FOUNDATIONS OF COMPUTING AND DECISION SCIENCES
Vol. 50 (2025) No. 1
ISSN 0867-6356
DOI: 10.2478/fcds-2025-0001 e-ISSN 2300-3405

Ranking of Efficient Fuzzy Portfolios by Hybrid MSBM-TOPSIS Technique

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Abstract. This study focuses on ranking of investment portfolios by integrating the Modified Slack Based Measure (MSBM) of Data Envelopment Analysis (DEA) with a multi-criteria decision-making method. Specifically, it extends the MSBM model to evaluate portfolios with positive and negative inputs and outputs in a fuzzy environment using possibilistic mean return of the assets as output and possibilistic variance and semi-variance as inputs. The ranking process involves two stages: first, portfolios are evaluated using an MSBM fuzzy portfolio model, followed by their ranking through the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method. This hybrid MSBM-TOPSIS approach provides a robust and reliable ranking system, enabling investors to identify efficient portfolios by leveraging the strengths of both methods. Detailed numerical illustrations are presented here to authenticate the proposed approach and the obtained results are compared with other existing DEA methods that validate the accuracy and feasibility of the proposed technique.

Keywords: Fuzzy Set, Modified Slack based Measure, Data Envelopment Analysis, Portfolio, TOPSIS method

1. Introduction

In financial terminology, a portfolio is simply a collection of financial assets such as stocks, bonds, cash, currency, derivatives, etc. Portfolio optimization employs a

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method to create an ideal asset mix, taking into account the goals and preferences of the investor. The major revolution in portfolio selection theory came in 1952 with the path-breaking work of [27]. His insight resulted in an optimization model now famous as the mean-variance model, where the portfolio's return and risk are quantified using the statistical measures of expected return and variance of the portfolio return distribution, respectively. Thus, an efficient portfolio has the minimum risk level for a given return or the maximum return for a given level of risk. The distance between the portfolio under evaluation and its projection on the efficient frontier can be used to calculate the efficiency of the portfolio by the M-V model [48].

Efficient frontiers are challenging, especially in large-scale evaluation scenarios, because of the many constraints imposed by real-world investment conditions [43]. To deal with this issue, the traditional DEA methodologies are widely used to approximate portfolio efficiency. DEA is one of the most prevalent approaches for evaluating the relative efficiency of concern portfolios based on multiple inputs and outputs. The first DEA model was initiated in 1978 by [9] to calculate the efficiency of an educational center and then further developed by [5] in 1984. Numerous researchers have utilized DEA models to assess portfolio efficiency. [14] addressed a portfolio performance evaluation problem by incorporating margin limitations, demonstrating through simulation how the DEA frontier increasingly aligns with the portfolio frontier as the sample size expands. [24] applied a DEA frontier improvement technique to suggest a re-balancing strategy for portfolios inside the MV framework. [48] presents a segmented DEA method that depends on data segment points to handle the problem of cardinality constraints in portfolio selection problems.

It is important to note that DEA models inherently assume non-negative input and output values. However, negative inputs or outputs may arise in many cases, such as losses relative to net profit, negative net income when expenses surpass revenues, or return rates on investments. [33] introduced the range directional model (RDM) in 2004, which is applicable in such circumstances. [37] proposed a modified slack-based measure (MSBM) capable of handling both negative outputs and negative inputs. Some other DEA-based models for dealing with negative inputs and outputs are the semi-oriented radial measure (SORM) model by [15], the Variant of the radial measure (VRM) model by [12], BP-SBM model by [39] etc.

The studies mentioned above rely on a probabilistic environment assuming future asset values are known ex-ante, but accurately predicting returns from historical data is impractical due to market complexity and volatility. Non-probabilistic factors like policies, regulations, and human behavior further complicate this. Fuzzy techniques, introduced by [44], offer a better fit for handling ambiguity in financial markets, with methods like possibility, credibility, and random fuzzy theories proposed for fuzzy portfolio selection. Possibility theory, introduced by [45] is an uncertainty framework designed to address incomplete and imprecise information. The concept of possibility theory used in fuzzy-set approaches is generally more applicable when examining an early stage of a decision process and when little information is available. [7] proposed the definition of lower and upper possibilistic mean values of a fuzzy number and considered them as possibility distributions. Besides, [46] defined the lower and upper possibilistic variances and covariances of fuzzy numbers. [8]

developed a possibilistic approach to portfolio selection with the highest utility score by assuming the returns of assets as trapezoidal fuzzy numbers. [25] formulated two possibilistic mean semi-variance models for portfolio selection, in which market risk and liquidity risk are measured by lower possibilistic semi-variance. [28] proposed a fuzzy multi-objective Markowitz-DEA cross-efficiency method in a possibilistic framework. [11] introduced DEA-based fuzzy portfolio estimation models to find the fuzzy portfolio efficiency. They construct three types of DEA-based fuzzy portfolio models assuming different risk indicators, i.e., possibilistic variance, possibilistic semi-variance, and possibilistic semi-absolute deviation. Later, possibility theory and credibility theory-based fuzzy portfolio optimization models were proposed by [10, 41, 29, 17].

The Multi-Criteria Decision-making (MCDM) method makes decisions when multiple alternatives are available for contradictory criteria. Its principal goal is to maintain control over the decision-making process in case of various alternative and criterion diversity. TOPSIS is a powerful MCDM approach developed by [20] to evaluate the performance of different alternatives based on the similarity degree with the ideal solution. According to this technique, the best option should have the shortest distance from the ideal solution and the farthest from the negative ideal.

Following the work presented in [11], we offer the hybrid MSBM-TOPSIS technique that considers efficient fuzzy portfolios as the available alternative and different inputs and outputs of efficient portfolios as the evaluation attributes. Similar to the work [11], randomly generated fuzzy portfolios are considered in this study, where the return from the asset is a trapezoidal fuzzy number. Two types of risk measures, possibilistic variance, and possibilistic semi-variance, are considered here with numerical examples which effectively verify the proposed approach.

1.1. Literature Review of Ranking models in DEA

In DEA, each DMU is self-evaluated with the most favorable weights for input and output to obtain relative efficiency. Consequently, DEA might assess too many DMUs as the best and cannot distinguish them further. To cope with this issue, [2] introduced a variable return to scale (VRS) based super efficiency (SE) DEA model in which the efficient DMU under SE evaluation is excluded and compared with a linear combination of the remaining DMUs in the production possibility set. Despite being infeasible in some cases, Super-efficiency models are widely applied and extended in literature. [23] developed a directional distance function (DDF) based super-efficiency DEA model, building on [2] approach, by designing a suitable directional vector. This model can accommodate negative values in the dataset and also resolves the infeasibility problem associated with the VRS radial super-efficiency DEA model.[40] Integrate the super-efficiency model with the SBM model. [32] applied the SE model to the water use efficiency of Shandong Province cities. [18] introduced RDM super efficiency model to provide a unique ranking for efficient DMUs. Although the DEA-super efficiency models provide a unique raking to the efficient DMUs, researchers still argue for developing more methods for ranking ef-

ficient DMUs due to the infeasibility of DEA-SE models in many problems.
It is important to note that each method is based on a specific theory and framework, which can lead to different ranking outcomes for the same DMU. In practice, selecting a ranking method that the Decision Maker (DM) can trust is a critical concern. The challenge of ranking efficient DMUs can be addressed by employing Multi-Criteria Decision-Making (MCDM) techniques, which, although distinct from traditional DEA models, offer a complementary approach for more reliable rankings. TOPSIS method is one of the MCDM methods widely used in the literature. Some studies have combined TOPSIS and DEA within a unified framework to generate a distinct ranking of DMUs. In these studies, one technique is typically used as the primary method, while the other serves as a supplementary tool. By leveraging both DEA and TOPSIS, decision-makers can effectively achieve the objectives of a corporate sourcing strategy through a comprehensive evaluation approach. [26] considered efficient DMUs from different DEA models and ranked them using the TOPSIS method. [4] proposed a new method for ranking efficient units and compared it with the other methods presented in the literature, such as the super-efficiency model in DEA, the concept of Shapley value in game theory, and the TOPSIS method in multi-criteria decision making. [34] utilized a hybrid approach based on TOPSIS and DEA methods to obtain a rational ranking of efficient DMUs. [21] presented two models for ranking efficient units. Both are based on MCDM methods; the first uses the goal programming technique for deriving a super-efficiency measure, and the second combines AHP and DEA principles. [13] employed the DEA method to evaluate the efficiency of 15 Indian bank branches, using the SE-CCR-DEA model and TOPSIS for ranking. The study compares the outcomes of the CCR-DEA, SE-CCR-DEA, and TOPSIS methods. [30] used the hybrid DEA-TOPSIS method for ranking of the 14 state water service providers in Malaysia. [31] used a three-step DEA model to obtain four types of efficiency (technical efficiency, social efficiency, environmental efficiency scores, and sustainable efficiency) for Iranian airlines. They integrate DEA models with the TOPSIS method to aggregate the final efficiency score. [47] presents a method to compare and rank the interval efficiencies of DMUs. [38] recently introduced an integrated framework combining multi-attribute decision-making with DEA models to address heterogeneous attributes in evaluating European countries' SDG progress. This method reduces computational complexity and eliminates the need for expert-defined weights, with results compared to standard MCDM techniques. [3] proposed a novel hybrid DEA-TOPSIS approach to identify the most sustainable refrigeration system technologies for the UK. [16] focused on evaluating the performance of distance education departments at public universities in Turkey during the 2018–2019 academic year, utilizing both the DEA and TOPSIS methods. The study also aimed to compare and rank the efficient decision-making units based on their efficiency measurements. [6] proposed an integrated interval DEA model that classifies DMUs into three categories—super-efficient, efficient, and inefficient in a single step, accommodating integer and negative data. A detailed review of DEA ranking models has been presented in [1].
From the detailed literature analysis, we conclude that using DEA and TOPSIS is a good idea for handling the problem of robust ranking of DMUs.

1.2. Contributions and Purpose

In this study, we have made an effort to add one more aspect to the DEA application, i.e. ranking of efficient fuzzy portfolios with two risk metrics in the presence of negative input-outputs. The following are the key highlights of the current study.
1. The MSBM model is commonly employed to assess the efficiency of DMUs in the presence of negative data. However, its application in evaluating the efficiency and ranking of portfolios with uncertainty has not been thoroughly explored in the existing literature. This research represents the first attempt to extend the MSBM model within a possibilistic framework, allowing it to address and manage fuzzy variables effectively. By incorporating fuzziness into the model, this study aims to fill the gap in portfolio efficiency analysis and provide a novel approach that enhances decision-making under uncertainty.
2. Based on previous discussions, researchers across different fields generally agree that utilizing DEA and TOPSIS is a valuable approach. Very few studies [19, 22] etc. address full ranking with negative input-output variables, so we combine the MSBM fuzzy portfolio model with TOPSIS to leverage the strengths of both methods.
3. Implementing the MSBM fuzzy portfolio estimation models with TOPSIS for ranking the efficient portfolios is a new application in the field of DEA.
4. We demonstrate the applicability and effectiveness of our methodology using actual data from the Stock Exchange in portfolio construction challenges.

The remainder of the paper is organized in the following manner. Section 2 presents some basic definitions and preliminaries regarding fuzzy numbers and their possibilistic mean, variance, and semi-variance for portfolio selection. Section 3 offers the MSBM fuzzy portfolio estimation model in a possibilistic framework. Section 4 presents the various steps of the TOPSIS method to rank the efficient portfolios. In Section 5, numerical examples are presented to justify the proposed approach. Finally, Section 6 presents the conclusions with some future directions.

2. Preliminaries

In this section, we present the basics of fuzzy sets and fuzzy numbers. Further, the mathematical models of [8], for computing the possibilistic mean, possibilistic variance, and possibilistic semi-variance of a fuzzy number are presented here.

Definition 1 *Let X be a universal set. A fuzzy set $\tilde{A}$ in X is denoted by*

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle \mid x \in X\},$$

where $\mu_{\tilde{A}} : X \to [0,1]$ describes the degree of belongings of an element $x \in X$ to the set $\tilde{A}$.

Definition 2 (γ-cut) *Let $\tilde{A}$ be a fuzzy set in X and $\gamma \in (0,1]$. The γ-cut of the fuzzy set $\tilde{A}$ is the crisp set A_γ given by*

$$A_\gamma = \{x \in X : \mu_{\tilde{A}}(x) \geq \gamma\}.$$

Definition 3 *A fuzzy set $\tilde{A}$ in $\mathbb{R}$ is called a fuzzy number if it satisfies the following conditions*

1. *$\tilde{A}$ is normal.*
2. *$\tilde{A}_\gamma$ is a closed interval for every $\gamma \in (0,1]$.*
3. *The support of $\tilde{A}$ is bounded.*

Definition 4 (Trapezoidal fuzzy number (TrFN)) *A fuzzy number $\tilde{A}$ represented by the quadruplet $\tilde{A} = [a,b,\alpha,\beta]$, where $[a,b]$ is the tolerance interval; α and β denote the left and right width of the TrFN- $\tilde{A}$, respectively, is called a trapezoidal fuzzy number if its membership function is given by*

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq a-\alpha, x \geq b+\beta, \\ \dfrac{x-a+\alpha}{\alpha}, & a-\alpha \leq x \leq a, \\ 1, & a \leq x \leq b, \\ \dfrac{b+\beta-x}{\beta}, & b \leq x \leq b+\beta, \end{cases}$$

Further, the γ-cut of the TrFN $\widetilde{A} = (a,b,\alpha,\beta)$ is the closed interval and given by

$$A^\gamma = [a_1^\gamma, a_2^\gamma] = [(a-(1-\gamma)\alpha,\ b+(1-\gamma)\beta],\ \ \gamma \in (0,1].$$

Definition 5 *Let $\tilde{A}$ be a fuzzy number with $A^\gamma = [a_1^\gamma, a_2^\gamma]$, $\gamma \in (0,1]$. The (crisp) possibilistic mean (or expected value) and possibilistic variance of a fuzzy number $\tilde{A}$, denoted by $E(\tilde{A})$ and $Var(\tilde{A})$, is defined as*

$$E(\tilde{A}) = \int_0^1 \gamma\left(a_1^\gamma + a_2^\gamma\right) d\gamma,$$

and

$$Var(\tilde{A}) = \frac{1}{2}\int_0^1 \gamma\left(a_2^\gamma - a_1^\gamma\right)^2 d\gamma.$$

respectively.

It is easy to see that for a TrFN $\tilde{A} = [a,b,\alpha,\beta]$

$$E(\tilde{A}) = \int_0^1 \gamma[a-(1-\gamma)\alpha + b + (1-\gamma)\beta]d\gamma = \frac{(a+b)}{2} + \frac{(\beta-\alpha)}{6},$$

and

$$Var(\tilde{A}) = \left[\frac{(b-a)}{2} + \frac{(\alpha+\beta)}{6}\right]^2 + \frac{(\alpha+\beta)^2}{72}.$$

Further, [36] defined the lower possibilistic semi-variance of a fuzzy number $\tilde{A}$ denoted by $Var^{-}(\tilde{A})$, as

$$Var^{-}(\tilde{A}) = \int_0^1 2\gamma \left(E(\tilde{A}) - a_1^{\gamma}\right)^2 d\gamma.$$

Thus for a TrFN $\tilde{A} = [a, b, \alpha, \beta]$, the lower possibilistic semi-variance is as follows

$$Var^{-}(\tilde{A}) = \left[\frac{(b-a)}{2} + \frac{(\alpha+\beta)}{6}\right]^2 + \frac{(\alpha)^2}{18}.$$

Assume there are n risky assets whose returns are TrFNs represented by $\tilde{r}_i = (a_i, b_i, \alpha_i, \beta_i)$, $i = 1, 2, \ldots, n$ with investment proportion w_i, $i = 1, 2, \ldots, n$, such that $w_i \geq 0$; $\sum_{i=1}^{n} w_i = 1$. Also, there is a fixed linear transaction cost k_i associated with each asset. As a result, the portfolio's probabilistic expected net return, variance, and lower semi-variance, denoted by R, σ^2 and $(\sigma^2)^{-}$ are given by as follows

$$R = E\left(\sum_{i=1}^{n} \tilde{r}_i w_i - \sum_{i=1}^{n} k_i w_i\right) = \sum_{i=1}^{n} \left[\left\{\frac{1}{2}\left(a_i + b_i + \frac{1}{3}(\beta_i - \alpha_i)\right) w_i\right\} - k_i w_i\right],$$

$$\sigma^2 = Var\left(\sum_{i=1}^{n} \tilde{r}_i w_i\right) = \left[\left(\frac{(b_i - a_i)}{2} + \frac{(\alpha_i + \beta_i)}{6}\right) w_i\right]^2 + \frac{1}{72}\left[\sum_{i=1}^{n} (\alpha_i + \beta_i) w_i\right]^2,$$

and

$$(\sigma^2)^{-} = Var^{-}\left(\sum_{i=1}^{n} \tilde{r}_i w_i\right) = \left[\left(\frac{(b-a)}{2} + \frac{(\alpha+\beta)}{6}\right) w_i\right]^2 + \frac{1}{18}\left[\sum_{i=1}^{n} (\alpha_i w_i)\right]^2,$$

respectively.

2.1. Modified Slack Based Model to handle negative data

To address the issue of negative data in conventional DEA models, [37] introduced a modified slack-based measure (MSBM) model. The MSBM model is modified by [42] for evaluating the performance of production units by taking direction different from [37].

Consider that there are P DMUs to be evaluated, each using m inputs to generate s outputs. Let $x_{tj}, (t = 1, \ldots m)$ represent the t^{th}, input for the j^{th} DMU, and $y_{rj}, (r = 1, \ldots s)$ denote the r^{th} output of the j^{th} DMU. The modified MSBM method is presented as follows:

$$(\textbf{Model I}) \qquad \min \ \theta_0 = \frac{1 - \frac{1}{m}\sum\limits_{t=1}^{m} \frac{s_{io}^{-}}{R_t^{-}}}{1 + \frac{1}{s}\sum\limits_{r=1}^{s} \frac{s_{ro}^{+}}{R_r^{+}}}$$

$$\text{s. t. } \sum_{j=1}^{P} \lambda_j x_{tj} + s_{t0}^{-} = x_{t0} \quad t = 1, 2, \ldots, m$$

$$\sum_{j=1}^{P} \lambda_j y_{rj} - s_{r0}^{+} = y_{r0} \qquad r = 1, 2, \ldots, s$$

$$\sum_{j=1}^{P} \lambda_j = 1 \ \ \lambda_j \geq 0, j = 1, 2...P. \ , s_{t0}^{-}, \ s_{r0}^{+} \geq 0$$

The objective function $\theta_0^{\star}$ represents the optimal efficiency score of DMU_0. The terms s_{t0}^{-} $(t = 1, \ldots, m)$ and s_{r0}^{+} $(r = 1, \ldots, s)$ represent the t^{th} input excess and r^{th} output shortfall of DMU_0, referred to as slacks. The λ_j $(j = 1, \ldots, P)$ are intensity variables used to link inputs and outputs in a convex combination. Also, R_t^{-} and R_r^{+} are the feasible directions to move and are defined as follows:

$$R_t^{-} = \max_{j=1,2,\ldots,P} x_{tj} - \min_{j=1,2,\ldots,P} x_{tj}, \quad t = 1, 2, \ldots, m$$

$$R_r^{+} = \max_{j=1,2,\ldots,P} y_{rj} - \min_{j=1,2,\ldots,P} y_{rj}, \quad r = 1, 2, \ldots, s$$

In the following section, we present the MSBM fuzzy portfolio estimation models based on [11].

3. MSBM fuzzy portfolio evaluation models in the framework of possibilistic theory

In this section, we present the MSBM fuzzy portfolio estimation model to handle the unpredictability of the variables.
Assume that there are P portfolios to be evaluated. To evaluate a portfolio performance, we consider the risk associated with the portfolio as the input indicator, and the corresponding return as the output indicator. In this study, we assume n risky assets with fuzzy return as $\tilde{r}_i = (a_i, b_i, \alpha_i, \beta_i)$, $i = 1, 2, ..., n$ with investment proportions w_{ij}, $i = 1, 2, ..., n$; $j = 1, 2, \ldots, P$, such that $\sum\limits_{i=1}^{n} w_{ij} = 1$, are considered for constructing j^{th} portfolio which is considered to be a DMU. Suppose k_i is the unit transaction cost

of a unit for the i^{th} risky asset. Then the expected net return, variance, and semi-variance of the j^{th} portfolio denoted by $\overline{R}_j, \sigma_j^2$ and $(\sigma^2)_j^-$ respectively for $(j = 1, 2, \ldots, P)$ are given respectively. To employ the MSBM models for the efficiency evaluation of all portfolios, in our study we consider one input and one output ($m = s = 1$) criteria as risk and return respectively.

3.1. MSBM fuzzy portfolio evaluation model in possibilistic mean-variance framework

To handle the problem of negative data and to provide a full ranking of all DMUs, [42] suggested a novel MSBM (modified slacks-based model) model. We extend the above-presented model I in the possibilistic mean-variance framework. The mathematical form of the MSBM fuzzy portfolio model with possibilistic variances input (risk) and possibilistic mean as output (return) is given as follows:

$$(\textbf{Model II}) \qquad \min \ \rho_0 = \frac{1 - \dfrac{s_{10}^-}{R_1^-}}{1 + \dfrac{s_{10}^+}{R_2^+}}$$

$$\text{s. t.} \ \sum_{j=1}^{P} \lambda_j \left(\left\{ \sum_{i=1}^{n} \frac{1}{2} \Big[(b_i - a_i) + \frac{\alpha_i + \beta_i}{3} \Big] w_{ij} \right\}^2 + \frac{1}{72} \Big[\sum_{i=1}^{n} (\alpha_i + \beta_i) w_{ij} \Big]^2 \right) + s_{10}^-$$

$$= \left(\left\{ \sum_{i=1}^{n} \frac{1}{2} \Big[(b_i - a_i) + \frac{\alpha_i + \beta_i}{3} \Big] w_{i0} \right\}^2 + \frac{1}{72} \Big[\sum_{i=1}^{n} (\alpha_i + \beta_i) w_{i0} \Big]^2 \right)$$

$$\sum_{j=1}^{P} \lambda_j \left(\sum_{i=1}^{n} \frac{1}{2} \Big[(b_i + a_i) + \frac{\beta_i - \alpha_i}{3} \Big] w_{ij} - \sum_{i=1}^{n} k_i w_{ij} \right) - s_{10}^+$$

$$= \sum_{i=1}^{n} \frac{1}{2} \Big[(b_i + a_i) + \frac{\beta_i - \alpha_i}{3} \Big] w_{i0} - \sum_{i=1}^{n} k_i w_{i0},$$

$$\sum_{j=1}^{P} \lambda_j = 1 \ \ \lambda_j \geq 0 \ , s_{10}^-, \ s_{10}^+ \geq 0, \quad j = 1, 2, \ldots, P$$

Here, R_1^- and R_2^+ can be modified as follows :

$$R_1^- = \max_{j=1,2,\dots,P}\left(\sum_{i=1}^{n}(\frac{1}{2}[(b_i - a_i) + \frac{\alpha_i + \beta_i}{3}]w_{ij})^2 + \frac{1}{72}[\sum_{i=1}^{n}(\alpha_i + \beta_i)w_{ij}]^2\right)$$
$$- \min_{j=1,2,\dots,P}\left(\sum_{i=1}^{n}(\frac{1}{2}[(b_i - a_i) + \frac{\alpha_i + \beta_i}{3}]w_{ij})^2 + \frac{1}{72}[\sum_{i=1}^{n}(\alpha_i + \beta_i)w_{ij}]^2\right)$$

$$R_2^+ = \max_{j=1,2,\dots,P}\left(\sum_{i=1}^{n}\frac{1}{2}[(b_i + a_i) + \frac{\beta_i - \alpha_i}{3}]w_{ij} - \sum_{i=1}^{n}k_i w_{ij}\right)$$
$$- \min_{j=1,2,\dots,P}\left(\sum_{i=1}^{n}\frac{1}{2}[(b_i + a_i) + \frac{\beta_i - \alpha_i}{3}]w_{ij} - \sum_{i=1}^{n}k_i w_{ij}\right)$$

Let $(\rho^*, \lambda_1^*, \lambda_2^* \dots, \lambda_P^*)$ be an optimum solution of (Model II). Further, ρ_0^* indicates the optimal efficiency score of the portfolio under evaluation (0^{th}) Here, s_{10}^- and s_{10}^+, respectively denote the excess of input (risk) and shortfall of output(return) of 0^{th} portfolio, and they are identified as slacks.

3.2. MSBM fuzzy portfolio evaluation model in possibilistic mean semi-variance framework

The mathematical form of the MSBM fuzzy portfolio model with possibilistic semi-variance input (risk) and possibilistic mean as output (return) is given as follows:

$$(\textbf{Model III}) \quad \min\ \eta_0 = \frac{1 - \dfrac{s_{10}^-}{R_3^-}}{1 + \dfrac{s_{10}^+}{R_2^+}}$$

$$\text{s. t.}\ \sum_{j=1}^{P}\lambda_j\left(\left\{\sum_{i=1}^{n}\frac{1}{2}\Big[(b_i - a_i) + \frac{\alpha_i + \beta_i}{3}\Big]w_{ij}\right\}^2 + \frac{1}{18}\Big[\sum_{i=1}^{n}(\alpha_i)w_{ij}\Big]^2\right) + s_{10}^-$$

$$= \left(\left\{\sum_{i=1}^{n}\frac{1}{2}\Big[(b_i - a_i) + \frac{\alpha_i + \beta_i}{3}\Big]w_{i0}\right\}^2 + \frac{1}{18}\Big[\sum_{i=1}^{n}(\alpha_i)w_{i0}\Big]^2\right)$$

$$\sum_{j=1}^{P}\lambda_j\left(\sum_{i=1}^{n}\frac{1}{2}\Big[(b_i + a_i) + \frac{\beta_i - \alpha_i}{3}\Big]w_{ij} - \sum_{i=1}^{n}k_i w_{ij}\right) - s_{10}^+$$

$$= \sum_{i=1}^{n} \frac{1}{2}\Big[(b_i + a_i) + \frac{\beta_i - \alpha_i}{3}\Big]w_{i0} - \sum_{i=1}^{n} k_i w_{i0},$$

$$\sum_{j=1}^{P} \lambda_j = 1 \quad \lambda_j \geq 0, \ ,s_{10}^{-}, s_{10}^{+} \geq 0, \quad j = 1, 2, \ldots, P$$

Here, R_2^+ is the same as described in the model II and R_3^- is defined as follows:

$$R_3^- = \max_{j=1,2,\ldots,P}\left(\sum_{i=1}^{n}(\frac{1}{2}[(b_i - a_i) + \frac{\alpha_i + \beta_i}{3}]w_{ij})^2 + \frac{1}{18}[\sum_{i=1}^{n}(\alpha_i)w_{ij}]^2\right)$$
$$- \min_{j=1,2,\ldots,P}\left(\sum_{i=1}^{n}(\frac{1}{2}[(b_i - a_i) + \frac{\alpha_i + \beta_i}{3}]w_{ij})^2 + \frac{1}{18}[\sum_{i=1}^{n}(\alpha_i)w_{ij}]^2\right)$$

The models mentioned above can determine the efficiency values of all fuzzy portfolios; however, they do not provide a unique ranking of the portfolios. It is important to note that if multiple portfolios are deemed efficient, establishing a complete and unique ranking becomes impossible. Therefore, we apply the TOPSIS method to rank all efficient portfolios and obtain a unique ranking. The TOPSIS method was introduced by [20]. It is a practical method for organizing the available options based on numerous criteria. It outperforms other MCDM methods for solving a wide range of decision-making problems in real-world, due to its simple computational approach and systematic procedure.

4. TOPSIS method to rank the efficient portfolio

TOPSIS is founded on the notion of selecting the best alternative (here DMU) that is as close to the positive ideal solution at the same time being as far away from the negative ideal solution as possible. The best performance values for each portfolio demonstrated by each criterion (here criterion as risk and return) in the decision matrix make up the perfect solution. As a result, we used a negative feature criterion for risk and a positive feature criterion for return.

Let L be the number of efficient portfolios. Suppose we have m inputs and s outputs for each portfolio, let D_1 and D_2 denotes respectively the input and output matrix corresponding to L efficient portfolios. Therefore

$$D_1 = \begin{bmatrix} x_{11} & x_{12} & \ldots & x_{1L} \\ x_{21} & x_{22} & \ldots & x_{2L} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \ldots & x_{mL} \end{bmatrix} \qquad D_2 = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1L} \\ y_{21} & y_{22} & \cdots & y_{2L} \\ \vdots & \vdots & \vdots & \vdots \\ y_{s1} & y_{s2} & \cdots & y_{sL} \end{bmatrix}$$

Now we have to follow the following steps:
Step 1. Normalization

Calculate the normalized decision matrix corresponding to the cost criterion (m inputs) and benefit criteria (s outputs). Let p_{tl} and q_{rl} ($t = 1, 2, \ldots, m;\ r = 1, 2, \ldots, s$) denotes the normalized value of t^{th} input and r^{th} output of l^{th} DMU respectively and calculated as follows:

$$p_{tl} = \frac{x_{tl}}{\sqrt{\sum_{k=t}^{L} x_{tk}^2}}, \quad \forall t = 1, 2, \ldots, m,\ l = 1, 2, \ldots, L$$

and

$$q_{rl} = \frac{y_{rl}}{\sqrt{\sum_{k=1}^{L} y_{rk}^2}}, \quad \forall r = 1, 2, \ldots, s, \quad l = 1, 2, \ldots, L.$$

Step 2. Determine the positive ideal and negative-ideal points: Let p_{tw} and q_{tw} denotes respectively be the t^{th} input and r^{th} output of the worst DMU (D_w). Similarly let p_{tb} and q_{rb} denotes respectively be the t^{th} input and r^{th} output of the best DMU (D_b). Then

$$p_{tw} = \max_{1 \le l \le L} p_{tl}, \quad t = 1, 2, \ldots, m,$$

and

$$q_{rw} = \min_{1 \le l \le L} q_{rl}, \quad r = 1, 2, \ldots, s.$$

Similarly

$$p_{tb} = \min_{1 \le l \le L} p_{tl}, \quad t = 1, 2, \ldots, m,$$

and

$$q_{rb} = \max_{1 \le l \le L} q_{rl}, \quad r = 1, 2, \ldots, s.$$

Step 3. Measuring the distance from the ideal and negative ideal points:
Let d_{lw} be the distance between the target DMU l and the worst DMU D_w (best DMU D_b), so

$$d_{lw} = \sqrt{\sum_{t=1}^{m} (p_{tl} - p_{tw})^2 + \sum_{r=1}^{s} (q_{rl} - q_{rw})^2}, \quad l = 1, 2, \ldots, L,$$

and

$$d_{lb} = \sqrt{\sum_{o=1}^{m} (p_{tl} - p_{tb})^2 + \sum_{r=1}^{s} (q_{rl} - q_{rb})^2}, \quad l = 1, 2, \ldots, L.$$

Step 4. Calculate the similarity to the worst condition:
Let s_{lw} be the relative closeness of the target DMU l to the worst DMU D_w.

$$s_{lw} = \frac{d_{lw}}{d_{lw} + d_{lb}}, \quad l = 1, 2, \ldots, L.$$

Here, $0 \le s_{lw} \le 1$, and

$$s_{lw} = \begin{cases} 1, & \text{if and only if the alternative solution has the best condition} \\ 0, & \text{if and only if the alternative solution has the worst condition.} \end{cases}$$

Step 5. Ranking criterion:
Rank the alternatives or portfolios in the descending order of s_{lw}, the alternative with a maximum value of s_{lw} is the best-performing alternative.
The working procedure of the proposed approach has been described in the algorithm 1.

Algorithm 1 Computation Procedure of the proposed method

Input: The Fuzzy returns of Stocks with the different proportions invested in the stocks.
Output: The full ranking of efficient portfolios.
Step 1. Calculate the Efficiency of portfolios by using MSBM fuzzy portfolio models.
Step 2. Rank the portfolios according to the efficiency score.
Step 3. If the no. of efficient portfolio ≥ 1, then apply TOPSIS method.
Step 4. Identification of attributes (Return, Risk) for all the alternatives (efficient portfolios).
Step 5. Form a weighted normalized decision matrix.
Step 6. Calculate the positive and negative ideal points.
Step 7. Evaluate the similarity to the worst condition as the measure of relative closeness s_{lw}.
Step 8. Rank the efficient portfolio in the decreasing order of s_{lw}.

5. Numerical Illustration

To demonstrate the applicability and validity of the proposed method, two numerical examples are presented. In the first example, the possibilistic mean is considered as the output, and the possibilistic variance as the input. In the second example, the possibilistic mean is again used as the output, with the possibilistic semi-variance as the input. All simulations were implemented using MATLAB software.

Example 1
In this example, the returns from risky assets/stocks are provided in Table 1 as trapezoidal fuzzy numbers and are the same as taken by [29] in their study. In the experimental analysis of [29], the historical data of 5 assets from April 2009 to March 2015, in terms of the weekly closing prices are used to simulate the future returns. Furthermore, we create the random sample portfolios using the weights as shown in Table 2. For convenience, we select only those 5 stocks whose returns are negative. We employ Model II to calculate the efficiency and ranking of all randomly generated portfolios. The results are presented in Table 3.

Table 1. The possibility distribution of the return rates and their respective transaction cost of 5 stocks

Stocks	a_i	b_i	α_i	β_i	k_i
S_1	-0.1071	-0.0703	0.0105	0.0541	0.003
S_2	0.0883	0.1057	0.057	0.0772	0.003
S_3	0.0874	0.0948	0.0132	0.0928	0.003
S_4	-0.0907	-0.082	0.0055	0.0336	0.003
S_5	-0.0954	-0.082	0.0093	0.013	0.003

Table 2. The weight of each stock in different randomly generated portfolios

w	1	2	3	4	5	6	7	8	9	10
w_1	0.1875	0.2660	0.2616	0.1956	0.2608	0.1677	0.0637	0.0948	0.3678	0.0891
w_2	0.1566	0.1102	0.2422	0.0962	0.3080	0.2472	0.2610	0.2547	0.2150	0.0561
w_3	0.3301	0.2483	0.1600	0.2975	0.2684	0.2437	0.1899	0.2408	0.0389	0.3180
w_4	0.1585	0.0805	0.2621	0.3064	0.1287	0.1964	0.2413	0.0050	0.2217	0.2013
w_5	0.1673	0.2905	0.0741	0.1043	0.0340	0.1450	0.2442	0.4048	0.1567	0.3355
w_p	11	12	13	14	15	16	17	18	19	20
w_1	0.0099	0.2206	0.3053	0.2608	0.1184	0.1714	0.0244	0.3602	0.0844	0.3571
w_2	0.2762	0.3457	0.2831	0.1016	0.3696	0.0105	0.2256	0.1147	0.0871	0.0746
w_3	0.2373	0.1664	0.0314	0.1039	0.0955	0.1306	0.2335	0.1934	0.2467	0.3654
w_4	0.2792	0.0005	0.1601	0.1654	0.2866	0.3579	0.1711	0.2879	0.2542	0.0149
w_5	0.1974	0.2669	0.2201	0.3684	0.1299	0.3296	0.1253	0.0438	0.3276	0.1879
w_p	21	22	23	24	25	26	27	28	29	30
w_1	0.2904	0.1345	0.0715	0.1815	0.1990	0.0479	0.2311	0.0904	0.2334	0.2311
w_2	01905.	0.0337	0.2241	0.1996	0.2874	0.2777	0.2503	0.0134	0.0418	0.1278
w_3	0.2333	0.3373	0.2388	0.4058	0.0994	0.2683	0.1133	0.2684	0.4218	0.0272
w_4	0.0792	0.2580	0.1773	0.0647	0.1409	0.2410	0.0752	0.2950	0.1474	0.3668
w_5	0.2066	0.2366	0.2884	0.1484	0.2734	0.1651	0.3301	0.3328	0.1556	0.2452

Table 3. The efficiency score and the ranking of portfolios in possibilistic mean-variance framework

portfolio	Return	Risk	Efficiency by model II	ranking by Model II
1	0.0043	0.00052	0.4714	16
2	-0.0198	0.00048	0.4396	18
3	-0.0112	0.00057	0.3138	25
4	-0.0124	0.00045	0.5188	13
5	0.0212	0.00072	0.3558	23
6	0.0048	0.00055	0.4168	19
7	-0.0033	0.00045	0.5696	11
8	0.0040	0.00049	0.5376	12
9	-0.0392	0.00056	0.272	29
10	-0.0174	0.00034	0.672	5
11	0.0085	0.00045	0.6361	7
12	0.0076	0.00065	0.2256	30
13	-0.0286	0.00058	0.2756	28
14	-0.0488	0.00039	0.4689	17
15	-0.0004	0.00055	0.3918	20
16	-0.0605	0.00027	1.0000	1
17	0.0177	0.00031	1.0000	1
18	-0.0280	0.00055	0.3207	2
19	-0.0249	0.00033	0.6474	6
20	-0.0042	0.00060	0.3018	27
21	-0.0078	0.00059	0.3122	26
22	-0.0172	0.00036	0.6346	8
23	-0.0012	0.00044	0.5908	9
24	0.0263	0.00061	1.0000	1
25	-0.0155	0.00053	0.3816	21
26	0.0148	0.00050	0.5745	10
27	-0.0200	0.00052	0.3782	22
28	-0.0343	0.00028	1.0000	1
29	0.0005	0.00049	0.5065	14
30	-0.0577	0.00036	0.4788	15

Table 4. Ranking of efficient portfolios by different methods in possibilistic mean-variance framework

Portfolio	Return	Risk	ranking by Model II	ranking by [42]	ranking by MSBM-TOPSIS method
16	-0.0605	0.00027	1	4	4
17	0.0177	0.00031	1	2	1
24	0.0263	0.00061	1	1	2
28	-0.0343	0.00028	1	3	3

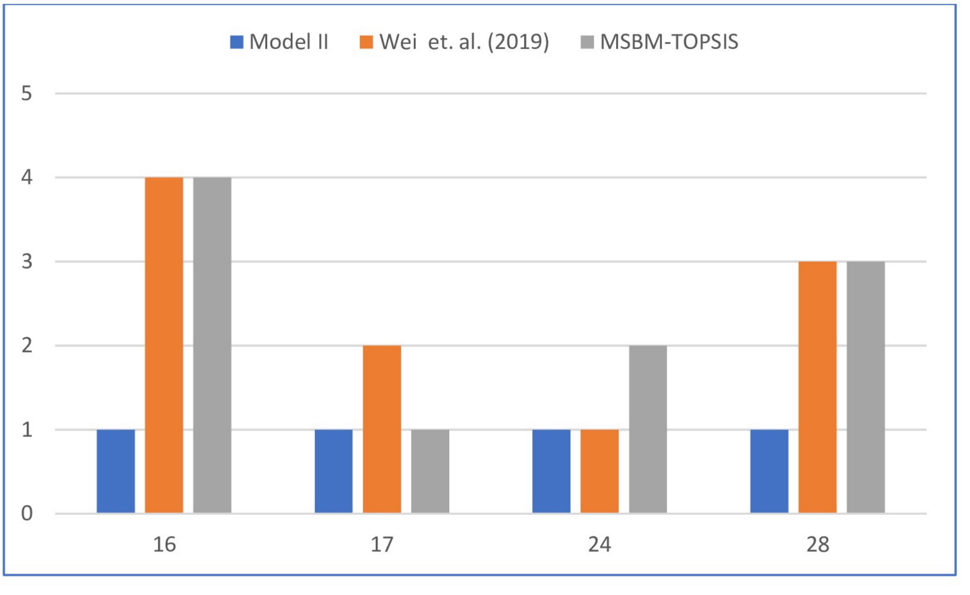

Figure 1. Rankings of efficient portfolios in possibilistic mean-variance framework

Remark 1 *Table 3 shows that the efficiency score of portfolios 16, 17, 24, and 28 is the same, and we cannot distinguish between efficient portfolios further. The MSBM-TOPSIS approach, as stated above, is used to determine the whole ranking. The fourth, fifth, sixth, and seventh column of Table 4 depicts the ranking of efficient portfolios by Model II, by [42], and by the proposed MSBM-TOPSIS method respectively. Figure 1 shows the ranking of efficient portfolios graphically. It should be highlighted that the process described in [42] gives us the whole ranking of all randomly produced portfolios. However, the Super efficiency model of [18] poses an in-feasibility concern. The ranking by the proposed method for Portfolio 16 and Portfolio 28 is the same as obtained by [42]. Compared to the method given by [42], the proposed approach is less calculative and easy to handle.*

If asset return distributions aren't symmetric, using variance as a risk indicator isn't recommended because it initiates to portfolio's projections drastically different from what would be expected in real-life situations. Excessively high and low returns are both regarded as undesirable by variance, which is one of the key disadvantages. Many academics have utilized a variety of techniques to deal with such problems. Risk can be classified using semi-variance as an alternative risk measure. Therefore, a downside risk metric called possibilistic semi-variance is utilized here instead of possibilistic variance as an input parameter. Table 5 shows the ranking, obtained by model III and Table 6 provides the complete ranking of efficient portfolios using different methods.

Table 5. The efficiency score and the ranking of portfolios in possibilistic mean semi-variance framework

Portfolio	Return	Risk	efficiency by Model III	ranking by Model III
1	0.0043	0.00045	0.4694	15
2	-0.0198	0.00043	0.4175	18
3	-0.0112	0.00051	0.2866	25
4	-0.0124	0.00039	0.5195	12
5	0.0212	0.00064	0.3272	23
6	0.0048	0.00049	0.4014	19
7	-0.0033	0.00040	0.5524	11
8	0.0040	0.00044	0.5082	14
9	-0.0392	0.00052	0.2289	28
10	-0.0174	0.00029	0.6719	5
11	0.0085	0.00039	0.6316	8
12	0.0076	0.00060	0.167	30
13	-0.0286	0.00053	0.2242	29
14	-0.0488	0.00036	0.4413	17
15	-0.0004	0.00050	0.353	20
16	-0.0605	0.00025	1.00	1
17	0.0177	0.00027	1.00	1
18	-0.0280	0.00049	0.2996	24
19	-0.0249	0.00029	0.6418	6
20	-0.0042	0.00053	0.2853	26
21	-0.0078	0.00052	0.2826	27
22	-0.0172	0.00031	0.638	7
23	-0.0012	0.00039	0.5777	9
24	0.0263	0.00053	1.00	1
25	-0.0155	0.00048	0.3387	21
26	0.0148	0.00043	0.5724	10
27	-0.0200	0.00047	0.335	22
28	-0.0343	0.00025	1.00	1
29	0.0005	0.00042	0.5146	13
30	-0.0577	0.00033	0.4547	16

Remark 2 *The suggested method's most significant advantage is its feasibility when compared to super-efficiency methods, which are in some situations infeasible.*

Table 6. Ranking of efficient portfolios by in possibilistic mean semi-variance framework

Portfolio	Return	Risk	ranking by Model III	ranking by [42]	ranking by MSBM-TOPSIS method
16	-0.0605	0.00025	1	3	4
17	0.0177	0.00027	1	1	1
24	0.0263	0.00053	1	2	2
28	-0.0343	0.00025	1	4	3

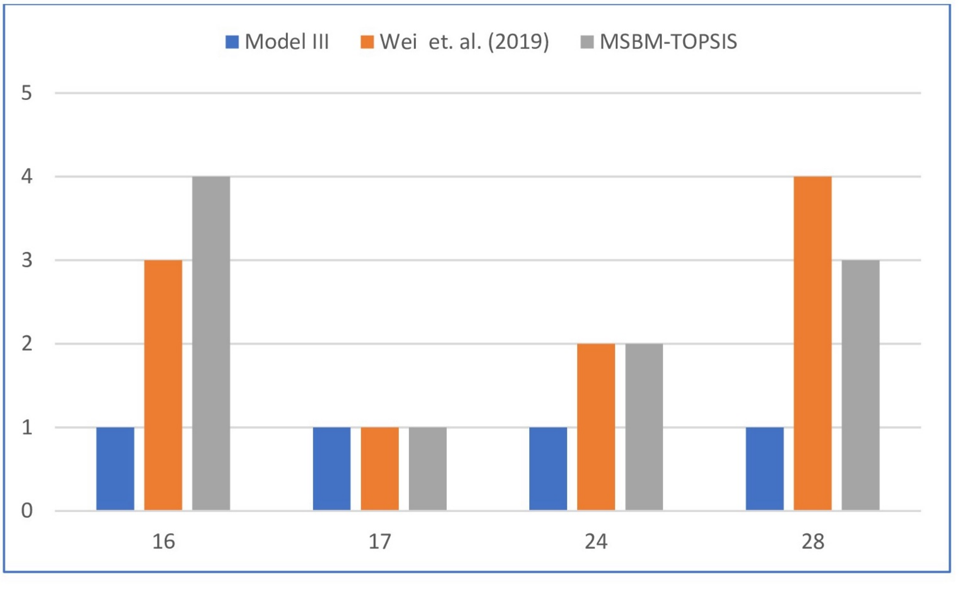

Figure 2. Rankings of efficient portfolios in possibilistic mean semi-variance framework

5.1. Computation Complexity

In this study, we compute the computational complexity by taking into account all logical-mathematical operations, such as addition, subtraction, multiplication, division, exponentiation and square root as described in [35]. The mathematical operations used for creating the return risk are the same for each strategy used in this study. As a result, they weren't considered while determining computational complexity. Suppose we have n efficient portfolio (to be considered as an alternative) and m criteria (return and risk) in the study, then it will take 3nm operations to reach at the weighted normalized matrix, 2(6nm) calculations required to determine the distance between efficient portfolio and the ideal and negative ideal points. nm-n calculations are required for calculating the positive ideal and negative ideal points. Lastly, 2n computations are used to calculate each portfolio's closeness coefficient. As a result, the TOPSIS's computational complexity to determine the full ranking of the efficient fuzzy portfolio in the second stage is determined as follows:

$$T_{n,m} = 3nm + 12nm + nm - n + 2n$$

.
The hybrid MSBM-TOPSIS requires 132 simple operations for the empirical analysis included in this analysis, whereas other DEA models need to solve the mathematical models repeatedly to get the whole ranking for efficient portfolios. Hence, Our method is substantially less intricate than the existing methods, so finding the most effective portfolio will take less time. In contrast to our approach, which uses basic operations to find the best portfolio in the second phase, the super efficiency and modified MSBM method given in[42] often solves at least three models for each portfolio to determine the best portfolio.

6. Conclusion and Future Research Directions

This work investigates the ranking problem of all efficient fuzzy portfolios with positive and negative input-output variables. We know that if a sufficiently large number of portfolios are designated efficient, the DEA's discrimination ability decreases. We extend the MSBM model for handling fuzzy portfolio evaluation problems in the possibilistic mean-variance and possibilistic mean semi-variance framework. By combining the proposed MSBM fuzzy portfolio models with the TOPSIS method, the work has established a hybrid strategy for ranking efficient portfolios in a fuzzy environment. By combining the strengths of MSBM in efficiency evaluation, fuzzy logic in handling uncertainty, and TOPSIS in ranking alternatives, this approach provides a comprehensive tool for investors looking to optimize their portfolios in uncertain market environments.

This approach will be explored in future work to investigate the classification of fuzzy portfolios having input as possibilistic semi-absolute deviation (SAD), possibilistic VaR, and CvaR. In this research, only 1st and 2nd-order moments (mean, variance, and semi-variance) have been used as output and input. The higher-order moments can be utilized as the input-output values for future work.

Acknowledgment

The authors extend their gratitude to the anonymous reviewers and editors for their valuable comments and suggestions on the earlier draft of this paper.

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Received 03.06.2024, Accepted 21.12.2024