



REVISITING THE 4% WITHDRAWAL RULE USING MONTE CARLO SIMULATIONS WITH RANDOM MARKET DECLINES

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ABSTRACT

This paper tracks the performance of a hypothetical retirement portfolio valued at \$1,000,000 by applying the popular 4% rule of thumb withdrawals. These withdrawals are adjusted annually to account for simulated inflation and market return rates. Additionally, we incorporate different market shocks that resemble “black swan” events into our analysis. Commencing at a retirement age of 64 and adopting a 30-year retirement planning horizon, we employ Monte Carlo simulations to compute the final value of the portfolio at age 93 under various market shocks. These events occur randomly within the 30-year planning horizon. The average ending portfolio balance and the probability of fund depletion are reported, considering diverse portfolios with a range of asset allocations divided between stocks and bonds. The study's results demonstrate that a portfolio with a higher allocation to equity can yield a superior average ending portfolio balance while reducing the risk of fund depletion.



Introduction

There has been a variety of research focusing on the amount an individual can withdraw from their retirement fund without risking running out of money. Bengen (1994) introduced the “4-percent rule,” which posits that retirees could safely spend about 4% of their retirement savings in the first year of retirement and increase this withdrawal amount by an inflation rate in each subsequent year. For example, with a retirement nest egg of \$1,000,000, a retiree can withdraw \$40,000 in the first year and increase this amount to \$41,200 based on an inflation rate of 3% the following year. Bengen’s findings show that using this 4% percent withdrawal rule, portfolios can last about 35 years or longer with an asset allocation of about 60% stocks and 40% bonds based on actual historical returns.

However, is the 4% withdrawal rule still a viable strategy, especially when ‘stress tested’ against random black swan events? In this study, we examine the performance of a hypothetical portfolio of a \$1,000,000 retirement fund, invested in equities and bonds, starting at age 64 and using the 4% rule of thumb yearly withdrawal adjusted for yearly inflation rates. Monte Carlo simulations are used to simulate historical market returns that include portfolio declines (i.e., black swan events) occurring randomly within the 30-year retirement planning horizon. The portfolio declines are intended to resemble “black swan” events (Taleb 2008) or market shocks that are unpredictable and can result in extreme consequences (e.g., the 9/11 Attacks, the 2008 Global Financial Crisis, and the COVID-19 Pandemic). In models analyzing retirement funds, it is prudent to include ‘black swan’ events, which are rare and unpredictable but often overlooked in existing studies. The reason for this inclusion is the unpredictable impact these events can have on financial outcomes. Black swan events, characterized by their extreme rarity and significant consequences, can cause substantial portfolio declines. This study, which uniquely incorporates black swan events—a key motivation of this study—addresses a gap in much of the existing literature. It contributes to a more comprehensive and realistic understanding of retirement planning and asset allocation strategies under uncertainty.

Reducing the risk of financial ruin in retirement is a fundamental objective of retirement planning. A key advantage of reducing the risk of financial ruin is the assurance of financial security throughout one’s retirement years. One crucial aspect of risk reduction involves diversifying the investment portfolio. Portfolio diversification plays a crucial role in mitigating the risk of financial ruin for investors, especially in enhancing the resilience of retirees’ investments to unforeseen events and market shocks. By examining the relationship between portfolio diversification, unpredictable market crashes, and the 4% withdrawal rate, we aim to provide valuable insights that can help retirees make informed decisions about their investment portfolios. Furthermore, the findings of this study aim to identify the most resilient portfolio allocations, specifically in scenarios involving simulated market crashes, which are frequently overlooked in traditional analyses. By incorporating these black swan events, the study seeks to provide a more comprehensive understanding of how different allocation strategies using the 4% rule perform.

In this study, we address the following research questions:

1. Is the 4% withdrawal rule still viable under simulated hypothetical market shocks, and relatedly, which asset allocation offers the lowest probability of financial ruin?
2. Which asset allocation, considering the 4% withdrawal rule and simulated hypothetical market shocks, yields the highest average ending value for the portfolio?



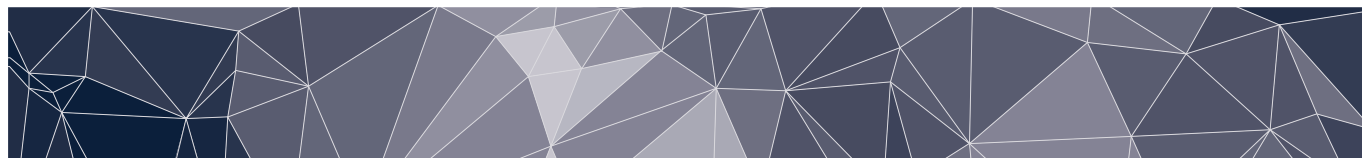
The two research questions are closely related but focus on different aspects of retirement portfolio management under the 4% withdrawal rule and in the context of hypothetical market shocks. The first research question examines the robustness of the 4% withdrawal rule and addresses a primary goal for many retirees, which is to ensure they have enough income to maintain their standard of living, cover healthcare expenses, and enjoy their retirement years without the risk of outliving their assets. This goal focuses on maximizing the utility or satisfaction retirees derive from their wealth during their lifetime. The second research question focuses on maximizing the potential value of the portfolio at the end of the retirement period. It inherently involves a different risk-return trade-off compared to the first question, especially for retirees who prioritize leaving an inheritance for their heirs. This objective often involves strategies to preserve and grow the principal amount of their retirement savings, sometimes at the expense of their current consumption. Using panel data of elderly households, Kupczuk (2007) found that, on average, households with a bequest motive spend about 25% less on personal expenditures.

While the first research objective focuses on prioritizing safety and preservation of capital, the second research question is more return-oriented, focusing on portfolio growth and legacy planning (i.e., leaving money to heirs or charities). For many retirees, ensuring that they can leave a legacy is as important as maintaining their lifestyle in retirement, thus allowing financial planners to take bequest intentions into account when advising clients on retirement planning (Ying and Russell 2023). Lee and Tan (2022) assert that even though depleting one's financial reserves is most beneficial as one approaches the latter stages of life, a significant number of retirees prefer to maintain substantial asset reserves to leave an inheritance. By shedding insights on these key questions, this study aims to enhance retirees' financial resilience in the face of market crashes and seeks to contribute to the field by offering a comprehensive analysis of the implications of financial ruin for retirees and the potential benefits of portfolio diversification in mitigating these risks.

The remainder of this paper is structured as follows: The subsequent sections consist of a literature review focusing on the 4% withdrawal rate and a detailed description of the methodology employed to construct the simulation model. The ensuing results section highlights key findings pertaining to the average ending value of the portfolio and the probability of fund depletion, encompassing different stock-bond allocations and portfolio declines. Finally, the concluding section offers recommendations from the study's outcomes and provides direction for future research endeavors.

Literature Review

The literature on retirees' portfolios, market crashes, and the 4% withdrawal rate reveals valuable insights into the challenges and strategies associated with retirement planning. One key area of focus has been the concept of the 4% withdrawal rate, which suggests that retirees can withdraw 4% of their initial portfolio balance annually, adjusted for inflation, to maintain a sustainable income. The appeal of the 4% withdrawal rule in retirement planning is its simplicity. It provides a straightforward method for individuals to estimate their retirement income needs without the need for complex calculations. The 4% withdrawal rule, a fixed self-managed drawdown strategy, offers several advantages. Level yearly withdrawals adjusted for inflation, along with convenient



budgeting, have been cited as key advantages to fixed self-managed withdrawal strategies (MacDonald et al. 2013). The 4% rule is analogous to a passive investment strategy, involving less frequent transactions and, consequently, lower transaction costs.

In the context of retirement planning, the 4% withdrawal rule has been subject to extensive examination, not only in its traditional, fixed format but also under dynamic withdrawal strategies. Research in this area suggests that adapting withdrawal rates in response to market performance and personal financial circumstances may enhance the sustainability and efficiency of retirement portfolios. This body of work indicates that dynamic strategies, which adjust withdrawals based on market conditions and individual retirement goals, may offer a more nuanced and flexible approach to retirement income planning compared to the fixed 4% rule. Salter (2013) suggests that a “buy and manage” strategy, in contrast to a “buy and hold” approach, offers improved control over spending. This adaptability is especially beneficial in response to significant economic shifts, enhancing the effectiveness of the 4% withdrawal rule in varying market conditions. Drew and Walk (2015, p. 31) assessed the 4% rule’s safety using historical international return data exploring different scenarios and asset allocations and contend that “the 4% Rule does present us with an opportunity to form a baseline which can dramatically improve our framing of expectations of what’s possible in retirement.” However, they suggest more dynamic approaches to withdrawal rates and retirement planning, considering the complex nature of financial markets and individual circumstances. Brown (2023) challenges the 4% rule’s reliability, highlighting its failure in 14.6% of scenarios over 35 years, based on global returns and a portfolio of 60% stocks and 40% bonds with 5% in precious metals. He proposes a dynamic withdrawal strategy, which adapts to market conditions and the retiree’s age, delivering better standards of living and fully utilizing the retirement portfolio. Brown (2023) recommends a highly aggressive strategy of 100% allocation to stocks, where 50% is allocated to U.S. stocks and 50% to non-U.S. stocks. Our approach is more conservative in that allocations are made to stocks and bonds and is supported by research from Sapra, Klein, and Martel (2023). They suggest that income-oriented investors should favor a portfolio with 60% allocated to stocks and 40% to bonds. Our view is that in retirement, it is appropriate for individuals to have an income bias.

A research study by Pfau (2011) highlighted that the sustainability of the 4% withdrawal rule depends on various factors, such as the length of the retirement period, asset allocation, and market conditions. Moreover, Pfau emphasized that the 4 percent rule might not be appropriate for all retirees and proposed a framework for determining the ideal savings rate based on these factors. Finke, Pfau, and Blanchett (2013) explored the challenges of the 4 percent rule in a low-interest-rate environment and suggested alternative withdrawal strategies that consider current market conditions and their impact on sustainable withdrawal rates. Their findings suggested that the 4 percent rule might be less effective in sustaining retirement income when interest rates are low and recommended alternative withdrawal strategies, such as dynamic withdrawal methods or adjusting the withdrawal rate based on prevailing interest rates, to address the limitations of the 4 percent rule in such conditions.

Stein (1998) emphasized that the amount withdrawn will likely not only be impacted by the inflation rate but also by lifestyle and proposes varying expenditures based on three phases: “active budget phase” (through age 75), “transition budget phase” (age 75 through age 85), and “passive budget phase” (beyond age 85). In the active budget phase, Stein argued that



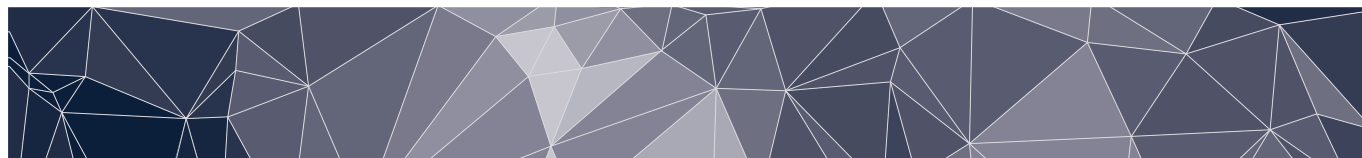
expenditures are at their peak since retirees would be most active, while during the transition budget phase expenses decrease due to possible worsening health and declined activity. In the passive budget phase, Stein stated that expenditures may be like those of the transition budget phase because of fewer discretionary expenditures. To allow for variable withdrawal rates, Bengen (2001) suggested alternative withdrawal strategies: a “Prosperous Retirement” model that utilizes larger withdrawals early in retirement and a “Performance-based” model that relates withdrawals to portfolio performance. Despite the popularity of the 4% rule, Coaching et al. (2013) argued that putting too much weight on wealth depletion ignores the lost potential enjoyment from spending more early in retirement. Chen, Schelling, and Sorensen (2023) explored the impact of low interest rates on retirement withdrawal strategies, emphasizing the need for retirees to manage their savings in ways that ensure long-term financial stability. It critically examines common self-managed withdrawal rules and proposes a mixed rule, combining fixed percentage and remaining lifetime approaches, as a superior option for enhancing retirees’ welfare within an expected utility framework. Notably, the utility model used excludes bequest motives, focusing solely on consumption as the generator of utility, thus not accounting for the utility derived from bequeathing wealth to heirs. This approach implies that the model’s primary concern is maximizing retirees’ lifetime consumption utility, rather than preserving wealth for inheritance purposes.

A significant gap in previous literature is the failure to include “random” market declines in the analysis. Market crashes or “black swan events” can have a significant impact on retirement portfolios and the success of withdrawal strategies as the occurrence of market crashes has been a recurrent theme in financial markets throughout history. Numerous studies have explored the causes of market crashes, highlighting factors such as speculative bubbles, financial imbalances, economic recessions, and bank failures (Aliber, Charles, and Robert 2015; Reinhart, and Rogoff’ 2009). This study introduces a novel approach to examining the 4% withdrawal rule by explicitly incorporating simulated market crashes into its analysis, a factor often overlooked in similar research. By integrating these simulations, the research provides a more comprehensive assessment of the rule’s viability under extreme financial conditions. This inclusion not only bridges a significant gap in the existing literature but also offers practical insights for financial planning in uncertain economic environments. The enhanced model aims to equip investors and advisors with a more resilient strategy, adapting to both theoretical expectations and real-world exigencies.

In the next section, we employ a rigorous methodology to analyze the performance and sustainability of a hypothetical retirement portfolio utilizing the popular 4% rule of thumb withdrawals. Our analysis incorporates various market shocks, simulating black swan events, to assess their impact on the portfolio’s final value.

Methodology

For each portfolio under consideration, we simulate the occurrence of two, three, and four market shocks, each representing a portfolio decline of 5% that occurs randomly within the 30-year planning horizon. We increase each market shock from 5% to 15% in increments of 5% to investigate worsening scenarios. Simulated inflation rates, rather than a fixed average rate,



are used in adjusting yearly retirement withdrawals. Simulating varying inflation rates provides a more realistic representation of understanding the potential risks and impacts of inflation on investments.

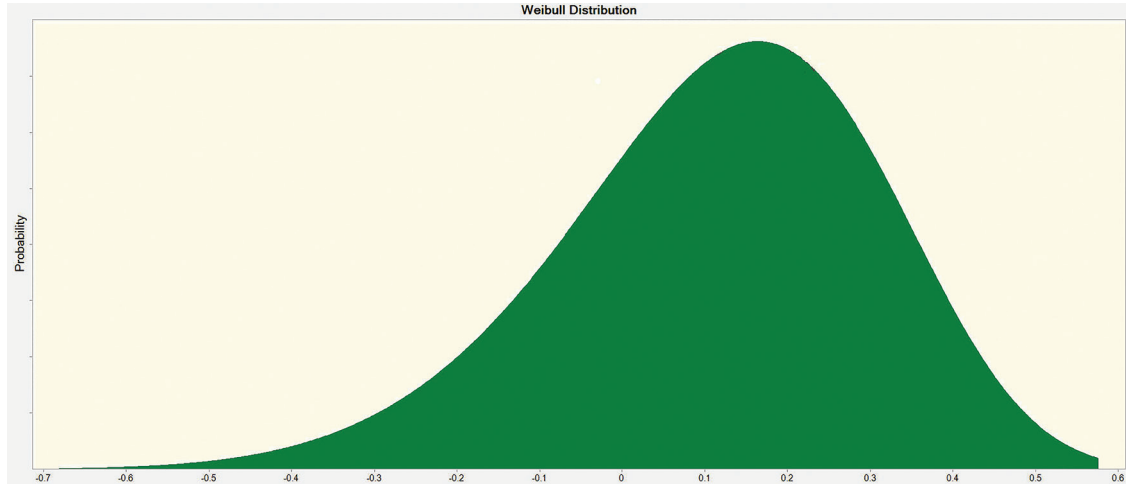
We initially assume 50% of the retirement fund is invested in equities (stocks) and 50% in fixed-income assets (e.g., bonds). The allocation is based on research findings by Bengen (1994) and Milevsky (2001) that recommend the portfolio should contain 50% to 75% common stocks to boost sustainability. We increase the percentage allocation of stocks from 50% to 70% in increments of 10% to create the different portfolios and report key performance metrics such as the average ending value of the retirement portfolio along with the probability of the fund running out of money (i.e., financial ruin). Based on each simulated model under consideration, we generate a probability distribution for the ending value of the retirement fund at age 93. Using this distribution, we compute the probability of the fund being depleted (i.e., zero balance). It should be noted that financial ruin depends on various factors such as the severity and frequency of market crashes, the allocation of investments, the withdrawal rate, and the length of retirement. Moreover, we assume a tax-deferred portfolio, such as a traditional IRA or 401(k), in which withdrawals are taxed and portfolio management fees are not considered.

To accurately reflect the uncertainties prevalent in financial markets, our approach employs Monte Carlo simulations. This method allows us to create a range of scenarios by incorporating market shocks of varying magnitudes and frequencies, which randomly occur throughout the 30-year planning horizon. Through these simulations, we determine the average ending balance of the portfolio and assess the probability of fund depletion under various asset allocation strategies between stocks and bonds with a focus on the efficacy of the 4% withdrawal rule. Our analysis will concentrate on fixed withdrawal strategies, specifically including considerations for black swan events, given that the 4% rule continues to be widely recommended by financial planners for its straightforwardness, rather than exploring dynamic withdrawal methods.

To identify the most appropriate distribution for each asset, we utilize the 'Batch Fit' feature in Oracle Crystal Ball software (available at www.oracle.com/applications/crystalball/). This tool conducts a range of statistical goodness-of-fit tests, including Kolmogorov-Smirnov, Anderson-Darling, and Chi-square tests, to ascertain the distribution that best fits the data. Based on S&P 500 historical returns (adjusted for dividends and obtained from <https://pages.stern.nyu.edu/~adamodar/>) from 1928 through 2022, we find the Weibull distribution (mean of 11.60% and a standard deviation of 18.77%) to be the best fitting model to simulate yearly equity growth rates based on the Anderson-Darling (A-D) goodness of fit test (see Figure 1). The Anderson-Darling (AD) test is considered superior to other goodness-of-fit tests like the Kolmogorov-Smirnov (KS) test and the Chi-square test as it is more effective in detecting extreme values in the tails of distributions, which is essential for accurate modeling in fields like finance and risk management where extremes are important (Razali and Wah 2011).

As illustrated in Figure 1, the distribution is skewed to the left which indicates the presence of a few large negative return events. This distribution better depicts the fit of the historical asset returns than a normal distribution that is symmetric around its mean and does not account for the possibility of extreme events such as crashes. Cont (2001, p. 227) argued that 'One of the important characteristics of financial time series is their high variability, as revealed by the

Figure 1: Weibull Distribution of S&P 500 Historic Returns.



Note: the Weibull distribution (mean = 11.60%, standard deviation = 18.77%) is used to simulate yearly equity growth rates. This distribution is skewed to the left, which allows for the simulation of large negative values, thereby mimicking historical returns.

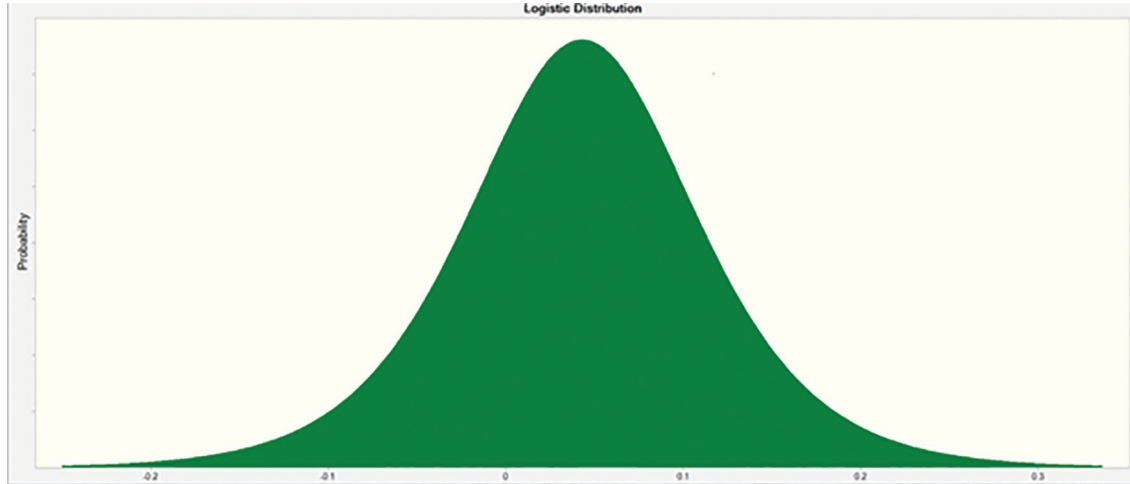
heavy-tailed distributions of their increments and the non-negligible probability of occurrence of violent market movements'. Although the normal distribution is commonly used in the industry due to its simplicity, the Weibull distribution better depicts the shape of the S&P 500 historic returns that are negatively skewed. The Weibull distribution allows for greater flexibility in capturing skewness and kurtosis in the data compared to the symmetric bell-shaped normal distribution. Market returns often exhibit fat tails, which the Weibull distribution can better capture by removing the symmetry constraint (Athavale and Goebel 2011). Moreover, market returns occasionally experience extreme events, such as market crashes or significant price jumps. These events are not well-captured by the normal distribution, which assumes that extreme events are highly unlikely, especially since financial data tend to display extreme price fluctuations (Longin 2005). The Weibull distribution can provide a better fit for such extreme events due to its ability to model heavy tails.

The bond allocations are equally distributed between the following investment grade bonds: US T-bonds and US Baa Corporate bonds. Using returns from 1928 to 2022, we find the Logistic distribution to be the best-fitting model for simulating returns in fixed-income assets (US T-Bonds: mean = 4.40%, standard deviation = 7.39%; US Baa Bonds: mean = 6.75%, standard deviation = 7.33%) based on the A-D goodness of fit test (see Figures 2 and 3). The shape of the logistic distribution is similar to that of the normal distribution but with relatively longer tails. A probability distribution with longer tails implies that there is a higher probability of observing extreme values in both directions.

To simulate inflation rates, we establish the following assumptions based on historical data spanning from 1928 to 2022. To avoid simulating negative random rates that would result in reduced withdrawal rates from previous years, we exclude negative inflation rates from our calculations. Based on these adjustments, we find the lognormal distribution to be the best-fitting model to simulate yearly inflation rates based on the A-D goodness of fit test (see Figure 4). As illustrated in Figure 4, the distribution is positively skewed, with a mean inflation rate of 3.70% and a standard deviation of 2.66%. Using simulated inflation rates rather than a fixed rate in adjusting retirement withdrawals offers several advantages. For example,

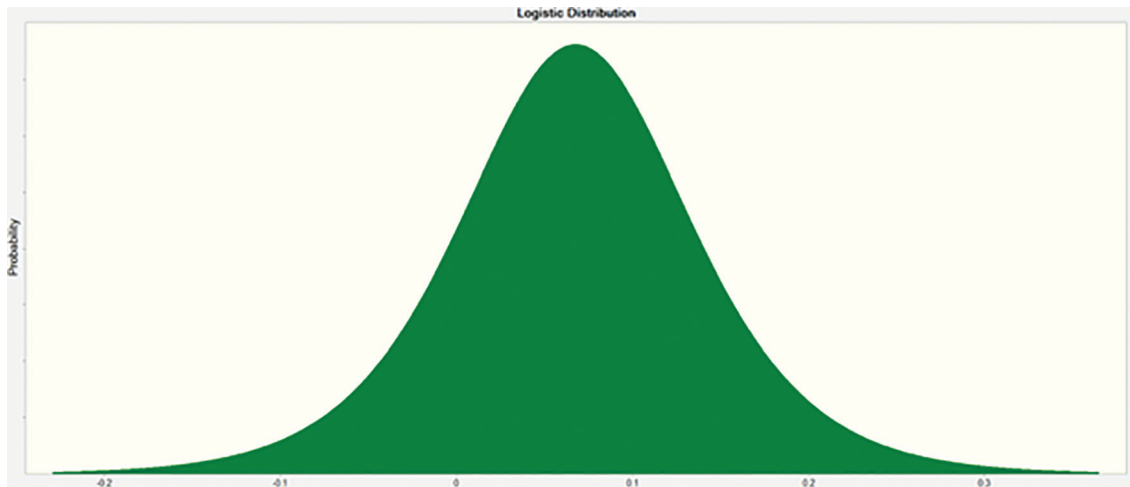


Figure 2: Logistic Distribution of US T-Bonds Historic Returns.



Note: the Logistic distribution (mean = 4.40%, standard deviation = 7.39%) is used to simulate US T-Bonds returns. The shape of the logistic distribution is similar to that of the normal distribution but with relatively longer tails.

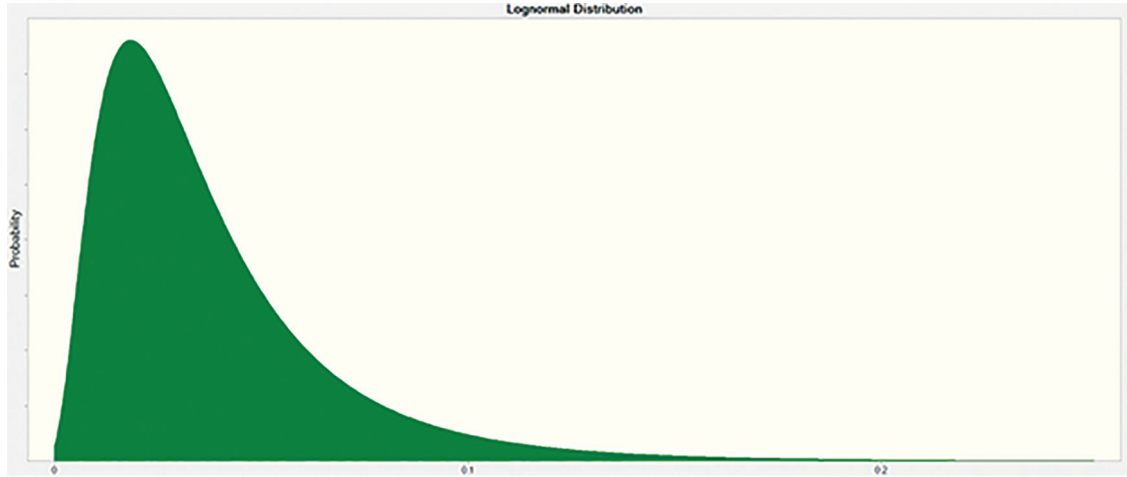
Figure 3: Logistic Distribution of Baa Bonds Historic Returns.



Note: Like the distribution of US T-bonds, the Logistic distribution (mean = 6.75%, standard deviation = 7.33%) is used to simulate Baa bonds returns.

simulated inflation rates provide a more realistic representation of the actual inflation experienced in the economy. Moreover, simulated inflation rates allow for an adaptive withdrawal strategy that responds to the prevailing inflation environment.

In our simulation process, we utilize bounded probability distributions when generating random values for equity, bonds, and inflation rates. These distributions are based on historical data and set to reflect the minimum and maximum values observed in the actual market data. For instance, when simulating the market returns of the S&P 500, we restrict the random values generated from a Weibull distribution to range between a minimum of -44% and a maximum of 53%. By incorporating these bounded distributions, we ensure that our simulations capture the realistic range of potential outcomes observed in historical market data. Creating a bounded probability distribution has several advantages over an unbounded distribution. Specifically,

Figure 4: Lognormal Distribution of Historic Inflation Rates.

Note: The lognormal distribution is used to simulate inflation rates. The distribution is positively skewed, with a mean inflation rate of 3.70% and a standard deviation of 2.66%. Simulated inflation rates allow for an adaptive withdrawal strategy that responds to the prevailing inflation environment.

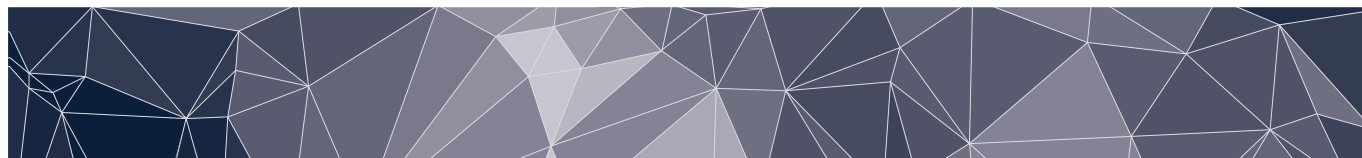
Table 1: Pairwise Pearson Correlation Coefficients

Sample 1	Sample 2	Correlation (r)	P-Value
UST. Bond	S&P500	0.02	0.87
Baa Bonds	S&P500	0.40	0**
Inflation	S&P500	-0.16	0.12
Baa Bonds	US T. Bond	0.60	0**
Inflation	US T. Bond	-0.09	0.42
Inflation	Baa Bonds	-0.20	0.05*

Note: *Significant at the 0.05 level, **significant the 0.01 level.

using a bounded distribution ensures that the generated simulated values align with the actual historical returns and reduces the risk of unrealistic extremes that may skew the analysis or predictions.

The correlation between equity returns, bond returns, and inflation rates is important in understanding the dynamics of financial markets. To explore these relationships, Table 1 presents Pearson's pairwise correlations among historical asset returns and inflation rates. Notably, we observe statistically significant positive correlations at the 0.01 level between Baa Bonds and S&P 500 returns ($r = 0.40$), as well as between Baa Bonds and US T-Bonds returns ($r = 0.60$). Additionally, we find a statistically significant negative correlation at the 0.05 level between inflation rates and Baa bonds ($r = -0.20$). Given the statistically significant dependencies among the aforementioned variables, we generate correlated simulated random returns based on the correlation coefficients observed in Table 1. By introducing correlation into the simulated random values, we enhance the accuracy of our simulations and effectively capture these interrelationships. Moreover, this alleviates a much-cited criticism (Nawrocki 2001, p. 92) that 'the typical assumption set used in Monte Carlo simulation assumes normal distributions and correlation coefficients of zero, neither of which are typical in the world of financial markets.'



To start, withdrawals of \$40,000, adjusted for inflation, are initiated at age 64, and they continue annually until age 93, representing a 30-year planning horizon. Using Monte Carlo simulations, we generate 10,000 retirement fund balances at the end of this period for different allocation scenarios. The simulations are based on nominal simulated returns of equities and fixed-income assets, employing the Weibull and Logistic distributions, respectively. Additionally, we introduce random market shocks, or portfolio declines, occurring with frequencies of 2, 3, and 4 using various asset allocations during the 30-year planning horizon. We examine different frequencies of portfolio declines, beginning with a 5% decline and subsequently increasing to 10% and 15%. These decline rates are applied to various asset allocations: 50% equity – 50% bonds, 60% equities – 40% bonds, and 70% equities – 30% bonds. The proportion allocated to bonds is evenly divided between Baa bonds and US T-bonds. The market shocks are intended to resemble unpredictable and impactful events and alleviate criticism that Monte Carlo analysis cannot accurately factor infrequent market crashes into its probability analysis. The simulated nominal returns are not adjusted to real returns because the withdrawal rates already consider inflation rates. Latin Hypercube design sampling is used as it provides a more efficient and representative sampling compared to simple random sampling. Latin Hyper Cube sampling is especially effective for uncertainty and sensitivity analyses in model predictions because it samples across the entire range of possible values. In specific, all parts of the distribution are likely to be represented in the sample, leading to a more thorough exploration of the space. The same sequence of random asset returns, and initial seed is used in running the simulation models. Such a setting offers several advantages, particularly in the context of reproducibility, comparability, and analysis. When comparing different simulation scenarios or models, using the same sequence of random numbers ensures that the differences in outcomes are due to the changes in the model or parameters and not due to variations in the random number sequence.

Figure 5 presents an illustrative instance of a single simulation run for a 70% equities – 30% bonds allocation. In this example, a 15% portfolio decline is applied to the ending balance at simulated ages 67 and 89, assuming the occurrence of two market crashes. The returns of the remaining years are based on simulated historic returns. The ending balance is calculated by deducting the withdrawal amount from the beginning balance and then adjusting it with the simulated returns drawn from the respective stipulated distribution (i.e., based on the reflected displayed mean of each distribution). However, this adjustment does not apply when randomly selected portfolio declines (i.e., black swan events) take place. In this example, which is based on a single trial, following the initial withdrawal at age 67, the portfolio experiences a 15% decrease in value, resulting in an ending balance of \$959,404.57 (i.e., $(\$1,173,315.74 - \$44,604.48) \times .85 = \$959,404.57$).

Results

Summary results for various portfolio declines, allocations, and market crashes based on a 30-year retirement planning horizon are shown in Table 2. As illustrated in Table 2, with an assumed two market crashes for a portfolio allocation of 50% stocks and 50% bonds, as the



Figure 5: Single Simulation Run with Two Market Crashes and 15% Portfolio Decline in Each Instance.

Age	Beginning Balance	Withdrawal	Ending Balance	SP500 Return	T-Bond Return	Baa Return	Inflation	Ending Balance Adjusted	Withdrawal	Portfolio % Decline
64	\$1,000,000.00	\$40,000.00	\$1,054,007.91	11.60%	4.40%	6.75%	3.70%	\$1,054,007.91	\$40,000.00	15%
65	\$1,054,007.91	\$41,479.43	\$1,111,680.24	11.60%	4.40%	6.75%	3.70%	\$1,111,680.24		
66	\$1,111,680.24	\$43,013.59	\$1,173,315.74	11.60%	4.40%	6.75%	3.70%	\$1,173,315.74		
67	\$1,173,315.74	\$44,604.48	\$959,404.57	11.60%	4.40%	6.75%	3.70%	\$959,404.57		
68	\$959,404.57	\$46,254.21	\$1,002,570.52	11.60%	4.40%	6.75%	3.70%	\$1,002,570.52		
69	\$1,002,570.52	\$47,964.97	\$1,048,085.22	11.60%	4.40%	6.75%	3.70%	\$1,048,085.22		
70	\$1,048,085.22	\$49,738.99	\$1,096,109.19	11.60%	4.40%	6.75%	3.70%	\$1,096,109.19		
71	\$1,096,109.19	\$51,578.63	\$1,146,816.13	11.60%	4.40%	6.75%	3.70%	\$1,146,816.13		
72	\$1,146,816.13	\$53,486.31	\$1,200,394.04	11.60%	4.40%	6.75%	3.70%	\$1,200,394.04		
73	\$1,200,394.04	\$55,464.54	\$1,257,046.62	11.60%	4.40%	6.75%	3.70%	\$1,257,046.62		
74	\$1,257,046.62	\$57,515.95	\$1,316,994.60	11.60%	4.40%	6.75%	3.70%	\$1,316,994.60		
75	\$1,316,994.60	\$59,643.22	\$1,380,477.40	11.60%	4.40%	6.75%	3.70%	\$1,380,477.40		
76	\$1,380,477.40	\$61,849.18	\$1,447,754.77	11.60%	4.40%	6.75%	3.70%	\$1,447,754.77		
77	\$1,447,754.77	\$64,136.72	\$1,519,108.72	11.60%	4.40%	6.75%	3.70%	\$1,519,108.72		
78	\$1,519,108.72	\$66,508.87	\$1,594,845.55	11.60%	4.40%	6.75%	3.70%	\$1,594,845.55		
79	\$1,594,845.55	\$68,968.76	\$1,675,298.13	11.60%	4.40%	6.75%	3.70%	\$1,675,298.13		
80	\$1,675,298.13	\$71,519.63	\$1,760,828.37	11.60%	4.40%	6.75%	3.70%	\$1,760,828.37		
81	\$1,760,828.37	\$74,164.84	\$1,851,829.90	11.60%	4.40%	6.75%	3.70%	\$1,851,829.90		
82	\$1,851,829.90	\$76,907.89	\$1,948,731.09	11.60%	4.40%	6.75%	3.70%	\$1,948,731.09		
83	\$1,948,731.09	\$79,752.40	\$2,051,998.26	11.60%	4.40%	6.75%	3.70%	\$2,051,998.26		
84	\$2,051,998.26	\$82,702.11	\$2,162,139.30	11.60%	4.40%	6.75%	3.70%	\$2,162,139.30		
85	\$2,162,139.30	\$85,760.91	\$2,279,707.55	11.60%	4.40%	6.75%	3.70%	\$2,279,707.55		
86	\$2,279,707.55	\$88,932.85	\$2,405,306.11	11.60%	4.40%	6.75%	3.70%	\$2,405,306.11		
87	\$2,405,306.11	\$92,222.11	\$2,539,592.55	11.60%	4.40%	6.75%	3.70%	\$2,539,592.55		
88	\$2,539,592.55	\$95,633.02	\$2,683,284.04	11.60%	4.40%	6.75%	3.70%	\$2,683,284.04		
89	\$2,683,284.04	\$99,170.09	\$2,196,496.85	11.60%	4.40%	6.75%	3.70%	\$2,196,496.85		
90	\$2,196,496.85	\$102,837.98	\$2,298,680.23	11.60%	4.40%	6.75%	3.70%	\$2,298,680.23		
91	\$2,298,680.23	\$106,641.53	\$2,406,693.89	11.60%	4.40%	6.75%	3.70%	\$2,406,693.89		
92	\$2,406,693.89	\$110,585.76	\$2,520,954.31	11.60%	4.40%	6.75%	3.70%	\$2,520,954.31		
93	\$2,520,954.31	\$114,675.87	\$2,641,913.04	11.60%	4.40%	6.75%	3.70%	\$2,641,913.04		

Note: This figure illustrates a single trial example for a 70% equities – 30% bonds allocation where two simulated market crashes (i.e., portfolio declines) occur, one at age 67 and the other at age 89. Each decline results in a 15% decrease in portfolio value. The simulated returns on assets and inflation rates are generated from their respective distributions, as indicated by their mean values. The ending balance is calculated by deducting the withdrawal amount from the beginning balance and then adjusting it with the simulated returns. The adjusted ending balance checks whether the ending balance column is negative. If the value is negative, then the adjusted ending column corrects the value to zero. Otherwise, a negative ending balance will be carried over to the next period's beginning balance.

portfolio decline increases from 5% to 15%, the average portfolio ending balance decreases from \$2,572,924 to \$1,684,465. Additionally, the probability of the fund running out of money at age 93 increases from 22% to 33%. This suggests that a larger portfolio decline leads to a lower average ending balance and a higher likelihood of exhausting the fund.

Moreover, Table 2 illustrates that for a 5% portfolio decline with a 50% stocks – 50% bonds portfolio, the average ending balance increases to \$3,440,807 for a 60% stocks – 40% bonds allocation and further to \$4,448,100 for a 70% stocks – 30% bonds allocation. This demonstrates that as the equity allocation increases, the average portfolio ending balance tends to rise for the same level of portfolio decline. A higher average ending balance provides several advantages associated with bequest planning. Bequests can provide financial security for heirs or beneficiaries and leave a philanthropic legacy.

Considering the probability of running out of money at age 93, we observe a similar trend. Table 2 shows a 22% probability for a 50% stocks – 50% bonds portfolio with a 5% decline, which decreases to 21% for a 60% stocks – 40% bonds allocation and further decreases to 20% for a 70% stocks – 30% bonds allocation. This suggests that as the stock allocation increases, the probability of running out of money tends to decrease for the same portfolio decline, but the decline is less pronounced as we move from more conservative to more aggressive allocations.

We observe similar patterns with the two hypothesized random market crashes for 10% and 15% portfolio declines. The average portfolio ending balances increase across the different allocations



Table 2: Projected Retirement Portfolio Value at Age 93: Average Balance and Probability of Fund Running Out of Across Different Asset Allocations and Market Crashes

Two Market Crashes								
Allocation: 50% Stocks - 50% Bonds			Allocation: 60% Stocks - 40% Bonds			Allocation: 70% Stocks - 30% Bonds		
Portfolio Decline	Average Ending Balance	Probability of Fund Running Out of Money at Age 93	Portfolio Decline	Average Ending Balance	Probability of Fund Running Out of Money at Age 93	Portfolio Decline	Average Ending Balance	Probability of Fund Running Out of Money at Age 93
5%	\$2,572,924	22%	5%	\$3,440,807	21%	5%	\$4,448,100	20%
10%	\$2,098,119	27%	10%	\$2,866,918	25%	10%	\$2,773,931	24%
15%	\$1,684,465	33%	15%	\$2,341,817	30%	15%	\$3,110,075	28%
Three Market Crashes								
Allocation: 50% Stocks - 50% Bonds			Allocation: 60% Stocks - 40% Bonds			Allocation: 70% Stocks - 30% Bonds		
Portfolio Decline	Average Ending Balance	Probability of Fund Running Out of Money at Age 93	Portfolio Decline	Average Ending Balance	Probability of Fund Running Out of Money at Age 93	Portfolio Decline	Average Ending Balance	Probability of Fund Running Out of Money at Age 93
5%	\$1,998,895	27%	5%	\$2,691,420	25%	5%	\$3,537,160	24%
10%	\$1,427,056	37%	10%	\$2,009,766	32%	10%	\$2,685,524	30%
15%	\$968,235	46%	15%	\$1,445,437	41%	15%	\$1,976,380	38%
Four Market Crashes								
Allocation: 50% Stocks - 50% Bonds			Allocation: 60% Stocks - 40% Bonds			Allocation: 70% Stocks - 30% Bonds		
Portfolio Decline	Average Ending Balance	Probability of Fund Running Out of Money at Age 93	Portfolio Decline	Average Ending Balance	Probability of Fund Running Out of Money at Age 93	Portfolio Decline	Average Ending Balance	Probability of Fund Running Out of Money at Age 93
5%	\$1,526,137	33%	5%	\$2,133,393	30%	5%	\$2,777,253	29%
10%	\$953,182	46%	10%	\$1,380,912	41%	10%	\$1,903,689	38%
15%	\$569,125	59%	15%	\$850,908	53%	15%	\$1,233,732	48%

Note: The findings demonstrate how each asset performs within a hypothetical portfolio that experiences both a decline and an increased frequency of market crashes.

as we move from more conservative to more aggressive allocations. Similarly, the probability of running out of money decreases for the same portfolio declines.

Overall, as the frequency of crashes increases, we observe that a higher allocation to stocks (70% stocks and 30% bonds) provides a higher average portfolio ending balance compared to the more conservative allocations (50% stocks and 50% bonds, and 60% stocks and 40% bonds) for the same level of portfolio declines (see Table 2). Moreover, it's important to note that the probability of running out of money modestly decreases as the stock allocation increases. However, in general, the probability of financial ruin at age 93 significantly increases as the frequency of market crashes and percentage of portfolio declines increase. Figure 6 provides additional statistics specific to the median and the standard deviation for the various portfolios under consideration. The data suggests that higher stock allocations

Figure 6: Projected Retirement Portfolio Value at Age 93: Median and Standard Deviation Across Different Asset Allocations and Market Crashes.



Note: The figure displays the median and standard deviation performance metrics for each asset within a hypothetical portfolio subject to both a decline and an increased frequency of market crashes.



lead to higher median portfolio values, albeit at a greater variability. A higher standard deviation (SD) means there is more variation in investment returns from the average, which indicates greater risk.

Discussion and Conclusions

This study on retirement planning models uniquely addresses the critical gap in existing literature by emphasizing the inclusion of black swan events, those rare but significantly impactful occurrences often overlooked in traditional retirement planning models. By incorporating simulated market crashes into retirement planning models, investors and financial planners can stress test their portfolios against extreme market conditions to provide a clearer insight into market uncertainties. Furthermore, the Monte Carlo methodology used in this study simulates a wide spectrum of market returns, inflation rates, and black swan events, exerting maximum pressure on the portfolios using the 4% withdrawal rule. Ameriks, Veres, and Warshawsky (2001) argue that Monte Carlo simulation is superior to using actual historical market returns, as it provides a broader range of possible future scenarios rather than relying on a single sequence of past events that may not be repeatable. Because we rely on Monte Carlo simulations, our analysis allows for situations where simulated market crashes coincide with the start of a client's retirement, alongside simulated periods of high inflation rates. Specifically, in conducting 10,000 simulation runs, it is quite probable that simulated market crashes and periods of elevated inflation rates may randomly occur at the start of a client's retirement. For the 4% rule, "random sequence of returns risk" refers to the chance that poor market performance in the initial years of retirement might exhaust a portfolio long before reaching the 30-year mark. Conversely, if a retiree begins their retirement during a bull market, the random sequence of returns can significantly boost the portfolio's value, potentially resulting in a substantial balance even after 30 years, despite adhering to the 4% withdrawal rule. While the average life expectancy at age 65 in the United States is about 18.9 years for both sexes combined (www.cdc.gov/nchs/fastats/older-american-health.htm), planning for a longer period (like 30 years) can help safeguard against the financial risks associated with living longer than expected especially where advancements in healthcare could potentially increase life expectancies.

This study's findings, which focus on the efficacy of the 4% rule, reveal that in the face of black swan events causing increased fund declines and frequencies, portfolios with a higher equity allocation not only achieve a superior median and average ending balance but also help in reducing the risk of fund depletion. As the frequency and magnitude of black swan events increase, we observe that a higher allocation to stocks (70% stocks and 30% bonds) provides a higher average portfolio ending balance compared to the more conservative allocations (50% stocks and 50% bonds, and 60% stocks and 40% bonds) for the same level of portfolio declines. Such findings are particularly pertinent for retirees who are not only concerned about avoiding financial ruin but also interested in maximizing their estate or leaving a larger financial legacy rather than focusing solely on consumption as the generator of utility.

The key new insight that our analysis uncovers is that a 70% stock and 30% bond allocation provides a more robust portfolio structure, particularly during simulated black swan events using the 4% rule. That is, the higher equity exposure allows for greater recovery potential



and long-term growth compared to more conservative fixed asset allocations. Our results align with Brown's (2023) findings, which indicate that a higher allocation to stocks within the 4% rule framework (5% in precious metals and 100% in stocks) results in a 2.2% probability of running out of money over a 35-year retirement horizon, using actual global historical returns rather than random market returns. Ameriks et. al. (2001) also find a 4.5% withdrawal rate can be sustained for 30 years with a high probability if the portfolio is heavily weighted toward stocks. Specifically, portfolios with 60-85% stock allocation were found to have a higher success rate (i.e., running out of money) compared to more conservative allocations. In specific, conservative portfolios (e.g., 20% stocks, 50% bonds, 30% cash) had a much higher probability of failure over long retirement periods (67.4% failure rate after 30 years at a 4.5% withdrawal rate) compared to more aggressive portfolios (8.4% failure rate). Moreover, their findings show that incorporating immediate annuities into the retirement portfolio can reduce the risk of running out of money.

Although the 4% rule is advantageous in terms of its simplicity and ease of use, it does not incorporate making dynamic adjustments to the withdrawal rate in response to market conditions and/or lifestyle changes. While Brown (2023, p. 92) cites the 4% rule as the "single most commonly referenced retirement withdrawal approach", however, he cautions that most analyses using this constant withdrawal rule suffer from two major deficiencies. First, they use past U.S. returns rather than global returns. Second, they rely on independent and identically distributed (iid) random variables. Although investment markets (e.g., stocks, bonds, etc.) may indeed exhibit time series patterns such as trends or cycles, we note that the "yearly" asset returns used in our analyses do not exhibit such patterns as reflected in the autocorrelation functions (ACFs) shown in Figure 7. The autocorrelation functions (ACFs) in Figure 7 analyze the correlation of a given time series (i.e., asset returns) with its own past values over different lag periods, ranging from 1 to 12 years. By examining these ACFs, we do not identify the persistence of patterns or trends within the market asset returns over time.

Brown's recommendation of a dynamic withdrawal rule relies on a 100% allocation to equities, with 50% allocated to U.S. stocks and 50% to international or non-U.S. stocks. Such an aggressive approach is not likely to appeal to most financial advisors. We use a more conservative approach that relies on varying allocations to stocks and bonds. This approach is supported by Sapra, Klein, and Martel (2023), who opine that income-oriented investors should favor portfolios with a more traditional allocation, such as 60% stocks and 40% bonds.

While the current study is based on the 4% rule for withdrawals incorporating black swan events, other amounts of less than \$40,000 (or greater than \$40,000) may be easily accommodated using Monte Carlo simulations to test the performance metrics of the portfolio (i.e., average ending value of the portfolio along with the probability of the fund running out of money) based on the retiree's risk tolerance. Moreover, the results do not incorporate social security distributions at the beginning of each year. For example, if a retiree receives \$20,000 a year through social security, this implies that instead of withdrawing \$40,000 from retirement savings each year, \$20,000, which is the difference between the \$40,000 withdrawal amount and the social security revenues, would be the amount that needs to be withdrawn. It should be noted that while the S&P500 index is not investable by itself, there are numerous Exchange-Traded Funds (ETFs) and index funds that mimic the performance of the S&P 500 index (e.g., Vanguard S&P 500 ETF, SPDR S&P 500 ETF Trust, etc.).

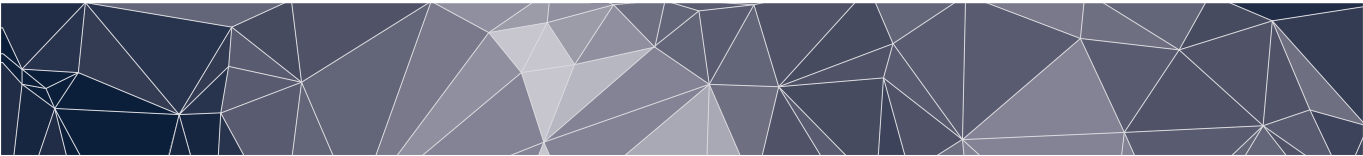
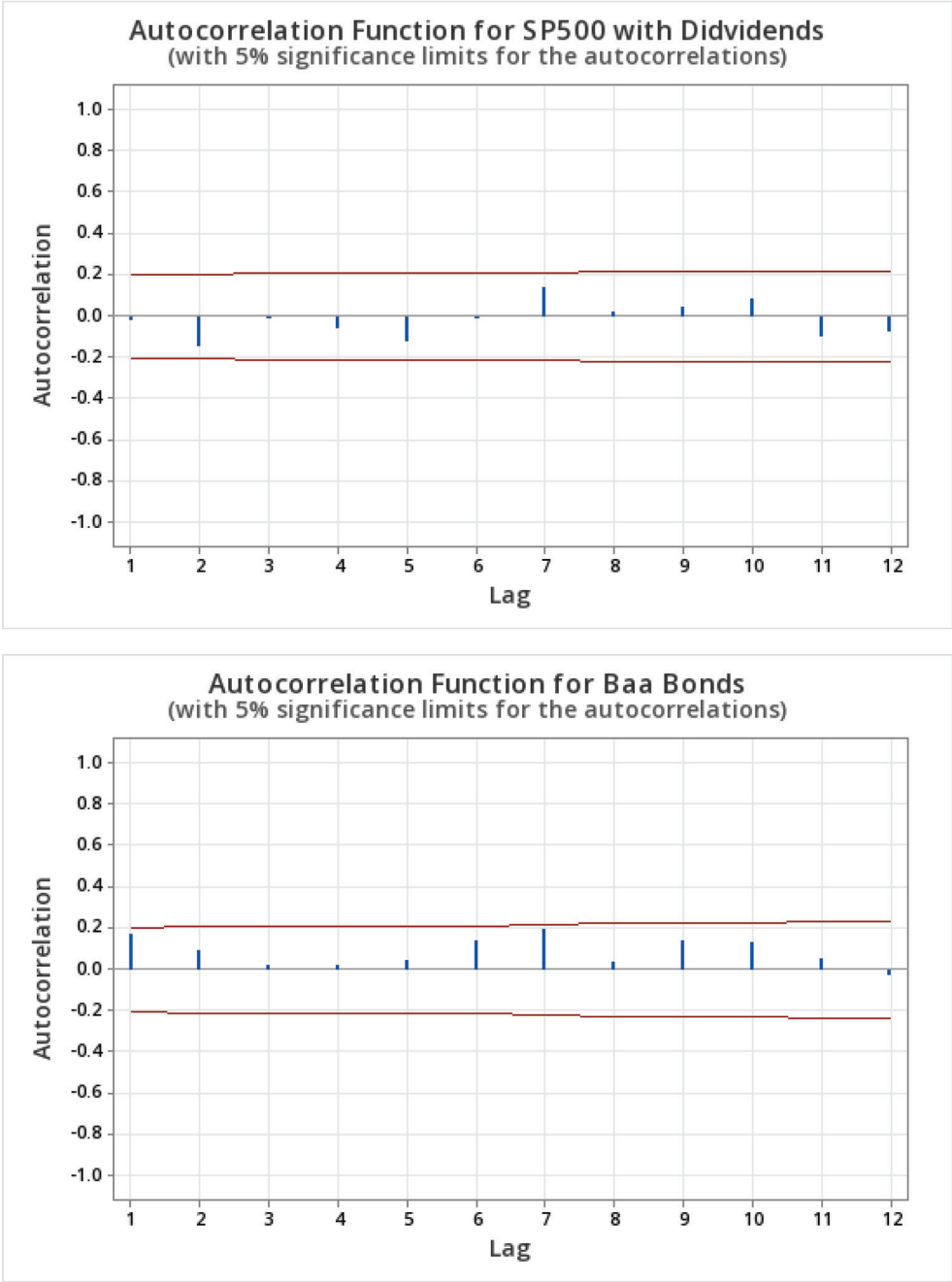


Figure 7: Autocorrelation Functions for SP 500.



Note: The figure displays the autocorrelation function for various market assets using lag periods ranging from 1 to 12 years.

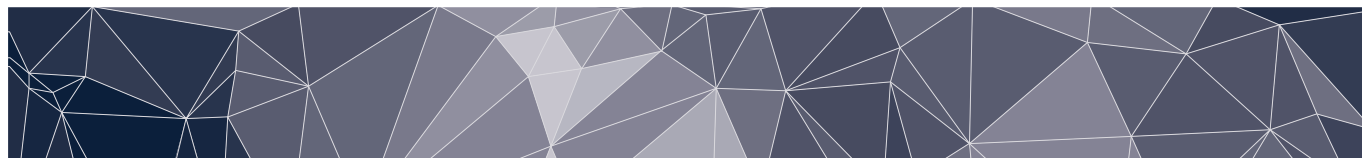
Future research may examine the performance of the 4% rule along with the hypothesized random portfolio declines using dynamic adjustments to the withdrawal rate in response to market performance or different lifestyles, including shorter planning horizons. Moreover, the performance metrics of the simulation model can be further explored by employing various ‘what-if’ scenarios, such as differing frequencies and magnitudes of black swan events,



including global assets. While black swan events are inherently unpredictable, incorporating control over the sequence and timing of such events would be an interesting extension to this study. Moreover, while reducing risk is a key tenant of portfolio optimization, financial planners are encouraged to include black swan events in their planning models, often ignored in many financial retirement models, through stress testing and scenario analyses. This involves evaluating how a portfolio might perform under extreme market conditions. Moreover, our study could prompt further research into retirement planning models, particularly those that incorporate unpredictable market events, with the aim of offering more resilient strategies for retirees.

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