequency.

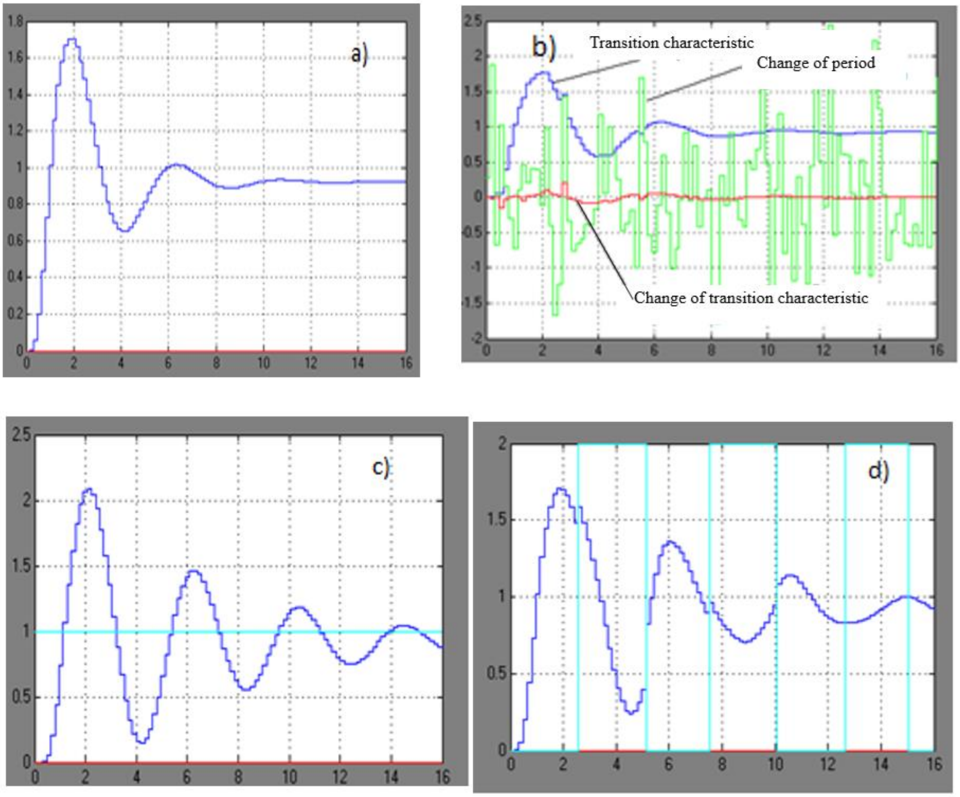


Figure 2. Transient response of the system

a) With the calculated sampling period T_0
b) With period changes of the white noise type
c) With increasing the sampling period to T_d=2T_0
d) Periodic change with a frequency of 0.5 Hz with an average value T_d=2T_0

## 4. DISCUSSION

This paper presents a glimpse into the field of digital control of real-time systems. This section deals with the issue of random changes in the sampling period and their impact on the stability and quality of regulation. Modeling and simulation were conducted to investigate dynamics with random events. This approach allows us to accurately track the behaviour of a system and helps to quantify its resistance to time deviations. Many studies have focused on deterministic disturbances or similar tightly defined interruption intervals, while not always considering the inherent randomness that is common in real systems. In the literature [Aboud et al., 2023, Bansal et al., 2013], theoretical approaches and analytical models have been used that are applied to robust control methods. These methods often rely on analytical models with precise parameters. This paper constrains random disturbances and their modelling through simulations, which offers a practical view of the problem. The paper [Tasoujian et al., 2019, Dezfuli et al., 2016] addresses issues related to the dynamic properties of the system itself, with the main topic being stability and performance from a closed-loop perspective. The next paper explores the impact of irregular sampling on real-time systems. As shown in [Silva, Silva, 2024, Prempreechakun et al., 2019, Pillai et al., 2018], it is important to ensure the computational complexity of algorithms for sampling period analysis. However, including the average interrupt processing time can improve the stability. While his approach may not cover the most important cases, it offers a simple and feasible solution to the upfront cost. Studies focused on dynamic task scheduling [Raji et al., 2022] have emphasised the need for an adaptive approach to task management in environments with variable time requirements. In total, the paper emphasizes minimizing the computational burden, which is important for embedded systems with limited resources. The study also emphasizes dealing with problems caused by random changes in the sampling period.

From a practical perspective, the use in the field of automation includes, for example, the control of robotic arms or production lines. In these cases, random timing variability can seriously disrupt the coordination and accuracy of movements. This is particularly problematic for processes running in cyclical intervals that are subject to interruption. This issue is also relevant to IoT devices, such as smart thermostats or security cameras. These devices often operate in environments with limited resources and must respond quickly to events in real-time. With high dynamics and limited computing capacity, time deviations in data processing can occur. Deviations subsequently threaten the stability and performance of the system. This paper offers practical solutions to maintain the stability of control algorithms in cases of unpredictable changes during the time of task processing.

## 5. CONCLUSION

Based on the above considerations, we conclude that the correct determination of the sampling period is a key factor in the design of digital controllers for real-time control systems. The selection of an appropriate sampling frequency ensures not only the stability of the system but also the required quality of control, which is essential for the efficient and accurate operation of the entire system. In real-time implementation, other factors must be considered, such as the time required to process events through the interrupt system. Experimental results have shown that adjusting the calculation period by the average value of the interrupt processing time can significantly contribute to reliability.

This paper presents a systematic view of the issue of determining the sampling period for digital control systems in real time, also considering practical aspects, such as the time required to process events through the interrupt system. The issue demonstrates how, based on a theoretical framework, it can provide a comprehensive overview of the influence of the sampling period on the stability and quality of digital control system regulation. This view allows a better understanding of the dynamics of control systems. The proposed model provides a recommendation in terms of practical use. When increasing the period by the average processing time value, the value of the processing time can ensure greater system reliability. Simulation shows that such an approach effectively solves the problem of the influence of the random value of interrupt processing.

Although the paper offers a solution for a single request, some questions remain. One of these is how to ensure stability in systems with high variability or unexpected events, where the interrupted processing time may not be sufficient. It would also be interesting to see how this approach can be combined with other techniques, such as dynamic task scheduling or prediction algorithms.

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Darja Gabriska
Institute of Computer Technologies and Informatics,
Faculty of Natural Sciences,
University of SS. Cyril and Methodius, 917 01 Trnava, Slovak Republic
Email: darja.gabriska@ucm.sk