Control chart for generalized exponential distribution based on modified chain sampling

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Abstract

In recent years, modern statistical process control has established higher concentration, and this approach is reinforced by new control charts to efficiently monitor the process. The goal of this article is to create a control chart based on a modified chain sampling plan (MchSP) when product quality dimensions are followed by a generalized exponential distribution. There was a brief overview of the generalized exponential distribution (GED) and the modified chain sampling strategy. The chart parameters were presented for several sample sizes at various average run length (ARL) values. A variety of tables were provided for the actual application of the produced control chart in industrial applications at various shift values of scale and form parameters. The real data set was used to demonstrate the application of the created MchSP control chart process. In addition, the proposed MchSP control chart process was developed using simulated data. The average run length (ARL) results of the established control chart for shifting both scale and shape parameters were computed for various sample sizes. The results show that when the shift constant (both scale and form) increases, the ARL values decrease. Also, it was discovered that ARL1 values tend to decrease as the shape parameter value increases.

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Key Words and Phrases: Generalized exponential distribution; modified chain sampling plan; Control chart; average run length; monitor the process.

1. INTRODUCTION

Control charts are essential tools for monitoring the manufacturing process and maintaining predetermined quality levels. Statistical process control (SPC) is a well-known way for measuring, surveying, and controlling processes using statistical analysis for the benefit of achieving manufacturing process steadiness and lowering process variability for enrichment competency (Montgomery, 2013). Quality control is a crucial component of the long-term production process of finished products. Certainly, process quality is the most significant thing that stimulates both the producer and the consumer's concentration. Control charts are a popular and widely used tool for examining the process average or variance in SPC. Shewhart-type control charts are most commonly used in implementations of different control charts due to their simplicity, which makes them easy to grasp, put into practice, and

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use. Regrettably, even though Shewhart control charts are resourceful for discovering outsized process changes, they are ineffective for discovering small shifts in a process. Broadly, to monitor the process control, it can be attributed as a variable and attribute control chart. Variable control charts are used in a process when the quality characteristic is quantifiable; on the other hand, attribute control charts are developed for non-measurable data that could be classified as conforming or non-conforming. The rationale behind the design of both charts is to quickly detect shifts in the process. A shift in the process is detected by the control charts, which helps the process engineer quickly rectify the abnormal process situation. In general, the control chart is prepared with the upper and lower control limits. The manufacturing process is said to be declared in-control (IC) when chart statistic values lie between the upper control lisinglemit (UCL) and lower control limit (LCL); if not, the process is called out-of-control (OOC).

Several control charts have been formulated to monitor processes based on normal and non-normal distributions. Numerous researchers have concentrated on time-truncated sampling control charts in recent years. The reason behind studying time-truncated sampling control charts based on a non-normal distribution is that the assumption of normality may mislead the process engineers if the process variable does not follow a normal distribution. In recent years, more authors in SPC have developed time-truncated sampling control charts based on single sampling, double sampling, recitative sampling, and multiple dependent sampling plans, for example: (Al-Oraini and Rahim, 2002, Ahmad et al., 2014, Ahmad et al., 2016, Aslam et al., 2017a, Aslam et al., 2015a, Aslam et al., 2015b, Aslam et al., 2015c, Aslam et al., 2020, Aslam et al., 2017b, Aslam et al., 2017a, De Araújo Rodrigues et al., 2011, Adeoti and Ogundipe, 2021, Adeoti and Rao, 2022, Ahmad et al., 2013, Ahmad et al., 2015, Aldosari et al., 2019, McCracken and Chakraborti, 2013, Wu and Wang, 2007, Rao, 2018, Rao et al., 2019a, Rao et al., 2019b, Rao et al., 2020, Rao et al., 2021, Rao and Al-Omari, 2022, Ha Lee and Khoo, 2021, Hossain et al., 2022, Khan et al., 2018).

The aforementioned control charts based on sampling plans for non-normal distributions are motivated to develop MchSP using time truncated life testing. The

chain sampling plans first initiated by (Dodge, 1977) later on, some authors studied various chain sampling plans for different situations for more details see (Govindaraju and Balamurali, 1998, Luca, 2018, Tripathi et al., 2021, Tripathi et al., 2022, Ramaswamy and Jayasri, 2015). By exploring the literature, we found that, to the best of our knowledge, no work has attempted control charts for GED based on the MchSP under time-truncated life tests. The aim of this work is to develop control charts for GED based on the MchSP for the time-truncated scheme. GED is the most celebrated probability distribution, and it has several advantages as compared with other lifetime distributions like exponential, gamma, and Weibull distributions.
The remaining article is prepared as: A concise explanation of the GED is given in Section 2. The proposed MchSP chart for GED and shift monitoring is discussed in Section 3. In Section 4, the demonstration of proposed control chart tables is provided. A real example illustration is provided in Section 5. The performance of the proposed control chart is established in Section 6, and in Section 7, concluding remarks and prospects of the proposed chart are discussed.

2. THE GENERALIZED EXPONENTIAL DISTRIBUTION

The aim of this article is to develop a modified chain sampling plan based on truncated life tests when the lifetime of a product follows the generalized exponential distribution (GED), which has been introduced and studied comprehensively by the authors (Gupta and Kundu, 2001, Gupta and Kundu, 2002, Gupta and Kundu, 2003, Gupta and Kundu, 2004, Gupta and Kundu, 2007, Kundu and Gupta, 2005, Kundu and Gupta, 2008, Rao, 2012). Let X is a lifetime that is distributed according to GED with two parameters $\alpha > 0$ and $\lambda > 0$. The probability density function (PDF) and cumulative distribution function (CDF) of the two-parameter GED respectively, are given by

$$f(x;\alpha,\lambda) = \left(\frac{\alpha}{\lambda}\right) e^{\frac{x}{\lambda}}(1 - e^{\frac{x}{\lambda}})^{\alpha-1} \text{ for } x > 0, \tag{1}$$

$$F(x;\alpha,\lambda) = (1 - e^{\frac{x}{\lambda}})^{\alpha}; \text{ for } x > 0 \tag{2}$$

Where α and λ are shape and scale parameters respectively. The average (median) of this distribution is given by

$\mu = -\lambda \, ln(1 - 0.5^{\frac{1}{\alpha}}) = -\lambda\tau$; where $\tau = ln(1 - 0.5^{1/\alpha})$. Single acceptance sampling plans based on the GED are developed by (Aslam et al., 2010).

3. THE CONTROL CHART BASED ON MODIFIED CHAIN SAMPLING PLAN

The following step-by-step procedure is adopted for the attribute control chart for GED based on the MchSP using time-truncated life testing:

STAGE 1: Draw at random a sample of size n in each subgroup from the process. Count the number of defective items (denoted by δ) which are failed before reaching a predefined time $t_0 = a\mu_0$.

STAGE 2: The process is out-of-control if $\delta < LCL \; or \; \delta > UCL$.

STAGE 3: we declare the process as in-control if $LCL \leq \delta \leq UCL$ provided that there is at most one statistic δ among the k preceding δ statistics falls inside LCL and UCL. Otherwise, process is out-of-control.

It should be remembered that the binomial distribution (BD) with parameters n and p_0 for the IC phase is accompanied by the number of failure items δ. Where p_0 is the probability that an item fails before time t_0. The control limits for attribute control chart based proposed chart will be

$$LCL = \max\left[0, np_0 - \mathcal{L}\sqrt{np_0(1 - np_0)}\right]. \tag{3}$$

$$UCL = np_0 + \mathcal{L}\sqrt{np_0(1 - np_0)}. \tag{4}$$

Where $\mathcal{L}$ is the control chart coefficient. Customarily the value of p_0 is not known, so to tackle that case we draw a preliminary sample from in-control process to assess the value of p_0. However, we can say the process is in control when $\mu = \mu_0$ (or the distribution parameters $\alpha = \alpha_0$ and $\lambda = \lambda_0$). Therefore, from Equation (2), the probability p_0 is obtained as follows:

$$p_0 = (1 - exp(a\tau_0))^{\alpha_0} \quad \text{where } \tau_0 = ln(1 - 0.5^{1/\alpha_0}) \tag{5}$$

Consequently the control limits which are designed for practical objective are

$$LCL = max\left[0, \bar{\delta} - \mathcal{L}\sqrt{\bar{\delta}\left(1 - \bar{\delta}/n\right)}\right]. \tag{6}$$

$$UCL = \bar{\delta} + \mathcal{L}\sqrt{\bar{\delta}\left(1 - \bar{\delta}/n\right)}. \tag{7}$$

Where $\bar{\delta} = \frac{\sum \delta}{n}$ denotes the mean failure time of items before time t_0 in a subgroup.

The probability that the process is declared as in-control when it is working smoothly is given below:

$$P_{in}^{0} = P_{a0}\{P_{a0}^{k} + kP_{a0}^{k-1}(1 - P_{a0})\}. \tag{8}$$

Where $P_{a0} = P(LCL \leq \delta \leq UCL|p_0)$.

$$= \sum_{j=LCL+1}^{UCL} \binom{n}{j} (p_0)^j (1 - p_0)^{n-j} \tag{9}$$

The competence of the proposed chart is assessed using ARL. ARL for in-control process is given below:

$$ARL_0 = \frac{1}{1-P_{in}^{0}}. \tag{10}$$

3.1 Control chart when Scale Parameter shifted

If the scale parameter is shifted from $\lambda_1 = c\lambda_0$, where c is shift constant then the process is said to be out-of-control. Here p_1 is the chance of an item that fails before the experiment time t_0 and it is given by

$$p_1 = (1 - exp(\,ac\tau_0))^{\alpha_0} \qquad \text{where } \tau_0 = ln(\,1 - 0.5^{1/\alpha_0}) \tag{11}$$

The chance of the process is in-control when process shifted as given by

$$P_{in}^{1} = P_{a1}\{P_{a1}^{k} + kP_{a1}^{k-1}(1 - P_{a1})\}. \tag{12}$$

Where $P_{a1} = P(LCL \leq \delta \leq UCL|p_1)$.

$$= \sum_{j=LCL+1}^{UCL} \binom{n}{j} (p_1)^j (1 - p_1)^{n-j} \tag{13}$$

In this case, the ARL for the shifted process is denoted as ARL_1 and it is given by

$$ARL_1 = \frac{1}{1-P_{in}^{1}}. \tag{14}$$

3.2 Control chart when Shape Parameter shifted

The proposed MchSP chart is examined in this subsection when the shape parameter is shifted due to some external factors. Consider that the shape parameter is shifted from α_0 to $\alpha_1 = \varepsilon\alpha_0$ where ε is shift constant. Hence, the chance of an item that fails before the experiment time t_0, denoted by p_2 and as given below:

$$p_2 = (1 - exp(\,a\tau_1))^{\alpha_1} \qquad \text{where} \quad \tau_1 = ln(\,1 - 0.5^{1/\alpha_1}) \tag{15}$$

The chance of the process is in-control when process shifted as given by

$$P_{in}^2 = P_{a2}\{P_{a2}^k + kP_{a2}^{k-1}(1 - P_{a2})\}. \tag{16}$$

Where $P_{a2} = P(LCL \leq \delta \leq UCL|p_2)$.

$$= \sum_{j=LCL+1}^{UCL} \binom{n}{j} (p_2)^j (1 - p_2)^{n-j} \tag{17}$$

In this case, the ARL for the shifted process is denoted as ARL_1 and it is given by

$$ARL_1 = \frac{1}{1 - P_{in}^2}. \tag{18}$$

The following step by step procedure is used to get the results of the proposed MchSP chart.

1) Prefix the ARL value say r_0, and shape parameters α_0, respectively.
2) Determine the chart parameters $\mathcal{L}$, a and n (sample size) for which the ARL_0 is almost close to r_0, i.e., $ARL_0 \geq r_0$.
3) Once obtained the chart parameters using Step 2, the out-of-control ARL say ARL1 are obtained based on shift of scale parameter c or shift of shape parameter constant ε.

The computed proposed chart parameters along with ARL_1 values for assorted values of α_0, r_0 and n are presented in Tables 1-6 for shifting of scale parameter and presented shifting of shape parameter in Tables 7-9.

The observations based on the tables are given below:

1. The ARL_1 shows a rapid decrease when the shift constants (either (c), or (ε)) diminish.
2. From Tables 1-6, it is noticed that ARL_1 values are decreasing tendency when the shape parameter α value increases.
3. Furthermore, from Tables 1-6, it is observed that when the constant multiplier (a) increases, the values of ARL_1 decreases.

4. In addition, it is comprehensible from Tables 7-9 that if the shape parameter α increments, the ARL_1 values are also increments.

Table 1: The ARL values for developed chart for shifts in scale parameter when $r_o = 300$ and $n = 15$.

α	1	1.5	2	2.5	3	3.5	4
$\mathcal{L}$	2.997	2.997	2.997	2.999	2.999	3.002	2.997
a	0.7814	0.8241	0.8473	0.8619	0.8725	0.880	0.887
c	k =1	k =1	k =1	k =2	k =2	k =3	k =1
1.00	300.18	300.03	300.04	300.25	300.23	300.32	300.19
0.95	214.57	194.31	179.85	168.50	159.55	151.38	146.34
0.90	148.01	120.68	103.19	90.31	81.10	72.61	68.90
0.85	99.82	73.60	58.55	48.05	41.22	34.70	33.15
0.80	66.31	44.59	33.33	25.73	21.25	16.71	16.60
0.75	43.59	27.00	19.18	13.95	11.19	8.17	8.77
0.70	28.43	16.43	11.24	7.70	6.07	4.16	4.94
0.65	18.44	10.10	6.76	4.39	3.45	2.31	3.00
0.60	11.94	6.32	4.22	2.64	2.12	1.48	2.00
0.55	7.74	4.06	2.76	1.73	1.46	1.14	1.47
0.50	5.07	2.71	1.92	1.28	1.15	1.02	1.19
0.40	2.31	1.44	1.18	1.01	1.00	1.00	1.01
0.30	1.29	1.05	1.01	1.00	1.00	1.00	1.00
0.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 2: The ARL values for developed chart for shifts in scale parameter when $r_o = 300$ and $n = 25$.

α	1	1.5	2	2.5	3	3.5	4
$\mathcal{L}$	3.111	3.111	3.027	3.029	3.042	3.111	3.109
a	0.4749	0.5627	0.731	0.756	0.7254	0.6925	0.7079
c	k =2	k =2	k =1	k =2	k =2	k =2	k =1
1.00	300.17	300.50	300.04	300.31	300.52	300.55	300.16
0.95	203.47	178.97	157.37	144.28	129.60	128.75	121.92
0.90	133.09	102.27	79.05	66.40	56.74	53.66	48.88
0.85	85.16	57.53	39.86	30.88	25.81	23.01	20.66
0.80	53.69	32.20	20.54	14.73	12.38	10.31	9.43
0.75	33.47	18.03	10.94	7.29	6.35	4.91	4.73
0.70	20.67	10.15	6.10	3.81	3.55	2.57	2.67
0.65	12.67	5.78	3.61	2.18	2.19	1.57	1.72
0.60	7.73	3.39	2.31	1.44	1.52	1.16	1.28
0.55	4.73	2.11	1.62	1.12	1.20	1.03	1.09
0.50	2.95	1.45	1.26	1.02	1.06	1.00	1.02
0.40	1.37	1.03	1.02	1.00	1.00	1.00	1.00
0.30	1.02	1.00	1.00	1.00	1.00	1.00	1.00
0.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 3: The ARL values for developed chart for shifts in scale parameter when $r_o = 300$ and $n = 35$.

α	1	1.5	2	2.5	3	3.5	4
$\mathcal{L}$	3.052	3.038	3.032	3.052	3.032	3.039	3.042
a	0.8817	0.9079	0.9213	0.9265	0.9347	0.9381	0.941
c	k =3	k =2	k =2	k =3	k =1	k =2	k =2
1.00	300.05	300.06	300.01	300.06	300.02	300.11	300.68
0.95	232.14	203.07	182.57	172.76	156.11	147.67	140.61
0.90	140.70	102.83	81.57	68.50	58.42	51.14	45.98
0.85	76.77	48.65	35.43	26.21	22.51	18.35	15.88
0.80	40.39	23.12	16.10	10.31	9.55	7.21	6.11
0.75	20.95	11.25	7.83	4.32	4.56	3.21	2.74
0.70	10.79	5.69	4.16	2.10	2.51	1.73	1.53
0.65	5.60	3.06	2.45	1.30	1.62	1.19	1.12
0.60	3.03	1.83	1.64	1.05	1.22	1.03	1.02
0.55	1.80	1.28	1.25	1.00	1.06	1.00	1.00
0.50	1.26	1.07	1.07	1.00	1.01	1.00	1.00
0.40	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.30	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 4: The ARL values for developed chart for shifts in scale parameter when $r_o = 370$ and $n = 15$.

α	1	1.5	2	2.5	3	3.5	4
$\mathcal{L}$	3.0980	3.0970	3.0980	3.0970	3.0980	3.0990	3.0980
a	0.7538	0.8016	0.8273	0.8443	0.8560	0.8649	0.8720
c	k =2	k =1	k =2	k =1	k =2	k =2	k =2
1.00	370.07	370.22	370.22	370.29	370.09	370.44	370.31
0.95	271.34	246.80	228.48	214.77	203.02	193.83	185.75
0.90	188.67	154.03	130.77	115.35	102.69	93.47	85.95
0.85	126.84	93.31	72.95	61.04	51.39	44.95	39.96
0.80	83.42	55.86	40.45	32.59	26.02	22.04	19.08
0.75	53.98	33.34	22.48	17.76	13.43	11.11	9.45
0.70	34.47	19.96	12.57	9.96	7.13	5.82	4.91
0.65	21.75	12.05	7.12	5.82	3.95	3.23	2.75
0.60	13.58	7.39	4.14	3.57	2.35	1.97	1.73
0.55	8.40	4.64	2.52	2.34	1.56	1.37	1.26
0.50	5.18	3.02	1.67	1.67	1.19	1.11	1.07
0.40	2.08	1.52	1.06	1.11	1.01	1.00	1.00
0.30	1.13	1.07	1.00	1.00	1.00	1.00	1.00
0.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 5: The ARL values for developed chart for shifts in scale parameter when $r_o = 370$ and $n = 25$.

α	1	1.5	2	2.5	3	3.5	4
$\mathcal{L}$	3.1330	3.1300	3.1290	3.1300	3.1300	3.1290	3.1290
a	0.6052	0.6770	0.7174	0.7436	0.7625	0.7770	0.7883
c	k =3	k =2	k =1	k =2	k =2	k =1	k =2
1.00	370.37	370.37	370.50	370.52	370.57	370.66	370.15
0.95	251.22	221.48	200.40	183.74	170.83	160.76	151.20
0.90	159.49	122.56	100.08	83.83	72.62	64.97	57.52
0.85	97.64	66.15	49.61	38.28	31.32	27.27	22.76
0.80	58.45	35.54	25.04	17.89	14.01	12.25	9.58
0.75	34.38	19.15	13.04	8.65	6.59	6.00	4.38
0.70	19.89	10.42	7.09	4.40	3.34	3.26	2.27
0.65	11.34	5.76	4.09	2.44	1.91	2.01	1.42
0.60	6.41	3.31	2.54	1.54	1.30	1.42	1.10
0.55	3.66	2.03	1.73	1.16	1.07	1.14	1.01
0.50	2.20	1.40	1.31	1.03	1.01	1.04	1.00
0.40	1.13	1.02	1.02	1.00	1.00	1.00	1.00
0.30	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 6: The ARL values for developed chart for shifts in scale parameter when $r_o = 370$ and $n = 35$.

α	1	1.5	2	2.5	3	3.5	4
$\mathcal{L}$	3.0320	3.0560	3.0570	3.0600	3.0570	3.0560	3.0480
a	0.7209	0.7785	0.8073	0.8264	0.8391	0.849	0.8559
c	k =4	k =2	k =2	k =1	k =2	k =2	k =3
1.00	370.01	370.11	370.04	370.14	370.04	370.19	370.06
0.95	295.69	253.26	228.74	208.55	194.13	181.66	172.98
0.90	184.39	130.06	102.28	84.23	71.40	62.13	54.41
0.85	101.69	61.86	43.58	33.71	26.39	21.85	17.42
0.80	53.20	29.36	19.02	14.39	10.43	8.38	6.02
0.75	27.00	14.19	8.66	6.72	4.52	3.61	2.45
0.70	13.39	7.07	4.20	3.50	2.25	1.86	1.35
0.65	6.59	3.71	2.26	2.08	1.39	1.23	1.05
0.60	3.35	2.12	1.43	1.43	1.08	1.04	1.00
0.55	1.89	1.40	1.11	1.15	1.01	1.00	1.00
0.50	1.27	1.11	1.02	1.03	1.00	1.00	1.00
0.40	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.30	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 7: The ARL values for developed chart for shifts in shape parameter when $r_o = 370$ and $n = 20$.

α	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$\mathcal{L}$	3.0640	3.0650	3.0670	3.0650	3.0650	3.0650	3.0640
a	0.7678	0.8126	0.8368	0.8531	0.8643	0.8728	0.8796
$\mathcal{E}$	k =1	k =2	k =3	k =2	k =2	k =2	k =1
1.00	370.20	370.27	370.15	370.19	370.40	370.27	370.53
0.95	349.57	351.78	353.00	354.11	355.08	355.57	356.35
0.90	328.73	332.98	335.47	337.65	339.36	340.45	341.77
0.85	307.85	313.97	317.65	320.86	323.28	324.97	326.80
0.80	287.09	294.87	299.62	303.81	306.91	309.16	311.49
0.75	266.58	275.78	281.47	286.57	290.28	293.06	295.86
0.70	246.46	256.81	263.28	269.20	273.46	276.72	279.96
0.65	226.84	238.05	245.14	251.77	256.51	260.18	263.82
0.60	207.82	219.59	227.12	234.34	239.47	243.49	247.49
0.55	189.48	201.52	209.30	216.98	222.40	226.71	230.99
0.50	171.90	183.90	191.73	199.73	205.34	209.86	214.37
0.45	155.14	166.80	174.50	182.67	188.36	192.99	197.66
0.40	139.23	150.29	157.66	165.84	171.50	176.15	180.90
0.35	124.22	134.40	141.25	149.29	154.79	159.36	164.11
0.30	110.15	119.20	125.33	133.06	138.28	142.66	147.31

0.25	97.04	104.72	109.95	117.19	121.99	126.08	130.52
0.20	84.96	91.03	95.16	101.72	105.95	109.61	113.74
0.15	73.97	78.22	81.03	86.71	90.20	93.28	96.95
0.10	64.17	66.47	67.73	72.26	74.79	77.09	80.13

Table 8: The ARL values for developed chart for shifts in shape parameter when $r_o = 370$ and $n = 30$.

α	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$\mathcal{L}$	3.0570	3.0570	3.0600	3.0570	3.0560	3.0570	3.0570
a	0.7224	0.7753	0.8041	0.8233	0.8368	0.8468	0.8548
$\mathcal{E}$	k =2	k =2	k =3	k =2	k =1	k =2	k =2
1.00	370.11	370.33	370.14	370.22	370.55	370.32	370.17
0.95	340.28	343.61	345.35	346.94	348.39	349.00	349.59
0.90	310.39	316.60	320.14	323.21	325.75	327.18	328.48
0.85	281.02	289.74	294.87	299.31	302.87	305.05	307.02
0.80	252.62	263.38	269.82	275.50	279.97	282.82	285.40
0.75	225.55	237.83	245.29	252.01	257.26	260.67	263.79
0.70	200.07	213.35	221.51	229.06	234.94	238.79	242.35
0.65	176.35	190.12	198.66	206.83	213.17	217.34	221.24
0.60	154.47	168.28	176.91	185.46	192.11	196.46	200.60
0.55	134.47	147.92	156.36	165.08	171.86	176.27	180.55
0.50	116.33	129.08	137.10	145.78	152.54	156.89	161.19
0.45	99.98	111.77	119.16	127.63	134.22	138.39	142.61
0.40	85.36	95.97	102.58	110.66	116.96	120.84	124.89
0.35	72.37	81.65	87.35	94.90	100.79	104.29	108.08
0.30	60.91	68.76	73.45	80.36	85.73	88.78	92.22
0.25	50.88	57.25	60.87	67.02	71.80	74.31	77.34
0.20	42.19	47.06	49.56	54.88	58.99	60.91	63.46
0.15	34.78	38.16	39.53	43.93	47.30	48.58	50.58
0.10	28.57	30.55	30.78	34.19	36.75	37.33	38.73

Table 9: The ARL values for developed chart for shifts in shape parameter when $r_o = 370$ and $n = 40$.

α	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$\mathcal{L}$	3.1160	3.1140	3.1160	3.1140	3.1140	3.1140	3.1130
a	0.6508	0.7156	0.7514	0.7751	0.7919	0.8046	0.8147
$\mathcal{E}$	k =2	k =1	k =2	k =1	k =1	k =1	k =1
1.00	370.22	370.34	370.29	370.39	370.27	370.34	370.06
0.95	338.42	341.90	344.15	345.76	346.83	347.87	348.36
0.90	302.67	309.92	314.65	318.00	320.42	322.55	323.92
0.85	265.48	276.20	283.21	288.30	292.06	295.27	297.55
0.80	228.99	242.40	251.22	257.84	262.77	266.94	270.07
0.75	194.72	209.84	219.86	227.65	233.50	238.44	242.26
0.70	163.58	179.45	190.04	198.59	205.05	210.51	214.84
0.65	136.03	151.80	162.39	171.28	178.03	183.76	188.40
0.60	112.10	127.13	137.26	146.13	152.87	158.64	163.38
0.55	91.65	105.49	114.82	123.36	129.85	135.44	140.12
0.50	74.37	86.76	95.06	103.04	109.08	114.34	118.79
0.45	59.92	70.72	77.87	85.13	90.60	95.39	99.51
0.40	47.94	57.13	63.07	69.52	74.33	78.58	82.27
0.35	38.08	45.72	50.47	56.07	60.17	63.82	67.03
0.30	30.03	36.21	39.83	44.58	47.97	51.01	53.71
0.25	23.51	28.38	30.95	34.87	37.56	40.00	42.18
0.20	18.29	21.98	23.61	26.76	28.78	30.64	32.32
0.15	14.16	16.84	17.64	20.06	21.48	22.79	24.00
0.10	10.96	12.78	12.88	14.65	15.51	16.31	17.07

4. PROPOSED CHART DEMONSTRATION

For real world application of the proposed MchSP control chart could be illustrated in the following way: let us assume that the lifetime of the product comes from the GED with specific shape parameterα_0=1.5. Suppose that the mean target lifetime of the product is μ_0= 1000 hours and r_0= 300. Consider a truncated life test containing sample size of (n =15), which will be kept for all subsection. Hence, from Equation (5) the value of p_0 = 0.4182. Furthermore, as of Table 1 we acquire a=0.8241, $\mathcal{L}$ =2.778, LCL=0, UCL=11 and t_0 =819.3 hours. Therefore, the implementation of the projected chart is given below:

i. Choose 30 random samples for each subgroup and conduct life testing experiment for 819.3 hours. Determine the number of failure items say D for the duration of the experiment time.

ii. Announce the process considered as in-control when $0 \leq D \leq 11$; otherwise, the process declared as out-of-control.

5. APPLICATIONS IN INDUSTRY

The developed control chart based on MchSP is demonstrated using a real data in this section. The data was collected from (Shen et al., 2022) the Reddit advertising data consists of 130 observations. The Reddit advertising data set could also be obtained from: https://www.kaggle.com/ammar111/reddit-top-1000-posts-analysis-for-18-sub-reddits/notebook. For ready reference, the Reddit advertising data is 11.340, 6.296, 5.136, 7.292, 6.700, 3.648, 7.272, 5.140, 2.980, 6.336, 5.128, 7.564, 6.180, 5.588, 8.760, 11.560, 5.192, 10.032, 6.660, 7.932, 7.536, 4.436, 17.580, 9.836, 5.580, 6.092, 7.912, 6.760, 10.096, 5.948, 11.156, 6.936, 5.424, 8.532, 4.980, 6.536, 13.012, 6.668, 5.540, 11.184, 7.052, 9.336, 10.624, 7.148, 4.892, 6.584, 4.436, 11.696, 7.868, 4.040, 6.748, 5.336, 9.056, 11.496, 11.548, 12.392, 3.636, 7.380, 8.940, 9.068, 4.536, 13.060, 8.940, 7.900, 9.972, 5.740, 5.560, 6.136, 10.904, 7.960, 9.380, 6.484, 3.512, 4.792, 8.980, 5.512, 2.392, 5.336, 3.440, 4.580, 6.704, 7.296, 4.600, 5.592, 9.292, 7.816, 7.068, 8.492, 7.556, 8.836, 5.960, 3.696, 9.816, 10.908, 5.636, 8.536, 6.260, 7.912, 12.492, 8.880, 6.188, 11.900, 7.692, 7.496, 10.340, 9.636, 3.784, 5.068, 2.940, 9.992, 7.252, 10.956, 7.512, 8.108, 7.796, 6.928, 6.236, 4.924, 8.056, 3.468, 7.904, 3.780, 5.912, 7.756, 9.900, 5.472, 3.956, 5.044, 12.676, 5.376.

It is established that the Reddit advertising data set follows the GED with a shape parameter $\hat{\alpha}$=14.0842 and scale parameter $\hat{\lambda}$=2.2801. Kolmogorov-Smirnov test for fitted data as 0.0423 and corresponding p-value is 0.9742. In addition, a pictorial presentation of the goodness of fit of the distribution in terms of histogram along with theoretical PDF and Q-Q plot is shown in Figure 1.

From the Kolmogorov-Smirnov test and Figure 1, the data set of Reddit advertising is well fitted to the GED. Table 10 shows the contrast between the planned MchSP control chart and the existing chain sampling plan control chart from real data examples for anticipated parameters of GED. It is observed from Table 10 that the developed MchSP control chart shows smaller ARL values compared to the existing chain sampling plan control chart.

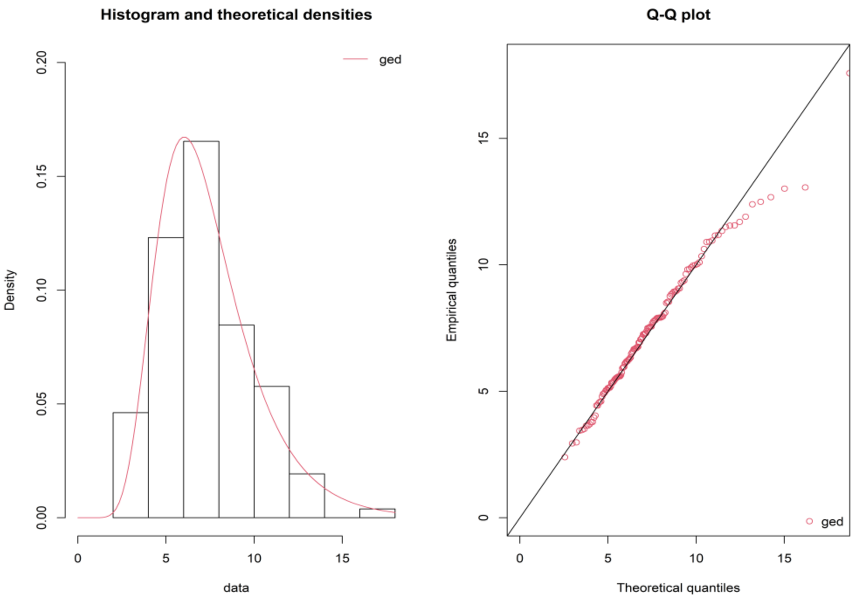


Figure 1: Pictorial presentation of various plots for the GED for Reddit advertising data.

From Reddit advertising data, n = 130, a= 0.9168, $\hat{\alpha}$= 14.0842, $\hat{\lambda}$=2.2801, and r_0= 370. using Equation (5) the p_0 value is 0.4071. The average value of Reddit advertising data is μ =1.3316 and the duration of test $t_0 = a\mu_0 = (0.9168)(1.3316) = 1.2207$. The control limits proposed chart are LCL=0.2130 and UCL=11.9986 for the chart coefficient $\mathcal{L}$=3.097. The proposed control chart for Reddit advertising data is presented in Figure 2 and on hand chain sampling plan control chart for $\mathcal{L}$=3.3310, LCL=0.7702 and UCL=13.6619 is given in Figure 3. From Figure 2, it is identified that the planned chart identified the six sample points out-of-control signal, whereas from Figure 3, the existing chain sampling plan control chart detected one sample point out-of-control. Thus, the proposed control chart shows six out-of-control signals, but the on-hand chain sampling plan control chart shows one out-of-control signal. Therefore, the developed chart is superior for examining the quality of the goods. In conclusion, the developed control chart is more competent than the existing control charts.

Table 10: Comparison of proposed control chart with existing chart when n=15, ARL0=370 for estimated parameters

	Proposed	Existing
α	14.0842	14.0842
$\mathcal{L}$	3.0970	3.3310
a	0.9168	0.9826
c	k=1	k=2
1.00	370.46	370.68
0.95	121.43	285.97
0.90	39.39	72.24
0.85	14.55	17.44
0.80	6.27	5.58
0.75	3.19	2.49
0.70	1.92	1.52
0.65	1.37	1.17
0.60	1.13	1.05
0.55	1.04	1.01
0.50	1.01	1.00

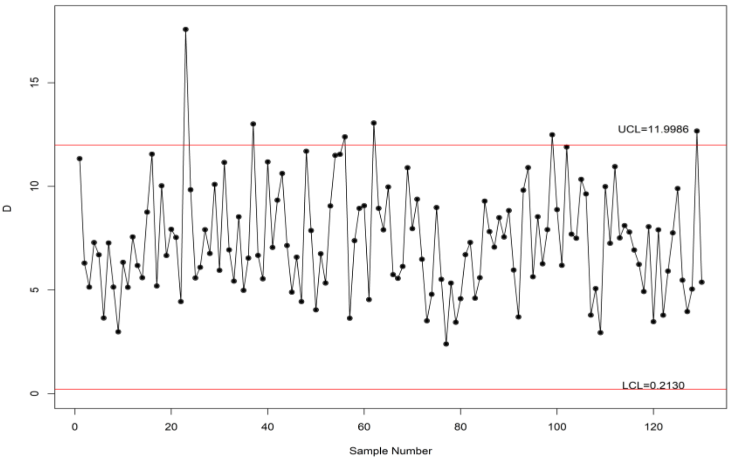


Figure 2: The proposed chart for real data.

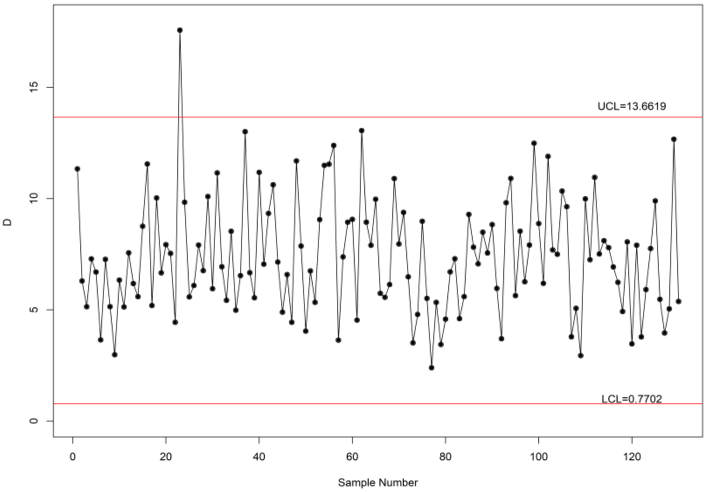


Figure 3: The existing chain sampling control chart for real data.

6. PROPOSED CHART MONITORING

A simulation technique is adopted to inspect the presentation of the developed MchSP chart. We adopt the below simulation procedure, which is applied to the production of data from GED and the construction of the MchSP control chart:

i. Draw a random sample of size *n*.

ii. A breed of size *n* from GED with parameter (α_0=2, λ =1).

iii. Obtain the chart statistics *D* from every subgroup.

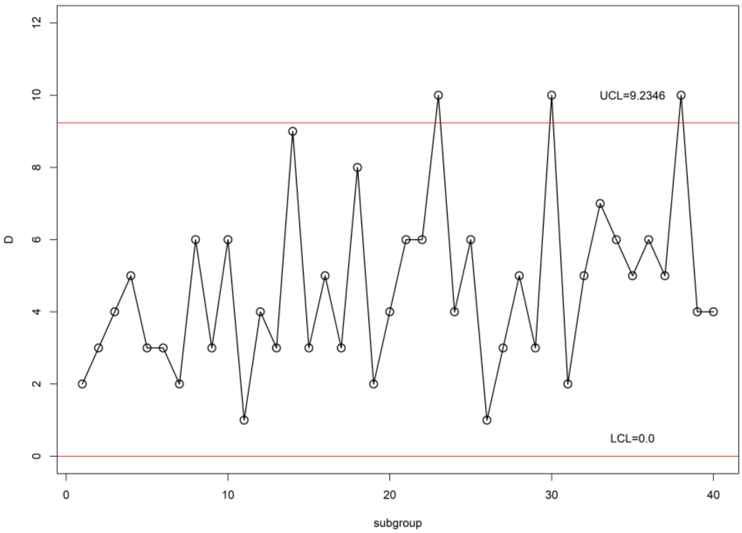
iv. Reproduce from (i) to (iii) up to get the preferred sample groups (*m*=20).

v. Compute the planned chart limits by applying Eqs. (6) and (7).

vi. Construct the MchSP control chart sample point corresponding to each subgroup.

To construct the developed control chart, breed the first 20 subgroups of size 15 are yielded from GED with in-control parameters (α_0=2,λ_0=1). The subsequent set of the 20 samples of subgroup size 15 are from GED with scale parameter λ_1=0.75, (i.e., scale parameters is shifted 0.75, i.e., *c*=0.75). Table 4 shows that when ARL at 370 and in-control parameters we find control chart constant $\mathcal{L}$ is 3.098, *a* value is 0.8273 and *n* is 15. The life test termination time is t_0=0.8273×1.2279=1.0159. In

Table 11 the chart statistics *D*, as well as sample values, are shown. From Table 11, we get $\bar{D}$ is 3.95; using Eqs. (6) and (7) the developed chart control limits are obtained as LCL=0 and UCL=9.2346. Figure 4 represents the proposed MchSP control chart for simulated data. The same D values, as well as the control limits shown in Figure 5, are shown on the current chain sampling plan control chart. Figure 4 indicates that the projected MchSP control chart detects the shift at the 23rd, 30th, and 38th samples, but Figure 5 shows that the existing chain sampling plan control chart does not detect the out-of-control condition. As a result, the planned control chart outperforms the available chain sampling plan control chart in



detecting when the process is moved.

Figure 4: The proposed chart for simulated data.

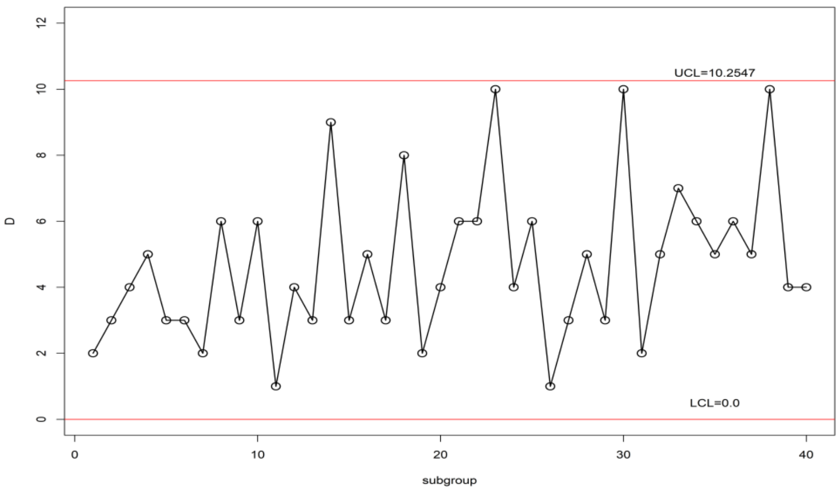


Figure 5: The existing chain sampling control chart for simulated data.

Table 11: The chart data points for simulated data

S No	Sample observations															D
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
1	1.14	0.62	11.66	4.49	4.11	6.27	3.49	7.16	1.37	2.00	2.77	0.72	6.06	3.13	2.03	2
2	0.90	71.62	1.43	22.66	1.53	0.17	94.68	2.03	7.09	1.29	2.28	9.29	4.44	25.98	0.39	3
3	5.35	0.43	2.82	2.77	16.65	0.63	1.79	0.43	1.32	5.00	0.85	3.89	5.68	2.41	26.80	4
4	8.87	1.46	0.63	0.53	2.45	82.83	1.91	1.18	4.78	0.91	0.62	17.41	1.10	10.93	0.52	5
5	3.62	0.60	0.35	38.07	6.42	1.58	13.19	4.74	3.53	2.10	0.39	6.49	1.04	8.00	4.95	3
6	1.38	2.31	3.91	0.55	2.20	4.56	3.37	0.16	0.89	3.18	40.16	1.30	14.85	5.39	2.18	3
7	1.82	1.09	15.32	1.15	5.08	19.15	1.87	14.89	4.29	2.73	7.82	2.15	1.50	0.21	0.56	2
8	0.41	0.88	0.09	1.26	23.57	1.87	0.12	2.32	0.46	22.99	4.04	0.71	1.91	2.52	3.93	6
9	0.83	54.99	5.25	11.98	4.49	5.84	1.58	2.24	9.99	0.77	14.70	2.28	38.80	0.98	3.12	3
10	1.42	70.44	0.23	81.56	3.88	12.45	1.59	1.00	0.55	0.85	0.51	20.50	2.48	1.89	0.86	6
11	1.14	1.99	1.08	12.22	3.63	5.08	1.70	4.79	2.03	51.81	13.71	18.49	0.96	1.20	7.43	1
12	11.34	2.86	1.47	0.55	1.72	2.73	15.74	0.87	1.19	35.66	0.39	7.56	1.99	0.75	3.38	4
13	1.59	0.70	4.85	2.01	1.18	18.10	2.24	1.45	4.15	1.99	0.31	2.17	0.28	1.86	2.97	3
14	0.73	0.89	13.62	6.07	0.52	1.29	0.86	0.30	1.00	0.71	9.52	1.20	0.11	0.29	3.47	9
15	50.27	3.55	2.15	4.70	11.34	0.52	1.55	1.80	0.58	12.83	0.93	1.45	9.57	35.22	5.11	3
16	0.75	7.46	0.33	4.70	1.89	6.92	5.90	15.82	1.11	9.29	12.51	0.68	0.50	1.01	8.38	5

17	1.36	1.29	0.23	18.49	2.19	2.50	0.33	36.34	1.52	9.01	2.63	0.84	1.58	3.00	1.52	3
18	11.43	0.61	0.52	6.79	0.44	0.91	0.22	1.51	0.24	2.34	2.66	3.30	0.99	6.35	0.49	8
19	1.65	3.25	0.80	4.26	2.53	4.56	31.31	13.85	3.70	4.98	12.32	8.34	2.07	0.30	2.06	2
20	0.72	0.18	12.82	1.60	2.69	3.69	5.54	0.78	1.96	0.73	31.71	2.47	2.40	1.12	6.69	4
21	5.36	0.18	3.19	2.69	0.62	0.35	10.39	2.50	0.21	0.37	1.69	3.39	0.12	1.12	2.26	6
22	1.04	5.50	8.10	0.37	1.66	2.18	1.66	5.47	1.28	0.35	0.90	0.77	0.80	0.24	19.63	6
23	0.67	3.96	0.48	0.74	0.13	0.90	0.50	4.53	0.64	1.31	2.75	0.13	0.11	1.38	1.00	10
24	5.25	1.37	0.84	0.13	0.21	4.18	0.65	5.80	30.66	3.41	2.52	1.82	28.30	8.38	8.78	4
25	3.33	34.61	12.84	10.06	0.59	1.56	0.39	2.25	0.98	0.59	0.97	1.24	0.60	6.64	12.39	6
26	4.42	18.73	69.94	3.07	1.27	0.23	12.45	1.16	1.36	2.16	5.98	19.52	7.03	6.77	3.23	1
27	0.79	1.66	2.91	4.07	1.14	2.80	9.82	34.18	32.99	7.33	0.24	0.50	5.50	5.43	3.60	3
28	0.39	0.28	31.94	2.73	6.36	0.12	10.00	2.46	0.57	4.74	3.95	0.71	13.03	5.45	1.26	5
29	1.13	2.26	1.30	1.65	53.85	0.33	5.63	1.22	1.57	0.64	1.92	26.30	5.57	0.96	2.26	3
30	0.29	0.95	1.65	4.91	1.32	2.71	0.14	1.01	0.73	1.39	0.67	0.48	0.12	0.66	0.15	11
31	2.50	1.44	5.60	0.28	1.12	1.06	1.49	0.17	20.06	1.50	1.40	3.43	4.42	2.24	3.15	2
32	8.19	0.70	0.97	0.35	2.11	1.15	2.09	1.75	0.49	4.43	1.07	80.32	1.60	0.26	3.03	5
33	2.46	9.87	0.81	27.36	1.08	0.79	0.77	18.32	0.18	0.21	0.51	0.58	3.20	7.28	1.34	7
34	0.25	10.55	23.15	0.52	2.98	6.18	0.70	1.13	0.60	1.51	10.14	1.39	0.08	0.07	4.54	6
35	6.19	0.13	0.39	0.50	2.03	4.52	3.30	0.44	6.02	4.66	2.20	1.96	0.15	37.54	9.22	5
36	0.23	4.32	1.14	15.43	0.54	0.82	0.33	2.40	4.50	0.37	11.79	1.86	0.29	1.74	2.71	6
37	0.48	4.29	0.41	2.46	1.15	0.13	1.15	1.71	31.89	1.17	0.53	0.99	16.36	2.00	3.81	5
38	3.28	0.22	2.05	0.53	0.34	0.28	7.13	0.90	0.82	0.22	0.19	0.27	0.82	2.18	0.23	11
39	1.19	0.17	17.99	0.90	35.35	25.41	85.52	2.55	0.21	4.98	2.65	0.28	1.48	60.00	2.48	4
40	0.10	2.52	1.52	1.13	5.70	1.59	1.30	1.35	0.76	3.75	0.37	4.20	2.23	6.53	0.23	4

7. CONCLUSIONS

When the quality dimensions of the product's lifetime follow a generalized exponential distribution, an attribute control chart based on the modified chain sampling plan is proposed in this article. The chart parameters are calculated for different sample sizes and ARL values. Extensive findings are provided for industrial applications of the suggested control chart at various scale and shape parameters. The created MchSP control chart methodology is supported by real-world examples as well as simulated data. The results show that as the shift constant (both scale and form) increases, the ARL values decrease. In a neutrosophic statistics context the modified multiple dependent sampling control charts, the prospect research would be used as an additional room of projected charts.

DECLARATIONS

Data Availability: The data is taken from (Castagliola et al., 2011)

Conflict of Interest: No conflict of interest regarding the paper

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Author's contributions: Rao contributed simulation work and methodology, Aslam presented concept and methodology, Kirigiti contributed manuscript preparation.

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