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Modelling problems in a regional labour market in Poland with MARS and CMARS — supported by optimisation

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A B S T R A C T

The study mainly aims to learn about job candidates' characteristics that influence the declared levels of their general skills *(G)*. The set of characteristics examined included common skills *(C)*, professional skills *(P)* and other economic and social characteristics of candidates. In this context, it is essential to observe the required knowledge and core skills on the market and to make this knowledge available to decision-makers from public institutions, entrepreneurs and educational institutions. The main research objective is also related to verifying the method used to analyse large multi-dimensional databases of competencies that can be used to improve vocational education and its fit to the labour market. An additional objective of the study is also to find the relationship between the characteristics of the offers (the time of visibility and publication of offers, the distance of the job from the place of residence) and the grouping of competencies in individual professions and occupational groups. It is linked to another problem: identifying factors that influence the design and availability of employee competencies. MARS and CMARS methods were used as neutral computing methodologies to determine the relationship between the response variable *(G)* and the input variables. Their use allows for commenting on how fulfilment can be determined between the requirements of employers and decision-makers in government. The duration of visibility and publication of the offer might be informative for job candidates and, thus, significantly influence the development of competences in the region. The innovative use of advanced statistical methods to achieve the goal in the area of competence management allows for high precision of the results, reducing risks in making management decisions.

K E Y   W O R D S

competence management, conic optimisation, multivariate adaptive regression splines, interior point method, green skills, digital skills

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Introduction

The "fourth revolution" caused by the creation and introduction of new technologies, the internet, the internet of things, computing clouds, interpersonal communication and machines, including cyber-physical systems (Lee et al., 2015), forces employees to continuously develop their understood competencies, comprehended as knowledge, as skills, and as attitudes (Hellström et al., 2000). Financial resources play an essential role in achieving corporate goals and maximising profits. However, more important skills emerge from the knowledge generated and

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accumulated by the company in the form of human capital and approaches, organisational procedures, processes, procedures, and norms (Soosay & Hyland, 2008). They help create opportunities to use all of the company's resources.

The contribution aims to work out and develop a mathematical model for the identification, analysis, and further use of the dependencies of selected competence categories among learners in professions, as well as other factors that influence the acquisition of these competencies. One of the competence categories was chosen as the basis for the analyses in this paper. This category includes the general skills *(G)* of learners in occupations. The studies were carried out using the Multivariate Adaptive Regression Splines *(MARS)* and *Conic MARS (CMARS)* methods.

This study introduced models based on Conic Multivariate Adaptive Regression Spline *(CMARS)* as the main achievement; an associated Tikhonov regularisation (TR) (Aster et al., 2018) problem was addressed using conic quadratic programming (CQP) methods (Taylan et al., 2007). The complexity of MARS has been penalised via the so-called TR, which expresses itself and is studied here as a problem of CQP. This results in the development of the novel approach *CMARS* (Weber et al., 2012), which employs continuous, well-structured convex optimisation that enables the application of interior point methods (Roos et al., 1997, 2005) and their codes, e.g., MOSEK (Mosek ApS. A Very Powerful Commercial Software for CQP., n.d.). This allows for the benefit of a more model-based approach founded on mathematical optimisation.

The problem of access to competencies has been present in the European market for a long time. The disparity between the existing competencies among workers and the needs of employers can be attributed to various factors, including the cost of living, transportation costs, and migration patterns (Zierahn, 2013). Recently, this problem has been exacerbated for reasons of digital transformation in the economy, sectoral differentiation related to wage levels, or migration issues (Fischer & Nijkamp, 2021). In many publications, this topic appears, while it does not comprehensively address a systemic solution based on comparing market needs and competency potential based on such large data amounts. Most often, it is related to examining the gap between the offered competencies and skills transferred to students upon graduation from higher education and the requirements of the labour market in a narrow area, e.g., marketing (Stimac & Tanasic, 2023) or in selected areas, e.g., knowledge economy (Muzam, 2023). In addition to selecting individual skill areas, the articles also include an analysis of skill gaps related to the career stage in the career advancement stage (Heimler et al., 2012). Most articles show a skills mismatch problem between the university and the labour market in various directions but rarely use data from the actual labour market (Luo & Teeraporn, 2024). The work also refers to corrective actions, e.g., developing training models that close competency gaps (Alonso-Garca, 2020), rather than actions that precede the creation of gaps, as in the research described in the article.

The article proposes an automated and innovative form of competency modelling, considering data on competency needs among employers and competency profiles of candidates, with a simultaneous proprietary method of reducing competencies to the analysed groups.

In the article, after the introduction, a literature review was conducted, the main purpose of which was to indicate the importance of the competence concept in resource management and to show the problem of competence gaps in the labour market. The research method discusses the development of a mathematical model to analyse and study the relationship of selected categories of competencies among students. Multivariate Adaptive Regression Spline *(MARS)* and conic *MARS (CMARS)* methods were chosen for the study due to the complex number of characteristics analysed. The scope and preparation of the data for the study are also presented. In the results section, the developed model, evaluation criteria, and related features are presented, highlighting the accuracy and stability of the model.

1. Literature review

Most often, the definition of competence includes the concept of "skills", especially in professional skills. Competencies are characteristics of employees that contribute to successful job performance and organisational results. These include knowledge, skills, and abilities, plus their characteristics, such as values, motivation, initiative, and self-control (O'Carroll et al., 2017). Since the term "skills" is used in the term "competence", these terms are often obliterated or applied interchangeably.

The terminological problem is not the only one in competence management. Others include the problem of classifying or grouping competences.

Many glossaries, classifications, and divisions of competences were developed in practice. For instance, in Europe, the European Dictionary of Skills and Competences (DISCO II) of the European Commission (European Dictionary of Skills and Competences (DISCO II), n.d.) is one of the standard glossaries. Other sources list other competence categories such as cross-sectional (Szafrański et al., 2017), key (Szafrański & Goliński, 2015), technological (Astuti et al., 2022), digital (Mattar et al., 2022), instrumental (personal), interpersonal (organisation and communication skills), systemic (transferable skills), behavioural and technical (Palacios-Marqués et al., 2016), personal, professional, and social (Naim & Lenka, 2017) competences.

For many years, scientists have pointed out the necessity of personal development of competences to make it possible for the employee to keep or obtain work (Coll & Zegwaard, 2006). The new economy imposes new requirements closely connected to the acquisition of new competences and, thus, lifelong learning and requalification (The Boston Consulting Group, 2018).

This paper analysed competences divided into general *(G)*, common *(C),* and professional *(P)* categories. This division is very pragmatic, based on the reflection of the actual needs of the labour market. In grouping competences, there is a significant correlation to existing typologies. Some of the similarities include a reference to the separation of, e.g., conceptual (creativity and problem-solving), interactive (communication and negotiation), and technical (making plans and items) competences.

Practice and theory show the dynamic approach to interpreting and using competence divisions. For example, the data-driven method was proposed to automatically extract soft skills from text (Fareri et al., 2021). In another competency analysis, the quantification scheme was used with a mathematical approach (Bohlouli et al., 2017), which can be used to evaluate employees towards optimal job assignment and vocational training or for recruitment processes. Meanwhile, the necessity to support the development of competences both in the area of vocational competences (Lee & Yan, 2019) as well as social competences (Engle et al., 2017) is quite explicit, and the area of research in this issue is still a challenge. Across Europe, employers see skills gaps in the local labour market as a major barrier to achieving business goals (Future of Jobs Report, 2023). Labour shortages and skills gaps in Europe are, among other things, the focus of work by the Directorate-General for Communication, which has identified five important areas for strengthening Europe's labour market potential: supporting underrepresented people to enter the labour market, providing support for skills development, training, and education; improving working conditions; improving fair intra-EU mobility for workers and learners and attracting talent from outside the EU (Tackling labour and skills..., 2024).

Since 2017, the OECD Skills for Jobs database has been in place as a tool for policymakers and practitioners to understand where gaps between skill supply and demand are emerging (OECD Skills for Jobs database, 2025).

The topic is also recognised by The Global Green Skills Report 2023, which identifies trends at the intersection of workforce and sustainability based on the activity of more than 930 million LinkedIn users worldwide (Global Green Skill Report, 2023). Among the many documents related to the labour market, attention can also be drawn to European trends that indicate ongoing challenges to sustainability, as well as opportunities for positive change. New technology can save resources and improve work, but it can also increase the demand for energy and natural resources (Production and consumption, 2024).

More and more companies see the need to get involved in the shaping of soft skills among students at the learning stage. The factors affecting employee competences are of significant importance to the management and human resources; therefore, their mutual dependencies were used to develop the model described in the further part of the article with the use of the *MARS* and *CMARS*.

Today, this type of data is modelled primarily by logistic regression (Cabero-Almenara et al., 2021) using a dichotomous scale. This scale did not match the analysed data, and it was unjustified to recode them into 1–0 variables. Considering the multiplicity of variables and both the nominal data (e.g., the region from which the job applicant comes) and the ordinal data (e.g., the level of acquired skills), methods were selected to build adaptive regression models of many variables. Two modelling methods, namely, *MARS* and *CMARS* algorithms, were selected for further analyses. The advantage of *MARS* and *CMARS* is that they make it easy to interpret the results. *MARS* and *CMARS* methods are applied and compared jointly in the scientific literature only in a few publications (Graczyk-Kucharska et al., 2020, 2022), while *CMARS* is newly introduced in the labour market in this study.

2. Research methods

2.1. Multivariate Adaptive Regression Splines

Multivariate Adaptive Regression Spline *(MARS)* (Friedman, 1991) is a special modelling method which is used for solving regressive and classification problems to find the values of dependent variables based on independent variables (Ju et al., 2021). A special strength that makes *MARS* stand out among other methods is its application, which does not require any special assumptions on the functional connections between dependent and independent variables. It is very useful with large amounts of diverse data (data mining). It establishes a multivariate additive model in a two-stage process: the forward and backward stages (Hastie et al., 2009).

In *MARS*, a denotation is adopted concerning its 1-dimensional piecewise linear BFs (basis functions), which is expressed as follows:

$$c^{+}(x,\tau) = [+(x-\tau]_{+}, c^{-}(x,\tau) = [-(x-\tau)]_{+} \quad (1)$$

with $[q]_{+} := max\{0, q\}$ *a*nd τ being a univariate knot $(x, \tau \in \mathbb{R})$. Therefore, the basic rules of the application of *MARS* can be described as follows:

- The two one-dimensional BFs show piecewise linearity with a knot τ, and together, they are called a reflected pair.
- As a function of vector $X = (X_1, X_2, \dots, X_l)^T$, the output or response variable is represented Y as:

$$Y = f(X) + \varepsilon, \quad (2)$$

where the vector X demonstrates each input variable or predictor X_v, and the stochastic component ε is the noise term, which is supposed to have 0 mean and a finite variance.

- The aim is to identify the reflected pairs for all inputs X_v $(v = 1, 2, \dots, l)$ with l-dimensional knots $\tau_\mu = (\tau_{\mu,1}, \tau_{\mu,2}, \dots, \tau_{\mu,l})^T$ and input data $\tilde{x}_\mu = (\tilde{x}_{\mu,1}, \tilde{x}_{\mu,2}, \dots, \tilde{x}_{\mu,l})T$ $(\mu = 1,2, \dots, N)$, where N is the number of observations (Hastie et al., 2009; Özmen & Weber, 2014).
- The purpose is to develop a set of 1-dimensional basis functions consisting of all the inputs X_v $(v = 1, 2, \dots, l)$*:*

$$S := \{[X_v - \tau]_{+}, [\tau - X_v]_{+} \mid \tau \in \{x_{1,v}, x_{2,v}, \dots, x_{N,v}\}, v \in \{1,2, \dots, l\}\}. \quad (3)$$

- Linear combination f (X) is built up based on the set S and takes the form:

$$Y = \beta_0 + \sum_{p=1}^{P} \beta_p T_p\,(X^p) + \varepsilon, \quad (4)$$

including constant term β_0.

- Given that the input values differ from each other, there are *2Np BFs* as follows:

$$T_p(x_\mu^p) := \prod_{k=1}^{K_p} [m_{\kappa_v^p} \cdot (x_{\mu\kappa_v^p} - \tau_{\kappa_v^p})]_{+} \quad (\mu = 1,2, \dots, N) \; (5)$$

Based on Formula (5), the number of truncated mappings, multiplied within the BF, $x_{\alpha\kappa_b^p}$ is the knot associated with $x_{\alpha\kappa_b^p}$ and $m_{\kappa_b^p}$ denotes a selected sign ±1 (Hastie et al., 2009; Özmen et al., 2014).

2.2. Conic Multivariate Adaptive Regression Splines

In *CMARS*, where “C” can also be read as Convex and Continuous, the golden middle between accuracy and stability is sought, where stability can also be interpreted by reduced complexity. This idea of maintaining the reasonable complexity level is delivered in two different ways: (i) by keeping the discretised integral of first- and second-order derivatives of the BFs within some tolerance (under some upper bound), and (ii) by using modern optimisation theory along with a model-based approach for more integrated treatment of the *MARS* forward step and its backward step. Therefore, it makes it closer to applying the power of differential and integral calculus and to future developments in calculus and optimisation theory. The complexity of regression and classification tool *MARS* was penalised via the so-called TR, which expresses itself and is studied here as a problem of CQP. This leads to the new method *CMARS* (Taylan et al., 2007), which is more model-based and applies continuous, well-structured convex optimisation that enables the employing of interior point methods (Roos et al., 1997, 2005) and their codes, e.g., MOSEK (Mosek ApS. A Very Powerful Commercial Software for CQP., n.d.).

In *CMARS*, the backward stepwise algorithm of *MARS* is not used to inquire the model *f(x)* in Eq. (2). More detailed characteristics of the method were widely described in the operational research literature (Taylan et al., 2007; Özmen et al., 2011; Weber et al., 2012; Özmen & Weber, 2014).

2.3. Scope and preparation of data for research

At the start of the modelling, the researchers found a big set of data from 2013–2015 with competence characteristics of nearly 20 000 students of technical secondary schools representing several dozen professions and demand for competences from nearly 1 200 companies described with 20 thousands of skills, and additionally, several dozen diverse variables (including time-based ones), describing the competence profile of the students and job offers published by the business owners.

In the first step (ordering the data for analyses), a data structure was made based on their source of origin, i.e., system.zawodowcy.org. It was necessary to specify which type of data can be used. The purpose was to analyse the possibility of using big datasets and data mining analyses to create mathematical and applicational models supporting decision-making for the availability of human resources to improve the competitiveness of the region. The available categories of data from the system which were reviewed are the following: competence, qualification, enterprise, job offer, user skills, skills on the job offer, type of qualification, skills, and users. After the analyses, data categories were created that apply to employers and jobs, internships, and training programme candidates, as well as an example of the type of data in separate categories. In reality, many more data categories exist in the system due to its extensive functionality. Most items belonging to the data categories are linked to one another with at least one attribute, thus creating a very complex network of relations among them.

It must be emphasised that n is 20 000 skills, and every one of those selected by an individual candidate might have been evaluated on a scale of 1 to 5, where 1 means the lowest level of mastering a given skill, and 5 means the highest. The candidates self-assessed the mastering of their skills during classes supervised by teachers. In the case of employers, they estimated skills required for job positions on the same scale — 1 being the lowest and 5 the highest.

The diversity of the entire dataset proved too extensive relative to the capacities of the MARS-support tool available to the researchers; therefore, in the next step, it was necessary to choose a specific data

Tab. 1. Dependent variable and independent variables selected for the applicative mathematical model of general skills (G)

	VARIABLE	SCALE
Student's total evaluation value of general skills for particular offer:	Y_G	1-5
Student's gender:	X_1	1-2
Student's birthday:	X_2	date
Student's competence profile creation date:	X_3	date
Starting time of job offer publication:	X_4	date
Ending time of job offer publication:	X_5	date
Time of commencement of work indicated in the job offer:	X_6	date
Date of creation job offer:	X_7	date
Part-time or full-time employment indicated in the job offer:	X_8	2-5
Type of employment (contract) indicated in the job offer:		1-3
Number of employment positions indicated in the job offer:	X_{10}	1-60
Shift work indicated in the job offer:	X_{11}	1-2
Job offer nonstationary work:	X_{12}	1-2
Compliance of the location between job offer and candidate:	X_{13}	1-2
Student's total evaluation value of common skills for a particular offer: X_C or	X_{14}	1-5
Student's total evaluation value of professional skills for a particular offer: X_P or	X_{15}	1-5

Tab. 2. Numbers of skills from the dictionary in system.zawodowcy.org indicated at least once in the competence profiles of IT specialists in the job, training period and internship offers. Data for the period 01.01.2013 – 31.12.2015.

SKILL CATEGORIES	NUMBER OF SKILLS INDICATED AT LEAST ONCE BY STUDENTS (IT TECHNICIAN) IN THEIR COMPETENCE PROFILES	NUMBER OF SKILLS INDICATED AT LEAST ONCE IN JOB, TRAINING PERIOD, AND INTERNSHIP OFFERS
G	148	109
C	335	444
P	3 324	2 568

group out of the whole set of 20 321 students and 1 206 employers. The data are chosen as follows due to the vastness of the group of variables named skills, according to the competence key, for a representative group of students, competence profiles in the profession of IT technician, as well as due to representativeness, credibility, and reliability of data, and the possibility of changing the quality description to quantity one with respect to variables $x_1,\ldots,x_{15}$ presented in Table 1.

The choice of the profession is based on two criteria. First is the vast demand for competences in the studied region, and second is the biggest sample representing the dataset in the analysed dataset. In this case, the analysis of the possible uses of that dataset for data mining analyses and the creation of applicative mathematical models is most justified. For further analyses, the researchers chose data connected with the profession of the IT technician, which is represented by 2 173 IT technicians in the database. In their competence profiles prepared with the use of the competence dictionary, the students selected from 3 874 skills at least once. Meanwhile, in the job, training programmes and internship offers posted in the system and employers pointed out 3 133 skills at least once in 619 job offers. Despite narrowing the data to the profession of the IT technician, the number and diversity of skills are still too extensive. Table 2 shows how many skills *G, C,* and *P* are pointed out to at least once by 2 173 IT technicians, and how many skills in each category are pointed out to at least once in the job, training programme and internship offers.

It is also worth pointing out the necessity of preprocessing the input, such as dates, city, type of work, and work time, which are significant for the designed data model. The data were going to be used for *MARS* and *CMARS* analyses; therefore, it was necessary to reduce them and transform them into numerical values.

Consequently, to scientifically clarify the aforementioned needs and goals, the article focuses on two modelling approaches which set up a frame to generate the response data as the forward problem: a general and a reduced (simplified) model. Then, the main modelling follows the inverse problem, formulated by optimisation problems, solved by models, and the estimation results. This approach is a part of the novelty work in this paper.

The thousands of skills attributed to student groups and job offers forced the researchers to group them. The data reduction made it possible to analyse the data in the context of all skills in each of the three categories (*G, C,* and *P*) and each relation: competence profile-R-offer, compliance, and gaps in skills when it comes to the needs of employers stemming from job offers and the possibility of their satisfaction by students learning to be IT technicians, via three values x_P ,x_C, y_G. The analysed independent variable y refers to skills in the category *G*, which means the examined influence of different independent variables on the shaping of the set of *G* category skills of students learning to be IT technicians, including whether the *G* category set of skills is affected by skill sets in the category *C* and *P*. To reduce the number of dependent variables referring to skills, a generalised model was developed, which was then simplified even more to a simplified model for further analyses. Due to the complexity of the model, its presentation was supplemented with examples.

The data source system.zawodowcy.org contains a list of skills (denoted by *k*), which is based on the core curriculum in vocational education, and which are linked with one of the *G, C* or *P* groups.

Student i has a skill (k) that has been assessed , $\propto_i^k$ e.g., G_1 is assessed at 3, G_2 at 3, G_3 at 5, C_1 at 4, C_2 at 4, P_1 at 2, P_2 at 3, and P_3 at 5. In the offer j, one of the entrepreneurs described job position skills that are required from potential employees. Additionally, employers estimated the significance of how much the chosen skills are needed. The generalised model includes all the data specified in Table 3; however, in the simplified model, the level of the chosen skills required by the entrepreneur β_j^k is skipped.

The compilation of data shows information about similarities and dissimilarities between the expectations of entrepreneurs and the possibilities of the students. Regarding each skill k, it is possible to describe those differences in one value/number which results from the multiplication of two factors in the simplified model (6) and three factors in the generalised model (7) (Table 2):

$$\vartheta_j^k \alpha_i^k \tag{6}$$

$$\vartheta_{ij}^k \alpha_i^k \beta_j^k \tag{7}$$

In this case, the model for calculating the data of the response variable Y_G for a given pair (i, j) looks as follows:

$$y_G = \sum_{k=1}^{K} k \vartheta_j^k \alpha_i^k, \tag{8}$$

$$y_G = \sum_{k=1}^{K} k \vartheta_j^k \alpha_i^k \beta_j^k. \tag{9}$$

In the same way, as the value y_G in Eq. (8) for the simplified model and in Eq. (9) for the generalised model was computed, the values x_c and x_p are calculated. In this case, by summing up multiplications in columns 5 and 6 of Table 3 for competences in category *G*, the total evaluation value about *G* of any given pair *(i,j)* is built. This is shown as the sum (y_G) in Table 3. Analogically, the values for skills in groups *C* and *P* are aggregated and denoted as (x_0) and (x_z), respectively (Table 3). The similarities and dissimilarities in the assessed values in terms of y and x follow from our statistical big data existing in the system, through which the relationships are modelled between general *(G)*, common *(C)*, and professional skills *(P)*, how strongly *C* and *P* impact *G*, and vice versa. In the presented example of the job offer, the sum of values calculated for each group of general skills *G*, a response is treated as a dependent or an output variable. It is a sum of values related to other groups of variables that are treated as predictor, independent, or input variables. For each possible student-offer combination *(i, j)*, the triple (y_G, x_C, x_P) was calculated. An example is presented in Tables 3–4. A programming environment was developed for research, consisting of several Matlab and object-oriented codes.

Tab. 3. Compatibility assessment and gaps between skills developed by student i and requirements of offer j for the simplified model and generalised model

SYMBOL OF SKILLS (κ)	STUDENTS' ASSESSMENT α_i^k	LEVELS OF CHOSEN SKILLS WHICH ARE REQUIRED BY THE ENTREPRENEUR β_j^k	SIGNIFICANCE FOR ENTREPRENEUR ϑ_{ij}^k	SIMPLIFIED MODEL $\vartheta_j^k \alpha_i^k$	GENERALISED MODEL $\vartheta_{ij}^k \alpha_i^k \beta_j^k$
G_1	3	4	2	6	24
G_2	3	5	2	6	30
G_3	5	-	-	0	0
G_5	-	2	1	0	0
Sum (y_G)				**12**	**54**
C_1	4	-	-	0	0
C_2	4	-	-	0	0
C_3	-	1	1	0	0
Sum (x_C)				**0**	**0**
P_1	2	3	1	2	6
P_2	3	2	2	6	12
P_3	5	-	-	0	0
Sum (x_P)				**8**	**18**

Tab. 4. Couple (i, j) and sums of values for particular categories of skills

COUPLE (*i, j*)	y_G	x_C	x_P
(1, 1)	12	0	8
(1, 2)	32	12	21
⋮	⋮	⋮	⋮
(126, 56)	12	0	3
⋮	⋮	⋮	⋮
(1650, 186)	125	234	0

3. Research results

In this *MARS-supported* approach, optimally estimated models with a reduced number of BFs and M_{max} are obtained after a forward and a backward step of *MARS* along its software (SPM 2018). Hereby, a notable benefit is derived from the help of the "generalised cross-validation" *(GCV)* given in Eq. (6), and eventually, the optimal model with the best predictive fit is chosen. At first, with the forward stage of *MARS,* the highest degree of interaction (M_{max}) and the number of BFs are assigned as 3 and 38, respectively. Then, through the backward stage of *MARS,* the number of BFs is cut down to 22. Herewith, the complexity of the *MARS* model was strongly reduced while its stability was enhanced. Consequently, after these efforts, at the end of the backward stage, the BFs are represented as follows:

$$
\begin{aligned}
&BF_1 = \max\{0, x_4 - 5281\}, && BF_2 = \max\{0, 5281 - x_4\},\\
&BF_3 = \max\{0, x_6 - 5292\}\cdot BF_1, && BF_4 = \max\{0, 5292 - x_6\}\cdot BF_1,\\
&BF_5 = \max\{0, x_{14} - 0\}\cdot BF_4, && BF_7 = \max\{0, 4 - x_{15}\},\\
&BF_8 = \max\{0, x_{11} - 1\}\cdot BF_7, && BF_{11} = \max\{0, x_6 - 5431\}\cdot BF_8,\\
&BF_{12} = \max\{0, 5431 - x_6\}\cdot BF_8, && BF_{14} = \max\{0, 5296 - x_6\}\cdot BF_7,\\
&BF_{15} = \max\{0, x_6 - 5115\}\cdot BF_2, && BF_{16} = \max\{0, 5115 - x_6\}\cdot BF_2,\\
&BF_{18} = \max\{0, 5752 - x_5\}\cdot BF_7, && BF_{19} = \max\{0, x_6 - 5407\}\cdot BF_8,\\
&BF_{21} = \max\{0, x_{14} - 0\}\cdot BF_2, && BF_{22} = \max\{0, x_4 - 5320\}\cdot BF_8,\\
&BF_{23} = \max\{0, 5320 - x_4\}\cdot BF_8, && BF_{24} = \max\{0, x_4 - 5269\}\cdot BF_8,\\
&BF_{26} = \max\{0, x_4 - 5395\}\cdot BF_8, && BF_{28} = \max\{0, x_6 - 5176\}\cdot BF_7,\\
&BF_{30} = \max\{0, x_4 - 5395\}\cdot BF_{28}, && BF_{34} = \max\{0, x_4 - 5377\}\cdot BF_{28},\\
&BF_{36} = \max\{0, x_4 - 5403\}\cdot BF_{28}, && BF_{38} = \max\{0, x_6 - 5187\}\cdot BF_8.
\end{aligned}
\tag{10}
$$

The optimal *MARS* model with the BFs above is presented in the subsequent form:

$$
\begin{aligned}
\hat{Y} = \beta_0 + \sum_{m=1}^{M} \beta_m T_m(\boldsymbol{X}^m) &= -5.076 + 0.0017\cdot BF_3 + 0.0056\cdot BF_4 + 0.00014\cdot BF_5\\
&+ 8.917\cdot BF_7 + 43.130\cdot BF_8 - 3.086\cdot BF_{11} - 0.230\cdot BF_{12} - 0.0405\cdot BF_{14} + 0.0015\cdot BF_{15}\\
&+ 0.012\cdot BF_{16} + 0.010\cdot BF_{18} + 2.363\cdot BF_{19} + 0.005\cdot BF_{21} - 2.051\cdot BF_{22} + 0.152\cdot BF_{23}\\
&+ 1.025\cdot BF_{24} + 1.475\cdot BF_{26} + 0.055\cdot BF_{28} - 0.031\cdot BF_{30} + 0.0094\cdot BF_{34} + 0.021\cdot BF_{36}\\
&- 0.235\cdot BF_{38}.
\end{aligned}
\tag{11}
$$

Random variables are congruent with the description in Table 2. For the *CMARS* model, the offer visible from (x_4), the job offer visible to (x_5), the offer date of work (x_6), the offer of shift work (x_{11}), a student's total evaluation value of common skills for the particular offer X_C (x_{14}), and a student's total evaluation value of professional skills for the particular offer x_p (x_{15}) are significant. Here, all of the parameter values estimated for the *MARS* model are represented in Table 2. In consequence, six inputs have effects on the response variable among 15 variables within our *MARS* model. Here, three of these six inputs are regarded as time. Thus, it means that three of our six time-variables (Table 2) are included in the *MARS* model. This proves that this model has a high time dependence.

In the *CMARS*, which is optimisation-supported, estimated models with the maximum number of *BFs*, M_{max}, are constructed after a forward step of *MARS* along its software (SPM, 2018). Hereby, a notable benefit is derived from the help of the *PRSS* and *CQP* given in Eq. (8) and Eq. (9), and eventually, the optimal model with the best predictive fit is chosen. At first, with the forward stage of *MARS*, the highest degree of interaction (M_{max}) and the number of BFs are assigned as three and 38, respectively. The complexity of *MARS* is penalised via the so-called TR, which expresses itself and is studied here as a problem of *CQP*. Herewith, the complexity of the *MARS* model was reduced while its stability was enhanced. Consequently, after these efforts, at the end of the forward stage, the BFs are represented as follows:

$$
\begin{aligned}
&BF_1 = \max\{0, x_4 - 5281\}, && BF_2 = \max\{0, 5281 - x_4\},\\
&BF_3 = \max\{0, x_6 - 5292\}\cdot BF_1, && BF_4 = \max\{0, 5292 - x_6\}\cdot BF_1,\\
&BF_5 = \max\{0, x_{14} - 0\}\cdot BF_4, && BF_6 = \max\{0, x_{15} - 4\},\\
&BF_7 = \max\{0, 4 - x_{15}\}, && BF_8 = \max\{0, x_{11} - 1\}\cdot BF_7,\\
&BF_9 = \max\{0, x_5 - 5715\}\cdot BF_8, && BF_{10} = \max\{0, 5715 - x_5\}\cdot BF_8,\\
&BF_{11} = \max\{0, x_6 - 5431\}\cdot BF_8, && BF_{12} = \max\{0, 5431 - x_6\}\cdot BF_8,\\
&BF_{13} = \max\{0, x_6 - 5296\}\cdot BF_7, && BF_{14} = \max\{0, 5296 - x_6\}\cdot BF_7,\\
&BF_{15} = \max\{0, x_6 - 5115\}\cdot BF_2, && BF_{16} = \max\{0, 5115 - x_6\}\cdot BF_2,\\
&BF_{17} = \max\{0, x_5 - 5725\}\cdot BF_7, && BF_{18} = \max\{0, 5752 - x_5\}\cdot BF_7,\\
&BF_{19} = \max\{0, x_6 - 5407\}\cdot BF_8, && BF_{20} = \max\{0, x_7 - 5407\}\cdot BF_8,\\
&BF_{21} = \max\{0, x_{14} - 0\}\cdot BF_2, && BF_{22} = \max\{0, x_4 - 5320\}\cdot BF_8,\\
&BF_{23} = \max\{0, 5320 - x_4\}\cdot BF_8, && BF_{24} = \max\{0, x_4 - 5269\}\cdot BF_8,\\
&BF_{25} = \max\{0, 5269 - x_4\}\cdot BF_8, && BF_{26} = \max\{0, x_4 - 5395\}\cdot BF_8,\\
&BF_{27} = \max\{0, 5395 - x_4\}\cdot BF_8, && BF_{28} = \max\{0, x_6 - 5176\}\cdot BF_7,\\
&BF_{29} = \max\{0, 5176 - x_6\}\cdot BF_7, && BF_{30} = \max\{0, x_4 - 5395\}\cdot BF_{28},\\
&BF_{31} = \max\{0, 5395 - x_4\}\cdot BF_{28}, && BF_{32} = \max\{0, x_4 - 5408\}\cdot BF_{28},\\
&BF_{33} = \max\{0, 5408 - x_4\}\cdot BF_{28}, && BF_{34} = \max\{0, x_4 - 5377\}\cdot BF_{28},\\
&BF_{35} = \max\{0, 5377 - x_4\}\cdot BF_{28}, && BF_{36} = \max\{0, x_4 - 5403\}\cdot BF_{28},\\
&BF_{37} = \max\{0, 5403 - x_4\}\cdot BF_{28}, && BF_{38} = \max\{0, x_6 - 5187\}\cdot BF_{28},\\
&BF_{39} = \max\{0, 5187 - x_6\}\cdot BF_{28}, && BF_{40} = \max\{0, x_5 - 5271\}\cdot BF_{28}.
\end{aligned}
\tag{12}
$$

The optimal CMARS model with the BFs above is presented in the subsequent form:

$$
\begin{aligned}
\hat{Y} = \beta_0 + \sum_{m=1}^{M_{\max}} \beta_m T_m(\boldsymbol{X}^m) = &-5.7471 - 0.1029\cdot BF_2 - 0.0203\cdot BF_3 + 0.0020\cdot BF_4 + 0.0063\cdot BF_5 + 0.00014\cdot BF_6\\
&+ 0.0102\cdot BF_7 + 0.0015\cdot BF_8 + 0.0024\cdot BF_9 - 0.0071\cdot BF_{10} - 0.0034\cdot BF_{11} - 2.1519\cdot BF_{12} - 1.6404\cdot BF_{13}\\
&- 0.0571\cdot BF_{14} + 0.0360\cdot BF_{15} + 0.0018\cdot BF_{16} + 0.0011\cdot BF_{17} + 0.014\cdot BF_{18} + 1.099\cdot BF_{19} + 1.553\cdot BF_{20}\\
&+ 0.005\cdot BF_{21} - 0.987\cdot BF_{22} - 1.138\cdot BF_{23} + 0.704\cdot BF_{24} + 0.431\cdot BF_{25} + 0.849\cdot BF_{26} + 0.878\cdot BF_{27}\\
&+ 0.00005\cdot BF_{28} - 0.086\cdot BF_{29} - 0.016\cdot BF_{30} - 0.016\cdot BF_{31} - 0.0019\cdot BF_{32} - 0.0011\cdot BF_{33} + 0.005\cdot BF_{34}\\
&+ 0.004\cdot BF_{35} + 0.0127\cdot BF_{36} - 0.0131\cdot BF_{37} - 0.0858\cdot BF_{38} - 0.1595\cdot BF_{39} + 0.000006\cdot BF_{40}.
\end{aligned}
\tag{13}
$$

Random variables for our *CMARS* model are congruent with the description in Table 2, the same as for our *MARS* model. In consequence, six inputs have effects on the response variable among 15 variables within the *CMARS* model. Thus, it means that three of our six time-variables (Table 2) are included in the *CMARS* model like in the MARS model.

Some of the most essential measures of statistical performance assessment and statistical comparison were used, as represented in Table 5.

Herewith, Average Absolute Error (AAE), Root Mean Square Error (RMSE), Multiple Coefficient of Determination (Adjusted R^2), and Correlation Coefficient *(r)* were addressed as the measurements of the accuracy to measure the predictive capability of each model. Therefore, the accuracy criteria show which model has a superior predictive ability to others and which is the best.

Here, adjusted R^2 likewise R^2 reflects the amount of data falling within the line or the regression curve. However, while R^2 presumes that every regarded variable explains the variation in the response variable, the adjusted R^2 provides the percentage of variation explained by those independent variables that affect the response variable. Moreover, adjusted R^2 causes a penalisation for adding independent variables which do not fit the model. Herewith, it suits this study with its use of generalised cross-validation or, in terms of optimisation theory, Tikhonov regularisation.

As far as the closeness to linearity is concerned, *MARS* models belong to the “nearest” elements on the portfolio of model classes since, eventually, in any single dimension or variable, *MARS* is based on piecewise linear functions. Nonlinearity only enters into the *MARS* model through the multiplication of those 1-dimensional basis functions, which has to serve for a more accurate and sufficiently stable data fitting. Hereby, the number of multiplications in *MARS* can be controlled by a (hyper-) parameter, which is the "maximal degree of interaction".

The measure r is the standardised regression coefficient beta of one variable by another (and vice versa), and it assesses the size of the mutual effect. This is rather clearly observed whenever the variables are dichotomous. In fact, r is very helpful when a priori investigating multi-dimensional data by pairs of variables and scatterplots of the (projected) data and figuring out some approximate linear dependencies. These scatterplots can be a modelling help for *MARS* by identifying areas ("regimes") of linear dependence and, overall, understanding the underlying piecewise structure of the *MARS* model. All of this can become a part of a GUI (Graphical User Interface), which aims to make it convenient for users by the end of the further study.

Tab. 5. Performance criteria and related measures

ABBREVIATION	MEASURE	EXPLANATION	FORMULA
R^2_{adj}	Multiple Coefficient of Determination	Percentage of variation in response explained by the model	$R^2_{adj} := 1 - \left(\frac{\sum_{k=1}^{N} (y_k - \hat{y}_k)^2}{\sum_{k=1}^{N} (y_k - \bar{y}_k)^2} \right) \times \left(\frac{N-1}{N-p-1} \right)$
AAE	Average Absolute Error	The average magnitude of errors	$AAE := \frac{1}{N} \sum_{j=1}^{N} \left\| y_j - \hat{y}_j \right\|$
RMSE	Root Mean Square Error	The square root of the mean squared error	$RMSE := \sqrt{\frac{1}{n} \sum_{j=1}^{n} (y_j - \hat{y}_j)^2}$
r	Correlation coefficient	The linear relation between the observed and predicted response	$r := \frac{\frac{\sum_{j=1}^{n} (y - \bar{y})(\hat{y} - \bar{\hat{y}})}{(n-1)}}{\sqrt{s(y)^2 s(\bar{y})^2}}$

Tab. 6. Accuracy performance criteria of MARS and CMARS models

	CMARS		MARS	
	TRAIN	TEST	TRAIN	TEST
R^2_{adj}	0.573	0.389	0.566	0.398
AAE	15.02	18.26	15.309	18.537
RMSE	25.4	35.87	25.742	36.071
r	0.764	0.655	0.757	0.650

Tab. 7. Accuracy performance criteria of MARS and CMARS models

	CMARS	MARS
R^2_{adj}	0.68	0.704
AAE	0.823	0.826
RMSE	0.708	0.714
r	0.857	0.858

Notes y_j: *jth* actual response value; $\hat{y}_j$: *jth* predicted (fitted) response value; $\bar{y}$: mean of the actual values; $\bar{\hat{y}}$: predicted response variable; $\bar{\hat{y}}$: mean of the predicted response variable; $s(y)^2$: standard deviation of the actual response variable; $s(\hat{y})^2$: standard deviation of the predicted response variable, n: number of observations; p: number of terms in the model. In any model, smaller values of for AAE and RMSE, close to 0, give better results, whereas for R^2_{adj} and r values closer to 1 are preferred.

The purpose is to achieve a potential predictive model by using the training dataset; the test dataset is also implied to measure the performance of the model (Hyndman & Koehler, 2006). For the intended and sought prediction of the general skills *(G),* the *MARS* and *CMARS* models, identified in the previous sections, were evaluated by applying the performance criteria formalised according to Table 6. That table reflects the matrices of performances in the training case and the testing case of the *MARS* and *CMARS* models obtained for the assessment or forecast of general skills *(G)*. Throughout this investigation, the authors prefer a rigorous approach from mathematics and statistics towards performance criteria or measures which is applied to the models. Hence, the scientific toolbox contains a truly operational, OR and optimisation-supported way to assess and quantify the uncertainty. The latter is given in the data and becomes translated into the models attained and rigorously compared by the authors. What is more, this study paid particular attention to the model aims and purposes of "accuracy" and "stability". In diverse and overall rich ways, these goals enter into our performance or comparison criteria and measures.

The values listed in Table 7 are compared with the stability performance for each comparison criterion on both train and test datasets. When the values are closer to one, they indicate a more stable model. Stable methods perform equally well on both training and test datasets.

4. Discussion of the results

The problem of competences in the context of the labour market is often addressed in the literature; however, not very often on a big sample of several thousand competence profiles and with statistical methods, including data mining such as *MARS* or *CMARS*. This pioneering research analysis presents a general skills G model for a group of students learning to be IT technicians in the Wielkopolska province. *MARS* and *CMARS* models were developed based on 2 173 competence profiles indicating 3 874 skills and 619 job offers, in which 3 133 skills were selected. The results were analysed for the influence of the dependent variables collected between 2012 and 2015 in the IT tool www.system.zawodowcy.org.

Human intelligence is an outcome of optimising the balance between robustification (stability), which can also be called routinisation, standardisation and others, and attentiveness to the important details in the observations. It is a standard for optimising the balance between socialisation and individualisation. Consequently, the article focused on two modelling approaches that set up a frame and delivered the response data: a general model and a reduced or simplified model. This step may be called the "forward problem". This helps to simplify large numbers of data (mostly connected with thousands of numbers of various skills) to use them for *MARS* and *CMARS* statistical analysis. The pre-processing model (simplified model), understood as a forward problem in this paper, was developed and implemented on a research sample during the analyses. The core part of the study consisted of the modelling work — the "inverse problem" — which gave the estimation results: *MARS* and *CMARS* models. During the analysis, the group of general skills (y_G) was chosen as the independent variable. After 1000s of variables in the form of diverse skills describing competence models of the candidates and the demand for competences among employers were reduced with the simplified model, 15 variables were selected. They included the aggregated group of skills *G, C* and *P*, gender, age of the candidate, date of creation of the candidate profile, offer, offer visibility time, type of work (shift-based, permanent, etc.), shifts, the proximity of the job offer to the candidate.

The strengths of the article are the research methods used, the exceptionally large amount of data on the supply and demand of competencies, and the fact that the data is up-to-date and detailed, allowing the analysis in many occupational areas. This undoubtedly fills the research gap that allows many areas of competence, rather than being limited to selected professions and small group sizes (Stimac & Tanasic, 2023). The inference that was carried out in the article allows the method to be applied systemically and to refer to a large scale. Valuable studies of competency gaps proposed in the literature are sometimes subject to the error of objectivity, referring, for example, to a group of experts (Muzam, 2023). Another value of the conducted research and the developed research model is the possibility of ongoing monitoring of the level of competencies in the educational market before graduates appear in the labour market. The method developed is preventive in nature, characterised by higher efficiency than corrective actions occurring at the stage of professional work (Heimler et al., 2012; Alonso-Garca et al., 2020; Jędrzejczyk 2013).

In this study on the inverse problem, a mathematical and statistical approach was pursued towards performance and comparison criteria, which were applied to the models. This approach is viewed as closely connected with the real-world application, featuring human factors and interpreted accordingly. Within that approach, operational research was presented to represent, quantify and model uncertainty coming from a real-life context. This was described in terms of the given and pre-processed data and further explained, evaluated and discussed by the models. Herewith, a rigorous and meaningful comparison and assessment of the models was made. The models were designed and compiled based on *MARS* and *CMARS*, respectively. This work paid more attention to the modelling goals of "accuracy" and "stability" and to their trade-offs.

Given the developed *MARS* and *CMARS* models, it can be concluded that the demand for general skills *G:*

- is not affected by factors such as gender, age, date of creation of the profile in www.system.zawodowcy.org, date of creation of the offer in the system, type of employment, working hours, number of job positions for the published offer, stationarity of the work, and proximity of work;
- the model is affected by dependent variables such as time of visibility of the offer (from-to), work commencement time, whether the job is shift-based, and accelerated index for skill groups C and P.

Conclusions

The research conclusions include the fact that the duration of visibility and publication of the offer might be informative for job candidates and thus be of significant influence on the development of competences in the region. Another group of significant variables includes an accelerated indicator for skill groups common for given professions and professional skills. The studies confirmed the significant dependence of all skill groups on the demand for competences among employers. Stationarity and proximity of work do not matter since competence shortages are then covered by other candidates who know the employer's expectations or develop competences in the nearby schools.

Further works connected with data and competence models on the labour market concern applying *MARS* and *CMARS*, and further data mining methods: RMARS (Robust *MARS*) (Özmen & Weber, 2014), *RCMARS* (Özmen et al., 2011), *GPLMs, CGPLMs* and *RCGPLMs* (Hastie et al., 2009; Özmen et al., 2013), to represent and understand P, S and *G,* comparing all models and systems by statistical performance criteria, error diagrams, and sensitivity analyses.

The important parts of future research studies are the representation of our challenges from human resource management and intelligent networking in terms of Artificial Intelligence. This is a pioneering contribution by new powerful methods from mathematics, statistics, natural sciences, and neutral computing, carefully evaluated for the problems studied. It is worth pointing out that the time factor is used very rarely in mathematical models. In the developed model, the time of the model is an independent variable as seasons may influence the number of job offers and the need for competences in the labour market. Time is thus important in the model. For the sake of brevity of the paper, the authors could neither work out the further rich details and facets of the approach nor state all of its interpretations and opportunities.

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