

Domination in Plithogenic product fuzzy graphs

T. BHARATHI, S. LEO AND B. DAVVAZ

Abstract

The notion of Plithogenic vertex domination and Plithogenic edge domination in Plithogenic product fuzzy graphs (PPFGs) has been newly introduced and discussed based on P-weights of P-vertices and P-edges respectively with properties, results and examples. Plithogenic vertex domination number and Plithogenic edge domination number for a Plithogenic product fuzzy graph (PPFG), and the bounds and algorithms for and the maximum and minimum values and algorithms for the same are also investigated. Perfect Plithogenic vertex domination and perfect Plithogenic edge domination are presented as special cases of Plithogenic vertex domination and Plithogenic edge domination respectively. It is observed that the P-vertex (P-edge) with the maximum P-weight exists in every Plithogenic vertex (edge) dominating set of a Plithogenic product fuzzy graph. The novelty in these ideas of domination is that a better perception of reality can be obtained through the analysis of both, Plithogenic vertex domination and Plithogenic edge domination, in any Plithogenic product fuzzy graphical structure.

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1. INTRODUCTION

The concept of domination in different graphical structures is a constantly growing area of research both in theory and applications. Domination in fuzzy graphs (FGs) and product fuzzy graphs were introduced by [Somasundaram and Somasundaram 1998] and [Somasundaram 2005] respectively. Nagoorgani et al. investigated strong and weak domination in FGs [Gani and Chandrasekaran 2006; Gani and Ahamed 2007]. Various other types of domination parameters in FGs were developed and their applications were discussed in [Akram et al. 2017; Arriola 2017; Gani and Devi 2013; Gani et al. 2021; Ismail and Mohamed 2010; Karunambigai et al. 2013; Karaaslan and Davvaz 2018; Manjusha and Sunitha 2014; 2014; 2015; Natarajan and Ayyaswamy 2010; Pal et al. 2020; Ponnappan et al. 2017; Prassanna 2019; Rana 2021; Selvam and Ponnappan 2020; Xavier et al. 2013]. In fact, there are more than 30 parameters to describe the concept of domination in FGs and they are very rich and noteworthy in both theory and applications [Rana 2021]. However, since human situations are always affected by multiple factors, [Bharathi and Leo 2021] introduced PPFGs which are different from the existing literature, and which with the

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help of multiple attributes display a better perception and interpretation of certain realities around us to find more suitable solutions to issues concerning them. [Bharathi et al. 2022] also investigated and studied the operations of complement, union and intersection in PPFs. In this paper, two types of dominations in PPFs are presented for the first time. We newly define Plithogenic vertex dominating set (PVDS) and Plithogenic edge dominating set (PEDS) in PPFs using the P-weights of P-vertices and P-edges respectively. Perfect PVDS and perfect PEDS are presented as special cases of PVDS and PEDS respectively. Plithogenic vertex domination number (PVDN) and Plithogenic edge domination number (PEDN) are generated to minimize the domination parameters in PPFs. Further, this paper is organized as follows. Section 2 contains basic concepts required for this study. Plithogenic vertex domination and Plithogenic edge domination in PPFs are analyzed in Section 3. It is proved that every minimal Plithogenic vertex (edge) dominating set is a Plithogenic vertex (edge) dominating set, but the converse need not be true. A lower bound and an upper bound for Plithogenic vertex (edge) domination number are obtained using the concepts of maximum P-weight of P-vertices (P-edges) and the maximum of the sums of P-weights of the elements of the minimal Plithogenic vertex (edge) dominating sets of the concerned PPF. We have also discussed the application of these domination parameters in the selection of a set of representatives in Section 4. Finally, the conclusion in Section 5 discloses the main ideas of the relation between Plithogenic vertex domination and Plithogenic edge domination in PPFs, and the scope for further research in this line.

2. PRELIMINARIES

Let $G : (\alpha, \beta)$ be a FG on V and $D \subseteq V$. The fuzzy cardinality of D is defined to be $\sum_{v \in D} \alpha(v)$. D is said to be a fuzzy dominating set if for every $v \in V - D$, there exists $u \in D$ such that (u, v) is strong arc. D is called a minimal dominating set of G , if for every $v \in D$, $D - v$ is not a fuzzy dominating set. The domination number $\gamma(G)$ is the minimum of the fuzzy cardinalities of all the minimal dominating sets of vertices of G [Gani and Vadivel 2011]. Let e_i and e_j be two edges of G . We say that e_i dominates e_j if e_i is a strong arc in G and adjacent to e_j . $S \subseteq E(G)$ is called an edge dominating set of G if for every $e_j \notin S$, there exists $e_i \in S$ such that e_i dominates e_j . S is called a minimal edge dominating set if no proper subset of S is an edge dominating set of G [Gani and Devi 2013]. A dominating set D of G is said to be a perfect dominating set if for each vertex $v \notin D$ is adjacent to exactly one vertex of D [Revathi et al.

2013]. An edge dominating set S of G is said to be a perfect edge dominating set if every edge $e_j \notin S$ is adjacent to exactly one edge in S [Ramya and Lavanya 2016].

Let $G_F = (V_F, E_F)$ be a Plithogenic fuzzy graph, where $V_F = \{V_1, V_2, \dots, V_n\}$ and $E_F = \{E_1, E_2, \dots, E_m\}$ are Plithogenic fuzzy sets of vertices and relevant edges respectively characterized by k attributes. G_F is said to be a PPFG denoted by $G_P = (V_P, E_P)$, if for any two adjacent $V_i, V_j \in V_F, i \neq j$, with the corresponding edge $E_c \in E_F, V_i = X_i(s_1, s_2, \dots, s_k), V_j = X_j(t_1, t_2, \dots, t_k)$ and $E_c = Y_c(c_1, c_2, \dots, c_k)$ where for any $c_d \in Y_c, c_d = s_a t_b$ with $s_a \in X_i$ and $t_b \in X_j; d = a = b; 1 \leq d, a, b \leq k; c_d, s_a, t_b \in [0, 1]$. Here X_i, X_j and Y_c are sets of attribute values from $[0, 1]$ characterizing V_i, V_j and E_c respectively in G_F and $k \geq 4$. The sum of the attribute values of any $V \in V_P$ is its P -weight denoted by $P_{VW}(V)$. The sum of the attribute values of any $E \in E_P$ is its P -weight denoted by $P_{EW}(E)$. The sum of the P -weights of all the P -vertices in G_P is denoted by $P_O(G_P)$. Any $V \in V_P$ with attribute values $s_1, s_2, \dots, s_k$ is said to be a strong P -vertex, if $(s_1 + s_2 + \dots + s_k) \geq A_{VW}(G_P)$, where $A_{VW}(G_P)$ is the average P -weight of P -vertices in G_P , i.e., $A_{VW}(G_P) = P_O(G_P)/|V_P(G_P)|$, where $|V_P(G_P)|$ is the number of P -vertices in G_P . Otherwise, V is called a weak P -vertex. A P -edge is said to be highly strong, if both of its adjacent P -vertices are strong. A P -edge is said to be strong, if one of its adjacent P -vertices is strong. Otherwise, it is a weak P -edge. A Plithogenic product fuzzy path (PPFP) is a sequence $V_0, E_1, V_1, E_2, \dots, V_{n-1}, E_n, V_n$ of distinct P -vertices and P -edges such that for any P -edge E_i with end points V_{i-1} and $V_i, P_{EW}(E_i) > 0$ where $1 \leq i \leq n$. Here n is the length of the PPFP. G_P is said to be a complete PPFG if every $V \in V_P$ is adjacent to the rest of the P -vertices in G_P [Bharathi and Leo 2021].

3. DOMINATION IN PLITHOGENIC PRODUCT FUZZY GRAPHS

3.1. Plithogenic vertex domination

DEFINITION 3.1. Let $G_P = (V_P, E_P)$ be a connected Plithogenic product fuzzy graph. For any adjacent $V_i, V_j \in V_P$ in G_P, V_i is said to dominate V_j if $V_i V_j$ is either a highly strong or strong P -edge in G_P such that $P_{VW}(V_j) \leq P_{VW}(V_i)$. Otherwise, the P -vertex with a greater P -weight is said to dominate the other whenever $V_i V_j$ is a weak P -edge.

DEFINITION 3.2. A subset R_P of V_P is called a Plithogenic vertex dominating set of G_P if for every $V_j \notin R_P$, there exists at least one $V_i \in R_P$ such that V_i dominates V_j .

DEFINITION 3.3. A Plithogenic vertex dominating set R_P is called a minimal

Plithogenic vertex dominating set, if there is no Plithogenic vertex dominating set R' of G_P such that $R' \subset R_P$.

DEFINITION 3.4. The sum of the P-weights of the elements of any Plithogenic fuzzy set V_P is said to be its Plithogenic fuzzy cardinality denoted by $|V_P|_f$.

DEFINITION 3.5. The Plithogenic vertex domination number $\gamma_v(G_P)$, and the Plithogenic vertex upper-domination number $D_v(G_P)$ are the minimum and the maximum Plithogenic fuzzy cardinalities taken over all minimal Plithogenic vertex dominating sets of G_P .

DEFINITION 3.6. A minimum Plithogenic vertex dominating set of G_P is a minimal Plithogenic vertex dominating set with minimum Plithogenic fuzzy cardinality.

NOTE 3.7. Plithogenic vertex domination doesn't exist in a PPFG, if there exists a weak P-edge such that the corresponding P-vertices are neither adjacent to at least one strong P-vertex nor the P-weights of both are distinct.

NOTE 3.8. In a disconnected PPFG, Plithogenic vertex domination doesn't exist.

EXAMPLE 3.9. Consider a Plithogenic product fuzzy graph, $G_P = (V_P, E_P)$, such that $V_P = \{V_1, V_2, V_3, V_4, V_5\}$ and $E_P = \{E_1, E_2, E_3, E_4, E_5, E_6\}$ as shown in Figure 1. Here,

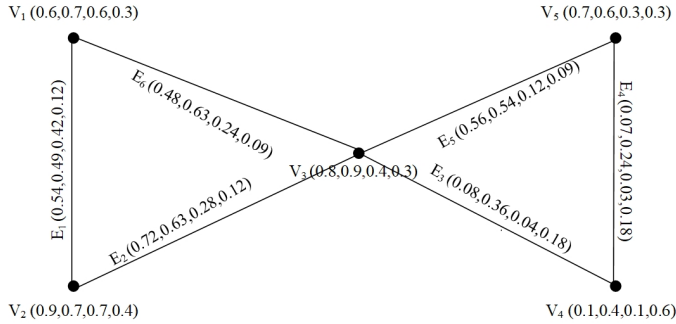


Fig. 1. Plithogenic vertex domination in PPFG

$P_{VW}(V_1) = 2.2$; $P_{VW}(V_2) = 2.7$; $P_{VW}(V_3) = 2.4$; $P_{VW}(V_4) = 1.2$; $P_{VW}(V_5) = 1.9$. Average P-weight of P-vertices, $A_{VW}(G_P) = 10.4/5 = 2.08$. Therefore, $\{V_2, V_3, V_5\}$, $\{V_2, V_5\}$ and $\{V_2, V_3\}$ are PVDSSs. $\{V_2, V_5\}$ and $\{V_2, V_3\}$ are minimal PVDSSs. Since the Plithogenic fuzzy cardinalities of $\{V_2, V_5\}$ and $\{V_2, V_3\}$ are 4.6 and 5.1 respectively, $\gamma_v(G_P) = 4.6$ and $D_v(G_P) = 5.1$. Thus, $\{V_2, V_5\}$ is the minimum PVDSS.

LEMMA 3.10. Let R_p be a Plithogenic vertex dominating set of G_p . Then for any $V_i \in R_p$ if $R_p - V_i$ is a Plithogenic vertex dominating set, then R_p is not a minimal Plithogenic vertex dominating set of G_p .

The proof is obvious by Definition 3.3.

LEMMA 3.11. Let R_p be a minimal Plithogenic vertex dominating set of G_p . Then for every $V_i \in R_p$, $R'_p = R_p - V_i$ is not a Plithogenic vertex dominating set of G_p .

The proof is obvious by Definition 3.3.

LEMMA 3.12. If $G_p = (V_p, E_p)$ is a Plithogenic product fuzzy graph with distinct P-weights of P-vertices, then every Plithogenic vertex dominating set of G_p contains $V_j \in V_p$ such that $P_{VW}(V_j)$ is the maximum.

PROOF. Let $G_p = (V_p, E_p)$ be a PPFG where $V_p = \{V_1, V_2, \dots, V_n\}$ and $E_p = \{E_1, E_2, \dots, E_m\}$ and $P_{VW}(V_i)$ is distinct for every $V_i \in V_p$ with $i = 1$ to n . Consider $D_p = \{D_{P_1}, D_{P_2}, D_{P_3}, \dots, D_{P_k}\}$ to be the collection of all PVDSs of G_p and $V_j \in V_p$ such that $P_{VW}(V_j)$ is the maximum in G_p . This implies that by Definition 3.1., V_j dominates at least one $V_i \in G_p$, and is dominated by no P-vertex of G_p . Suppose for any $D_{P_\alpha} \in D_p$, $V_j \notin D_{P_\alpha}$, then there exists at least one $V_r \in V_p$ such that V_r dominates V_j in G_p . This is a contradiction to the fact that $P_{VW}(V_j)$ is the maximum in G_p . Therefore, $V_j \in D_{P_\alpha}$ where $\alpha = 1$ to k .

REMARK 3.13. If $G_p = (V_p, E_p)$ is a PPFG with distinct P-weights of P-vertices, every minimal PVDS of G_p contains $V_j \in V_p$ such that $P_{VW}(V_j)$ is the maximum.

THEOREM 3.14. Every minimal Plithogenic vertex dominating set of a Plithogenic product fuzzy graph is a Plithogenic vertex dominating set, but the converse need not be true.

PROOF. Let R_p be a minimal PVDS of G_p . Suppose R_p is not a PVDS of G_p , then there exists at least one P-vertex $V_j \notin R_p$, which is not dominated by any element of R_p . This contradicts the fact that R_p is a minimal PVDS of G_p , i.e., for every $V_j \notin R_p$, there exists $V_i \in R_p$ such that V_i dominates V_j , and for any $V_i \in R_p$, $R_p - V_i$ is not a PVDS of G_p by Lemma 3.11. Therefore, R_p must be a PVDS of G_p . Conversely, let $D_p = \{D_{P_1}, D_{P_2}, D_{P_3}, \dots, D_{P_k}\}$ be the collection of all PVDS of G_p .

Case (i) Suppose, for any $D_{P_i} \in D_p$, if there exists at least one $V_i \in D_{P_i}$ such that $D_{P_i} - V_i$ is a PVDS of G_p . Then D_{P_i} is not a minimal PVDS of G_p by Lemma 3.10.

Case (ii) Suppose, for any $D_{P_i} \in D_p$ if for every $V_i \in D_{P_i}$, $D_{P_i} - V_i$ is not a PVDS of G_p . Then D_{P_i} is a minimal PVDS of G_p by Lemma 3.11.

THEOREM 3.15. Every minimal Plithogenic vertex dominating set of G_P need not be a minimum Plithogenic vertex dominating set of G_P .

PROOF. Let $D_{PM} = \{D_{P_1}, D_{P_2}, D_{P_3}, \dots, D_{P_m}\}$ be the collection of all minimal PVDSs of G_P .

Case (i) If $\sum_{V_i \in D_{P_\delta}} P_{VW}(V_i) = c$ (constant) for every $D_{P_\delta} \in D_{PM}$ with $\delta = 1$ to m , then $\gamma_v(G_P) = c$. Therefore, every D_{P_δ} is a minimum PVDS of G_P .

Case (ii) If $\sum_{V_i \in D_{P_\delta}} P_{VW}(V_i)$ is distinct for every $D_{P_\delta} \in D_{PM}$ with $\delta = 1$ to m , then by Definition 3.6., $\gamma_v(G_P) = \min \{\sum_{V_i \in D_{P_\delta}} P_{VW}(V_i)\}$ where $D_{P_\delta} \in D_{PM}$ with $\delta = 1$ to m . Therefore the corresponding D_{P_δ} is the only minimum PVDS of G_P .

Case (iii) Suppose that there exists more than one D_{P_δ} 's having the same Plithogenic fuzzy cardinalities. Let's say, $\sum_{X_i \in D_{P_1}} P_{VW}(X_i) = \sum_{Y_i \in D_{P_2}} P_{VW}(Y_i) = \sigma$, where $D_{P_1}, D_{P_2} \in D_{PM}$. Then if $\sigma = \min \{\sum_{V_i \in D_{P_\delta}} P_{VW}(V_i) : D_{P_\delta} \in D_{PM}\}$ with $\delta = 1$ to m , then both D_{P_1} and D_{P_2} are minimum PVDSs. Otherwise, D_{P_δ} corresponding to the minimum of $\{\sum_{V_i \in D_{P_\delta}} P_{VW}(V_i)\}$ where $D_{P_\delta} \in D_{PM}$ with $\delta = 1$ to m , is the minimum PVDS of G_P .

LEMMA 3.16. If a Plithogenic product fuzzy graph $G_P = (V_P, E_P)$ is complete with $P_{VW}(V_i) = c$ (constant) for every $V_i \in V_P$, then its minimal Plithogenic vertex dominating sets are singleton sets containing every element of V_P .

PROOF. Since every P-vertex is adjacent to the rest of the P-vertices in G_P , and $P_{VW}(V_i) = c$ for every $V_i \in V_P$, it is obvious that every element of V_P dominates the rest of the P-vertices in G_P . Therefore, all minimal Plithogenic vertex dominating sets of G_P are singleton sets containing every element of V_P .

LEMMA 3.17. If a Plithogenic product fuzzy graph $G_P = (V_P, E_P)$ is complete and $P_{VW}(V_i)$ is distinct for every $V_i \in V_P$, then there exists exactly one minimal Plithogenic vertex dominating set which is a singleton set.

PROOF. Since every P-vertex is adjacent to the rest of the P-vertices in G_P , and $P_{VW}(V_i)$ is distinct for every $V_i \in V_P$, it is obvious that the Plithogenic fuzzy set that contains $V_i \in V_P$ such that $P_{VW}(V_i)$ is the maximum is the only minimal PVDS, since it is the only P-vertex that dominates the rest of the P-vertices in G_P .

PROPOSITION 3.18. If a Plithogenic product fuzzy graph $G_P = (V_P, E_P)$ is complete with $P_{VW}(V_i) = c$ (constant) for every $V_i \in V_P$, then $\gamma_v(G_P) = c$.

PROOF. Let $G_P = (V_P, E_P)$ be a complete PPFG, where $V_P = \{V_1, V_2, \dots, V_n\}$ and $E_P = \{E_1, E_2, \dots, E_m\}$ characterized by k attribute values. Since $P_{VW}(V_i) = c$ for every $V_i \in V_P$, every V_i dominates the rest of the P-vertices in G_P . This implies that $D_{P_i} = \{V_i\}$

where $V_i \in V_P$ and $i = 1$ to n , is a minimal PVDS of G_P by Lemma 3.16. Therefore, $\gamma_v(G_P) = P_{VW}(V_i) = c$.

PROPOSITION 3.19. If a Plithogenic product fuzzy graph $G_P = (V_P, E_P)$ is complete and $P_{VW}(V_i)$ is distinct for every $V_i \in V_P$, then $\gamma_v(G_P) = \max\{P_{VW}(V_i) : V_i \in V_P\}$.

PROOF. Let $G_P = (V_P, E_P)$ be a complete PPFG where $V_P = \{V_1, V_2, \dots, V_n\}$ and $E_P = \{E_1, E_2, \dots, E_m\}$ characterized by k attribute values, and $P_{VW}(V_i)$ be distinct for every $V_i \in V_P$. Since every V_i is adjacent to the rest of the P-vertices in G_P , $R_P = \{V_i : \text{where } P_{VW}(V_i) \text{ is the maximum}\}$ is the only minimal PVDS of G_P by Lemma 3.17. This implies that the Plithogenic vertex domination number, $\gamma_v(G_P) = \max\{P_{VW}(V_i) : V_i \in V_P \text{ and } i = 1 \text{ to } n\}$.

THEOREM 3.20. For any connected Plithogenic product fuzzy graph G_P , $\Delta_{VW}(G_P) \leq \gamma_v(G_P) \leq D_v(G_P)$.

PROOF. Let $G_P = (V_P, E_P)$ be a PPFG with $V_P = \{V_1, V_2, \dots, V_n\}$ and $E_P = \{E_1, E_2, \dots, E_m\}$, and $\gamma_v(G_P)$ be the PVDN of G_P , $\Delta_{VW}(G_P)$ be the maximum of P-weights of P-vertices in G_P , $D_v(G_P)$ be the maximum Plithogenic fuzzy cardinality taken over all minimal Plithogenic vertex dominating sets of G_P , and $D_{PM} = \{D_{P_1}, D_{P_2}, D_{P_3}, \dots, D_{P_m}\}$ be the collection of all minimal PVDSs of G_P . Suppose that G_P is a complete PPFG. Then every P-vertex is adjacent to the rest of the P-vertices in G_P .

Case (i) When $P_{VW}(V_i) = c$ (constant) for every $V_i \in V_P$, then $D_{P_i} = \{V_i\}$ where $D_{P_i} \in D_{PM}$ and $i = 1$ to m , and $m = n$ by Lemma 3.16, and therefore, $\gamma_v(G_P) = c$ by proposition 3.18. This implies $\gamma_v(G_P) = \Delta_{VW}(G_P) = D_v(G_P)$.

Case (ii) When $P_{VW}(V_i)$ is distinct for every $V_i \in V_P$, then there exists only one minimal PVDS of G_P , i.e., $D_{P_i} = \{V_i : P_{VW}(V_i) \text{ is the maximum in } G_P\}$ by Lemma 3.17. Therefore, $\gamma_v(G_P) = \max\{P_{VW}(V_i) : V_i \in V_P\}$ by Proposition 3.19. This implies $\Delta_{VW}(G_P) = \gamma_v(G_P) = D_v(G_P)$.

Case (iii) When there are more than one minimal PVDSs with equal Plithogenic fuzzy cardinalities. Let's say, $|D_{P_1}|_f = |D_{P_2}|_f$. Suppose $|D_{P_1}|_f = |D_{P_2}|_f = \Delta_{VW}(G_P)$, then by Proposition 3.19., $\Delta_{VW}(G_P) = \gamma_v(G_P) = D_v(G_P)$. Otherwise $D_{P_i} = \{V_i : P_{VW}(V_i) \text{ is the maximum in } G_P\}$ is the minimum PVDS by the same proposition. This implies $\Delta_{VW}(G_P) = \gamma_v(G_P) = D_v(G_P)$.

Suppose that G_P is not a complete PPFG. Then by Lemma 3.12, every D_{P_i} contains $V_j \in V_P$ such that $P_{VW}(V_j)$ is the maximum in G_P .

Case (iv) If there exists exactly one minimal Plithogenic vertex dominating set of G_P

with only one element, then it implies $\Delta_{VW}(G_P) = \gamma_v(G_P) = D_v(G_P)$.

Case (v) If there exists exactly one minimal PVDS of G_P with more than one element, then it implies $\Delta_{VW}(G_P) < \gamma_v(G_P) = D_v(G_P)$.

Case (vi) If there exists more than one minimal PVDSs of G_P , then it implies $\Delta_{VW}(G_P) < \gamma_v(G_P) = D_v(G_P)$ if $\sum_{V \in D_{P_i}} P_{VW}(V) = c$ (constant) for every $D_{P_i} \in D_{PM}$. Otherwise $\Delta_{VW}(G_P) < \gamma_v(G_P) < D_v(G_P)$, since by Definition 3.6, $\min\{\sum_{V_i \in D_{P_i}} P_{VW}(V_i)\} = \gamma_v(G_P)$. Hence the proof.

3.2. Plithogenic edge domination

DEFINITION 3.21. Let $G_P = (V_P, E_P)$ be a PPFG. For any adjacent $E_i, E_j \in E_P$ in G_P , we say that E_i dominates E_j provided E_i is either a highly strong or strong P-edge such that $P_{EW}(E_j) \leq P_{EW}(E_i)$. Otherwise, the P-edge with a greater P-weight is said to dominate the other whenever both E_i and E_j are weak.

DEFINITION 3.22. A subset S_P of E_P is called a Plithogenic edge dominating set of G_P if for every $E_j \notin S_P$, there exists at least one $E_i \in S_P$ such that E_i dominates E_j .

DEFINITION 3.23. A Plithogenic edge dominating set S_P is called a minimal Plithogenic edge dominating set, if there is no Plithogenic edge dominating set S'_P of G_P such that $S'_P \subset S_P$.

DEFINITION 3.24. The Plithogenic edge domination number $\gamma_e(G_P)$, and the Plithogenic edge upper-domination number $D_e(G_P)$ are the minimum and the maximum Plithogenic fuzzy cardinalities taken over all minimal Plithogenic edge dominating sets of G_P .

DEFINITION 3.25. A minimum Plithogenic edge dominating set of G_P is a minimal Plithogenic edge dominating set with minimum Plithogenic fuzzy cardinality.

NOTE 3.26. Plithogenic edge domination doesn't exist in a PPFG, if there are adjacent weak P-edges such that they are neither adjacent to at least one highly strong or strong P-edge, nor the P-weights of both are distinct.

NOTE 3.27. In a disconnected PPFG, Plithogenic edge domination doesn't exist.

EXAMPLE 3.28. Let $G_P = (V_P, E_P)$ be a PPFG as shown in Figure 2, where $V_P = \{V_1, V_2, V_3, V_4, V_5, V_6\}$ and $E_P = \{E_1, E_2, E_3, E_4, E_5, E_6, E_7\}$.

By computation, $P_{VW}(V_1) = 2.6$; $P_{VW}(V_2) = 2.1$; $P_{VW}(V_3) = 2.4$; $P_{VW}(V_4) = 2.2$; $P_{VW}(V_5) = 2$ and $P_{VW}(V_6) = 1.9$. Average P-weight of P-vertices,

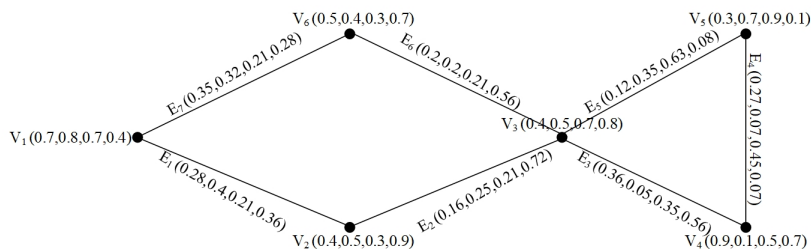


Fig. 2. Plithogenic edge domination in PPFG

$A_{VW}(G_P) = 13.2/6 = 2.2$. $P_{EW}(E_1) = 1.25$; $P_{EW}(E_2) = 1.34$; $P_{EW}(E_3) = 1.32$; $P_{EW}(E_4) = 0.86$; $P_{EW}(E_5) = 1.18$; $P_{EW}(E_6) = 1.17$; $P_{EW}(E_7) = 1.16$. Therefore, $\{E_1, E_2, E_3, E_5, E_6\}$, $\{E_1, E_2, E_3, E_5\}$, $\{E_1, E_2, E_5, E_6\}$, $\{E_1, E_2, E_3, E_6\}$, $\{E_2, E_3, E_6\}$, $\{E_2, E_5, E_6\}$, $\{E_1, E_2, E_5\}$ and $\{E_1, E_2, E_3\}$ are PEDSs of G_P . $\{E_2, E_3, E_6\}$, $\{E_2, E_5, E_6\}$, $\{E_1, E_2, E_5\}$ and $\{E_1, E_2, E_3\}$ are the minimal PEDSs of G_P .

The Plithogenic fuzzy cardinalities of $\{E_2, E_3, E_6\}$, $\{E_2, E_5, E_6\}$, $\{E_1, E_2, E_5\}$ and $\{E_1, E_2, E_3\}$ are 3.83, 3.69, 3.77 and 3.91 respectively. Thus, $\gamma_e(G_P) = 3.69$ and $D_e(G_P) = 3.91$. Thus $\{E_2, E_5, E_6\}$ is the minimum PEDS of G_P .

LEMMA 3.29. Let S_P be a Plithogenic edge dominating set of G_P . Then for any $E_i \in S_P$ if $S_P - \{E_i\}$ is a Plithogenic edge dominating set of G_P , then S_P is not a minimal Plithogenic edge dominating set of G_P .

The proof is obvious by Definition 3.23.

LEMMA 3.30. Let S_P be a minimal Plithogenic edge dominating set of G_P . Then for every $E_i \in S_P$, $S'_P = S_P - \{E_i\}$ is not a Plithogenic edge dominating set of G_P .

The proof is obvious by Definition 3.23.

LEMMA 3.31. If $G_P = (V_P, E_P)$ is a Plithogenic product fuzzy graph with distinct P-weights of P-edges, then every Plithogenic edge dominating set of G_P contains $E_j \in E_P$ such that $P_{EW}(E_j)$ is the maximum.

The proof is similar to that of Lemma 3.12 based on Definition 3.22.

REMARK 3.32. If $G_P = (V_P, E_P)$ is a Plithogenic product fuzzy graph with distinct P-weights of P-edges, every minimal Plithogenic edge dominating set of G_P contains $E_j \in E_P$ such that $P_{EW}(E_j)$ is the maximum.

THEOREM 3.33. Every minimal Plithogenic edge dominating set of a Plithogenic product fuzzy graph is a Plithogenic edge dominating set, but the converse need not be true.

PROOF. Let S_p be a minimal PEDS of G_p . Suppose S_p is not a PEDS of G_p , then there exists at least one P-edge $E_j \notin S_p$, which is not dominated by any element of S_p . This contradicts the fact that S_p is a minimal PEDS of G_p , i.e., for every $E_j \notin S_p$, there exists $E_i \in S_p$ such that E_i dominates E_j , and for every $E_i \in S_p$, $S_p - \{E_i\}$ is not a PEDS of G_p by Lemma 3.30. Therefore, S_p must be a PEDS of G_p . Conversely, let $E'_p = \{S_{p_1}, S_{p_2}, S_{p_3}, \dots, S_{p_k}\}$ be the collection of all PEDS of G_p .

Case (i) Suppose, for any $S_{p_i} \in E'_p$, if there exists at least one $E_i \in S_{p_i}$ such that $S_{p_i} - \{E_i\}$ is a PEDS of G_p . Then S_{p_i} is not a minimal PEDS of G_p by Lemma 3.29.

Case (ii) Suppose, for any $S_{p_i} \in E'_p$, if for every $E_i \in S_{p_i}$, $S_{p_i} - \{E_i\}$ is not a PEDS of G_p . Then S_{p_i} is a minimal PEDS of G_p by Lemma 3.30.

THEOREM 3.34. Every minimal Plithogenic edge dominating set of G_p need not be a minimum Plithogenic edge dominating set of G_p .

PROOF. Let $S_{PM} = \{S_{p_1}, S_{p_2}, S_{p_3}, \dots, S_{p_m}\}$ be the collection of all minimal PEDSs of G_p .

Case (i) If $\sum_{E_i \in S_{p_\delta}} P_{EW}(E_i) = c$ (constant) for every $S_{p_\delta} \in S_{PM}$ with $\delta = 1$ to m , then $\gamma_e(G_p) = c$. Therefore, every S_{p_δ} is a minimum PEDS of G_p .

Case (ii) If $\sum_{E_i \in S_{p_\delta}} P_{EW}(E_i)$ is distinct for every $S_{p_\delta} \in S_{PM}$ with $\delta = 1$ to m , then by Definition 3.25, $\gamma_e(G_p) = \min\{\sum_{E_i \in S_{p_\delta}} P_{EW}(E_i)\}$ where $S_{p_\delta} \in S_{PM}$ with $\delta = 1$ to m . Therefore the corresponding S_{p_δ} is the only minimum PEDS of G_p .

Case (iii) Suppose that there exists more than one S_{p_δ} 's having the same Plithogenic fuzzy cardinality. Let's say, $\sum_{W_i \in S_{p_1}} P_{EW}(W_i) = \sum_{Z_i \in S_{p_2}} P_{EW}(Z_i) = \tau$, where $S_{p_1}, S_{p_2} \in S_{PM}$. Then if $\tau = \min\{\sum_{E_i \in S_{p_\delta}} P_{EW}(E_i) : S_{p_\delta} \in S_{PM} \text{ with } \delta = 1 \text{ to } m\}$, then both S_{p_1} and S_{p_2} are minimum PEDSs of G_p . Otherwise, S_{p_δ} corresponding to the minimum of $\{\sum_{E_i \in S_{p_\delta}} P_{EW}(E_i)\}$ where $S_{p_\delta} \in S_{PM}$ with $\delta = 1$ to m , is the minimum PEDS of G_p .

THEOREM 3.35. For any connected Plithogenic product fuzzy graph G_p , $\Delta_{EW}(G_p) \leq \gamma_e(G_p) \leq D_e(G_p)$.

PROOF. Let $G_p = (V_p, E_p)$ be a connected PPFG with $V_p = \{V_1, V_2, \dots, V_n\}$ and $E_p = \{E_1, E_2, \dots, E_m\}$, $\gamma_e(G_p)$ be the PEDN of G_p , $\Delta_{EW}(G_p)$ be the maximum of P-weights of P-edges in G_p , $D_e(G_p)$ be the maximum Plithogenic fuzzy cardinality taken over all minimal Plithogenic edge dominating sets of G_p .

Case (i) When there exists only one minimal PEDS with exactly one P-edge, it is obvious that $\Delta_{EW}(G_p) = \gamma_e(G_p) = D_e(G_p)$ by Lemma 3.31.

Case (ii) When there exists only one minimal PEDS with more than one P-edge, then $\Delta_{EW}(G_p) < \gamma_e(G_p) = D_e(G_p)$ by Lemma 3.31.

Case (iii) Suppose that there exist more than one minimal PEDSs. Let $S_{PM} = \{S_{P_1}, S_{P_2}, S_{P_3}, \dots, S_{P_r}\}$ be the collection of all minimal PEDSs where every S_{P_i} contains E_j such that $P_{EW}(E_j)$ is the maximum in G_P . If $\sum_{E_i \in S_{P_i}} P_{EW}(E_i) = c$ (constant) for every $S_{P_i} \in S_{PM}$, then $\Delta_{EW}(G_P) < \gamma_e(G_P) = D_e(G_P)$. Otherwise $\Delta_{EW}(G_P) < \gamma_e(G_P) < D_e(G_P)$, since by Definition 3.25., $\min\{\sum_{E_i \in S_{P_i}} P_{EW}(E_i)\} = \gamma_e(G_P)$. Hence the proof.

3.3. Perfect Plithogenic domination

Perfect Plithogenic vertex domination and perfect Plithogenic edge domination are special cases of Plithogenic vertex domination and Plithogenic edge domination respectively.

DEFINITION 3.36. A Plithogenic vertex dominating set R_P is said to be a perfect Plithogenic vertex dominating set of $G_P = (V_P, E_P)$ if for every $V_j \in V_P - R_P$, there exists exactly one $V_i \in R_P$, such that V_i dominates V_j .

EXAMPLE 3.37. Consider a Plithogenic product fuzzy graph, $G_P = (V_P, E_P)$ as shown in Figure 3 where $V_P = \{V_1, V_2, V_3, V_4, V_5, V_6\}$ and $E_P = \{E_1, E_2, E_3, E_4, E_5\}$.

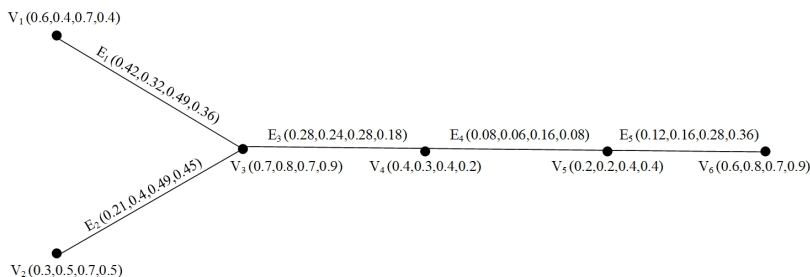


Fig. 3. Perfect Plithogenic vertex domination in PPFG

By computation, $P_{VW}(V_1) = 2.1$; $P_{VW}(V_2) = 2$; $P_{VW}(V_3) = 3.1$; $P_{VW}(V_4) = 1.3$; $P_{VW}(V_5) = 1.2$ and $P_{VW}(V_6) = 3$. Here, $R_P = \{V_3, V_6\}$ is a PVDS of G_P , and $V_P - R_P = \{V_1, V_2, V_4, V_5\}$. V_1, V_2 and V_4 are dominated by V_3 . V_5 is dominated by V_6 . Therefore, $R_P = \{V_3, V_6\}$ is a perfect PVDS set of G_P .

DEFINITION 3.38. A Plithogenic edge dominating set S_P is called a perfect Plithogenic edge dominating set of $G_P = (V_P, E_P)$ if for every $E_j \in E_P - S_P$, there exists exactly one $E_i \in S_P$, such that E_i dominates E_j .

EXAMPLE 3.39. Consider a Plithogenic product fuzzy graph, $G_P = (V_P, E_P)$ as shown in Figure 4 where $V_P = \{A, B, C, D, E, F, G, H, I\}$ and $E_P = \{K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8\}$. By computation, $P_{VW}(A) = 1.7$; $P_{VW}(B) = 3.2$; $P_{VW}(C) = 3$; $P_{VW}(D) = 1.4$;

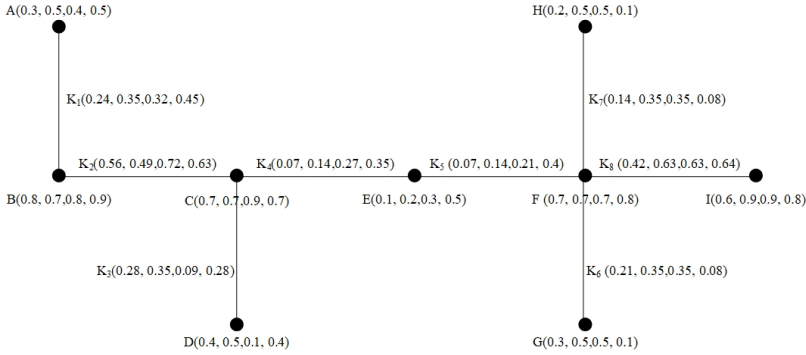


Fig. 4. Perfect Plithogenic edge domination in PPFG

$P_{VW}(E) = 1.1$; $P_{VW}(F) = 2.9$; $P_{VW}(G) = 1.4$; $P_{VW}(H) = 1.3$ and $P_{VW}(I) = 3.2$. Similarly, $P_{EW}(K_1) = 1.36$; $P_{EW}(K_2) = 2.4$; $P_{EW}(K_3) = 1$; $P_{EW}(K_4) = 0.83$; $P_{EW}(K_5) = 0.82$; $P_{EW}(K_6) = 0.99$; $P_{EW}(K_7) = 0.92$ and $P_{EW}(K_8) = 2.32$. Here, $S_P = \{K_2, K_8\}$ is a PEDS of G_P , and $E_P - S_P = \{E_1, E_3, E_4, E_5, E_6, E_7\}$. Since K_1, K_3 and K_4 are dominated by K_2 , and K_5, K_6 and K_7 are dominated by K_8 , $S_P = \{K_2, K_8\}$ is a perfect PEDS of G_P .

THEOREM 3.40. A perfect Plithogenic vertex dominating set is always a minimal Plithogenic vertex dominating set.

PROOF. Let $G_P = (V_P, E_P)$ be a PPFG and R_P be its perfect PVDS. i.e., for every $V_j \in V_P - R_P$, there exists exactly one $V_i \in R_P$ such that V_i dominates V_j . Suppose, R_P is not minimal PVDS of G_P . Then there exists at least one $V_r \in R_P$, such that $R_P - \{V_r\}$ is a PVDS by Lemma 3.10. This implies that there exists at least one element $V_s \in V_P - R_P$, such that V_s is dominated by more than one element of R_P including V_r . This is contradiction to our assumption, since R_P is a perfect PVDS of G_P . Therefore, R_P is a minimal PVDS.

REMARK 3.41. Converse of the theorem 3.40 need not be true. For example, consider the PPFPG in Figure 1. $D_{P_1} = \{V_2, V_3\}$ is its minimal PVDS. Since $V_1 \in V_P - D_{P_1}$ is dominated by both V_2 and V_3 , D_{P_1} is not a perfect Plithogenic vertex dominating set.

THEOREM 3.42. A perfect Plithogenic edge dominating set is always a minimal Plithogenic edge dominating set.

PROOF. Let $G_P = (V_P, E_P)$ be a PPFPG and S_P be its perfect PEDS, i.e., for every $E_j \in E_P - S_P$, there exists exactly one $E_i \in S_P$ such that E_i dominates E_j . Suppose, S_P is not minimal PEDS of G_P . Then there exists at least one $E_r \in S_P$, such that $S_P - \{E_r\}$ is a PEDS by Lemma 3.29. This implies that there exists at least one element $E_s \in E_P - S_P$, such that E_s is dominated by more than one element of S_P including E_r . This is a contradiction to our assumption, since S_P is a perfect PEDS of G_P . Therefore, S_P is a minimal PEDS.

REMARK 3.43. Converse of the theorem 3.42 need not be true. For example, consider the PPFPG in Figure 2. $S_{P_1} = \{E_1, E_2, E_3\}$ and $S_{P_2} = \{E_2, E_3, E_6\}$ are its minimal PEDSs. $E_P - S_{P_1} = \{E_4, E_5, E_6, E_7\}$ and $E_P - S_{P_2} = \{E_1, E_4, E_5, E_7\}$. Since E_5 and E_6 are dominated by both E_2 and E_3 , S_{P_1} and S_{P_2} are not perfect PEDSs.

An Algorithm to find the Plithogenic vertex domination number

Step 1: Let G_P be a PPFPG.

Step 2: Fix the attribute values to P-vertices and P-edges of G_P .

Step 3: Find the P-weights of P-vertices.

Step 4: Select any P-vertex V_i .

Step 5: If V_i dominates V_j which is any P-vertex adjacent to V_i , then collect V_i in D_{P_1} . If not, choose any other P-vertex and repeat the same process.

Step 6: Continue the process in step 5 till a PVDS (D_{P_1}) of G_P is obtained. If not, go to step 11.

Step 7: Follow the process in steps 4, 5 & 6 till all possible PVDSs (D_{P_i} 's) are obtained and collected in D_P .

Step 8: Collect all minimal PVDSs of G_P in D_{PM} from D_P .

Step 9: Find $|D_{P_i}|_f$ where $D_{P_i} \in D_{PM}$ and $i = 1$ to m .

Step 10: $\gamma_v(G_P) = \min\{|D_{P_i}|_f : D_{P_i} \in D_{PM} \text{ and } i = 1 \text{ to } m\}$, where the corresponding D_{P_i} is the minimum PVDS.

Step 11: Write that Plithogenic vertex domination doesn't exist.

NOTE 3.44. A similar algorithm can be formulated to find the PEDN of a PPFPG.

4. APPLICATION: SELECTION OF A SET OF REPRESENTATIVES

The following is an analysis of educational institutions of higher learning based on availability of transportation, infrastructure, competent personnel, accountability and social responsibility using the concept of Plithogenic domination in Plithogenic product fuzzy graphical model.

Suppose that a government wants to select a few Arts and Science Colleges from a particular district to implement its skill-development schemes for the benefit of all college students of that district in the shortest possible timeframe with the maximum efficiency. So, in order to make the best suitable selection, the government decides to analyse the Arts and Science Colleges in that district based on the following criteria: availability of transportation (A), infrastructure (B), competent personnel (C), accountability (D) and social responsibility (E). This problem is presented as a PPFG model as in Figure 5 with P-weights allotted to its units and the connectivities.

Let $V_1, V_2, V_3, V_4, V_5, V_6, V_7$ and V_8 represent Arts and Science colleges characterized by A, B, C, D and E respectively. These attributes associated with $V_1, V_2, V_3, V_4, V_5, V_6, V_7$ and V_8 , and the corresponding connectivities are assigned fuzzy values. The presence of a P-edge between a pair of institutions indicates that they are already networking and collaborating with each other. The P-weight of a P-edge displays the quality of the relation and the connectivity strength of the corresponding adjacent units in the network calculated based on the definition of PPFG. Table I shows the computation of the P-weights of P-vertices based on the attribute values allotted to the P-vertices for the corresponding attributes.

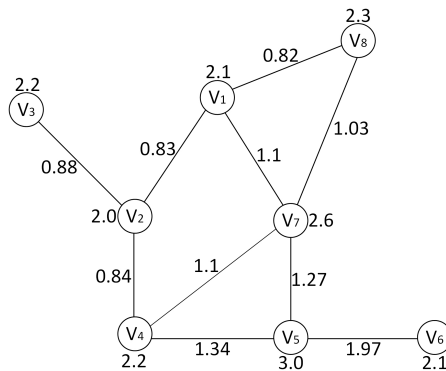


Fig. 5. Plithogenic product fuzzy graph with P-weights

Minimal Plithogenic vertex dominating sets represent the possible combinations of minimum number of colleges with the necessary requirements and potential to handle the needs and the demands of the schemes such that maximum number of students of that area benefit from the government welfare schemes.

Table I. Attribute values of P-vertices and their P-weights

Attributes	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
A	0.3	0.5	0.4	0.3	0.8	0.5	0.4	0.9
B	0.6	0.4	0.4	0.6	0.9	0.7	0.2	0.4
C	0.1	0.3	0.5	0.4	0.8	0.8	0.5	0.6
D	0.4	0.5	0.5	0.3	0.2	0.3	0.8	0.1
E	0.7	0.3	0.4	0.6	0.3	0.8	0.7	0.3
P-weights	2.1	2	2.2	2.2	3	3.1	2.6	2.3

Table II displays the calculation of attribute values for the P-edges in Figure 5 as well as their P-weights.

Table II. Attribute values of P-edges and their P-weights

Attributes	V_1V_2	V_1V_7	V_1V_8	V_2V_3	V_2V_4	V_4V_5	V_4V_7	V_5V_6	V_5V_7	V_7V_8
A	0.15	0.12	0.27	0.20	0.15	0.24	0.12	0.40	0.32	0.36
B	0.24	0.12	0.24	0.16	0.24	0.54	0.12	0.63	0.18	0.08
C	0.03	0.05	0.06	0.15	0.12	0.32	0.20	0.64	0.40	0.30
D	0.20	0.32	0.04	0.25	0.15	0.06	0.24	0.06	0.16	0.08
E	0.21	0.49	0.21	0.12	0.18	0.18	0.42	0.24	0.21	0.21
P-weights	0.83	1.1	0.82	0.88	0.84	1.34	1.1	1.97	1.27	1.03

Average P-weight of P-vertices is $= 19.5/8 = 2.4375 = 2.4$. Therefore, V_5, V_6 and V_7 are strong P-vertices. V_5V_6 a highly strong P-edge. V_4V_5, V_4V_7, V_1V_7 and V_7V_8 are strong P-edges. V_2V_4, V_2V_3, V_1V_2 and V_1V_8 are weak P-edges. As per the data in Tables I and II, minimal Plithogenic vertex dominating sets for Figure 5 are $\{V_3, V_5, V_7\}$ and $\{V_3, V_5, V_8\}$ with 7.8 and 7.5 as their Plithogenic fuzzy cardinalities respectively. Similarly, minimal Plithogenic edge dominating sets with their Plithogenic fuzzy cardinalities are as follows: $\{V_5V_6, V_4V_7, V_2V_3, V_1V_2\}$ with 4.78, $\{V_5V_6, V_4V_7, V_1V_7, V_2V_3\}$ with 5.05, $\{V_5V_6, V_5V_7, V_2V_3, V_1V_2\}$ with 4.95, $\{V_5V_6, V_5V_7, V_2V_3, V_1V_7\}$ with 5.22, $\{V_5V_6, V_4V_7, V_2V_3, V_7V_8\}$ with 4.98, $\{V_5V_6, V_1V_7, V_2V_3, V_7V_8\}$ with 4.98, $\{V_5V_6, V_5V_7, V_2V_3, V_7V_8\}$ with 5.15 and $\{V_5V_6, V_4V_5, V_2V_3, V_7V_8\}$ with 5.22.

This shows that $\{V_5V_6, V_5V_7, V_2V_3, V_1V_7\}$ and $\{V_5V_6, V_4V_5, V_2V_3, V_7V_8\} = 5.22$ have the best strength of connectivities. $\{V_3, V_5, V_7\}$ is the most suitable choice for a better networking and collaboration with the necessary requirements and potential to handle the needs and demands for the implementation of government schemes with the maximum success and performance in less amount of time, since the Plithogenic fuzzy cardinality of $\{V_3, V_5, V_7\}$ is greater than that of $\{V_3, V_5, V_8\}$.

5. CONCLUSION

Certain real-life situations are complex as they are affected by multiple factors. To understand and make decisions in such scenarios, we require a suitable tool that analyses them from multiple perspectives. Domination in Plithogenic product fuzzy graphical structures is one such useful mathematical tool in that regard. Depending upon the issues under consideration, a researcher may opt for either of the two types of dominations or both discussed in this paper. This would give a better analysis of the graph such as minimum number of dominating connectivities with minimum strength, minimum number of dominating objects with minimum strength, minimum number of dominating connectivities with maximum strength, minimum number of dominating objects with maximum strength, number of all possible dominating sets of objects, number of all possible dominating sets of connectivities, more significant and influential dominating objects or connectivities and so on. By comparing all these, more suitable and optimal solutions to the issues under consideration can be arrived at. Further research can be explored in this line by extending these domination parameters to the operations on PPFs and identifying the possible relationships between them.

REFERENCES

- AKRAM, M., WASEEM, N. AND DAVVAZ, B. 2017. Certain Types of domination in m-polar fuzzy graphs. *Journal of Multiple-Valued Logic & Soft Computing*, 29(6), 619-646.
- ARRIOLA, B. H. 2017. A note on some domination of fuzzy graphs. *Far East Journal of Applied Mathematics*, 113-123.
- BHARATHI, T. AND LEO, S. 2021. A Study on Plithogenic Product Fuzzy Graphs with Application in Social Network. *Indian Journal of Science and Technology*, 14(40), 3051-3063.
- BHARATHI, T., LEO, S. AND ANTONY VINOTH, S. 2022. Operations in Plithogenic Product Fuzzy Graphs. *South East Asian J. of Mathematics and Mathematical Sciences*, 21, 233-248.
- GANI, A.N. AND CHANDRASEKARAN, V.T. 2006. Domination in fuzzy graph, *Advances in Fuzzy Sets and Systems*, 1(1), 17-26.

-
- GANI, A. N. AND AHAMED, M. B. 2007. Strong and weak domination in fuzzy graphs. *East Asian mathematical journal*, 23(1), 1-8.
- GANI, A. N. AND DEVI, K. P. 2013. Edge domination and independence in fuzzy graphs. *Advances in Fuzzy Sets and systems*, 15(2), 73-84.
- GANI, A.N., MURUGANANTHAM, P. AND NAFIUNISHA, A. 2021. A new type of dominating fuzzy graphs. *Advances and Applications in Mathematical Sciences*, 20(6), 1085-1091.
- GANI, A. N. AND VADIVEL, P. 2011. A study on domination, independent domination and irredundance in fuzzy graph. *Applied Mathematical Sciences*, 5(47), 2317-2325.
- ISMAIL MOHIDEEN, S. AND MOHAMED ISMAYIL, A. 2010. Domination in fuzzy graph: a new approach. *International Journal of Computational Science and Mathematics*, 2(3), 101—107.
- KARUNAMBIGAI, M. G., M. AKRAM, K. PALANIVEL AND S. SIVASANKAR. 2013. Domination in bipolar fuzzy graphs, *IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, 1-6.
- KARAASLAN, F. AND DAVVAZ, B. 2018. Properties of single-valued neutrosophic graphs. *Journal of Intelligent & Fuzzy Systems*, 34(1), 57-79.
- MANJUSHA, O. T. AND SUNITHA, M. S. 2014. Notes on domination in fuzzy graphs. *Journal of Intelligent & Fuzzy Systems*, 27(6), 3205-3212.
- MANJUSHA, O. T. AND SUNITHA, M. S. 2014. Total domination in fuzzy graphs using strong arcs. *Annals of Pure and Applied Mathematics*, 9(1), 23-33.
- MANJUSHA, O. T. AND SUNITHA, M. S. 2015. Strong domination in fuzzy graphs. *Fuzzy Information and Engineering*, 7(3), 369-377.
- NATARAJAN, C. AND AYYASWAMY, S. K. 2010. On strong (weak) domination in fuzzy graphs. *International Journal of Mathematical and Computational Sciences*, 4(7), 1035-1037.
- PAL, M., SAMANTA, S. AND GHORAI, G. 2020. *Modern trends in fuzzy graph theory*, Berlin: Springer.
- PONNAPPAN, C. Y., MUTHURAJ, R. AND AHAMED, S. B. 2017. Some new kinds of domination parameters in fuzzy graph. *International Journal of Computational and Applied Mathematics*, 12(1), 156-167.
- PRASSANNA D JEYANTHI. 2019. Domination in fuzzy graphs. *International Journal of Advanced Research in Engineering and Technology (IJARET)*, 10(6), 442-447.
- RANA, A. 2021. A survey on the domination of fuzzy graphs. *Discrete Mathematics, Algorithms and Applications*, 13(01).
- RAMYA, S. AND LAVANYA, S. 2016. Perfect vertex (edge) domination in fuzzy graphs. *Indian Journal of Science and Technology*, 9(4),1-5.
- REVATHI, S., JAYALAKSHMI, P. J. AND HARINARAYANAN, C. V. R. 2013. Perfect dominating sets in fuzzy graphs. *IOSR Journal of Mathematics*, 8(3), 43-47.
- SOMASUNDARAM, A. AND SOMASUNDARAM, S. 1998. Domination in fuzzy graphs–I. *Pattern Recognition Letters*, 19(9), 787-791.
- SOMASUNDARAM A. 2005. Domination in products of fuzzy graphs, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 13(02), 195-204.
- SELVAM, A. AND PONNAPPAN, C. Y. 2020. A note on edge domination in fuzzy graphs. *Malaya Journal of Matematik (MJM)*, 8(4), 2000-2004.
- XAVIOR, D. A., ISIDO, F. AND CHITRA, V. M. 2013. On domination in fuzzy graphs. *International Journal of Computing Algorithm*, 2(2), 81-82.

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