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A Heuristic Cutting Plane Algorithm For Budget Allocation of Large-scale Domestic Airport Network Protection

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Abstract. It is well known that airport security is an important component of homeland security, since airports are highly vulnerable to terrorist attacks. In order to improve the overall security of the domestic airport network, some work studied the budget allocation of domestic airport network protection. They established a minimax optimization model and designed an exact cutting plane algorithm to solve the problem. However, the exact algorithm can not solve large-scale problems in an acceptable time. Hence, this paper designs a heuristic cutting plane algorithm for solving budget allocation of large-scale domestic airport network protection. Finally, numerical experiments are carried out to demonstrate the feasibility and effectiveness of the new algorithm.

Keywords: airport security, large-scale, heuristic cutting plane algorithm, budget allocation

1. Introduction

It is well known that densely populated or high-value areas such as ports, airports, stations, and squares in cities are easy targets for terrorist attacks. In particular, airports are favored by terrorists because of the large volume of people and the security checks are difficult to cover, which has been proved by many facts [7, 11]. For example, on the morning of September 11, 2001, the largest terrorist attack in the history of the United States took place, which is also known as "9/11" [11]. Terrorists hijacked two passenger planes and crashed them into America's proud twin towers. The event killed nearly three thousand people and had a significant and lasting impact on the

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economy of the United States and the world. These events illustrate how important it is to improve airport security.

After the famous terrorists' attacks of 9/11[11], the Transportation Security Administration (TSA) was created under the Aviation and Transportation Security Act passed by the 107th Congress on November 19, 2001. The TSA protects the nation's transportation systems to ensure freedom of movement for people and commerce. Although the TSA has made a lot of efforts to improve different layers of defense for airport security, terrorists are continuing interested in doing harm to U.S. citizens through penetrating the aviation security system. For example, On December 25, 2009, passengers aboard a Northwest Airlines flight from Amsterdam heard a loud noise on the plane as it descended toward Detroit. Witnesses described it as sounding much like a firecracker. The device failed, but AbdulMutallab became enveloped in a fireball that spread to the wall and carpeting of the plane. Therefore, how to avoid and reduce terrorists' attacks in airport security and improve airport security is a very important subject in our management science field.

In fact, with the rapid development of economy and civil aviation industry, in order to save costs, large civil airlines have adopted the hub-and-spoke route network[18]. Hub-and-spoke route network refers to the route network in which an airline distributes all its routes to the hub airport as the starting point and each destination airport and other hub airports as the end point. This route layout will eventually present the route network that flies to other airports with the hub airport as the axis. Under the hub-and-spoke structure, a non-direct domestic flight usually has one or two pit stops, namely two or three segments. Each segment includes an originating airport and a destination airport. Due to screening time and economic issues, it is not possible to screen passengers and their checked baggage at the originating airport for every segment covered by all flights. Most airports currently check passengers and their checked baggage only once at the originating airport of the flight. As a result, under the current practice, if a terrorist successfully passes through the security system at the originating airport of the flight, the passenger will not be subject to security checks in the entire journey. Airports are connected by air routes and are connected in terms of resources. The entry of terrorists into the initial airport may result in the security of the entire air route and airports on the ground. Therefore, we need to look at the whole airport network from a holistic perspective which will help the government allocate budget to each airport reasonably. In total, there are more than 500 commercial service airports in the United States[4]. How to protect such a huge domestic airport network remains a challenging task. This will strengthen the construction of airports, reduce terrorist attacks, so as to improve the security of the entire airport network.

In the literature above airport security research, game models are often used to formulate airport security problem. Pita et al. (2009) [15] developed a Bayesian Stackelberg game model for random patrol and surveillance problems at airports and proposed a software package called ARMOR(Route Random Monitoring Assistant). They studied relevant information, design options, challenges, and evaluations about ARMOR design. Nie et al. (2010) [12] developed a bi-level optimization model by describing the airport air police scheduling problem as an incomplete information game between the defender (the TSA) and the attacker (the terrorist group). The first level is that the TSA determines how to deploy air marshals on flights of different risk levels to maximize the expected rate of terrorist arrests. The second level is that the terrorist group decides which class of risk level flights to attack in order to maximize the expected air marshal detection rate and minimize the terrorist apprehension rate. Bagchi and Paul (2014) [1] analyzed the role of intelligence gathering and security screening in improving airport security by using the game between TSA and terrorists. By investing in intelligence gathering, the TSA can improve the accuracy of its information. By contrast, security checks can be used to search passengers and thus deter terrorist attacks to ensure maximum airport security and minimum social costs. Guo et al. (2016) [6] introduced a novel coalitional security game (CSG) model. A CSG consists of a set of attackers connected by a (communication or trust) network who can form coalitions as connected subgraphs of this network so as to attack a collection of targets. Janssen et al. (2020) [10] proposed a novel empirical game theory approach in which payoffs are estimated using agent-based modeling, while a security game is used to find optimal security patrols. Hunt et al. (2021) [8] established a piece of information game model between attacker and defender. The defender is the TSA that consider adopting a new security technology to expose and deter terrorist attacks. The attacker is the terrorist group that think about which airline to attack. It helped fill significant gaps in airport security technology adoption and disclosure strategies, and provided governments with more information on how to keep airport security. All of the above work focuses on the security protection of one airport. As we all know, there are few works on the whole airport network. Yan et al. (2022) [17] used the Stackelberg game model to consider the optimal allocation of resources for the whole airport network, and proposed a $0-1$ bi-level integer optimization model, in which the TSA was the leader, whose goal was to allocate the budget for the airports so as to minimize the losses caused by the terrorist group, while the terrorist group was the follower, who seeks to choose attack strategies to maximize the losses.

As for practice problems in airport security, accurate algorithms can only solve small-scale problems. Therefore, heuristic algorithms become important methods to solve large-scale airport problems. For example, Nikolaev et al. (2012) [13] proposed a heuristic algorithm based on Monte Carlo simulation to solve the multi-stage continuous passenger screening problem in airport security, and compared it with the accurate algorithm of random order assignment and feedback control. They studied the passenger and carry-on luggage screening operation so as to improve the overall safety performance of passenger security. Daniel and Megan (2017) [3] proposed a neighborhood search heuristic algorithm for establishing the optimal temporary hub strategy after large-scale airport shutdown. Ozlem et al. (2021) [14] designed a wave-velocity search heuristic algorithm and a filtered wave-velocity search heuristic algorithm to solve the gate assignment problem of large-scale airports. Du et al. (2023)[5] developed a column generation-based heuristic (CDBH) to solve robust gate assignment problem (RGAP). Huo et al. (2023)[9] used a meta-heuristic simulated annealing algorithm combined with the rolling horizon approach to investigate based on the arrival traffic of the Paris Charles de Gaulle (CDG) airport.

As our closely related work, Yan et al. (2022) [17] proposed a $0-1$ bi-level integer optimization model and developed an exact cutting plane algorithm for the protection of the whole domestic airport network. However, due to the large number of airports in a typical domestic airport network and the complex number of related flights, the exact algorithm cannot solve the problem within an acceptable time. Therefore, it is very necessary to design a fast, simple and satisfactory algorithm for the budget allocation problems of large-scale domestic airport. Based on the work of Yan et al.(2022) [17], the paper develop a heuristic cutting plane algorithm to solve the budget allocation problem of large-scale domestic airport network security.

2. Problem description and model formulation

This paper considers how to allocate budget to each airport effectively and improve the security of the entire airport network in the existing domestic airport network. Borrowing the ideas of work [17], we regard this problem as a Stackelberg game and propose a bi-level optimization model, in which the leader is the TSA and the follower is the terrorist group. The TSA first determines the amount of budget allocated to each airport, and accordingly, the probability that terrorists can successfully pass through each airport can be obtained. After obtaining information about the probability of successful pass through the airport through intelligence, the terrorist group decide which flight segments to attack with a limited budget. The TSA's goal is to minimize the maximum expected damage caused by the terrorist group, while the terrorist group's goal is to maximize the expected damage.

Define I as the set of all domestic airports, indexed by i. J as the set of all possible flights, indexed by j. H as the set of all segments, indexed by h. We know that a flight is either a direct flight with only one segment, or a non-direct flight with two or more segments. Then we represent each segment h corresponding to flight j as the flight segment jh.

For flight j, define j_S as the originating airport of the first flight segment. P_i^0 indicates the probability that a terrorist will successfully pass through airport i if the terrorist group chooses the airport i as the originating airport before the government allocates the budget. Define the decision variable x_i as the TSA's budget allocation to airport i. Assume that the budget allocated to each airport i has a lower limit and an upper limit. That is to say, x_i has a lower bound and an upper bound, denoted as L_i and U_i, respectively. Thus, we can obtain the probability that terrorist through an airport successfully after the budget allocation, defined as $P_i^0 e^{-\lambda_i x_i}$ [2, 16], where λ_i is the instantaneous pass rate associated with the airport i.

Segments have different characteristics, such as aircraft type, origin and destination cities, departure times, duration, etc.. Different segments may have different values to the terrorist group. Define d_{jh} as the projected loss caused by the terrorist group choosing to attack the flight segment jh. Define c_{jh} as the expenditure required for terrorists to attack the flight segment jh. Define B_T as the total cost of the terrorist group. Define B_G as the TSA's total budget for all airports. In addition, for terrorist attacks, the $0-1$ decision variable y_{jh} is defined. If the flight segment jh is attacked by the terrorist group, y_{jh} is 1, otherwise it is 0.

According to the above description, we establish the following $0-1$ bi-level integer optimization model:

$$
\begin{aligned}
\min_{W,x_i} \quad & W && (1)\\
\text{s.t.} \quad & \textstyle\sum_{i\in I} x_i \le B_G, && (2)\\
& L_i \le x_i \le U_i, \forall i \in I, && (3)
\end{aligned}
$$

where,

$$
\begin{aligned}
W = \max_{y_{jh}} \quad & \textstyle\sum_{j\in J}\sum_{h\in H} P^0_{js} e^{-\lambda_{js}x_{js}} d_{jh} y_{jh} && (4)\\
\text{s.t.} \quad & \textstyle\sum_{j\in J} y_{jh} \le 1, \forall h \in H, && (5)\\
& \textstyle\sum_{j\in J}\sum_{h\in H} c_{jh} y_{jh} \le B_T, && (6)\\
& y_{jh} \in \{0,1\}, \quad \forall j \in J, h \in H. && (7)
\end{aligned}
$$

In the upper level, the goal of the TSA is to minimize the damage W caused by terrorists. Constraint (2) ensures that the total budget allocated to all airports is within the government's budget limit B_G. Constraints (3) are the boundary constraints. In the lower level, the goal of the terrorist group is to maximize the damage W by specifying an attack strategy. Constraints (5) ensure that each flight is attacked by terrorists at most once. Constraint (6) ensures that the total cost of the attack is within the terrorist group's budget limit B_T. Constraints (7) are $0-1$ constraints.

3. Algorithm

In the above $0-1$ bi-level integer program $(1)-(7)$, we can see that the lower level $(4)-(7)$ is a large-scale $0-1$ integer optimization problem. As we all know, solving the large-scale $0-1$ integer optimization problem needs a lot of cost. Fortunately, we can regard the lower level program as a $0-1$ knapsack problem and solve it effectively by a heuristic algorithm.

We consider the flight segments in our problem as items in the knapsack problem. The set of items is $G = \{jh|\ flight\ segment\ jh\}$. Meanwhile, the cost c_{jh} of flight segment jh in our problem is considered as the weight of item jh, the damage $P^0_{js} e^{-\lambda_{js}x_{js}} d_{jh}$ of flight segment jh in our problem is considered as the profit of item jh, and the total cost of the terrorist group B_T in our problem is considered as the capacity of the knapsack. The objective of the lower level $(4)-(7)$ is to find a subset of items with the maximum profit sum (that is, the uppermost damage caused by terrorists' attacks) within the knapsack capacity B_T. Binary variable y_{jh} is 1 if item jh is chosen, otherwise 0. We consider the same segment and all segments on the same flight to be "Similar Items". Constraints (5) mean that at most one of the "Similar Items" can be chosen.

In this section, we introduce a greedy heuristic to quickly obtain solutions for large-scale problems. The idea is to sequentially choose item jh such that the total expected damage is maximized given that the previously chosen items are fixed. The procedure of the greedy adding heuristic is as follows:

Greedy Heuristic For The Lower Level

Step 0. (Initial value) Set the initial objective function value $O = 0$, initial attack strategy $Z = \emptyset$.

Step 1. (Termination condition) Check if $B_T > 0$ or $G \neq \emptyset$, if so, go to Step 2. Otherwise, stop the procedure and obtain the optimal attack strategy Z and the corresponding objective function value O.

Step 2. (Sort) Select $j^{'}h^{'} \in \arg\max_{jh}\{P^0_{js}e^{-\lambda_{js}x_{js}}d_{jh}/c_{jh}|jh \in G, c_{jh} \leq B_T\}$.

Step 3. (Update) $Z = Z \cup \{j^{'}h^{'}\}$, $O = O + P^0_{j^{'}_S}e^{-\lambda_{j^{'}_S}x_{j^{'}_S}}d_{j^{'}h^{'}}$, $B_T = B_T - c_{j^{'}h^{'}}$.

Step 4. (Delete) $M = \{jh|j = j^{'}$ or $h = h^{'}\}$. $G = C_G M$ and go to Step 1.
where M is a new set of "Similar Items" with the item $j^{'}h^{'}$.

Now, we are ready for solving the bi-level integer program (1) – (7). We borrow the cutting plane algorithm [17] where the developed greedy heuristic for solving the lower lever program. We call this new algorithm as the heuristic cutting plane algorithm (HCPA).

Heuristic Cutting Plane Algorithm (HCPA)

Step 0. Set an optimality tolerance $\varepsilon > 0$, an iterations counter $k = 0$. Choose $\widehat{y}^0_{jh}$ $= 0$.

Step 1. Based on the previously obtained attack strategy $\widehat{y}^{\tau}_{jh}$ for $\tau \in \{0, \ldots, k\}$, solve the following upper level (8) – (11) to determine a current optimal budget allocation $\widehat{x}^k_i$. Obtain the corresponding objective function values $\widehat{W}^k$.

$$\min_{W,x_i} \quad W \tag{8}$$

$$s.t. \quad \textstyle\sum_{jh\in G} P^0_{js}e^{-\lambda_{js}x_{js}}d_{jh}\widehat{y}^{\tau}_{jh} - W \leq 0, \tau \in \{0, \cdots, k\}, \tag{9}$$

$$\textstyle\sum_{i\in I} x_i \leq B_G, \tag{10}$$

$$L_i \leq x_i \leq U_i, \forall i \in I. \tag{11}$$

Step 2. According to the obtained airport resource budget allocation $\widehat{x}^k_i$, using the above greedy heuristic to solve the lower level (4) – (7) to determine the current optimal attack strategy $\widehat{y}^{k+1}_{jh}$. Obtain the corresponding objective function values $\widehat{O}^{k+1}$.

Step 3. If $\widehat{O}^{k+1} - \widehat{W}^k < \varepsilon$, stop the procedure and obtain optimal solutions. For the TSA, $\widehat{x}^k_i$ is an optimal protection solution, while $\widehat{y}^{k+1}_{jh}$ is an optimal attack strategy for the terrorist group. Otherwise, set $k = k + 1$ and go to Step 1.

4. Numerical experiments

In this section, in order to verify the effectiveness of the proposed heuristic algorithm, numerical experiments are conducted to compare the heuristic cutting plane algorithm (HCPA) with the exact cutting plane algorithm (ECPA) [17]. All the experiments are implemented in MATLAB (R2020b) running on a PC with an Intel(R) Core(TM) i5-1035G7 cpu @ 1.20GHz 1.50GHz and 8GB. For convex nonlinear optimization problems solved in Step 1, we use the FMINCON function in MATLAB, while for $0-1$ bi-level integer optimization programs solved in Step 2, we use the developed greedy heuristic algorithm. We set the budget allocation lower and upper bounds of all airports as 0 and infinity, respectively, and the optimality tolerance is set to 10^{-8} .

For convenience, we express the exact cutting plane algorithm [17] as follows:

Exact Cutting Plane Algorithm [17](ECPA)

Step 0. Set an optimality tolerance $\varepsilon > 0$ and an iteration counter $k = 0$. Choose $\widehat{y}_j^0 = 0$ and $\widehat{z}_{jl_j}^0 = 0$.

Step 1. Based on the previously obtained attack strategy $\widehat{y}_j^\tau$ and $\widehat{z}_{jl_j}^\tau$ for $\tau \in \{0, \cdots, k\}$, solve the following upper-level optimization problem $(12)-(15)$ to determine a current optimal budget allocation $\widehat{x}_i^k$. Obtain the corresponding objective function values $\widehat{W}^k$.

$$\min_{W, x_i} \quad W \tag{12}$$

$$s.t. \quad \textstyle\sum_{j\in J}\sum_{h\in H} P_{jS}^0 e^{-\lambda_{jS} x_{jS}} d_{jl_j} \widehat{z}_{jl_j}^\tau - W \le 0, \tau \in \{0, \cdots, k\}, \tag{13}$$

$$\textstyle\sum_{i\in I} x_i \le B_G, \tag{14}$$

$$L_i \le x_i \le U_i, \forall i \in I. \tag{15}$$

Step 2. Based on the obtained airport budget allocation $\widehat{x}_i^k$, solve the following lower-level optimization problem $(16)-(21)$ to determine a current optimal attack plan $\widehat{y}_j^{k+1}$ and $\widehat{z}_{jl_j}^{k+1}$. Obtain the corresponding objective function value $\widehat{O}^{k+1}$, defined as $\sum_{j\in J}\sum_{l_j=1}^{n_j} P_{jS}^0 e^{-\lambda_{jS}\widehat{x}_{jS}^k} d_{jl_j} \widehat{z}_{jl_j}^{k+1}$.

$$\max_{y_j, z_{jl_j}} \quad \textstyle\sum_{j\in J}\sum_{l_j=1}^{n_j} P_{jS}^0 e^{-\lambda_{jS}\widehat{x}_{jS}^k} d_{jl_j} z_{jl_j} \tag{16}$$

$$\text{s.t.} \quad \textstyle\sum_{j\in J} a_{jh} y_j \le 1, \forall h \in H, \tag{17}$$

$$\textstyle\sum_{j\in J} c_j y_j \le B_T, \tag{18}$$

$$\textstyle\sum_{l_j=1}^{n_j} z_{jl_j} \le y_j, \forall j \in J, \tag{19}$$

$$y_j \in \{0,1\}, \forall j \in J, \tag{20}$$

$$z_{jl_j} \in \{0,1\}, \forall j \in J, l_j \in \{1, \ldots, n_j\}. \tag{21}$$

Step 3. If $\widehat{O}^{k+1} - \widehat{W}^k < \varepsilon$, stop the procedure and obtain optimal solutions. For the TSA, $\widehat{x}_i^k$ is an optimal protection solution, while $\widehat{y}_j^{k+1}$ and $\widehat{z}_{jl_j}^{k+1}$ are a best attack strategy for the terrorist group. Otherwise, set $k = k + 1$ and go to Step 1.

In the experiment, 45 test instances of large-scale budget allocation problems are tested by the exact cutting plane algorithm and the heuristic cutting plane algorithm. We consider the airport network composed of 10 airports, 20 airports, 30 airports, 35 airports, 40 airports, 45 airports, 50 airports, respectively. When the airport network

Table 1. Numerical experiments

Airports	Flights	ECPA		HCPA		
		Obj.($\times 10^3$)	Time	Obj.($\times 10^3$)	Time	GR(%)
10	50	3.687	2.863	3.677	1.868	0.271
	60	3.149	3.016	3.119	0.970	0.953
	70	2.645	2.820	2.464	0.891	6.843
	80	3.687	2.887	3.677	1.587	0.271
	90	3.879	3.619	3.812	1.325	1.727
	100	3.879	4.507	3.812	1.775	1.727
20	90	28.868	7.278	28.248	6.897	2.148
	100	28.868	10.247	28.365	4.577	1.741
	110	71.076	15.536	68.433	8.156	3.718
	120	30.494	13.755	28.667	5.893	5.991
	130	32.661	23.303	31.982	20.625	2.079
	140	28.868	13.316	28.420	9.507	1.550
30	130	106.029	58.922	105.249	19.855	0.736
	150	111.849	143.904	110.512	36.857	1.195
	170	125.888	139.345	124.738	56.327	0.914
	190	126.183	95.260	125.624	50.104	0.443
	210	124.420	131.299	122.240	29.695	1.752
	230	123.116	171.859	120.723	21.063	1.944
35	150	128.568	127.278	127.2388	36.897	1.034
	170	135.861	210.221	131.355	30.111	3.316
	190	151.066	115.768	148.043	58.532	2.001
	210	143.403	335.001	135.336	90.800	5.625
	230	162.632	290.879	154.008	50.665	5.302
	250	160.324	350.656	153.111	63.112	4.499
40	170	228.157	248.386	219.527	61.833	3.782
	180	226.215	613.617	217.770	179.578	3.733
	200	185.353	188.681	178.033	61.952	3.949
	220	227.409	224.948	216.238	106.528	4.912
	240	251.778	303.832	241.149	68.307	4.221
	260	259.191	177.636	242.488	48.109	6.444
45	230	245.278	802.990	233.928	169.645	4.628
	250	251.961	291.860	243.482	66.264	3.365
	280	254.275	1417.403	247.231	112.162	2.770
	320	247.397	378.050	237.828	135.022	3.868
	360	332.661	523.303	321.982	100.625	3.210
	400	328.868	613.316	328.420	90.507	0.136
50	380	365.721	852.113	364.345	232.156	0.376
	420	389.111	1300.349	386.156	234.687	0.759
	460	346.236	590.879	340.690	119.167	1.602
	500	352.879	1500.780	343.011	189.053	2.796
	540	411.754	678.479	398.879	100.086	3.126
	580	458.657	956.018	451.955	157.709	1.461
	620	490.125	1491.687	487.125	300.139	0.612
	660	506.968	1806.563	490.156	245.363	3.316
	700	551.036	2600.136	524.334	290.156	4.846

is 10 airports, the number of flights is considered from 50 to 100 in 6 cases, and the terrorist group's budget is 600 dollars. When the airport network is 20 airports, the number of flights is considered from 90 to 140, and the terrorist group's budget is 600 dollars. When the airport network is 30 airports, the number of flights is considered from 130 to 230, and the terrorist group's budget is 800 dollars. When the airport network is 35 airports, the number of flights is considered from 150 to 250, and the terrorist group's budget is 800 dollars. When the airport network is 40 airports, the number of flights is considered from 170 to 260, and the terrorist group's budget is 900 dollars. When the airport network is 45 airports, the number of flights is considered from 230 to 400, and the terrorist group's budget is 900 dollars. when the airport network is 50 airports, the number of flights is considered from 380 to 700, and the terrorist group's budget is 1000 dollars.

In each test instance, the initial pass rate and the instantaneous pass rate are randomly even distributed between $0.001 - 0.008$. The TSA's total budget is 200,000 dollars. The cost of each segment is randomly even distributed between $100 - 600$ dollars. The damage of attacking each segment is randomly even distributed between $35 - 80$ million dollars. GR represents the relative deviation of the heuristic cutting plane algorithm (HCPA) from the exact cutting plane algorithm (ECPA), which is computed by the following formula:

$$GR = \frac{ECPA\ Value - \ HCPA\ Value}{ECPA\ Value} \times\ 100\%.$$

From Table 1, we can see that the average relative error is 2.704%, the maximum relative error is 6.843%, and the minimum relative error is 0.136%. For large-scale problems, the time advantage of the algorithm HCPA is obvious, and better feasible solutions can be obtained in a shorter time. The larger the airport network is, the more obvious the advantage of the algorithm HCPA is for all instances. The average running time of the algorithm HCPA is 81.714 seconds, and the average running time of the algorithm ECPA is 359.054 seconds. The running time of the algorithm HCPA was on average 277.34 seconds shorter than that of the algorithm ECPA. The longest running time of the algorithm HCPA is about 5 minutes, while the longest running time of the algorithm ECPA is about 43 minutes. It can be seen that the heuristic cutting plane algorithm (HCPA) proposed in this paper can obtain superior solutions in a shorter time, which shows its feasibility and superiority.

5. Conclusions and future work

Based on the work [17], in order to solve the budget allocation problem of large-scale domestic airport network security, this paper develops a heuristic cutting plane algorithm according to the special structure of the formulated $0 - 1$ bi-level integer optimization model. It compares the heuristic cutting plane algorithm with the exact cutting plane algorithm [17] through numerical experiments. The experimental results show that the heuristic cutting plane algorithm has an excellence ability and can obtain a more satisfactorily feasible solution in a short time. The heuristic cutting plane algorithm is a more effective algorithm to the large scale domestic airport budget allocation problem.

There are three future research directions. First, this paper designs the heuristic algorithm only for the lower level, so designing a heuristic algorithm for the whole problem to solve the problem more quickly will be a focus of our later research work. Secondly, this paper assumes that all TSA's information, such as TSA's budget, each airport's initial pass rate and instantaneous pass rate are known to the terrorist group, meanwhile, the TSA also knows information related to the terrorist group. This is not in line with the actual situation. How to take asymmetric or partial information into consideration deserves further investigation. Thirdly, This paper does not give relevant theoretical results, which will be one of our future research topics.

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