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FINANCIAL ENGINEERING OF A REVERSE TWIN-WIN CERTIFICATE THROUGH REPLICATION METHOD

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ABSTRACT

This paper examines a rarely studied type of structured product—reverse twin-win certificates—focusing on their design, pricing, and potential for risk management and portfolio diversification. It introduces two versions of the product: capped and uncapped, each offering different risk-return profiles depending on market conditions. These certificates are applied to the case of a Bulgarian technology company, representing an emerging market context where such instruments are largely unexplored. The pricing is carried out using a widely accepted method that replicates the product's return using a combination of basic financial instruments and financial options. This approach allows for transparent and flexible valuation. Numerical simulations show that uncapped reverse twin-win certificates perform well in strongly declining markets, offering high upside when the underlying asset falls sharply. Capped versions are more conservative, limiting both risk and return—making them more appropriate for cautious investors or moderately volatile markets. The study highlights how structured products like these can be adapted to different investment strategies, from speculation to hedging. It also contributes to the literature by extending structured product analysis to the underrepresented setting of emerging markets.

Keywords: *structured product, twin-win certificate, replication method, option pricing*

1. INTRODUCTION

In recent years, financial innovation has facilitated the development of increasingly sophisticated investment instruments aimed at addressing the evolving objectives and risk preferences of investors. Among these, structured products have become increasingly popular due to their ability to offer customized risk-return profiles. Structured products are investment instruments whose return is contingent on the return of their underlying asset (UA) and derivatives, primarily options and various option strategies. These products provide exposure to a wide range of underlying assets-including stocks, bonds, indices, interest rates, commodities, and currencies-making them attractive tools for portfolio diversification.

What sets structured products apart is their flexibility. By combining conventional financial instruments with financial derivatives, these products can be tailored to meet specific investment objectives. This flexibility allows investors to align their strategies with individual risk appetites and market views. Structured products pose significant design, pricing, and risk management challenges due to their non-linear and path-dependent payoff structures.

One such innovative product is the twin-win certificate, which enables investors to profit from both upward and downward movements in the price of the underlying asset, subject to predefined conditions. This dual-sided return potential makes twin-win certificates especially attractive in volatile or range-bound market environments. The design of twin-win certificates is based on the structure of barrier options. These certificates use a barrier mechanism to provide investors with the opportunity to benefit from market movements, as long as the barrier is not breached during the term. This structure enables higher yield potential while maintaining conditional capital protection.

Barrier options are a type of path-dependent derivative, meaning their value depends not only on the price of the underlying asset at maturity but also on whether that price reaches a specific barrier level during the life of the option. Depending on the type, crossing the barrier can either activate the option (knock-in) or cancel it (knock-out). Because of this, barrier options are influenced by the entire path the asset's price takes over time-not just where it ends up (Broadie, Glasserman, & Kou, 1997; Ritchken, 1998; Kim, Yoon, & Park, 2021). In contrast, standard options—also called vanilla options-are not path-dependent. Their payoff depends only on the price of the underlying asset at the time the option expires, not on how the price moved during the option's life (Hull, 2021). A call option gives the holder the right to buy the asset at a fixed price, referred to as the strike price, while a put option gives the holder the right to sell the asset at that same price. Because they depend only on the final price, standard options are easier to understand and value. Barrier options, on the other hand, are more complex because their value depends on the entire price path, but they can offer more flexible risk and return features.

Rusnáková, Gordiaková, and Harčariková (2014) provided the analysis for capped bonus and capped twin-win certificates to explore how are applied the principles of financial engineering to the creation of structured products. Timková and Bobriková (2023) presented the proposal of two new types of investment certificates, i.e. twin-win outperformance certificates and capped twin-win outperformance certificates, which belong to the segment of partially guaranteed investment tools due to a security buffer. The advantage the proposed certificates lies in the combination of the features of two certificates into one product.

This paper introduces a systematic framework for the construction and valuation of reverse twin-win certificates, with a particular focus on their capped and uncapped variants. These instruments are not yet fully conceptually developed and remain insufficiently examined in both academic literature and practical implementation. The central research question addressed in this study is: How can reverse twin-win certificates be efficiently designed and priced through option replication techniques, and how do their performance characteristics differ under various market scenarios?

The methodology relies on the replication-based option pricing in certificate construction. This approach made it possible to investigate the parameters that determine the prices of investment certificates. The authors used the Black-Scholes model for standard options and a closed-form solution for barrier options based on Haug's framework. All simulations and valuations were implemented in the R statistical computing environment, ensuring reproducibility and flexibility in modelling complex payoff structures.

The original contribution of the paper lies in demonstrating how structured products with asymmetric payoff profiles-such as reverse twin-win certificates-can be systematically engineered using a replication-based approach that combines standard and barrier options. This approach not only improves pricing transparency but also illustrates the adaptability of option-based strategies.

A practical application of the proposed framework was conducted using the Bulgarian capital market. This market was selected as the empirical setting due to its relevance as a representative emerging economy with increasing interest for innovative investment instruments but limited market penetration of structured products. This application provides meaningful insights into both the opportunities and constraints associated with introducing sophisticated financial products in similar markets.

The empirical findings highlight that uncapped reverse twin-win certificates offer strong potential in bearish markets, where they allow investors to profit from substantial declines in the underlying asset's value. In contrast, capped versions deliver more conservative and predictable outcomes, making them more suitable for moderately volatile or uncertain environments. These findings emphasize the need to customize structured products in response to market dynamics and investor demands. By bridging theoretical modelling with practical application, this study aims to fill a gap in the structured product literature and inform both academics and practitioners on how to extend the use of replication techniques to support innovation in portfolio construction.

The structure of the paper is as follows: Section 1 presents a review of the relevant literature. Section 2 outlines the methodological framework employed in the study. Section 3 provides the empirical application, including data analysis, results, and discussion.

2. LITERATURE REVIEW

Structured products now play a key role in the portfolios of both retail and institutional investors around the world, offering customized solutions that address diverse investment objectives. Reflecting their growing relevance in financial markets, the European Structured Investment Products Association (EUSIPA, 2025) reported that the structured products market in Europe reached an open interest of approximately €471 billion in the first quarter of 2025, marking a substantial 24 percent increase compared to the previous year. This significant growth highlights the increasing demand for structured investments that can respond flexibly to changing market dynamics and investor expectations.

To provide clarity and standardization, EUSIPA offers a comprehensive framework for classifying structured investment products into four primary categories: capital-protected products, yield enhancement products, participation products, and leverage products. The first three categories are commonly referred to as investment certificates, whereas the fourth comprises leverage certificates. Each category encompasses several different types, which may be issued with additional features and specifications. For a further detail, we refer to the official EUSIPA website.

As mentioned in the study (Vigaò, Vitali, Moriggia, & Zanotti, 2019), structured products are similar to exchange traded funds (ETFs). Both provide passive investment strategies and are traded on exchanges. ETFs and structured products differ because the latter do not pay dividends but do have a maturity; moreover, the buyer of a structured product assumes the risk of default but does not pay any management fee.

Structured products are a class of financial instruments that consist of at least two components to create a payoff profile that may not be achievable through conventional investments (Yuan & Rieger, 2021). As a result of having multiple elements in their payoff design, the pricing of these products often relies on replication-based methods, in which the structured product's payoff is replicated by a portfolio of simpler instruments. This approach allows issuers and investors to value structured products consistently by decomposing them into basic building blocks, typically combining stocks or bonds with options. As noted, Benet, Giannetti, and Pissaris (2006), structured products combine the contingent-claim features of options with characteristics akin to fixed-income cash flows. With the growth of the exchange-traded certificates market, issuers have introduced a broad range of offerings that reflect diverse payoff profiles. The complexity of these payoffs can vary widely-from highly sophisticated designs to very straightforward combinations of primary securities. In fact, many structured products can be replicated by simple portfolios of classical calls, puts, and zero-coupon bonds. Halme, Dehghan, and Aspara (2022) look at whether a very simple, structured product (a basic collar-buy the stock plus buy a put and sell a call) can be a good choice for retail investors. The authors Das and Statman (2013) examined how optimal behavioural portfolios are constructed. They found that such portfolios often include put options (downside protection), call options (upside potential for gains), and other financial products.

Replication-based methods can be analytical when closed-form solutions exist, or numerical-such as Monte Carlo simulation-when payoffs are too complex for direct valuation. The replicating portfolio price provides a fair value estimate for the structured product. Various types of structured products have been examined in academic research using these approaches.

The authors Younis and Rusnáková (2014) found a replicating portfolio to the purchase of bonus certificate without a cap and with a cap. Baule and Tallau (2011) examined the valuation of bonus certificates by comparing fair values derived from various pricing models. Their analysis emphasizes the calibration of the standard Black-Scholes framework and assesses model risk relative to Heston's stochastic volatility model. In contrast, Baule and Shkel (2021) explore three distinct modeling approaches beyond Black-Scholes-stochastic volatility, local volatility, and jump diffusion.

Reverse bonus certificate design and pricing using a replicating portfolio performed Bobriková and Harčariková (2015). Replication of the payoffs for bonus and twin-win certificate presented the authors in the paper (Rusnáková et al., 2014).

Outperformance certificates are also attracting growing interest (Harčariková, 2015; Hernández, Lee, Liu, & Dai, 2013). The proposal and valuation of new modifications of existing certificates, i. e. twin-win outperformace and barrier outperformance certificates, was provided by Timková and Bobriková (2023) and Šoltés and Harčariková (2017).

The paper (Bobriková & Harčariková, 2017) shows the replication-based methods of a new discount basket certificate using two-asset correlation options. Baule, Entrop, and Wilkens (2008) focused on analysing and pricing discount certificates in the German secondary market. They compared market prices with theoretical fair values, taking into account both market and credit risk. The pricing behaviour in the market for discount certificates has been examined in the studies (Baule, 2011; Entrop, Fischer, McKenzie, Wilkens, & Winkler, 2016; Idris & Rahman Razak, 2025).

Express certificates are structured financial instruments that offer investors the potential for early redemption and predefined returns based on the performance of an underlying asset (Harčariková & Bánociová, 2015; Hernández, Tobler, & Brusa, 2010; Šoltés, Timková, & Gičová, 2019; Zaghdoud, 2025). The pricing of express certificates typically involves modeling the underlying asset's price dynamics and the probability of early redemption.

Tomaszewski (2013) analysed a wide variety of structured financial products (capital protection, bonus, tracker, turbo and factor certificates) linked to precious metals on the Warsaw Stock Exchange, focusing on their design, risk exposure profiles, and market activity as alternative instruments for financial investors seeking exposure to commodity markets (silver and gold).

Ammann, Arnold, and Straumann (2023) replicate various investment certificates, including discount certificates, barrier discount certificate, reverse convertibles, barrier reverse convertibles, bonus certificates, outperformance certificates and capital protection certificates, by creating equivalent portfolios using fixed-income instruments and options that match the products' payoff structures. Based on their analysis of a large sample of term sheets, they find that issuers often fail to disclose critical information such as the volatility and dividend yield of the underlying assets. This lack of transparency contributes to the presence and variation of mark-ups across products. The study concludes that asymmetric information, where issuers possess more knowledge than retail investors, is a key factor enabling the exploitation of investor mistakes.

Cao (2017) employs a Monte Carlo simulation to investigate how the statistical characteristics of underlying assets influence the risk and return profiles of structured financial products. Employing regression models, the authors Arnold, Schuette, and Wagner (2021) analyse the relationship between structured product pricing, issuer credit risk, and investor attention. Avdiu and Unger (2023) constructed replication portfolios to match the payoffs of ten different structured products and compared their theoretical fair values with observed market prices. Their analysis revealed substantial mark-ups over the fair values, ranging from 0.9% to 2.9%.

Table 1 presents a summary of existing studies on structured products, with a particular focus on replication-based option valuation methods.

Table 1. Summary of studies on investment certificates

Study	Structured product	Replication method	Type of approach
Baule et al. (2008)	Discount certificate	Structural valuation approach	Analytical
Hernández et al., 2010	Express certificate	Contingent claim valuation technique	Numerical
Baule (2011)	Discount certificate	Regression model	Numerical
Baule and Tallau (2011)	Bonus certificate	Option pricing models	Numerical
Hernández et al., 2013	Outperformance certificate	Contingent claim valuation technique	Numerical
Tomaszewski (2013)	Capital protection certificate Bonus certificate Tracker certificate Turbo certificate Factor certificate	Empirical analysis	Analytical
Younis and Rusnáková (2014)	Bonus certificate	Option pricing models	Analytical
Rusnáková et al. (2014)	Bonus certificate Twin-win certificates	Option pricing models	Analytical
Bobriková and Harčariková (2015)	Reverse bonus certificates	Option pricing models	Analytical
Harčariková and Bánociová (2015)	Express certificate	Option pricing models	Analytical
Harčariková (2015)	Outperformance certificate	Option pricing models	Analytical
Entrop et. al. (2016)	Discount certificate	Regression model	Numerical
Bobriková and Harčariková (2017)	Discount basket certificate	Option pricing models	Analytical
Šoltés and Harčariková (2017)	Barrier outperformance certificates	Option pricing models	Analytical
Cao (2017)	Structured products	Monte Carlo simulation	Numerical
Šoltés et al. (2019)	Express certificate	Option pricing models	Analytical
Arnold et al. (2021)	Structured products	Regression model	Numerical
Baule and Shkel (2021)	Bonus certificates	Option pricing models	Numerical
Timková and Bobriková (2023)	Twin-win outperformance certificate	Option pricing models	Analytical
Ammann et al. (2023)	Discount certificate Barrier discount certificate Capital protection certificate Reverse convertibles certificate Barrier reverse convertibles certificate Bonus certificate Outperformance certificates Capital protection certificates	Regression model	Numerical
Avdiu and Unger (2023)	Structured products	Option pricing models	Analytical

Source: Authors' overview

Option pricing models used to determine the fair value of options based on various assumptions about the underlying asset, time, volatility, interest rates, and other factors. Black and Scholes (1973) introduced theoretical price of European vanilla call and put options on the stocks without dividends. Black-Scholes formula was modified by Merton (1973), where he considered the pricing of vanilla call and put options on the stocks with dividends. Extended versions of classical option pricing models have incorporated stochastic volatility to better capture market dynamics. Notable contributions include models by Stein and Stein (1991), Heston (1993), Ball and Roma (1994), and Lee (2005) each of which enhances pricing accuracy by accounting for volatility that evolves randomly over time. Carr and Wu (2014) further extended this framework by examining pricing under stochastic skew and jump processes. More recently, Buehler, Gonon, Teichmann, and Wood (2018) developed a novel approach using deep reinforcement learning (DRL) to price a portfolio of derivatives in environments modelled by stochastic processes, specifically incorporating transaction costs in Heston-driven synthetic markets. Building on this line of research, Du et al. (2020) proposed a DRL-based model for option replication, demonstrating its potential to learn optimal hedging strategies in complex, dynamic market settings.

Merton (1973) modified Black-Scholes model about the barrier level and derived the first relationship for European down and out call option price. Later Rubinstein and Reiner (1991) applied Black-Sholes-Merton formula on 8 basic types of barrier options and Haug (2007) on all 16 types of standard European barrier options. Finally, the mathematical structure of barrier options value was derived by Rich (1994). Tian, Yang, and Zhang (2019) investigated barrier options for uncertain mean-reverting stock models. Yao and Oin (2021) derived some pricing formulas for barrier options of the Liu's uncertain stock model (Liu, 2008, 2009).

3. METHODOLOGY

This study builds upon prior research and focused on analytical replication-based option valuation of reverse twin-win certificates. It contributes to the existing literature in several distinct ways. First, it addresses a notable gap by examining reverse twin-win certificates—an instrument that has not yet been comprehensively analysed. Second, the analysis is applied to a Bulgarian company, introducing market-specific dynamics that differ from those typically explored in similar studies. Third, the study includes a profitability assessment under various price development scenarios, along with a sensitivity analysis to evaluate how changes in key parameters influence the product's financial performance.

In the absence of market-traded options for European companies, both vanilla and barrier options were priced theoretically using the Black-Scholes model and Haug's option pricing framework for barrier options, implemented in the statistical program R, utilizing the *pracma* and *RQuantLib* packages.

This paper introduces the **novel concept of reverse twin-win certificates** and derives their corresponding profit functions. To our knowledge, this specific structure has not been thoroughly examined in the academic literature. Furthermore, we develop the structural design of these financial instruments using financial engineering principles and replication techniques based on option pricing theory, thereby providing a rigorous analytical framework for their valuation and analysis.

The profit function of a **capped reverse twin-win certificate** is defined as follows:

$$P(S_T) = \begin{cases} m * (2S_0 - C - k_0) & if\ S_T < C \\ m * (2S_0 - S_T - k_0) & if\ C \leq S_T < S_0 \\ m * (2S_0 - S_T - k_0) & if\ \max_{0 \leq t \leq T} S_t \geq B\ and\ S_T \geq S_0 \\ m * (S_T - k_0) & if\ \max_{0 \leq t \leq T} S_t < B\ and\ S_T \geq S_0 \end{cases} \quad (1)$$

where:

m denotes the participation rate (multiplier),

S_T represents the price of the underlying asset at maturity (time *T*),

S_0 is the initial price of the underlying asset,

k_0 refers to the issue price of the certificate,

C denotes the predetermined cap level of the certificate, and

B is the predefined barrier level.

The profit function of a capped twin-win certificate can be replicated through a structured alternative portfolio. This portfolio comprises the following positions:

1) A short position in *m* units of the put options on the underlying asset, with a strike price equal to the cap level *C*, option premium *p*, and a time to maturity identical to that of the certificate *T*. The profit function of this short put position at maturity is given by:

$$P_1(S_T) = \begin{cases} m * (S_T - C + p) & if\ S_T < C \\ m * p & if\ S_T \geq C \end{cases} \quad (2)$$

2) A short position in *m* units of the underlying asset at the initial price S_0, yielding the following profit at maturity:

$$P_2(S_T) = m * (S_0 - S_T) \quad (3)$$

3) A long position in *2m* up and knock-out call options on the underlying asset, with the following characteristics:

- strike price identical to the initial price of the underlying asset S_0,
- barrier level identical to the barrier level of the certificate *B*,
- option premium c_{UO},
- maturity identical to the certificate's maturity *T*.

The corresponding profit function is:

$$P_3(S_T) = \begin{cases} -m * 2c_{UO} & if\ S_T < S_0 \\ -m * 2c_{UO} & if\ \max_{0 \leq t \leq T} S_t \geq B\ and\ S_T \geq S_0 \\ m * 2(S_T - S_0 - 2c_{UO}) & if\ \max_{0 \leq t \leq T} S_t < B\ and\ S_T \geq S_0 \end{cases} \quad (4)$$

The profit from the alternative portfolio, expressed as the sum of the individual components $P_1(S_T)$, $P_2(S_T)$ and $P_3(S_T)$ is given by:

$$P(S_T) = \begin{cases} m * (S_0 - C + p\ - 2c_{UO}) & if\ S_T < C \\ m * (-S_T + S_0 + p\ - 2c_{UO}) & if\ C \leq S_T < S_0 \\ m * (-S_T + S_0 + p\ - 2c_{UO}) & if\ \max_{0 \leq t \leq T} S_t \geq B\ and\ S_T \geq S_0 \\ m * (S_T - S_0 + p\ - 2c_{UO}) & if\ \max_{0 \leq t \leq T} S_t < B\ and\ S_T \geq S_0 \end{cases} \quad (5)$$

Let us define the issue price of the certificate as:

$$k_0 = m * (S_0 - p + 2c_{UO}) \tag{6}$$

By substituting k_0 into the profit function, we derive the explicit formulation of the profit function for an uncapped reverse twin-win certificate, as defined in equation (1).

The profit function of an **uncapped reverse twin-win certificate** is defined as follows:

$$P(S_T) = \begin{cases} m * (-S_T + 2S_0 - k_0) & if\ S_T < S_0 \\ m * (-S_T + 2S_0 - k_0) & if \max\limits_{0 \le t \le T} S_t \ge B\ and\ S_T \ge S_0 \\ m * (S_T - k_0) & if \max\limits_{0 \le t \le T} S_t < B\ and\ S_T \ge S_0 \end{cases} \tag{7}$$

The profit function of an uncapped twin-win certificate can be replicated through a combination of the positions described by functions (3) and (4). The corresponding alternative portfolio is given by:

$$P(S_T) = \begin{cases} m * (-S_T + S_0 - 2c_{UO}) & if\ S_T < S_0 \\ m * (-S_T + S_0 - 2c_{UO}) & if \max\limits_{0 \le t \le T} S_t \ge B\ and\ S_T \ge S_0 \\ m * (S_T - S_0 - 2c_{UO}) & if \max\limits_{0 \le t \le T} S_t < B\ and\ S_T \ge S_0 \end{cases} \tag{8}$$

Substituting k_0 as shown in equation (9) we obtain the expression given in equation (7).

$$k_0 = m * (S_0 + 2c_{UO}) \tag{9}$$

4. APPLICATION

Shelly Group SE (SLYG.DE), a Bulgaria-based technology firm listed on the Frankfurt Stock Exchange since November 2021, specializes in intelligent solutions for residential and industrial automation. The company is best known for its Internet of Things (IoT) portfolio, including smart relays, sensors, and home automation devices that integrate with mobile platforms and voice assistants.

We selected SLYG.DE as the empirical underlying due to its distinctive profile: an emerging market technology issuer with a focused IoT product portfolio, moderate historical volatility (31.11%), and limited liquidity in listed options markets. These features make it particularly well-suited for analyzing the behavior of replication-based structured products, specifically Reverse Twin-Win (referred as RTW) and Capped Reverse Twin-Win (referred as CRTW) certificates, where theoretical pricing models must be employed in the absence of a deep and liquid options market. Although the stock has exhibited consistent upward momentum, the possibility of short-term corrections strengthens the case for employing reverse payoff structures aimed at capitalizing on stable or moderately adverse price movements.

4. 1. DATA

Due to the predominance of over-the-counter (OTC) trading in vanilla and barrier options for European equities, we computed theoretical values for the embedded options, specifically, European call options and up-and-out barrier call options. All share and option prices are denominated in EUR, with a uniform time to maturity of one year. We selected four barrier levels (EUR 45, 50, 55, and 60) and three cap levels (EUR 25, 27, and 30) to represent both conservative and aggressive market scenarios. These levels correspond to commonly used round-number thresholds observed in market practice and are fixed at the time of certificate issuance.

The input parameters, historical volatility (σ = 31.11%), dividend yield (δ = 0.354%), and the one-year risk-free interest rate (r = 2.15%), were sourced from Bloomberg and ECB bond data as of 18 March 2025. For the purposes of this analysis, transaction costs are assumed to be zero, and the issuer's margin is also set to zero. Accordingly, the theoretical issue price is equal to the purchase price of the certificates. An overview of the key design parameters and conditions at issuance is provided in Table 2.

Table 2. Key information about the designed certificates

Underlying asset	Underlying price	Issue date	Maturity date	Multiplier	Historical volatility	Risk-free interest rate	Dividend yield
Shelly Group SE (SLYG.DE)	EUR 36.7	18th March 2025	18th March 2026	1:1	31.11%	2.15%	0.354%

Source: processed based on data from Yahoo Finance (2025)

4. 2. RESULTS

The proposed certificates have a barrier set at EUR 45; EUR 50; EUR 55 and EUR 60, with continuous monitoring throughout the entire period until the maturity date of March 18, 2026. The capped reverse twin-win certificate has a cap level set at EUR 25; EUR 27 and EUR 30. These parameters influence both the issue price and the investor's potential profit. For the purposes of this analysis, we consider the purchase of one certificate.

The issue price of each certificate, denoted as k_0, is calculated as the total cost of its replicating components, as specified in Equations (2), (3), and (4) for CRTW certificates, and in Equations (3) and (4) for RTW certificates. A representative numerical example, specified by Equation (10) is provided for the Capped Reverse Twin-Win certificate (CRTW3), which features a barrier of EUR 45 and a cap of EUR 25. The premiums for the short European put option (p=0.4275) and two up-and-out call options (c_{UO}=0.2127) are first determined. These are then combined with the proceeds from a short position in the underlying asset to compute the theoretical issue price as follows

$$k_0 \text{ (CRTW3)} = 36.70 + 2 \text{ x } 0.2127 - 0.4275 = 36.70 \qquad (10)$$

This example demonstrates the additive structure of the replication strategy, where the certificate's value is constructed from a portfolio of derivative instruments and the underlying asset, assuming zero transaction costs and no issuer markup. A summary of all designed certificates, along with the relevant parameters, is provided in Table 3.

Table 3. Parameters of selected RTW a CRTW certificates (in EUR per 1 certificate) with the maturity date of March 18, 2026

Denotation of investment certificate	Barrier (B)	Actual price (S0)	Call barrier option c_{UO}(S0)	Cap level (C)	Put option p (C)	Issue price (k_0)
RTW1	45	36.7	0.2127	-	-	37.13
RTW2	50	36.7	0.8247	-	-	38.35
RTW3	55	36.7	1.6912	-	-	40.08
RTW4	60	36.7	2.5555	-	-	41.81
CRTW1	45	36.7	0.2127	30	1.4326	35.69
CRTW2	45	36.7	0.2127	27	0.7341	36.39
CRTW3	45	36.7	0.2127	25	0.4275	36.70
CRTW4	50	36.7	0.8247	30	1.4326	36.92
CRTW5	50	36.7	0.8247	27	0.7341	37.62
CRTW6	50	36.7	0.8247	25	0.4275	37.92
CRTW7	55	36.7	1.6912	30	1.4326	38.65
CRTW8	55	36.7	1.6912	27	0.7341	39.35
CRTW9	55	36.7	1.6912	25	0.4275	39.65
CRTW10	60	36.7	2.5555	30	1.4326	40.38
CRTW11	60	36.7	2.5555	27	0.7341	41.08
CRTW12	60	36.7	2.5555	25	0.4275	41.38

Source: own calculations in statistical program R

For illustration, we consider designed certificates with the barrier level EUR 45 and EUR 60. For capped certificate, the cap level is EUR 30 and EUR 25. The profit profile of RTW is based on the relation (8) and CRTW on the relation (5). Different scenarios of UA price development at the maturity date are considered. The profit profile of reverse linear certificate (referred as RLC) is showed as:

$$36.70 - S_T \tag{10}$$

We compare these certificates with each other, and the results of the illustrated certificates are presented in Table 4 and Table 5.

Table 4. The profit profile of RTW certificates at the maturity date of March 18, 2026

Scenarios of UA price development at the maturity date	RTW1	RTW2	RTW3	RTW4
$if\ S_T < 36.7$	$- S_T + 36.27$	$- S_T + 35.05$	$- S_T + 33.32$	$- S_T + 31.59$
$if \max\limits_{0\leq t\leq T}(S_t) < 45 \wedge S_T \geq 36{,}7$	$S_T - 37.13$	-	-	-
$if \max\limits_{0\leq t\leq T}(S_t) \geq 45 \wedge S_T \geq 36{,}7$	$- S_T + 36.27$	-	-	-
$if \max\limits_{0\leq t\leq T}(S_t) < 50 \wedge S_T \geq 36{,}7$	-	$S_T - 38.35$	-	-
$if \max\limits_{0\leq t\leq T}(S_t) \geq 50 \wedge S_T \geq 36{,}7$	-	$- S_T + 35.05$	-	-
$if \max\limits_{0\leq t\leq T}(S_t) < 55 \wedge S_T \geq 36{,}7$	-	-	$S_T - 40.08$	-
$if \max\limits_{0\leq t\leq T}(S_t) \geq 55 \wedge S_T \geq 36{,}7$	-	-	$- S_T + 33.32$	-
$if \max\limits_{0\leq t\leq T}(S_t) < 60 \wedge S_T \geq 36{,}7$	-	-	-	$S_T - 41.81$
$if \max\limits_{0\leq t\leq T}(S_t) \geq 60 \wedge S_T \geq 36{,}7$	-	-	-	$- S_T + 31.59$

Source: own calculations

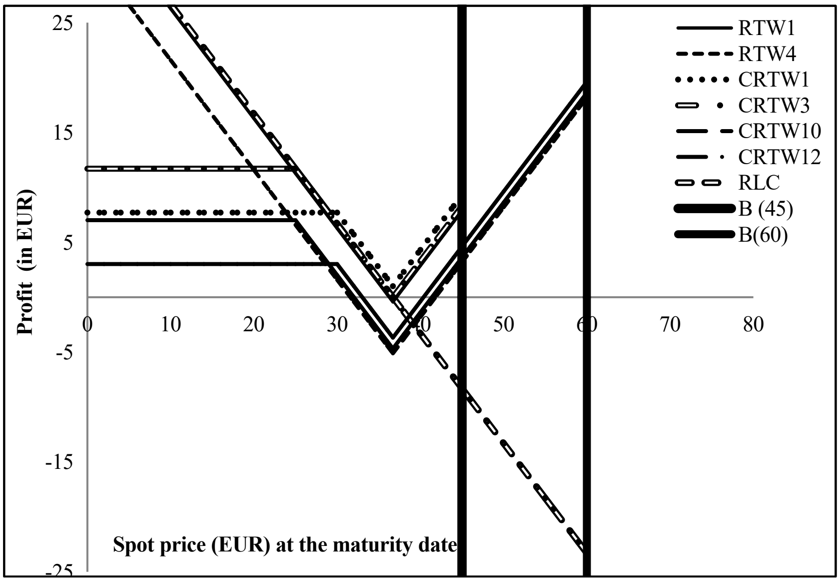
Table 5. The profit profile of selected CRTW certificates at the maturity date of March 18, 2026

Scenarios of UA price development at the maturity date	CRTW1	CRTW3	CRTW10	CRTW12
$if\ S_T < 30$	7.71	-	3.02	-
$if\ 30 \leq S_T < 36.7$	$- S_T + 37.71$	-	$- S_T + 33.02$	-
$if\ S_T < 25$	-	11.70	-	7.02
$if\ 25 \leq S_T < 36.7$	-	$- S_T + 36.70$	-	$- S_T + 32.02$
$if \max\limits_{0\leq t\leq T}(S_t) < 45 \wedge S_T \geq 36{,}7$	$- S_T + 37.71$	$S_T - 36.70$	-	-
$if \max\limits_{0\leq t\leq T}(S_t) \geq 45 \wedge S_T \geq 36{,}7$	$- S_T + 37.71$	$- S_T + 36.70$	-	-
$if \max\limits_{0\leq t\leq T}(S_t) < 60 \wedge S_T \geq 36{,}7$	-	-	$S_T - 40.38$	$S_T - 41.38$
$if \max\limits_{0\leq t\leq T}(S_t) \geq 60 \wedge S_T \geq 36{,}7$	-	-	$- S_T + 33.02$	$- S_T + 32.02$

Source: own calculations

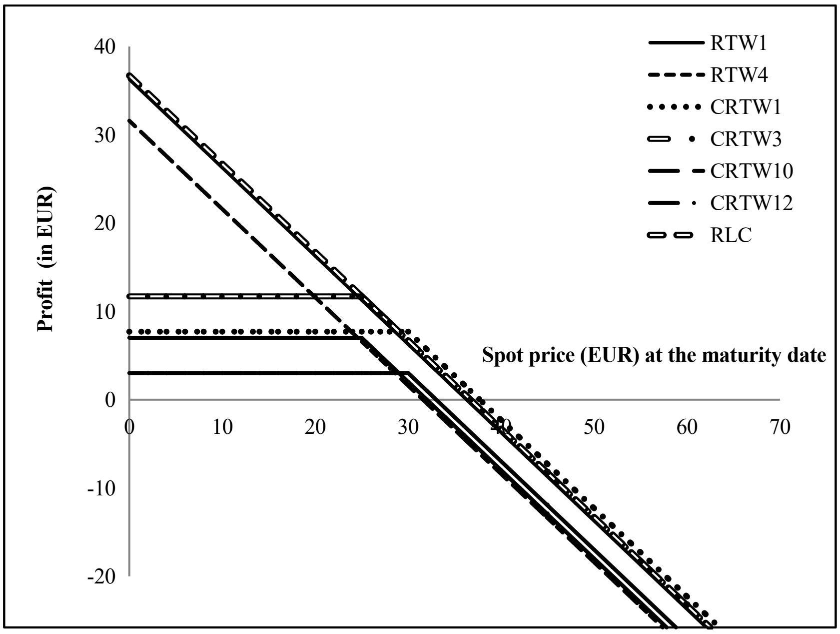
To facilitate a clearer presentation of the results from Tables 4 and 5, a graphical comparison of the designed certificates is provided. Figure 1 illustrates the case where the barrier level is not reached. Figure 2 depicts the case where the barrier level is reached. Only selected certificates are presented graphically.

Figure 1. Profit/Loss (in EUR per certificate) for the proposed RTW and CRTW certificates at maturity, assuming the barrier level is not reached during the time to maturity



Source: Own design

Figure 2. Profit/Loss (in EUR per certificate) for the proposed RTW and CRTW certificates at maturity, assuming the barrier level is reached during the time to maturity



Source: Own design

The analysed results indicate that the profit or loss from our designed certificates depends not only on the value of Shelly Group SE shares at the maturity date, but also on its development throughout the entire observed period. It is also important to monitor whether the barrier level has been breached or not at any point during the maturity period.

Once the barrier level is reached, the following statements apply. Unlimited profit potential in the event of a decline in Shelly Group SE price can be achieved through RLC certificates, starting from the price level of EUR 36.70 and below. These are followed by RTW1 certificates (from EUR 36.27) and RTW4 certificates (from EUR 31.59). In contrast, capped certificates offer a limited maximum profit, which is compensated by lower issuance costs or, in some cases, the possibility of issuing them at zero cost. Among the capped certificates, CRTW3 exhibits the highest maximum profit (EUR 11.70 per certificate), followed by CRTW1 (EUR 7.71), CRTW12 (EUR 7.02), and CRTW10 (EUR 3.02). The profit potential of CRTW1 increases when Shelly Group SE price falls below EUR 37.71; for CRTW3, the threshold is EUR 36.70; for CRTW10, EUR 33.02; and for CRTW12, EUR 32.02. Conversely, if Shelly Group SE price rises above these levels, the respective certificates begin to incur losses, which escalate with further price increases.

For comparative purposes, RLC certificates yield the highest profitability down to the level of EUR 25.00. In the price range of EUR 25.00 to EUR 28.99, CRTW3 and RLC certificates provide equivalent profit levels. Above EUR 28.99, CRTW1 becomes the most profitable; however, its profitability diminishes with increasing Shelly Group SE price, transitioning into a loss beyond EUR 36.70. CRTW10 and CRTW12 certificates are comparatively less profitable due to their higher barrier level (EUR 60). Additionally, their performance characteristics do not intersect with those of RLC, RTW1, CRTW1 or CRTW3 certificates.

Our proposed certificates are suitable if the barrier is not exceeded, at which point the barrier option is activated, which ensures its special payout profile. The most profitable certificates are those without a cap, specifically, the RLC certificate delivers the highest return, followed by RTW1 and RTW4, particularly in scenarios where a significant decline in the share price of Shelly Group SE is anticipated. Conversely, in cases of moderate declines or slight increases up to the barrier level, capped certificates tend to offer greater profitability. Certificate RTW1 [respectively RTW4] generates increasing profits as the share price of Shelly Group SE falls below EUR 36.27 [EUR 31.59] or rises above EUR 37.13 [EUR 41.81]. Losses occur when the share price remains within the range of EUR 36.27 to EUR 37.13 [EUR 31.59 to EUR 41.81].

Certificates CRTW1 and CRTW3 can be structured at zero initial cost while still delivering a positive return. CRTW1 [respectively CRTW3] yields increasing gains as the share price declines from EUR 37.71 [EUR 36.70], and continues to generate higher profits as the price rises up to the barrier level of EUR 45. Certificates CRTW10 [CRTW12] incur initial costs due to their higher predefined barrier level of EUR 60. These instruments begin to generate increasing profits as the share price falls below EUR 33.02 [EUR 32.02]. However, within the range of EUR 33.02 to EUR 40.38 [EUR 32.02 to EUR 41.38], both certificates result in losses, with the maximum loss amounting to EUR -3.68 per CRTW10 certificate and EUR -4.68 per CRTW12 certificate, occurring at the share price of EUR 36.70. Beyond this point, profits increase again as the share price rises toward the EUR 60 barrier. Among the capped certificates, CRTW3 proves to be the most profitable in the event of a sharp decline below EUR 28.99. Above this threshold, CRTW1 becomes more advantageous, up to the barrier level of EUR 45. Certificates with higher barriers (EUR 60) generally offer lower returns compared to the aforementioned alternatives. CRTW12 becomes more profitable when the share price drops significantly below EUR 28.99, while CRTW10 outperforms if the price remains above this level. When comparing the proposed capped certificates with the uncapped RLC certificate, the capped structures become more profitable if the share price of Shelly Group SE exceeds the following thresholds: EUR 36.91 (RTW1), EUR 39.26 (RTW4), EUR 25.00 (CRTW1), EUR 28.99 (CRTW3), EUR 38.54 (CRTW10), and EUR 39.04 (CRTW12) and is higher.

The following conclusions apply to the comparative analysis of capped versus uncapped certificates when the barrier is not breached. RTW1 outperforms CRTW1 (and CRTW3) in cases of a sharp price decline below EUR 28.57 (EUR 24.57). In contrast, CRTW1 (and CRTW3) are more advantageous in scenarios involving moderate declines or modest price increases up to the EUR 45 barrier. When comparing RTW1 with CRTW10 and CRTW12, RTW1 remains the most profitable certificate across all possible share price outcomes at maturity. Overall, RTW1 is the most profitable structure in the event of a significant price drop below EUR 24.57. CRTW3 performs best within the range of EUR 24.57 to EUR 28.99, while above this interval, CRTW1 offers the highest return, supported by its lower barrier level of EUR 45 and a higher cap of EUR 30.

4. 3. DISCUSSION

Based on the results from Table 3, there is possible to determine parameters which influence on the issue price and the profit/loss of designed certificates. There were identified parameters as the barrier level *B* and the cap level *C* (valid for capped certificates). There is a positive effect of the barrier level's growth on the certificate's issue price and a negative effect of the cap level's growth on the certificate's issue price and vice versa. A summary of these effects is provided in Table 6.

Table 6. Parameters' influence on the certificates' issue price and the profit/loss

Parameters	Designed certificates	Impact on certificates' issue price		Impact on the certificates' profit/loss	
		Drop	Growth	Drop	Growth
Barrier level (B)	RTW1-RTW2-RTW3-RTW4 CRTW1- CRTW4-CRTW7-CRTW10 CRTW2- CRTW5-CRTW8-CRTW11 CRTW3- CRTW6-CRTW9-CRTW12	↓	↑	↑/↓	↓/↑
Cap level (C)	CRTW1-CRTW2-CRTW3 CRTW4-CRTW5-CRTW6 CRTW7-CRTW8-CRTW9 CRTW10-CRTW11-CRTW12	↑	↓	↑	↓

Source: own design

This section presents a profitability analysis from the viewpoint of a potential investor. It evaluates possible outcomes based on assumed percentage changes in the market price of Shelly Group SE at the maturity date (March 18, 2026), relative to the current spot price S_0 = EUR 36.70. The results, summarized in Table 7, aim to identify the most suitable certificate for different market expectations, considering only a selected set of proposed certificates.

The findings indicate that if the price of Shelly Group SE remains unchanged (0% change, i.e., EUR 36.70), all proposed certificates result in a loss, with the largest loss observed in the RTW4 certificate. The only exception is CRTW3, which yields no loss (EUR 0). The reason for the initial losses lies in the structure of the certificates: due to the presence of a security buffer (barrier level), the issue prices are higher, while cap levels limit the maximum potential gains. When the price slightly decreases or increases, within the range bounded by the barrier, all proposed certificates become profitable. This is attributable to their structural advantage of lower creation costs, which make them more resilient to minor market movements. Investors begin to realize profits if the Shelly Group SE price changes by approximately ±5% or more (for RTW1 and CRTW3) or ±15% or more (for RTW4 and CRTW12) from the current spot price.

In the event of a significant price decline (e.g., -50%), RTW1 certificate delivers the highest return (EUR 17.92), comparable to RLC (EUR 18.35), the lower return RTW4 (EUR 13.24). However, in scenarios where the price increases beyond the initial level (e.g., +30%), RTW1 may incur a loss of up to EUR -11.44 if the barrier is breached. If the barrier remains intact, RTW1 gradually moves into positive territory, achieving a maximum return of EUR 5.08 at a

15% price increase. Conversely, under moderately bullish market conditions, RTW4 outperforms RTW1, provided the barrier is not breached, making it a more flexible option for neutral to slightly bullish scenarios. CRTW3 certificate, with a cap set at EUR 25 and a barrier at EUR 45, delivers identical returns regardless of whether the barrier is breached. This structure offers enhanced stability, making it suitable for investors expecting limited price movements. CRTW3 reaches its maximum return (EUR 11.70) in the case of a price decline of 40% or more. For price increases, returns improve only if the barrier remains unreached, with a maximum of EUR 5.51 at a 15% increase. Thus, CRTW3 is appropriate for stagnant to moderately declining or moderately rising markets. CRTW12 certificate, with a cap of EUR 25 and a higher barrier at EUR 60, behaves similarly to CRTW3 in down markets but offers a lower maximum return (EUR 7.02). Its returns increase under rising market conditions only if the barrier is not breached, making it less sensitive to market volatility. Therefore, CRTW12 is suitable for investors seeking lower but more stable returns in exchange for reduced market sensitivity.

Table 7. The profit profile of selected RTW and CRTW certificates at the maturity date of March 18, 2026

SLYG.DE change (%)	RTW1		RTW4		CRTW3		CRTW12		RLC
	P/L (in EUR) if B is reached	P/L (in EUR) if B is not reached	P/L (in EUR) if B is reached	P/L (in EUR) if B is not reached	P/L (in EUR) if B is reached	P/L (in EUR) if B is not reached	P/L (in EUR) if B is reached	P/L (in EUR) if B is not reached	P/L (in EUR)
-50%	17.92	17.92	13.24	13.24	11.70	11.70	7.02	7.02	18.35
-40%	14.25	14.25	9.57	9.57	11.70	11.70	7.02	7.02	14.68
-30%	10.58	10.58	5.90	5.90	11.01	11.01	6.33	6.33	11.01
-20%	6,91	6,91	2,23	2,23	7,34	7,34	2,66	2,66	7,34
-15%	5.08	5.08	0.39	0.39	5.51	5.51	0.82	0.82	5.51
-10%	3.24	3.24	-1.44	-1.44	3.67	3.67	-1.01	-1.01	3.67
-5%	1.41	1.41	-3.28	-3.28	1.84	1.84	-2.85	-2.85	1.84
0%	-0.43	-0.43	-5.11	-5.11	0	0	-4.68	-4.68	0.00
5%	-2.26	1.41	-6.95	-3.28	-1.83	1.84	-6.52	-2.85	-1.84
10%	-4.10	3.24	-8.78	-1.44	-3.67	3.67	-8.35	-1.01	-3.67
15%	-5.93	5.08	-10.62	0.39	-5.50	5.51	-10.19	0.82	-5.51
20%	-7,77	6,91	-12,45	2,23	-7,34	7,34	-12,02	2,66	-7,34
30%	-11.44	-	-16.12	5.90	-11.01	-	-15.69	6.33	-11.01
50%	-15.11	-	-19.79	9.57	-14.68	-	-19.36	10.00	-14.68
40%	-18.78	-	-23.46	13.24	-18.35	-	-23.03	13.67	-18.35
70%	-26.12	-	-30.80	-	-25.69	-	-30.37	-	-25.69

Source: own calculations

The following Table 8 provides a summary of a sensitivity comparison analysis, showing the percentage change in SLYG.DE relative to its initial price at issuance, along with recommended structured certificate types for investors based on their anticipated market outlook.

Table 8. Summary of a sensitivity analysis comparing the designed investment certificates

Market View	Price Change	Investor Profile	Recommended Certificates)	Key rationale
Strong bearish	> –30 %	Aggressive bearish speculator	RLC RTW1 (Uncapped) RTW4	Leverage downside potential; RTW1 barrier level near S_0; Maximizes decline payoff.
Moderate Bearish	–10% to –20%	Bearish Opportunist	RTW1 CRTW3 (Capped)	CRTW3 limits downside and secures profits (~€5.08–7.34); Barrier still near S_0 and the cap level distant from S_0.
Flat Market	≈ 0 %	Defensive or hedging oriented	CRTW3	Offers capital preservation with neutral payoff at maturity.
Moderate Bullish	+5% to +15%	Momentum Bull / Volatility Trader	RTW1 CRTW3	RTW1 benefits from mild uptrend; CRTW3 offers stable capped returns (in both case the barrier is not reached).
Strong Bullish	≥ +20 %	Momentum Bull	RTW4 CRTW12	Captures upside if barrier is intact; otherwise avoid due to loss risk.

Source: own design

Notes: $B > S_0$; $C < S_0$

Ultimately, selecting the right certificate depends on investors' outlook for Shelly's share price at maturity, and crucially, whether the barrier has been breached during the term. The optimal choice hinges on the precise configuration of structural parameters, especially barrier and cap levels, as these define each certificate's risk-return profile and suitability across varying market scenarios.

4. 4. PERFORMANCE COMPARISON AND PRACTICAL IMPLICATIONS

Our results indicate that uncapped Reverse Twin-Win (RTW) certificates and the Reverse Linear Certificate (RLC) generate the highest returns in scenarios of significant price declines in the UA. This finding is consistent with Bobriková and Harčariková (2015), who observed that removing caps from reverse bonus certificates enhances payoffs in strongly bearish markets. Conversely, under moderate upward movements, capped structures such as Capped Bonus and Capped Twin-Win certificates, introduced by Rusnáková et al. (2014), demonstrate superior performance by offering controlled upside potential within a defined risk profile. In particular, during periods of moderate price fluctuations (±5–15%), capped certificates like CRTW3 outperform their uncapped counterparts due to their structured, limited upside and lower initial cost, an effect similarly documented by Baule and Shkel (2021) and Ammann et al. (2023) for bonus and discount certificate designs. Additionally, we find that even marginal adjustments to the cap and barrier levels can lead to significant changes in both the issue price and break-even thresholds. This sensitivity aligns with prior findings by Baule et al. (2008) on discount certificates and by Cao et al. (2017), who employed Monte Carlo simulations to explore the impact of volatility on certificate pricing.

By applying our replication based pricing framework to Shelly Group SE, an emerging-market mid cap issuer lacking liquid exchange traded options, we extend the work of Tomaszewski (2013) on precious metals certificates in illiquid markets and of Harčariková and Bánociová (2015) on express certificates in OTC environments. These findings confirm that in the absence of liquid option markets, theoretical pricing methods remain both practical and effective for the valuation of structured financial instruments.

Our findings underscore the practical relevance of replication based pricing for structured products in European markets with limited derivatives trading. Issuers can leverage this approach to design both capped (CRTW) and uncapped Reverse Twin-Win (RTW) certificates, expanding product offerings despite market incompleteness. For investors, uncapped RTW or Reverse Linear Certificates (RLC) offer leveraged downside exposure suitable for bearish outlooks, while capped CRTW variants provide more stable returns and downside protection for conservative strategies.

Despite its contributions, the study has several limitations. Option prices are generated using theoretical models in R and have not been validated against actual market quotes, which may affect the realism of the results. Moreover, the analysis assumes idealized conditions, namely zero transaction costs and continuous monitoring of barrier levels, that may not align with real-world trading environments. Future research should address these limitations by incorporating empirical market data, accounting for trading frictions, and modeling discrete monitoring. Additionally, examining investor behavior and decision-making through experimental or empirical studies would further enhance the practical relevance and robustness of the proposed replication-based framework.

5. CONCLUSION

This paper extends replication based option pricing techniques to design and value Reverse Twin Win certificates, both uncapped and capped, and deriving analytical profit functions that directly link barrier and cap parameters to investor payoffs. Theoretically, our closed form results enrich the literature on path dependent payoff structures by showing how combinations of vanilla and barrier options can generate asymmetric exposure suited to bearish market views. Practically, we demonstrate how issuers in emerging market contexts (exemplified by Shelly Group SE) can engineer these certificates where exchange traded derivatives are absent, and how investors can tailor their choice, favoring uncapped variants for aggressive bearish strategies and capped variants for moderate, risk buffered returns, making them appropriate for conservative investors.

Our sensitivity analysis confirms the robustness of these designs across realistic shifts in volatility, barrier, and cap levels, underscoring the importance of transparent parameter disclosure and standardized monitoring protocols. For policymakers and market practitioners, this implies a need for clear term sheet transparency, risk tier classification, and oversight of issuance costs to ensure investor protection and product integrity. Limitations of our study include reliance on theoretical option prices without live market validation, omission of transaction costs and bid–ask spread effects, and the assumption of continuous barrier monitoring.

While our findings highlight the practical relevance and theoretical coherence of Reverse Twin-Win certificates, several avenues remain for future exploration. These include the incorporation of empirical option data, robustness testing under stochastic volatility or jump diffusion models, the inclusion of trading frictions, and behavioral analysis through experimental or market-based studies. Such extensions would further strengthen the academic rigor and real-world applicability of this approach. Moreover, the methodology may be readily adapted to other asset classes-such as currencies, indices, or exchange rates, and extended to alternative option structures, including American-style derivatives, thereby broadening its utility in both academic research and product development.

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