FOUNDATIONS OF COMPUTING AND DECISION SCIENCES
Vol. 50 (2025) No. 1
ISSN 0867-6356
DOI: 10.2478/fcds-2025-0004 e-ISSN 2300-3405

A Novel Scale for Inconsistency Reduction in the Pair-Wise Comparison Matrices

Amukta MalyadaVommi[1], Vijaya babu Vommi[2]

Abstract. Linguistic pairwise comparison of the attributes forms the basis for prioritization of the attributes and also aids in calculating the weights for each attribute for final decision making in the Analytical Hierarchy Process (AHP). This process may sometimes lead to inconsistencies in the pairwise comparison matrix, and the acceptability of the method is checked with respect to a measure known as Consistency Ratio (CR), whose upper limit is fixed as 0.1. The present work attempts to develop a new methodology in which the pairwise attribute comparison matrix is formed in such a manner that the decision maker can safely eliminate the process of consistency check. In this endeavor, a new scale called as 'Relative Percentage Supremacy' (RPS) scale with three variations namely, High, Moderate and Low is introduced and employed. The proposed methodology is successfully applied to Saaty's ‘Distance’, 'Optics', 'National Wealth' and 'Weights Estimation' problems for which the actual weights are available. Also, the method has been applied on Saaty's 'Buying a House' problem and a comparison of the results with the results of already existing scales is done. The Relative Percentage Supremacy scale with Moderate value is found to yield results close to the Saaty's actual values in the majority of the time and the applicability of the other variations are also discussed.

Keywords: Analytical Hierarchy Process (AHP), Pairwise comparison, Consistency Ratio (CR), Relative Percentage Supremacy (RPS).

1. Introduction

The availability of several alternatives for a specific application leads to the problem of properly choosing the best alternative for the purpose. Multiple Attribute Decision Making

[1]ECE department, Jawaharlal Nehru Technological University, Kakinada, 533005, A.P., India, malyada1212@gmail.com
[2]Department of Mechanical Engineering, Andhra University, Visakhapatnam, 530003, A.P., India, prof.vvbabu@andhrauniversity.edu.in

(MADM) methods aid the decision maker in choosing the best available alternative from a set of feasible choices. The literature is very rich in a number of MADM methods and the choice of the method would be based on the decision maker's exposure and conviction about a particular method. Since its introduction by Saaty [20], the Analytic Hierarchy Process (AHP) has gained a lot of importance in multiple attribute decision making, and many selection problems have been solved using this technique. To name a few, a software selection problem [15], a competitive bidding problem [2], a drug selection problem [25], a supplier selection problem [3], a weapon selection problem [4], an energy selection problem (Kahraman and Kaya 2010) [14] etc., show a wide range of fields in which AHP was employed. Recently, Siregar et al. [24] employed AHP in the development of efficient strategies in the pine chemical industry. Munizu and Riyadi [18] applied the AHP technique to enhance the competitiveness of the creative industry.

Apart from AHP, a number of popular techniques exist in multi-criteria decision making. Some of the popular methods include VIKOR (VIsekriterijumsko KOmpromisna Rangiranjen) [28], TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) [11], PROMITHEE (Preference Ranking Organization Method for Enhancement Evaluations) [1], ELECTRE (ELimination Et Choix TraduisantIa REalite) [19] etc., The list is increasing chronologically with the inclusion of new methods and concepts [26][27]. Irrespective of the MADM technique employed in solving the problems, AHP has been employed by many researchers to obtain the weights of the attributes. For example, in the selection of optimum plant layouts, Shivade and Saptkal [23] employ TOPSIS, but use AHP methodology to find the weights of the attributes.

Prioritization in the AHP method relies on forming the pairwise comparison matrix. A decision maker has to perform linguistic pairwise comparisons first and then obtain numerical pairwise comparisons by selecting certain numerical scale to quantify them [7]. Different numerical scales exist in literature. Saaty [20] proposed a linear scale i.e., S = x where x is a set of positive integers from 1 to 9. This linear scale has been criticized by many researchers [9][6]. Apart from Saaty's linear scale many different forms of scales came into existence. Harker and Vegas [10] introduced power and root scales in which the values are just squared values or the positive roots of the Saaty's linear scale values respectively. To be more clear, the scale values are S = x^2 in power scale and S = $\sqrt{x}$ in root scale. Lootsma [16] proposed a geometric scale with S = 2^{x-1}, Ma and Zheng [17] proposed an inverse linear scale with S = $9/(10-x)$, Dodd and Donegan [5] proposed an asymptotic scale with S = $tanh^{-1}\left(\frac{\sqrt{3}\,(x-1)}{14}\right)$ and Ishikaza et al. [12] introduced a logarithmic scale with S = $log_2(x+1)$ wherein for all scales x = $\{1,2,3....9\}$. Salo and Hamalainen [22] proposed a balanced scale S = w/(1-w) where w = {0.5, 0.55. 0.6, ...,0.9). The linguistic expressions used by Saaty [20] have been followed by all. Unfortunately, there are no guidelines on the usage of these different scales. In all the cases, to obtain a better solution, the consistency check has to be performed and the consistency ratio (CR) should be less than 0.1.

In the present work, an attempt has been made to suggest a new methodology which eliminates the need for a consistency check of the pairwise comparison matrix. A new scale called as Relative Percentage Supremacy (RPS) scale has been introduced at three different levels which are classified as Low, Moderate and High. The pairwise comparison matrix formed using the proposed method will always have a maximum Eigen value equal to the size of the matrix.

The remainder of the paper is organized as follows: Section 2 introduces the proposed method. Section 3 introduces the Relative Percentage Supremacy scale, and its application in forming pairwise comparison matrix. Section 4 presents the application of the proposed RPS scale for the four problems (viz., the distance problem, the optics problem, national wealth problem and the relative weights problem) and a comparison with the actual weights as given in [20]. In order to compare the weights obtained by the proposed method with those obtained using different existing scales, the 'Buying a House' problem of Saaty [21] has been considered in Section 5 and finally Section 6 concludes the paper with a discussion on the proposed work.

2. Proposed Methodology

The present work concentrates on eliminating the need for a consistency check of the pairwise comparison matrix, which is the basis for estimating the weight of each attribute in the AHP method. Even though the present method depends on pairwise comparisons of the attributes, the numerical scales are not allotted during pairwise comparisons. Instead, by pairwise comparisons, the most dominant attribute is found among the available attributes and a similar search is continued till all the attributes are arranged in their order of preference. At this stage using a new scale called as the Relative Percentage Supremacy (RPS) scale is suggested in the present work (explained in detail in Section 3), and the numerical scales are obtained. The following steps explain the proposed methodology in detail.

Step 1: Recognize the attributes possessed by the different alternatives.

Step 2: Using the pairwise comparisons, arrange the attributes recognized in Step 1 in the descending order of their importance. This arrangement places the most important attribute in the first place and the least important attribute in the last place. At this stage, there is no need to give numerical scales simultaneously. Emphasis should be given on arranging the attributes according to their preferences. For example, the p attributes should be arranged in the order based on their importance shown as under:

$$A_1 \quad > \quad A_2 \quad > \quad A_3 \quad > \ldots\ldots\ldots\ldots\ldots\ldots > \quad A_{p-1} \quad > \quad A_p$$

In the above arrangement A_1 is the attribute with top preference and hence A_1 is the most dominant attribute. While arranging the attributes, the attributes with equal importance can be placed arbitrarily at any position among themselves.

While comparing the attributes, there is no need to assign any scale. At this stage, it is required only to find the order of importance among the attributes.

This procedure can be explained as under by considering three attributes, A_1, A_2 and A_3, whose order of preference is unknown. Initially, a pair-wise comparison is made between A_1 and A_2.

if (A_1 is inferior to A_2)
        Dominant = A_2
else
        Dominant = A_1
if (Dominant is inferior to A_3)
        Dominant = A_3

The above pair-wise comparisons recognize the dominant attribute among the three. After recognizing the dominant attribute, the remaining two attributes can be compared to find the second and third attributes in the order.

The above procedure of comparing the attributes can be extended to any number of attributes. The algorithm presented below can be implemented to take the preferences of attributes interactively from the decision maker by using pair-wise comparisons.

Algorithm

```
Consider the list L of attributes in any order
for i in range (0, length(L)):
    for j in range (i+1, length(L)):
        assign  c  →'Y' if attribute[i] is inferior to attribute[j]
        assign c→'N' otherwise
        if c =='Y':
            temp = L[i]
            L[i] = L[j]
            L[j] = temp
print(L)
```

Step 3: The relative importance scale for each attribute with respect to the dominant attribute is obtained in this step. Initially, the dominant (first in the arrangement) attribute is given a scale of $S_1 = 1$. The scales given to all p attributes are designated as $S_1, S_2, \dots \dots S_p$ and are obtained in the sequence of their preferences. Based on the scale of the first (dominant) attribute, all other attributes are given their scales, i.e., with respect to the first attribute. A new scale known as 'Relative Percentage Supremacy' has to be used to give scales to the attributes. The scales and their numerical values are based on the linguistic terms which are provided in Table 1. The procedure to obtain the scales $S_2, S_3 \dots \dots S_p$ is explained in detail in Section 3. Using the RPS scale, if dominant attribute A_1 is assumed to be 100% superior to the attribute A_2, then the scale $S_2 = 2$ which compares A_1 with respect to A_2 . Likewise, the supremacy of the attribute A_2 over the attribute A_3 has to be judged, and the scale S_3 has to be obtained to show the supremacy of $A_1$with respect to the attribute A_3. This process continues till all the scale values are obtained. By this process, one would obtain the first row of the pairwise comparison matrix, which would be similar to the first row of the pairwise comparison matrix obtained in the usual procedure (Table 2, which is explained in detail in Section 3).

Step 4: After obtaining the first row of the pairwise comparison matrix, the next step is to obtain the remaining rows of the matrix. This can be accomplished very easily by simple manipulation. For example, the second row in the pairwise matrix is obtained by dividing the first row by the second element in the first row. The third row is obtained by dividing the second row by the third element in the second row. The scale values for each attribute can be obtained as indicated in Table 3.

Step 5: After deducting the pair wise comparison matrix for the given problem, the Logarithmic Least Squares Method (LLSM) was employed to find the weights of attributes.

In the proposed method, there is no need to check for the consistency of the pairwise matrix. Always the maximum Eigen value will be equal to the size of the pair wise comparison matrix. The method of allocating the scale values and obtaining the complete pairwise matrix from the first row of the pair wise matrix are explained in detail in the following sections.

Table 1: Linguistic terms and corresponding supremacy values (c) expressed as percentages

Linguistic comparison between attributes	Saaty's scale	'c' values for different supremacy scales		
		Low (200%)	*Moderate* (300%)	*High* (400%)
s_0 Equally important	1	c_0= 0	c_0= 0	c_0 = 0
s_1 Weakly more important	2	c_1 = 25	c_1 = 37.5	c_1 = 50
s_2 Moderately more important	3	c_2 = 50	c_2 = 75	c_2 = 100
s_3 Moderately plus more important	4	c_3 = 75	c_3= 112.5	c_3 = 150
s_4 Strongly more important	5	c_4 = 100	c_4 = 150	c_4 = 200
s_5 Strongly plus more important	6	c_5 = 125	c_5 = 187.5	c_5 = 250
s_6 Demonstratedly more important	7	c_6 = 150	c_6 = 225	c_6 = 300
s_7 Very very strongly more important	8	c_7 = 175	c_7 = 262.5	c_7 = 350
s_8 Extremely more important	9	c_8= 200	c_8 = 300	c_8 = 400

3. The Relative Percentage Supremacy (RPS) Scale

After arranging the attributes in the descending order of their importance, the initial attribute is given a scale of one ($S_1 = 1$) and the immediate attribute next to it should be given a scale (S_2) on the basis of supremacy of the dominant attribute relative to the second attribute. In general, the numerical scale for an attribute can be obtained by finding the supremacy of the preceding attribute over the present. For example, if the j^{th} attribute has a scale of S_j, the scale for the next attribute $S_{j+1}$is given below:

$$S_{j+1} = K_{(j)(j+1)} S_j \tag{1}$$

Equation (1) shows that the scale of $(j+1)^{th}$attribute is $K_{(j)(j+1)}$ times the scale of the j^{th} attribute. The $K_{(j)(j+1)}$ is the supremacy factor of the j^{th} attribute over the $(j+1)^{th}$ attribute.

The supremacy factor $K_{(j)(j+1)}$ depends on the resulting linguistic description of importance as obtained in the pair wise comparison between j^{th} and $(j+1)^{th}$ attributes. Based on the linguistic description the supremacy value of the j^{th} attribute over $(j+1)^{th}$ attribute, expressed as percentage can be obtained (Table 1). It can be noted that the linguistic descriptions used are similar to those used by Saaty [20]. Based on the supremacy value, the supremacy factor $K_{(j)(j+1)}$ can be calculated by following equation (2).

$$K_{(j)(j+1)} = (1 + c/100) \tag{2}$$

where c is the supremacy of the j^{th} attribute over the $(j+1)^{th}$ attribute expressed as percentage. The value of c depends on the linguistic term obtained by comparing the $j^{th}$and $(j+1)^{th}$attributes. Based on linguistic terms already in use, three scales have been suggested. The three scales are classified as *High, Moderate* and *Low* whose lowest value is 0 for all scales and highest values are 200%, 300% and 400% respectively. The percentage supremacy (c) values for the given linguistic terms for all the three proposed scales along with Saaty's scale are provided in Table 1.

The application of RPS-Low scale is illustrated in the following steps:

Step 1: Provide a scale for the first attribute
Starting with the first attribute in the list of attributes arranged in the descending order of their importance, the scale for the first attribute is taken as S_1=1 and $c = c_0 = 0$ since the supremacy factor of any attribute over the same attribute should be equal to one.

Step 2: Obtain the scale for the second attribute with respect to the first attribute
To find the scale for the second attribute in the order, one has to find the supremacy of the first attribute over the second in linguistic terms and choose the corresponding c value from Table 1. On comparison of the first and second attributes, if the linguistic description, for

example, weakly more important' is appropriate, a value of 50 for c i.e., $c = c_2$ (in case RPS-Low) has to be chosen, implying that the first attribute is 50% better than the second attribute. The supremacy factor for the first attribute over the second attribute is calculated as follows:

$$K_{12} = 1 + 50/100 = 1.5 \tag{3}$$

Hence, using the relation given in equation (1), the relation between the scales of the first and second attributes is given as:

$$S_2 = K_{12}S_1 = 1.5 \times 1 = 1.5 \tag{4}$$

Step 3: Obtain the scale for the third attribute with respect to the first attribute
Similarly, assuming supremacy of the second attribute over the third attribute as 100% (i.e., with linguistic description 'Strongly more important', c_4= 100) the supremacy factor K_{23} is calculated as below:

$$K_{23} = 1 + 100/100 = 2.0 \tag{5}$$

The scale for the third attribute is obtained using equations (4) and (5) as under:

$$S_3 = K_{23}S_2 = K_{23}\, K_{12}S_1 = 2.0 \times 1.5 = 3.0 \tag{6}$$

Step 4: Obtain the scale for the remaining attributes with respect to the first attribute
Proceeding in the similar way the scale for each attribute with respect to the dominant attribute can be computed as presented in Table 2.

Table 2: Attributes and their scales with respect to the most dominant attribute

Attribute	Scale	Equivalent scales in terms of S_1
1	$S_1 = 1$	$S_1 = 1$
2	$S_2 = K_{12}S_1$	$S_2 = K_{12}S_1$
3	$S_3 = K_{23}S_2$	$S_3 = K_{23}K_{12}\, S_1$
4	$S_4 = K_{34}S_3$	$S_4 = K_{34}K_{23}K_{12}S_1$
.	....	....
.	....	....
p-1	$S_{p-1} = K_{(p-2)(p-1)}S_{p-2}$	$S_{p-1} = K_{(p-2)(p-1)}K_{(p-3)(p-2)}K_{(p-4)(p-3)}\ldots K_{23}K_{12}S_1$
P	$S_p = K_{(p-1)(p)}S_{p-1}$	$S_p = K_{(p-1)(p)}K_{(p-2)(p-1)}K_{(p-3)(p-2)}\ldots. K_{23}K_{12}S_1$

3.1. Formation of Pairwise Attribute Comparison Matrix

The scales allotted to all attributes with respect to the dominant or the first attribute give the first row of the pairwise comparison matrix. The mathematical relations between the scales are presented in Table 2. The next step in the proposed methodology is to obtain the complete pairwise comparison matrix from the first row of the pairwise comparison matrix. The pairwise comparison matrix deducted from the first row will have the mathematical relations between all the attributes as presented in Table 3.

Table 3: Pairwise comparison matrix deduced from the Table 2

Attrib -utes	1	2	3	4	...	p-1	p
1	1	$\prod_{j=1}^{1} K_{(j)(j+1)}$	$\prod_{j=1}^{2} K_{(j)(j+1)}$	$\prod_{j=1}^{3} K_{(j)(j+1)}$	...	$\prod_{j=1}^{p-2} K_{(j)(j+1)}$	$\prod_{j=1}^{p-1} K_{(j)(j+1)}$
2	$\frac{1}{\prod_{j=1}^{1} K_{(j)(j+1)}}$	1	$\prod_{j=2}^{2} K_{(j)(j+1)}$	$\prod_{j=2}^{3} K_{(j)(j+1)}$	...	$\prod_{j=2}^{p-2} K_{(j)(j+1)}$	$\prod_{j=2}^{p-1} K_{(j)(j+1)}$
3	$\frac{1}{\prod_{j=1}^{2} K_{(j)(j+1)}}$	$\frac{1}{\prod_{j=2}^{2} K_{(j)(j+1)}}$	1	$\prod_{j=3}^{3} K_{(j)(j+1)}$	...	$\prod_{j=3}^{p-2} K_{(j)(j+1)}$	$\prod_{j=3}^{p-1} K_{(j)(j+1)}$
4	$\frac{1}{\prod_{j=1}^{3} K_{(j)(j+1)}}$	$\frac{1}{\prod_{j=2}^{3} K_{(j)(j+1)}}$	$\frac{1}{\prod_{j=3}^{3} K_{(j)(j+1)}}$	1	...	$\prod_{j=4}^{p-2} K_{(j)(j+1)}$	$\prod_{j=4}^{p-1} K_{(j)(j+1)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	$\vdots$
p-1	$\frac{1}{\prod_{j=1}^{p-2} K_{(j)(j+1)}}$	$\frac{1}{\prod_{j=2}^{p-2} K_{(j)(j+1)}}$	$\frac{1}{\prod_{j=3}^{p-2} K_{(j)(j+1)}}$	$\frac{1}{\prod_{j=4}^{p-2} K_{(j)(j+1)}}$	...	1	$\prod_{j=(p-1)}^{p-1} K_{(j)(j+1)}$
p	$\frac{1}{\prod_{j=1}^{p-1} K_{(j)(j+1)}}$	$\frac{1}{\prod_{j=2}^{p-1} K_{(j)(j+1)}}$	$\frac{1}{\prod_{j=3}^{p-1} K_{(j)(j+1)}}$	$\frac{1}{\prod_{j=4}^{p-1} K_{(j)(j+1)}}$	...	$\frac{1}{\prod_{j=p-1}^{p-1} K_{(j)(j+1)}}$	1

4. Application of the proposed Relative Percentage Supremacy (RPS) scale for Saaty's example problems [20]

Saaty [20] provided four example problems to evaluate the performance of the proposed linear scale. The same example problems are considered by Ji and Jiang [13] in discussing the parameter of the geometrical scale. Dong et al. [7] also used the same example problems in their comparative study of numerical scales. The first example problem deals with the 'Distances' between of Cairo, Tokyo, Chicago, San Francisco, London and Montreal from Philadelphia. The real relative distances are given by the weight vector W^T = (0.2777, 0.3611, 0.0320, 0.1324, 0.1773,0.0194). The second example deals with 'Optics' problems with two linguistic pairwise comparisons for which the actual relative brightness values are W^T= (0.6072, 0.2186, 0.1115, 0.0627). The third problem deals with the relative 'Wealth' of seven countries whose actual GNP fractions are given by W^T = (0.4134, 0.2249, 0.0425, 0.0694, 0.0546, 0.1041, 0.0910). Fourth problem deals with the estimation of 'Relative Weights' of five objects whose real relative weights are W^T = (0.10, 0.39, 0.20, 0.27, 0.04). Present section deals with these problems using RPS scales.

Example 1: The Distance Problem:

The linguistic pairwise comparison matrix for the distances of Cairo (A), Tokyo (B), Chicago(C), San Francisco (D), London (E) and Montreal (F) from Philadelphia is given below:

$$\begin{bmatrix} & A & B & \mathrm{C} & \mathrm{D} & \mathrm{E} & \mathrm{F} \\ Cairo\ (A) & s_0 & s_{-2} & s_7 & s_2 & s_2 & s_6 \\ Tokyo\ (B) & s_2 & s_0 & s_8 & s_2 & s_2 & s_8 \\ \text{Chicago (C)} & s_{-7} & s_{-8} & s_0 & s_{-5} & s_{-4} & s_1 \\ \text{San Francisco (D)} & s_{-2} & s_{-2} & s_5 & s_0 & s_{-2} & s_5 \\ \text{London (E)} & s_{-2} & s_{-2} & s_4 & s_2 & s_0 & s_5 \\ \text{Montreal (F)} & s_{-5} & s_{-8} & s_{-1} & s_{-5} & s_{-5} & s_0 \end{bmatrix}$$

On substituting the values of relevant scale values, the pair-wise comparison matrix takes the form given below [20]:

$$\begin{bmatrix} & A & B & \mathrm{C} & \mathrm{D} & \mathrm{E} & \mathrm{F} \\ Cairo\ (A) & 1 & 1/3 & 8 & 3 & 3 & 7 \\ Tokyo\ (B) & 3 & 1 & 9 & 3 & 3 & 9 \\ \text{Chicago (C)} & 1/8 & 1/9 & 1 & 1/6 & 1/5 & 2 \\ \text{San Francisco (D)} & 1/3 & 1/3 & 6 & 1 & 1/3 & 6 \\ \text{London (E)} & 1/3 & 1/3 & 5 & 3 & 1 & 6 \\ \text{Montreal (F)} & 1/6 & 1/9 & 1/2 & 1/6 & 1/6 & 1 \end{bmatrix}$$

Representing the countries of the distance problem as A, B, C, D, E and F and following the step 2 of the proposed methodology, the countries can be arranged in the following order based on their dominance over each other. In the present case, since the Saaty’s scale has already been formed, the order of preference can be found by taking the row product of the pair-wise comparison scales and arranging the countries based on the product in descending order.

B > A > E > D > C > F

In order to facilitate easy understanding of the problem, both Saaty's scales and RPS-scales equivalents are given in the sequence below:

$$B \underset{c_2}{\overset{s_2}{>}} A \underset{c_2}{\overset{s_2}{>}} E \underset{c_2}{\overset{s_2}{>}} D \underset{c_5}{\overset{s_5}{>}} C \underset{c_5}{\overset{s_5}{>}} F$$

The above arrangement shows that the most dominant among the countries is B and the rest follows in the order. Now, the scale for B in comparison with B (S_1) is given as 1. Comparing the country, A with B, it is observed from the pairwise matrix, that the Saaty's scale is s_2 and in the proposed RPS-Low scale (200%) it has a value of c equal to 50%. This gives the factor K_{12} a value equal to (1+50/100) i.e., 1.5. By multiplying the scale of B (S_1) by 1.5, the scale of B with respect to A (S_2) is obtained as 1.5. Now comparing the country, A with E, the Saaty's scale is observed to be s_2. The factor K_{23} can be calculated as 1.5 for the RPS-Low (200%) scale using a 'c' value of 50%. The scale for B with respect to E (S_3) can be obtained as the product of $K_{12}K_{23}$which gives a scale of 2.25 for B with respect to E. Similarly, comparing E with D, from the pairwise comparison matrix, the Saaty's scale is observed to be s_2. Hence, the factor K_{34} also becomes 1.5. The scale of B with respect to D (S_4) which is the product of $K_{12}K_{23}K_{34}$ is now 3.375. Comparing D and C, the Saaty's scale can be found as $s_5$and the K_{45} factor is (1+125/100) which is 2.25. This leads to a scale of B with respect to C (S_5) equal to $K_{12}K_{23}K_{34}K_{45}$ with a numerical value of 7.5938. Finally, comparing C with F and observing Saaty's scale as s_1 gives K_{56} as 1.25. The scale of B with respect to F (S_6) is given as the product of $K_{12}K_{23}K_{34}K_{45}K_{56}$ which is 9.4922. This completes the first row of the pair wise comparison matrix in the proposed method, which is depicted in Table 4.

Table 4: First row of the pair-wise comparison matrix with RPS-Low scale (200%)

Countries	B	A	E	D	C	F
B	1	1.5	2.25	3.375	7.5938	9.4922

The pair wise comparison matrix deducted from the first row of the pair-wise (Table 4) is given below:

$$\begin{array}{c} \\ B \\ A \\ E \\ D \\ C \\ F \end{array} \begin{bmatrix} B & A & E & D & C & F \\ 1.0000 & 1.5000 & 2.2500 & 3.3750 & 7.5938 & 9.4922 \\ 0.6667 & 1.0000 & 1.5000 & 2.2500 & 5.0625 & 6.3281 \\ 0.4444 & 0.6667 & 1.0000 & 1.5000 & 3.3750 & 4.2188 \\ 0.2963 & 0.4444 & 0.6667 & 1.0000 & 2.2500 & 2.8125 \\ 0.1317 & 0.1975 & 0.2963 & 0.4444 & 1.0000 & 1.2500 \\ 0.1053 & 0.1580 & 0.2370 & 0.3556 & 0.8000 & 1.0000 \end{bmatrix}$$

The weights obtained from the above pairwise comparison matrix using LLSM are shown in Table 5.

Table 5: Weights obtained for each attribute using a RPS-Low scale (200%)

Countries	B	A	E	D	C	F
Weight	0.3782	0.2521	0.1681	0.1120	0.0498	0.0398

In order to find the weights under the RPS-Moderate (300%) scale, the scale values for each country with respect to the dominant country B can be obtained as follows:

$S_1 = 1$

$S_2 = K_{12} S_1$

$K_{12} = (1 + c_3/100) = 1 + 75/100 = 1.75$

$S_2 = 1.75$

$K_{23} = (1 + c_3/100) = 1 + 75/100 = 1.75$

$S_3 = K_{23} S_2 = K_{23} K_{12} S_1 = 3.0625$

$K_{34} = (1 + c_3/100) = 1 + 75/100 = 1.75$

$S_4 = K_{34} K_{23} K_{12} S_1 = 5.394$

$K_{45} = (1 + c_6/100) = 1 + 187.5/100 = 2.875$

$S_5 = K_{45}K_{34}K_{23}K_{12}S_1 = 15.4082$

$K_{56} = (1 + c_2/100) = 1.375$

$S_6 = K_{56}K_{45}K_{34}K_{23}K_{12}S_1 = 21.1863$

The first row of the pairwise comparison matrix with RPS-Moderate scale is shown in Table 6.

Table 6: First row of the pair-wise comparison matrix using RPS-moderate scale (300%)

Countries	B	A	E	D	C	F
B	1	1.75	3.0625	5.394	15.4082	21.1863

Using the first row of the pair-wise comparison matrix with moderate scale (300%), the pairwise matrix formed is given as under:

$$\begin{array}{c} \\ B \\ A \\ E \\ D \\ C \\ F \end{array}\begin{bmatrix} B & A & E & D & C & F \\ 1.0000 & 1.7500 & 3.0625 & 5.3940 & 15.9082 & 21.1863 \\ 0.5714 & 1.0000 & 1.7500 & 3.0823 & 9.0904 & 12.1065 \\ 0.3265 & 0.5714 & 1.0000 & 1.7613 & 5.1945 & 6.9180 \\ 0.1854 & 0.3244 & 0.5678 & 1.0000 & 2.9492 & 3.9278 \\ 0.0629 & 0.1100 & 0.1925 & 0.3391 & 1.0000 & 1.3318 \\ 0.0472 & 0.0826 & 0.1446 & 0.2546 & 0.7509 & 1.0000 \end{bmatrix}$$

The weights obtained from the above pair wise matrix using LLSM are shown in Table 7.

Table 7: Weights obtained for each country using RPS-Moderate scale (300%)

Countries	B	A	E	D	C	F
Weight	0.4559	0.2605	0.1489	0.0845	0.0287	0.0215

Finally, employing the RPS-High scale (400%), the first row of the pair-wise matrix obtained is given in Table 8.

Table 8: First row of the pair-wise comparison matrix using RPS-High scale (400%)

Countries	B	A	E	D	C	F
B	1	2	4	8	28	56

Using the first row of the pair-wise comparison matrix, the deducted pair-wise matrix is given as under:

$$\begin{array}{c} \\ B \\ A \\ E \\ D \\ C \\ F \end{array} \begin{bmatrix} B & A & E & D & C & F \\ 1.0000 & 2.0000 & 4.0000 & 8.0000 & 28.0000 & 56.0000 \\ 0.5000 & 1.0000 & 2.0000 & 4.0000 & 14.0000 & 28.0000 \\ 0.2500 & 0.5000 & 1.0000 & 2.0000 & 7.0000 & 14.0000 \\ 0.1250 & 0.2500 & 0.5000 & 1.0000 & 3.5000 & 7.0000 \\ 0.0357 & 0.0714 & 0.1429 & 0.2857 & 1.0000 & 2.0000 \\ 0.0179 & 0.0357 & 0.0714 & 0.1429 & 0.5000 & 1.0000 \end{bmatrix}$$

The weights obtained from the above pair wise matrix using LLSM are shown in Table 9:

Table 9: Weights obtained for each attribute using an RPS-High scale (400%)

Countries	B	A	E	D	C	F
Weight	0.5185	0.2593	0.1296	0.0648	0.0185	0.0093

A comparison between the weights obtained from three suggested scales and the actual weights as given by Saaty [20] are provided in Table 10.

Table 10: A comparison of weights obtained using different scales for 'Distance' problem

Countries	B	A	E	D	C	F
Weight (RPS-Low)	0.3782	0.2521	0.1681	0.1120	0.0498	0.0398
Weight (RPS-Moderate)	0.4559	0.2605	0.1489	0.0845	0.0287	0.0215
Weight (RPS-High)	0.5185	0.2593	0.1296	0.0648	0.0185	0.0093
Actual Weights (Saaty)	0.3611	0.2777	0.1773	0.1324	0.0320	0.0194

Example 2: The Optics problem:

In this problem, the comparison of illumination intensity was performed by two sets of people and two linguistic pair-wise comparison matrices are formed. The experiment was conducted using four identical objects, namely C_1 (A), C_2 (B), C_3 (C)and C_4 (D). Both of the pair-wise matrices yield the same order of preference with different linguistic descriptions and, hence, different scales.

$$Pair-wise\ Comparison^{(1)} = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{bmatrix} A & B & C & D \\ s_0 & s_4 & s_5 & s_6 \\ s_{-4} & s_0 & s_3 & s_5 \\ s_{-5} & s_{-3} & s_0 & s_3 \\ s_{-6} & s_{-5} & s_{-3} & s_0 \end{bmatrix}$$

The objects are arranged in their descending order of importance as given under:

A > B > C > D

Using the Saaty's scale and the proposed RPS scales, the order can be written as:

$$A \underset{C_4}{\overset{S_4}{>}} B \underset{C_3}{\overset{S_3}{>}} C \underset{C_3}{\overset{S_3}{>}} D$$

With RPS-Low scale (200%):
Pair wise comparison matrix obtained using a Low scale (200%) is given below:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{bmatrix} A & B & C & D \\ 1.0000 & 2.0000 & 3.5000 & 6.1250 \\ 0.5000 & 1.0000 & 1.7500 & 3.0625 \\ 0.2857 & 0.5714 & 1.0000 & 1.7500 \\ 0.1633 & 0.3265 & 0.5714 & 1.0000 \end{bmatrix}$$

The weights obtained from the above pair wise matrix using LLSM are shown in Table 11.

Table 11: Weights for optics problem using RPS-Low scale (200%)

Objects	A	B	C	D
Weight	0.5131	0.2565	0.1466	0.0838

With RPS-Moderate scale (300%):
Pair wise comparison matrix obtained using a Moderate scale (300%) is given below:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{bmatrix} A & B & C & D \\ 1.0000 & 2.5000 & 5.3125 & 11.2891 \\ 0.4000 & 1.0000 & 2.1250 & 4.5156 \\ 0.1882 & 0.4706 & 1.0000 & 2.1250 \\ 0.0886 & 0.2215 & 0.4706 & 1.0000 \end{bmatrix}$$

The weights obtained using LLSM are shown in Table 12.

Table 12: Weights of the objects in optics problem with RPS-Moderate scale (300%)

Objects	A	B	C	D
Weight	0.5964	0.2385	0.1123	0.0528

With RPS-High scale (400%):
Pairwise comparison matrix obtained using a High scale (400%) is given below:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{bmatrix} A & B & C & D \\ 1.0000 & 3.0000 & 7.5000 & 18.7500 \\ 0.3333 & 1.0000 & 2.5000 & 6.2500 \\ 0.1333 & 0.4000 & 1.0000 & 2.5000 \\ 0.0533 & 0.1600 & 0.4000 & 1.0000 \end{bmatrix}$$

The weights obtained using LLSM are shown in Table 13. A comparison of weights obtained using the three RPS-scales with the actual weights as provided by Saaty are shown in Table 14.

Table 13: Weights of the objects in optics problem with RPS-Moderate scale (300%)

Objects	A	B	C	D
Weights	0.6579	0.2193	0.0877	0.0351

Table 14: A comparison of weights obtained using different scales for Optics (1) problem

Objects	A	B	C	D
Weights (RPS-Low)	0.5131	0.2565	0.1466	0.0838
Weights (RPS-Moderate)	0.5964	0.2385	0.1123	0.0528
Weights (RPS-High)	0.6579	0.2193	0.0877	0.0351
Actual Weights (Saaty)	0.6072	0.2186	0.1115	0.0627

The second linguistic pairwise decision matrix for the optics problem is given as under:

$$Pair-wise\ Comparison^{(2)} = \begin{bmatrix} & A & B & C & D \\ A & s_0 & s_3 & s_5 & s_6 \\ B & s_{-3} & s_0 & s_2 & s_3 \\ C & s_{-5} & s_{-2} & s_0 & s_1 \\ D & s_{-6} & s_{-3} & s_{-1} & s_0 \end{bmatrix}$$

Using the Saaty's scale, the order of preference can be written as:

$$A \underset{C_3}{\overset{S_3}{>}} B \underset{C_3}{\overset{S_3}{>}} C \underset{C_1}{\overset{S_1}{>}} D$$

Using the RPS-Low scale (200%):
The pairwise comparison matrix using RPS-Low scale is given below:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1.0000 & 1.7500 & 2.6250 & 3.2812 \\ 0.5714 & 1.0000 & 1.5000 & 1.8700 \\ 0.3810 & 0.6667 & 1.0000 & 1.2500 \\ 0.3048 & 0.5333 & 0.8000 & 1.0000 \end{bmatrix} \end{array}$$

The weights of the objects obtained using LLSM are shown in Table 15.

Table 15: The weights obtained for optics (2) problem using RPS-Low scale (200%)

Objects	A	B	C	D
Weight	0.4430	0.2532	0.1688	0.1350

Using RPS-Moderate scale (300%):
The pairwise comparison matrix for optics (2) problem with RPS-Moderate scale is given below:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1.0000 & 2.1250 & 3.7188 & 5.1133 \\ 0.4706 & 1.0000 & 1.7500 & 2.4062 \\ 0.2689 & 0.5714 & 1.0000 & 1.3750 \\ 0.1956 & 0.4156 & 0.7273 & 1.0000 \end{bmatrix} \end{array}$$

The weights obtained using LLSM for the optics (2) problem with RPS-Moderate are provided in Table 16.

Table 16: The weights obtained for Optics (2) problem using RPS-Moderate scale (300%)

Objects	A	B	C	D
Weights	0.5168	0.2432	0.1390	0.1011

Using RPS-High scale (400%):
The pairwise comparison matrix for optics (2) problem with RPS-Moderate scale is given below:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1.0000 & 2.5000 & 5.0000 & 7.5000 \\ 0.4000 & 1.0000 & 2.0000 & 3.0000 \\ 0.2000 & 0.5000 & 1.0000 & 1.5000 \\ 0.1333 & 0.3333 & 0.6667 & 1.0000 \end{bmatrix} \end{array}$$

The weights obtained using LLSM for the Optics (2) problem with RPS-High are provided in Table 17.

Table 17: The weights obtained for optics (2) problem using RPS-High scale (400%)

Objects	A	B	C	D
Weight	0.5769	0.2308	0.1154	0.0769

The comparison of the weights obtained using LLSM for the Optics (2) problem with different RPS scales along with Saaty's actual weights is provided in Table 18.

Table 18: Comparison of weights obtained for Optics (2) problem

Objects	A	B	C	D
Weights (RPS-Low)	0.4430	0.2532	0.1688	0.1350
Weights (RPS-Moderate)	0.5168	0.2432	0.1390	0.1011
Weights (RPS-High)	0.5769	0.2308	0.1154	0.0769
Actual Weights (Saaty)	0.6072	0.2186	0.1115	0.0627

Example 3: The Nation Wealth Problem

The linguistic pairwise matrix for the wealth of nations is given below:

	US (A)	USSR (B)	China (C)	France (D)	UK (E)	Japan (F)	W.Germany (G)
US(A)	S_0	S_3	S_8	S_5	S_5	S_4	S_4
USSR(B)	S_{-3}	S_0	S_6	S_4	S_4	S_2	S_3
China(C)	S_{-8}	S_{-6}	S_0	S_{-4}	S_{-4}	S_{-6}	S_{-4}
France(D)	S_{-5}	S_{-4}	S_4	S_0	S_0	S_{-2}	S_{-2}
UK(E)	S_{-5}	S_{-4}	S_0	S_0	S_0	S_{-2}	S_{-2}
Japan(F)	S_{-4}	S_{-2}	S_6	S_2	S_2	S_0	S_1
W.Germany(G)	S_{-4}	S_{-3}	S_4	S_2	S_2	S_{-1}	S_0

With relevant scales, the pair-wise comparison matrix takes the form shown below [20].

	US (A)	USSR (B)	China (C)	France (D)	UK (E)	Japan (F)	W.Germany (G)
US(A)	1	4	9	6	6	5	5
USSR(B)	1/4	1	7	5	5	3	4
China(C)	1/9	1/7	1	1/5	1/5	1/7	1/5
France(D)	1/6	1/5	5	1	1	1/3	1/3
UK(E)	1/6	1/5	1	1	1	1/3	1/3
Japan(F)	1/5	1/3	7	3	3	1	2
W.Germany(G)	1/5	1/4	5	3	3	1/2	1

The countries are arranged following the preferences given in the linguistic descriptions as under:

A > B > F > G > D = E > C
A > B > F > G > E = D > C

It can be observed that the countries D and E are equally good. Hence, they can be arranged in any of the manner shown above. Using the linguistic descriptions of Saaty's linear and proposed RPS-scales, the arrangement of the attributes can be given as below:

$$A \underset{c_3}{\overset{s_3}{.>}} B \underset{c_2}{\overset{s_2}{>}} F \underset{c_1}{\overset{s_1}{>}} G \underset{c_2}{\overset{s_2}{>}} D \underset{c_0}{\overset{s_0}{=}} E \underset{c_4}{\overset{s_4}{>}} C$$

$$A \underset{c_3}{\overset{s_3}{.>}} B \underset{c_2}{\overset{s_2}{>}} F \underset{c_1}{\overset{s_1}{>}} G \underset{c_2}{\overset{s_2}{>}} E \underset{c_0}{\overset{s_0}{=}} D \underset{c_4}{\overset{s_4}{>}} C$$

Using the RPS-Low scale (200%), the pairwise matrix formed is given as under:

$$\begin{bmatrix} & A & B & F & G & D & E & C \\ A & 1.0000 & 1.7500 & 2.6250 & 3.2812 & 4.1016 & 4.1016 & 8.2031 \\ B & 0.5714 & 1.0000 & 1.5000 & 1.8750 & 2.3438 & 2.3438 & 4.6875 \\ F & 0.3810 & 0.6667 & 1.0000 & 1.2500 & 1.5625 & 1.5625 & 3.1250 \\ G & 0.3048 & 0.5333 & 0.8000 & 1.0000 & 1.2500 & 1.2500 & 2.5000 \\ D & 0.2438 & 0.4267 & 0.6400 & 0.8000 & 1.0000 & 1.0000 & 2.0000 \\ E & 0.2438 & 0.4267 & 0.6400 & 0.8000 & 1.0000 & 1.0000 & 2.0000 \\ C & 0.1219 & 0.2133 & 0.3200 & 0.4000 & 0.5000 & 0.5000 & 1.0000 \end{bmatrix}$$

The weights obtained using RPS-Low scale are given in Table 19

Table 19: The weights obtained for the 'Nation Wealth' problem using RPS-Low scale

Nations	A	B	F	G	D	E	C
Weights	0.3488	0.1993	0.1329	0.1063	0.0850	0.0850	0.0425

Similarly, for RPS-Moderate scale (300%), the pairwise comparison matrix is given as below:

$$
\begin{array}{c} \\ A \\ B \\ F \\ G \\ D \\ E \\ C \end{array}
\begin{bmatrix}
A & B & F & G & D & E & C \\
1.0000 & 2.1250 & 3.7188 & 5.1133 & 8.9482 & 8.9482 & 22.3700 \\
0.4706 & 1.0000 & 1.7500 & 2.4063 & 4.2109 & 4.2109 & 10.5271 \\
0.2689 & 0.5714 & 1.0000 & 1.3750 & 2.4062 & 2.4062 & 6.0154 \\
0.1956 & 0.4196 & 0.7273 & 1.0000 & 1.7500 & 1.7500 & 4.3749 \\
0.1118 & 0.2375 & 0.4156 & 0.5714 & 1.0000 & 1.0000 & 2.4999 \\
0.1118 & 0.2375 & 0.4156 & 0.5714 & 1.0000 & 1.0000 & 2.4999 \\
0.0447 & 0.0950 & 0.1662 & 0.2286 & 0.4000 & 0.4000 & 1.0000
\end{bmatrix}
$$

Using the pairwise matrix above, the weights obtained for the Nation Wealth problem is provided in Table 20.

Table 20: The weights obtained for the 'Nation Wealth' problem using RPS-Moderate scale

Nations	A	B	F	G	D	E	C
Weights	0.3488	0.1993	0.1329	0.1063	0.0850	0.0850	0.0425

The pairwise matrix formed for the Nation Wealth problem with RPS-High scale is given as below:

$$
\begin{array}{c} \\ A \\ B \\ F \\ G \\ D \\ E \\ C \end{array}
\begin{bmatrix}
A & B & F & G & D & E & C \\
1.0000 & 2.5000 & 5.0000 & 7.5000 & 15.0000 & 15.0000 & 45.0000 \\
0.4000 & 1.0000 & 2.0000 & 3.0000 & 6.0000 & 6.0000 & 18.0000 \\
0.2000 & 0.5000 & 1.0000 & 1.5000 & 3.0000 & 3.0000 & 9.0000 \\
0.1333 & 0.3333 & 0.6667 & 1.0000 & 3.0000 & 3.0000 & 6.0000 \\
0.0667 & 0.1667 & 0.3333 & 0.5000 & 1.0000 & 1.0000 & 3.0000 \\
0.0667 & 0.1667 & 0.3333 & 0.5000 & 1.0000 & 1.0000 & 3.0000 \\
0.0222 & 0.0556 & 0.1111 & 0.1667 & 0.3333 & 0.3333 & 1.0000
\end{bmatrix}
$$

The weights of the objects using the RPS-High scale for 'Nation Wealth' are given in Table 21.

Table 21: The weights obtained for the 'Nation Wealth' problem using RPS-High scale

Nations	A	B	F	G	D	E	C
Weights	0.5294	0.2118	0.1059	0.0706	0.0353	0.0353	0.0118

A comparison of weights obtained using the three RPS-scales with the actual weights as provided by Saaty [20] are shown in Table 22.

Table 22: A comparison of the weights for 'Nation Wealth' with Saaty's actual weights

Nations	A	B	F	G	D	E	C
Weights with RPS-Low	0.3488	0.1993	0.1329	0.1063	0.0850	0.0850	0.0425
Weights with RPS-Moderate	0.4578	0.2067	0.1231	0.0895	0.0512	0.0512	0.0205
Weights with RPS-High	0.5294	0.2118	0.1059	0.0706	0.0353	0.0353	0.0118
Saaty's Actual Weights	0.4134	0.2249	0.1041	0.0910	0.0694	0.0546	0.0425

Example 4: The Weight Estimation Problem: In this problem, five objects, namely Radio(A), Type writer(B), Large attaché case(C), Projector (D), and Small attaché case (E) are compared and the pair-wise comparison matrix is given below:

$$Pairwise\ comparison\ matrix = \begin{bmatrix} & A & B & C & D & E \\ A & s_0 & s_{-4} & s_{-2} & s_{-3} & s_3 \\ B & s_4 & s_0 & s_1 & s_1 & s_7 \\ C & s_2 & s_{-1} & s_0 & s_{-1} & s_3 \\ D & s_3 & s_{-1} & s_1 & s_0 & s_6 \\ E & s_{-3} & s_{-7} & s_{-3} & s_{-6} & s_0 \end{bmatrix}$$

Depending on the linguistic pair-wise comparison the matrix, the preference order of the objects can be obtained as:

$$B \underset{c_1}{\overset{s_1}{>}} D \underset{c_1}{\overset{s_1}{>}} C \underset{c_2}{\overset{s_2}{>}} A \underset{c_3}{\overset{s_3}{>}} E$$

The pairwise comparison matrix for the 'Weight Estimation' problem using the RPS-Low scale is given below:

$$\begin{bmatrix} & B & D & C & A & E \\ B & 1.0000 & 1.2500 & 1.5625 & 2.3438 & 4.1016 \\ D & 0.8000 & 1.0000 & 1.2500 & 1.8750 & 3.2813 \\ C & 0.6400 & 0.8000 & 1.0000 & 1.5000 & 2.6250 \\ A & 0.4267 & 0.5333 & 0.6667 & 1.0000 & 1.7500 \\ E & 0.2438 & 0.3048 & 0.3809 & 0.5714 & 1.0000 \end{bmatrix}$$

The weights obtained for the 'Weight Estimation ' problem with RPS-Low are provided in Table 23.

Table 23: The weights of obtained for 'Weight Estimation' problem using RPS-Low

Objects	B	D	C	A	E
Weights	0.3215	0.2572	0.2058	0.1372	0.0784

The pairwise comparison matrix for the 'Weight Estimation' problem using the RPS-Moderate scale is given below:

$$\begin{array}{c|ccccc} & B & D & C & A & E \\ \hline B & 1.0000 & 1.3750 & 1.8906 & 3.3086 & 7.0308 \\ D & 0.7273 & 1.0000 & 1.3750 & 2.4063 & 5.1133 \\ C & 0.5289 & 0.7273 & 1.0000 & 1.7500 & 3.7188 \\ A & 0.3022 & 0.4156 & 0.5714 & 1.0000 & 2.2150 \\ E & 0.1422 & 0.1956 & 0.2689 & 0.4706 & 1.0000 \end{array}$$

Table 24: The weights obtained for the Weights Estimation' problem using RPS-Moderate scale

Objects	B	D	C	A	E
Weights	0.3703	0.2693	0.1959	0.1119	0.0527

The pairwise comparison matrix for the 'Weight Estimation' problem using the RPS-High scale is given below:

$$\begin{array}{c|ccccc} & B & D & C & A & E \\ \hline B & 1.0000 & 1.5000 & 2.2500 & 4.5000 & 11.2500 \\ D & 0.6667 & 1.0000 & 1.5000 & 3.0000 & 7.5000 \\ C & 0.4444 & 0.6667 & 1.0000 & 2.0000 & 5.0000 \\ A & 0.2222 & 0.3333 & 0.5000 & 1.0000 & 2.5000 \\ E & 0.0889 & 0.1333 & 0.2000 & 0.4000 & 1.0000 \end{array}$$

Table 25: The weights obtained for the Weights Estimation' problem using RPS-High scale

Objects	B	D	C	A	E
Weights	0.4128	0.2752	0.1835	0.0917	0.0367

The weights obtained for the Weights Estimation' problem using RPS-High scale are given in Table 25. A comparison of weights obtained using the three RPS-scales with the actual weights as provided by Saaty [20] are shown in Table 26.

Table 26: A comparison of the weights for 'Weight Estimation' with Saaty's actual weights

Objects	B	D	C	A	E
Weights with RPS-Low	0.3215	0.2572	0.2058	0.1372	0.0784
Weights with RPS-Moderate	0.3703	0.2693	0.1959	0.1119	0.0527
Weights with RPS-High	0.4128	0.2752	0.1835	0.0917	0.0367
Saaty's Actual relative Weights	0.3900	0.2700	0.2000	0.1000	0.0400

4.1. A comparison of the priorities obtained using RPS scales with those of existing scales

In this section, a house buyer's problem, illustrated by Saaty [21], with consistency is considered for comparison of the priorities obtained using the proposed RPS scale with other scales proposed in literature. For the same house buyer's problem, Franek and Kresta [8] have compared the priorities obtained with different scales existing in literature, namely, linear, power, root square, geometric, inverse linear, asymptotical, balanced and logarithmic scales. The pair-wise comparison with the proposed RPS scale is obtained for the same problem and a comparison of the weights is given by appending the weights obtained using RPS scales. In order to obtain the pair-wise matrix in the RPS scale, the same preferences as considered in Saaty [21] are taken as given in the matrix below:

$$
\begin{array}{cc}
 & \begin{array}{cccccccc} S & T & N & A & Y & M & C & F \end{array} \\
\begin{array}{c} S \\ T \\ N \\ A \\ Y \\ M \\ C \\ F \end{array} &
\begin{bmatrix}
s_0 & s_4 & s_2 & s_6 & s_5 & s_5 & s_{-2} & s_{-3} \\
s_4 & s_0 & s_{-2} & s_4 & s_2 & s_2 & s_{-4} & s_{-6} \\
s_{-2} & s_2 & s_0 & s_5 & s_2 & s_3 & s_5 & s_{-4} \\
s_{-6} & s_{-4} & s_{-5} & s_0 & s_{-2} & s_{-3} & s_{-6} & s_{-7} \\
s_{-5} & s_{-2} & s_{-5} & s_2 & s_0 & s_{-1} & s_{-4} & s_{-5} \\
s_{-5} & s_{-2} & s_{-3} & s_3 & s_1 & s_0 & s_{-4} & s_{-5} \\
s_2 & s_4 & s_{-5} & s_6 & s_4 & s_4 & s_0 & s_{-1} \\
s_3 & s_6 & s_4 & s_7 & s_5 & s_5 & s_1 & s_0
\end{bmatrix}
\end{array}
$$

In the above linguistic pairwise comparison matrix, the attributes considered are Size (S), Transport (T), Neighborhood (N), Age (A), Yard (Y), Modern (M), Conditions (C) and Finance (F).

Following the linguistic descriptions, the order of preference for the attributes is obtained as under:

$$F \overset{s_1}{\underset{c_1}{>}} C \overset{s_2}{\underset{c_2}{>}} S \overset{s_2}{\underset{c_2}{>}} N \overset{s_2}{\underset{c_2}{>}} T \overset{s_2}{\underset{c_2}{>}} M \overset{s_1}{\underset{c_1}{>}} Y \overset{s_2}{\underset{c_2}{>}} A$$

Applying the RPS-Low scale, the pairwise comparison matrix formed is given as under:

$$\begin{array}{c} \\ F \\ C \\ S \\ N \\ T \\ M \\ Y \\ A \end{array}\begin{bmatrix} F & C & S & N & T & M & Y & A \\ 1.0000 & 1.2500 & 1.8750 & 2.8125 & 4.2188 & 6.3281 & 7.9102 & 11.8652 \\ 0.8000 & 1.0000 & 1.5000 & 2.2500 & 3.3750 & 5.0625 & 6.3282 & 9.4922 \\ 0.5333 & 0.6667 & 1.0000 & 1.5000 & 2.2500 & 3.3750 & 4.2188 & 6.3281 \\ 0.3556 & 0.4444 & 0.6667 & 1.0000 & 1.5000 & 2.2500 & 2.8125 & 4.2187 \\ 0.2370 & 0.2963 & 0.4444 & 0.6667 & 1.0000 & 1.5000 & 1.8750 & 2.8125 \\ 0.1580 & 0.1975 & 0.2963 & 0.4444 & 0.6667 & 1.0000 & 1.2500 & 1.8750 \\ 0.1264 & 0.1580 & 0.2370 & 0.3556 & 0.5333 & 0.8000 & 1.0000 & 1.5000 \\ 0.0843 & 0.1054 & 0.1580 & 0.2370 & 0.3556 & 0.5333 & 0.6667 & 1.0000 \end{bmatrix}$$

Weights of the attributes for the house buying problem obtained with the RPS-Low scale using LLSM are provided in Table 27.

Table 27: Weights obtained using RPS-Low scale for the house buying problem

Attributes	F	C	S	N	T	M	Y	A
Weights	0.3035	0.2428	0.1619	0.1079	0.0719	0.0480	0.0384	0.0256

Applying the RPS-Moderate scale, the pairwise comparison matrix formed is given below:

$$\begin{array}{c} \\ F \\ C \\ S \\ N \\ T \\ M \\ Y \\ A \end{array}\begin{bmatrix} F & C & S & N & T & M & Y & A \\ 1.0000 & 1.3750 & 2.4062 & 4.2109 & 7.3691 & 12.8960 & 17.7320 & 31.0310 \\ 0.7273 & 1.0000 & 1.7500 & 3.0625 & 5.3593 & 9.3789 & 12.8960 & 22.5680 \\ 0.4156 & 0.5714 & 1.0000 & 1.7500 & 3.0625 & 5.3595 & 7.3693 & 12.8963 \\ 0.2375 & 0.3265 & 0.5714 & 1.0000 & 1.7500 & 3.0625 & 4.2110 & 7.3692 \\ 0.1357 & 0.1866 & 0.3265 & 0.5714 & 1.0000 & 1.7500 & 2.4063 & 4.2110 \\ 0.0775 & 0.1066 & 0.1866 & 0.3265 & 0.5714 & 1.0000 & 1.3750 & 2.4062 \\ 0.0564 & 0.0775 & 0.1357 & 0.2375 & 0.4156 & 0.7273 & 1.0000 & 1.7500 \\ 0.0322 & 0.0443 & 0.0775 & 0.1357 & 0.2375 & 0.4156 & 0.5714 & 1.0000 \end{bmatrix}$$

Weights of the attributes for the house buying problem obtained with the RPS-Moderate scale using LLSM are provided in Table 28.

Table 28: Weights obtained using RPS-Moderate scale for the house buying problem

Attributes	F	C	S	N	T	M	Y	A
Weights	0.3728	0.2711	0.1549	0.0885	0.0506	0.0289	0.0210	0.0120

Applying the RPS-High scale, the pairwise comparison matrix formed is given as under:

$$\begin{array}{c} \\ F \\ C \\ S \\ N \\ T \\ M \\ Y \\ A \end{array} \begin{bmatrix} F & C & S & N & T & M & Y & A \\ 1.0000 & 1.5000 & 3.0000 & 6.0000 & 12.0000 & 24.0000 & 36.0000 & 72.0000 \\ 0.6667 & 1.0000 & 2.0000 & 4.0000 & 8.0000 & 16.0000 & 24.0000 & 48.0000 \\ 0.3333 & 0.5000 & 1.0000 & 2.0000 & 4.0000 & 8.0000 & 12.0000 & 24.0000 \\ 0.1667 & 0.2500 & 0.5000 & 1.0000 & 2.0000 & 4.0000 & 6.0000 & 12.0000 \\ 0.0833 & 0.1250 & 0.2500 & 0.5000 & 1.0000 & 2.0000 & 3.0000 & 6.0000 \\ 0.0417 & 0.0625 & 0.1250 & 0.2500 & 0.5000 & 1.0000 & 1.5000 & 3.0000 \\ 0.0278 & 0.0417 & 0.0833 & 0.1667 & 0.3333 & 0.6667 & 1.0000 & 2.0000 \\ 0.0139 & 0.0208 & 0.0417 & 0.0833 & 0.1667 & 0.3333 & 0.5000 & 1.0000 \end{bmatrix}$$

Weights of the attributes for the house buying problem obtained with the RPS-High scale using LLSM are provided in Table 29.

Table 29: Weights obtained using RPS-High scale for the house buying problem

Attributes	F	C	S	N	T	M	Y	A
Weights	0.4286	0.2857	0.1429	0.0714	0.0357	0.0179	0.0119	0.0060

Finally, a comparison of the weights obtained for all the attributes of the 'Buying a House' problem using different scales along with the weights obtained by employing the proposed three RPS scales is provided in the Table 30.

In Table 30, the numbers in bold show the lowest and highest values of weights obtained for each attribute using different scales. Except for the attribute 'Conditions', the weights obtained for all attributes using RPS scales are found to fall between the extreme values obtained using all the scales. The weights obtained using RPS-Low and RPS-Moderate scales are found to be well within the low and high values obtained using all different scales. It is observed that the values obtained using RPS-moderate scales are much closer to the weights obtained using Saaty's linear scale.

Table 30: Comparison of the weights obtained for different scales [8] with proposed RPS-Low, Moderate and High scales

Type of Scale	F	C	S	N	T	M	Y	A
Linear	0.345	0.221	**0.175**	0.103	0.062	0.041	0.034	0.019
Power	0.592	0.227	**0.116**	**0.042**	**0.013**	0.006	**0.003**	**0.001**
Root-square	0.242	0.190	0.161	0.124	**0.094**	0.075	0.066	0.048
Geometric	**0.614**	0.199	0.129	0.038	0.010	**0.005**	**0.003**	0.001
Inverse linear	0.270	0.181	0.172	0.120	0.083	0.071	0.066	0.038
Asymptotical	**0.240**	**0.174**	0.164	**0.124**	0.093	**0.081**	**0.076**	0.049
Balanced	0.300	0.196	0.176	0.115	0.073	0.058	0.053	0.042
Logarithmic	0.260	0.199	0.162	0.122	0.089	0.069	0.058	**0.073**
RPS (Low)	0.304	0.243	0.162	0.108	0.072	0.048	0.038	0.026
RPS (Moderate)	0.379	0.271	0.153	0.089	0.051	0.029	0.021	0.012
RPS (High)	0.429	**0.286**	0.143	0.071	0.036	0.018	0.012	0.006

5. Summary and Conclusion

In the present work, a simple methodology is suggested in order to avoid consistency check of the pairwise comparison matrix, which is essential in the AHP method for better results. In the AHP method, the attributes are compared pairwise and the comparison matrix is formed by assigning certain numerical values from the selected scales. This procedure may lead to a possibility of inconsistencies in the pairwise comparison. In the suggested method, pairwise comparisons are made among the attributes to arrange them in the order of their importance. After recognizing the most dominant attribute which will be assigned a numerical scale of unity, the comparison is made with the next dominating attribute and the numerical values are assigned with respect to the preceding attribute. Hence, there will be p-1 comparisons only while assigning the numerical values where p is the number of attributes. Even though it requires many pairwise comparisons to arrange the attributes in the descending order of their importance, this number cannot exceed the number of comparisons needed to form the pairwise comparison matrix. i.e. this process may need less than or equal to ${}^{p}C_{2}$ comparisons. Initial comparison among attributes arranged in the descending order forms the first row of the pairwise comparison matrix in the usual procedure, and this first row is used in deducting the remaining rows of the pairwise comparison matrix, which avoids consistency check.

The present methodology is applied on the problems suggested by Saaty [20], with three different scales suggested, viz., high, moderate and low. It can be observed that in the "Distance' problem (example no.1), the weights obtained using RPS-Low scale are very close to Saaty's real relative distances. In the 'Optics' problem (example no.2), the weights obtained using an RPS-High scale are nearer to the actual relative brightness values given by Saaty. For the 'National Wealth' and 'Weight Estimation' problems, the weights obtained using the RPS-Moderate scale are closer to the actual GNP and relative weights as given in [20]. It can be stated that the decision maker can choose a suitable scale by comparing the most dominant attribute with other attributes. If the most dominant attribute is highly important compared to the other attributes in the list, a High scale (400%) can be used. If the attributes do not differ much from each other, but a hierarchy still exists, a Low scale (200%) can be chosen. In general, a Moderate scale can be chosen.

References

[1] Brans J. P., Vincke P., Mareschal B. How to select and how to rank projects: The PROMETHEE method, *European Journal of Operations Research*, 24, 228-238, 1986.

[2] Cagno E., Caron F., Pereg A. Multicriteria assessment of the probability of winning in competitive bidding process, *International Journal of Production Management*, 19, 313-324, 2001.

[3] Chamodrakas I., Batis D., Martakos D. Supplier selection in electronic market places using satisficing and fuzzy AHP, *Expert Systems with Applications*, 37, 490-498, 2010.

[4] Dagdeviren M., Yavuz S., Killinc N. Weapon selection using the AHP and TOPSIS methods under fuzzy environments, *Expert Systems with Applications*, 36, 8143-8151, 2009.

[5] Dodd F. J., Donegan H. A. Comparison of prioritization techniques using hierarchy mappings, *Journal of the operational research society*, 46(40), 492-498, 1995.

[6] Donegan H. A., Dodd F. J. A note on Saaty's random indexes, *Mathematical Computer Modelling*, 15, 135-137, 1991.

[7] Dong Y. Xu Y., Li H., Dai M., A comparative study of the numerical scales and the prioritization methods in AHP, *European Journal of Operational Research*, 186, 229-242, 2008.

[8] Franek J., Kresta A., Judgment Scales and Consistency Measure in AHP, *Procedia Econ. Financ.*, 12, 164–173, 2014.

[9] French S. *Decision Theory - an Introduction to the Mathematics off rationality*, Chapter 10, Section 10.3, Ellis Horwood Ltd, 1988.

[10] Harker P., Vegas L. The theory of ratio scale estimation: Saaty's Analytic Hierarchy Process, *Management Science*, 33(11), 1383-1403, 1987.

[11] Hwang C. L., Yoon K. P. *Multiple attribute decision making methods and applications*, Springer, New York, 1981.

[12] Ishikaza A., Balkenborg D., Kaplan T. Influence of aggregation and measurement scale on ranking a compromise alternatives in AHP, *Journal of the Operational Research Society*, 62(4), 700-710, 2010.

[13] Ji P., Jiang R. Scale sensitivity in the AHP, *Journal of the Operational Research Society*, 54, 896-905, 2003.

[14] Kahraman C., Kaya İ. A fuzzy multicriteria methodology for selection among energy alternatives. *Expert Syst. Appl.* 37, 6270–6281, 2010.

[15] Lai V. Wong B. K., Cheng W. Group decision making in a multiple criteria environment. A case of using AHP in software selection, *European Journal of Operations Research*, 137, 134-144, 2002.

[16] Lootsma, F. Conflict resolution via Pairwise comparison of concessions, *European Journal of Operational Research*, 40(1), 109-116, 1989.

[17] Ma D., Zheng X. Scale method of AHP, In *2nd International Symposium on (Vol.I), University of Pittsburgh*, 197-202,1991.

[18] Munizu M., Riyadi S. An application of analytical hierarchy process (AHP) in formulating priority strategy for enhancing creative industry competitiveness, *Decision Science Letters*, 10(3), 443-450, 2021.

[19] Roy B. The outranking approach and the foundations of ELECTRE methods., *Theory Decis*, 31, 49-73, 1991.

[20] Saaty T. A scaling method for priorities in hierarchical structures, *Journal of Mathematical Psychology*, 15, 234-281, 1977.

[21] Saaty T. Decision making with the AHP. Why is the principal eigenvector necessary, *European Journal of Operational Research*, 145, 85-91,2003.

[22] Salo A., Hamalaimen R. On the measurement of preference in the Analytical Hierarchy Process, *Journal of Multi-Criteria Decision Analysis*, 6 (6), 309-319, 1997.

[23] Shivade A. S., Sapkal, S. U. Selection of optimum plant layout using AHP-TOPSIS and WASPAS approaches coupled with entropy method, *Decision Science Letters*,11(4), 545-562, 2022.

[24] Siregar H., Suroso A. I., Siregar H., Djohar S. Development of efficient strategies to optimize production efficiency: Evidence from pine chemical industry, *Decision Science Letters*, 11(4), 419-430, 2022.

[25] Vidal L., Sahin E., Martelli N., Berhoune M., Bonan B. Applying AHP to select drugs to be produced by anticipation in a chemotherapy compounding unit, *Expert Systems with Applications*, 37, 1528-1534, 2010.

[26] Vommi V. B. TOPSIS with statistical distances: A new approach to MADM, *Decision Science Letters*, 6(1), 49-66, 2017.

[27] Vommi V. B., Kakollu S. A simple approach to multiple attribute decision making using loss function approach, *Journal of Industrial Engineering International*, 13, 107-116, 2017.

[28] Yu P. L. A class of solutions for group decision problems, *Management Science*, 19, 936-946, 1973.

Received 11.03.2024, Accepted 21.12.2024