DOI: 10.2478/eces-2024-0033 ECOL CHEM ENG S. 2024;31(4):507-525

Honghu GAO[1,3], Jialin Karen TANG[2], Daqing GONG[3*], Xiaoxia ZHAO[3,4] and Xiaojie YAN[3]

OPTIMISING LOW CARBON INVENTORY DECISIONS AND COORDINATION FOR USED CLOTHING UNDER CARBON TAX AND CARBON LABELLING SYSTEM

Abstract: This paper investigates the trade-off between carbon emissions and profitability in used clothing inventory management under carbon labelling and taxation policies. The study analyses various decision models within a two-tier supply chain comprising a single supplier and retailer, deriving optimal solutions for inventory carbon emission levels, sales prices, wholesale prices, and profit maximisation for each supply chain participant. Our findings reveal several key patterns: For environmentally conscious suppliers, both centralised and decentralised decision-making models show that increasing carbon tax rates lead to decreased inventory carbon emission levels of used clothing. When carbon tax remains constant, heightened consumer awareness results in reduced inventory carbon emission levels. For pollution-intensive suppliers under decentralised decision-making, the inventory carbon emission levels follow a U-shaped trend. Similarly, when carbon tax is fixed, increasing consumer awareness of carbon labels also produces a U-shaped trend in suppliers' inventory carbon emission levels. Under centralised decision-making, higher carbon tax rates consistently lead to decreased inventory carbon emission levels. Additionally, stronger consumer preferences for low-carbon labels result in reduced used clothing inventory carbon emission levels. When the baseline of low carbon levels is identical, carbon tax and carbon regulation intensity demonstrate similar effects on inventory carbon emission levels, demand, supply chain profits, and coordination mechanisms, while exhibiting opposite effects on sales prices. The study achieves supply chain coordination through an optimal wholesale price contract based on Nash bargaining, with numerical analysis validating the proposed model.

Keywords: carbon tax, carbon labelling system, low-carbon inventory, used clothing, wholesale price contract

Introduction

As a global leader in textile production, China processes over 50 % of the world's textile fibers, generating substantial annual textile waste. The environmental impact is significant: each kilogram of textile waste effectively recycled reduces carbon emissions by 3.6 kg. Countries including China, the United Kingdom, and Japan have implemented textile recycling programs to promote sustainability in the garment industry and reduce carbon emissions [1-3]. Resale represents the most common method of recycled clothing utilisation [4]. While recycling and reselling used clothing reduces manufacturing-related

[1] School of Management Engineering, Zhengzhou University of Aeronautics, Henan Province, China, email: hhgao2020@zua.edu.cn, ORCID: 0009-0000-8061-822X
[2] Class 2025, New Canaan High School, New Canaan, CT, 06840, USA, email: jialinkaren.tang@ncps-k12.org
[3] International Center for Informatics Research, Beijing Jiaotong University, Beijing, China, email: dqgong@bjtu.edu.cn, Xjyan@bjtu.edu.cn
[4] Shijiazhuang Posts and Telecommunications Technical College, China, email: zhaoxx@sjzpc.edu.cn
[*] Corresponding author: dqgong@bjtu.edu.cn

carbon emissions, the environmental impact of inventory management remains significant [5]. Effective inventory emission control in used clothing resale can facilitate closed-loop, low-carbon management throughout the garment supply chain. Globally, nations have implemented various carbon reduction initiatives. China's 2015 "Enhanced Actions on Climate Change-China's Nationally Determined Contribution" plan exemplifies this trend, targeting a 60 % - 65 % reduction in carbon emissions per GDP unit by 2030 compared to 2005 levels. Carbon taxation has emerged as an effective emissions reduction strategy [6], with countries such as Canada, Sweden, and South Africa already implementing such policies. Concurrent with policy developments, consumer environmental consciousness has grown, increasing willingness to pay premium prices for eco-friendly products. Carbon labelling systems enable consumers to assess products' environmental impact by indicating lifecycle carbon emissions [7]. The United Kingdom pioneered carbon labelling in 2007, with retailers like Tesco and Marks & Spencer displaying carbon emissions on everyday products. For example, potato chip packaging might display "CO_2 75 g." Following the UK's example, the United States and Canada have adopted similar labelling systems. China introduced the "Interim Measures for the Administration of Low-Carbon Product Certification" in 2013, mandating carbon labels on relevant products to inform consumer choices. Research indicates that carbon labelling significantly influences consumer behaviour, with many consumers willing to pay premiums for carbon-labeled products [8]. This confluence of carbon taxation and labelling systems holds particular relevance for used clothing resale operations and management, offering both challenges and opportunities for sustainable business practices.

We examine the low-carbon inventory decision-making and coordination in used clothing operations under carbon tax and labelling systems. The relevant literature encompasses three main areas:

First, low-carbon operations management has emerged as a significant focus of scholarly research [9-12]. Zhao [13] analyse the impact of emissions trading on environmental responsibility. Xu et al. [14] employed evolutionary game theory to investigate how fairness concerns impact resource-sharing decisions. Feng et al. [15] examined multi-supplier, multi-cycle low-carbon management under carbon emission contracts within cap-and-trade schemes. Sebatjane [16] developed a sustainable production-inventory integration model for three-tier supply chains under various carbon emission regulations, addressing both inventory replenishment and circular economy indicators. The same author later provided a comprehensive review of sustainable inventory models under carbon emission regulations [17]. Wang et al. [18] constructed a theoretical model examining how carbon trading policy optimises safety stock in multilevel supply chains, simulating the impact of carbon trading parameters and prices on emissions and safety stock levels.

Second, research has explored consumer purchasing behaviour and corporate decision-making under carbon labelling systems. Empirical study of Young et al. [19] of UK consumers revealed increased willingness to pay premium prices for environmentally friendly products following government-mandated carbon labelling. Using system dynamics methodology, Zhao and Zhong [20] analysed purchasing preferences for carbon-labelled versus non-labelled milk across different consumer segments, identifying various factors influencing purchasing habits. Fan et al. [21] investigated manufacturers' carbon labelling criteria development under incomplete consumer knowledge. Fischer and Lyon [22] compared environmental effects of NGO versus industry carbon labelling standards in perfect competition models, expanding analysis from binary to multi-layer labelling approaches. Jiang et al. [23] developed an evolutionary game model examining interactions among government regulators, carbon-labelling enterprises, and consumers, analysing green research and development effort levels, government oversight, and consumer purchasing preferences. Yang and Lin's [24] dynamic analysis of consumer purchase intentions under carbon labels (2019-2024) identified textiles and electronic products as optimal pilot products for consumer acceptance. Liu et al. [25] demonstrated carbon labelling's effectiveness in reducing cold chain inventory emissions and optimising supply chain node inventory.

Third, literature addresses low-carbon supply chain coordination. Wang et al. [26] examined decision-making and coordination in low-carbon supply chains considering retailer altruism, proposing cost-sharing contract-based Pareto improvements to benefit all members. Gao et al. [27] investigated supply chain coordination under carbon tax. Wang et al. [28] demonstrated how warehousing and option contracts can enhance supply chain emission reduction efficiency under consumer low-carbon preferences, achieving supply chain coordination. Pu et al. [29] applied game theory to analyse emission reduction in automobile supply chains under double-integral policy, developing a coordination model incorporating vertical spillover effects and Nash bargaining. Zhou et al. [30] examined low-carbon technology selection, investment, pricing decisions, and coordination in supply chains under carbon tax policies.

In summary, existing literature has made notable contributions to understanding low-carbon inventory management and emission reduction strategies within the frameworks of carbon tax and carbon labelling systems. However, there is limited research addressing low-carbon inventory management in the context of clothing recycling and resale. Furthermore, few studies consider the impact of low-carbon inventory practices on consumer purchasing behaviour. Similarly, research on supply chain coordination under carbon tax and carbon labelling systems remains scarce. We focus on balancing carbon emissions and profitability within a two-tier supply chain for used clothing-an area that is relatively underexplored. To address this gap, we examine the combined effects of carbon tax and carbon labelling systems on corporate emission reduction and operational decision-making. We investigate how these systems influence inventory carbon emission level, sales price, consumer demand, and supply chain profitability. Additionally, we explore strategies for rational profit distribution in the presence of double marginalisation within the supply chain, enabling suppliers and retailers to collaboratively reduce carbon emissions associated with used clothing inventory.

Methods

This paper examines the carbon tax and carbon labelling systems within a two-tier used garment supply chain structure, comprising one supplier and retailer. It analyses supply chain decision-making models under both decentralised and centralised decision-making frameworks. In the centralised decision-making framework, the supplier and retailer operate as a unified entity, jointly aiming to maximise overall supply chain profits. Together, they determine the inventory carbon emission level and the sales price. In contrast, under the decentralised decision-making framework, the supplier independently decides the inventory carbon emission level and sets the wholesale price, while the retailer determines the optimal sales price based on the supplier's wholesale price. In this scenario, both parties seek to maximise their individual profits. This paper explores the optimal decision-making strategies for the supply chain under these two frameworks, considering the dynamic effects of changes in carbon tax and carbon labelling. The notations and descriptions used in this study are presented in Table 1.

Table 1

Notations and descriptions

Notations	Descriptions
e	The inventory carbon emission level of used clothing
e_0	The baseline of low carbon level of used clothing inventory per unit
t	Per tonne (tonne = t =10^6 g = Mg) of CO_2/yuan
w	The supplier's wholesale price of used clothing
c	Inventory cost of per unit used clothing
a	The basic market demand
p	The sales price of the retailer
η	The consumers' sensitivity to carbon emissions
Q	The basic market demand of the used clothing
$c(e)$	The emission reduction cost of the supplier
k	The supplier's emission reduction cost coefficient
π_m	The profit of the supplier
π_r	The profit of the retailer
π_{SC}	The profit of the supply chain
D	The decentralised decision-making mode
C	The centralised decision-making mode

Assumption 1: Because companies differ in environmental properties, the government sets a baseline for supplier, that is, the supplier's minimum low carbon standard of used clothing per unit should be e_0. The government imposes a carbon tax on the supplier's CO_2 emissions at a rate of t [yuan/t] of CO_2.

Assumption 2: The supplier's wholesale price per unit used clothing and inventory cost are denoted as w and c, respectively, where the wholesale price is the supplier's decision variable.

Assumption 3: The market demand function is given by $Q = a - bp - \eta e\ (a, b > 0)$, where a represents the basic market demand scale and b denotes the price sensitivity coefficient of market demand. To simplify calculations, in this study, $b = 1$ is assumed, that does not affect the results. The coefficient η indicates consumers' sensitivity to carbon emissions, where p is the retailer's decision variable, and it is assumed that $p > w > c > 0$.

Assumption 4: To increase profits under the constraints of the carbon tax and carbon labelling systems, the supplier adopts environmentally friendly storage technologies to reduce CO_2 emissions per unit product during the inventory process. After investing in carbon emission reduction, the carbon emission level of used clothing inventory per unit product becomes e, where $0 \leq e \leq e_0$, and e is the supplier's decision variable. Referring to Gao et al. [27], the supplier's emission reduction cost after making investments is represented as $c(e) = k(e_0 - e)^2$, where k is the supplier's emission reduction cost coefficient. The function $c(e)$ satisfies $c'(e) < 0$ and $c''(e) < 0$, indicating that in the actual storage process, as the inventory carbon emission level of used clothing e decreases, the marginal cost increases. This implies that emission reduction is easier initially but becomes more challenging later.

Other notations: let π_m, π_r and π_{SC} denote the profits of the supplier, the retailer, and the supply chain, respectively. Superscript $*$ indicates the optimal solution, superscript D represents the decentralised decision-making mode, and superscript C indicates the centralised decision-making mode.

Decentralised decision-making in the supply chain

In the decentralised decision-making mode (as shown in Figure 1), the supplier determines the inventory carbon emission level of used clothing e based on its profits and costs. At the same time, the supplier also decides the used clothing's wholesale price w based on its profit and cost considerations. The retailer, after accepting the wholesale price set by the supplier, determines the final sales price of the used clothing. This interaction forms a Stackelberg game, where the supplier is the leader and the retailer is the follower. According to the above assumptions, the retailer's profit function is given by:

$$\pi_r = (p - w)(a - bp - \eta e) \tag{1}$$

Assume that the government imposes a carbon tax at a rate of t yuan per ton of CO_2 emissions. Therefore, the supplier's marginal profit from storing used clothing and selling it to the retailer is $w - c - te$, where te is the tax amount per unit product paid by the supplier. The supplier's profit function can then be expressed as:

$$\pi_m = (w - c - te)(a - bp - \eta e) - k(e_0 - e)^2 \tag{2}$$

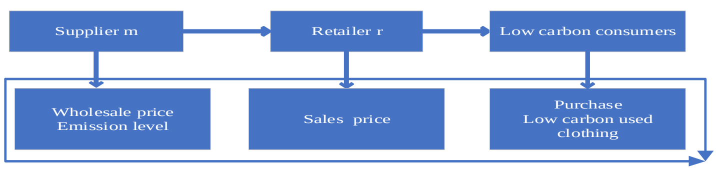

Fig. 1. Decentralised decision-making in the supply chain

Here, the first part on the right side of the equation represents the supplier's revenue from selling used clothing to the retailer, while the second part denotes the cost incurred by reducing the used clothing inventory emissions. The second derivative of the retailer's profit function with respect to p from equation (1) is $\partial^2\pi_r/\partial p^2 = -2 < 0$. Therefore, the retailer's profit function is strictly concave with respect to the optimal sales price p, ensuring the existence of an optimal sales price p that maximises the retailer's profit. Taking the first derivative of π_r with respect to p and setting it to zero yields $p = (a + w - \eta e)$. Substituting this value into equation (2), the supplier's profit function becomes:

$$\pi_m = (w - c - te)(a + w - \eta e) - k(e_0 - e)^2 \tag{3}$$

It is known that the optimal solution for π_m exists. Setting $\partial\pi_m/\partial w = 0$, we obtain $w = [a + c + (t - \eta)e]/2$. Substituting w into equation (3), the supplier's profit optimisation problem becomes:

$$\pi_m = [a - c + (t + \eta)e]^2/8 - k(e_0 - e)^2, s.t. 0 \le e \le e_0 \tag{4}$$

If $0 \le t \le t_1$, since $\partial^2\pi_m/\partial e^2 = (t + \eta)^2/4 - 2k$, when $\partial^2\pi_m/\partial e^2 < 0$, $\pi_m(e)$ is a concave function with respect to e. At this point, setting $\partial\pi_m/\partial e = 0$, we obtain:

$$e^{D*} = \frac{8k\boldsymbol{e_0} - (a - \boldsymbol{c})(t + \eta)}{8k - k(t + \eta)^2} \tag{5}$$

Then, we know that if $e^{D*} \geq 0 \Leftrightarrow e_0 \geq E_1$ and $e^{D*} \leq e_0 \Leftrightarrow e_0 \leq E_2$, thus satisfying $0 \leq t \leq t_1$, which implies $E_1 < E_2$, the maximum profit point for $\pi_m(e)$ is e_0. Thus, if this condition is not met, there is no optimal solution. If $E_1 < e_0 < E_2$, the optimal solution is e^{D*}. If $e_0 < E_1$, the optimal solution is $e^* = 0$. If $t > t_1$, $\partial^2\pi_m/\partial e^2 = (t+\eta)^2/4 - 2k > 0$, indicating that $e = e^{D*}$ is the minimum value point for $\pi_m(e)$. If $e = 0$ or $e = e_0$, $\pi_m(e)$ achieves its maximum value, thus, $\pi_m(0) - \pi_m(e_0) = [2(a-c)(t+\eta) - 8ke_0 - (t+\eta)^2 e_0]e_0/8$. If $\pi_m(0) - \pi_m(e_0) \geq 0$, $e^* = 0$. is the maximum point. If $\pi_m(0) > 0$ and $\pi_m(e_0) > 0$, it can be determined that $e_0 \leq E_3 = (a-c)/2\sqrt{2k}$. Additionally, since $E_3 - E_2 > 0 \Rightarrow E_3 > E_2$. If $\pi_m(0) - \pi_m(e_0) \leq 0$, the maximum point is $e^* = e_0$. At this point, if $E_2 \geq e_0 \geq E_3$ is not satisfied, then an optimal solution does not exist.

Where $E_1 = (a-c)(t+\eta)/8k$, $E_2 = (a-c)/(t+\eta)$, $e^{D*} = [8ke_0 - (a-c)(t+\eta)]/[8k - (t+\eta)^2]$. The decision regions for the supplier under the decentralised decision-making mode are shown in Figure 2.

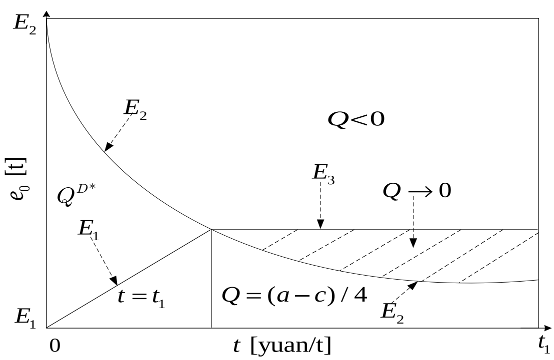

Fig. 2. Decision regions under the decentralised mode

Under the decentralised decision-making mode in the supply chain, if $0 \leq t \leq t_1$ and $E_1 < e_0 < E_2$, that is $e^* = e^{D*}$, the optimal decision for the supplier and retailer is $(e^{D*}, w^{D*}p^{D*})$, with market demand Q^{D*}. The profits of the supplier and retailer are π_m^{D*} and π_r^{D*} respectively. Where:

$$e^{D*} = \frac{8ke_0 - (a-c)(t+\eta)}{8k - (t+\eta)^2}, w^{D*} = \frac{4k[a + c + (t-\eta)e_0] - (t+\eta)at - (t+\eta)\eta c}{8k - (t+\eta)^2}$$

$$p^{D*} = \frac{2k[3a + c + (t - 3\eta)e_0] - (t+\eta)at - (t+\eta)\eta c}{8k - (t+\eta)^2}, Q^{D*} = \frac{2k[a - c + (t+\eta)e_0]}{8k - (t+\eta)^2}$$

$$\pi_m^{D*} = \frac{[a - c + (t+\eta)e_0]^2 k}{8k - (t+\eta)^2}, \pi_r^{D*} = \frac{[a - c + (t+\eta)e_0]^2 4k^2}{8k - (t+\eta)^2}.$$

Theorem 1: (1) a) $0 < e_0 < (a-c)/2\sqrt{2k}$, $0 < t < 2\sqrt{2k} - \eta$, then $\partial e^{D*}/\partial t < 0$; b) if $(a-c)/2\sqrt{2k} < e_0 < (a-c)/(t+\eta)$, $0 \leq t \leq t_b$, then $\partial e^{D*}/\partial t < 0$, means the optimal inventory carbon emission level is equal to the government carbon tax is negatively correlated. (2) If $(a-c)/2\sqrt{2k} < e_0 < (a-c)/(t+\eta)$, $t_a \le t \le 2\sqrt{2k}-\eta$, then $\partial e^{D*}/\partial t < 0$, means the optimal inventory carbon emission level is positively correlated with the government carbon tax. (3) a) If $0 < e_0 < (a-c)/2\sqrt{2k} - t$, then $\partial e^{D*}/\partial \eta < 0$; b) if $(a-c)/2\sqrt{2k} < e_0 < (a-c)/(t+\eta)$, $0 \le \eta \le \eta_a$, then $\partial e^{D*}/\partial \eta < 0$, that is, the optimal inventory carbon emission level of used clothing is negatively correlated with the government carbon label. (4) If $(a-c)/2\sqrt{2k} < e_0 < (a-c)/(t+\eta)$, $\eta_a < \eta < 2\sqrt{2k} - t$, then $\partial e^{D*}/\partial \eta < 0$, that is, the optimal inventory carbon emission level is positively correlated with the government carbon label. where:

$$t_a = \frac{-(a-c)\eta + 8ke_0 - 2\sqrt{8k^2e_0^2 - k(a-c)^2}}{a-c}, \eta_a = \frac{-(a-c)t + 8ke_0 - 2\sqrt{8k^2e_0^2 - k(a-c)^2}}{a-c}$$

Proof: (1), (2) Find $\partial e^{D*}/\partial t = [-(a-c)t^2 + (16ke_0 - 2a + 2c)t - (a-c)^2\eta^2 + (16ke_0 - 2a + 2c)\eta - 4k(a-c)]/[4bk - (t+\eta)^2]^2$. For the convenience of research, let $E_1(t) = -(a-c)t^2 + (16ke_0 - 2a + 2c)t - (a-c)^2\eta^2 + (16ke_0 - 2a + 2c)\eta - 4k(a-c)$. It can be seen that $E_1(t)$ is a quadratic function of t. Since $-(a-c)$ is a negative number, it can be seen that the function opens downward. Let $E_1(t) = 0$. Then the equation $E_1(t)$ is an equation of t. At this time, the two solutions of the equation are:

$$t_a = \frac{-(a-c)\eta + 8ke_0 - 2\sqrt{8k^2e_0^2 - k(a-c)^2}}{a-c},\ t_{a1} = \frac{-(a-c)\eta + 8ke_0 + 2\sqrt{8k^2e_0^2 - k(a-c)^2}}{a-c}$$

Since the equation has a solution, it satisfies $8k^2e_0^2 - k(a-c)^2 > 0$, which implies $8k^2e_0^2 - k(a-c)^2 > 0 \Rightarrow 8ke_0/(a-c) > (a-c)/e_0$, thus, $8ke_0/(a-c) - \eta > (a-c)/e_0 - \eta$. Given that the consumer demand is $Q = (a-c) - (t+\eta)e_0 > 0 \Rightarrow t \le (a-c)/e_0 - \eta$, we can see that t_{a1} contradicts the assumption, so we discard the point t_a. Thus, if $0 \le t \le t_a, E_1(t) < 0 \Rightarrow \partial e^{D*}/\partial t < 0$; if $t_a \le t \le 2\sqrt{2k} - \eta$, $E_1(t) > 0 \Rightarrow \partial e^{D*}/\partial t > 0$. If $t > 2\sqrt{2k} - \eta$, it can be seen that equation $E_1(t)$ has no solution, and $E_1(t) < 0$. It is evident that if $0 < t < 2\sqrt{2k} - \eta$, $\partial e^{D*}/\partial t < 0$. Similarly, it can be proven that conditions (3) and (4) hold.

When the supplier's per-unit carbon emissions baseline of the used clothing inventory are relatively low, such suppliers are classified as environmentally friendly used clothing suppliers.

Theorem 1 indicates that when the supplier is environmentally friendly an increase in government carbon taxes and carbon labelling regulations can reduce the carbon emission of the used clothing inventory for such suppliers. This aligns with real-world situations, as stronger carbon taxes prompt environmentally friendly suppliers to adopt emission reduction technologies and measures to maintain a good corporate image. However, carbon taxes and regulations can only effectively encourage enterprises to reduce emissions when they are below a certain threshold; otherwise, they fail to incentivise emission reduction.

On the other hand, for polluting suppliers, increased carbon taxes and regulations may not necessarily lead to proactive emission reductions. This could be due to the higher costs associated with emission reduction efforts, which diminish corporate profits and reduce the incentive to invest in emission reduction. As a result, stricter carbon taxes and regulations may decrease emission reduction efforts by such enterprises.

Centralised decision-making in the supply chain

In the integrated mode of centralised decision-making for the supply chain (as shown in Figure 3), the supplier and retailer operate as a unified system to maximise the entire supply chain's profit in a centralised manner. Together, they jointly determine the product's sales price p and the inventory carbon emission level of used clothing e. The total profit function for the supply chain in the centralised decision-making model is given by:

$$\pi_{SC} = (p - c - te)(a - p - \eta e) - k(e_0 - e)^2 \tag{6}$$

Here, the first part on the right side of the equation represents the revenue from selling used clothing under the supply chain's centralised decision-making mechanism, while the second part denotes the cost incurred for reducing the used clothing inventory emissions after the supply chain's emission reduction efforts.

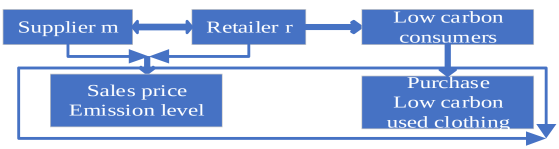

Fig. 3. Centralised decision-making in the supply chain

The second derivative of the supply chain's profit function with respect to p from equation (6) is $\partial^2\pi_{SC}/\partial p^2 = -2 < 0$. Therefore, the supply chain's profit function is strictly concave with respect to the optimal sales price p, ensuring the existence of an optimal sales price p that maximises the supply chain's profit. Taking the first derivative of π_{SC} with respect to p and setting it to zero yields $p = [a + c + (t - \eta)e]/2$. Substituting this p into equation (6), the supply chain's profit optimisation problem becomes:

$$\pi_{SC} = [a - c + (t + \eta)e]^2/4 - k(e_0 - e)^2, s.t. 0 \le e \le e_0 \tag{7}$$

Theorem 2: (1) If $0 \le t \le t_2$ and $0 < e_0 < E_4$, then $e^* = 0$. (2) If $0 \le t \le t_2$ and $E_4 < e_0 < E_6$, then $e^* = e$. (3) If $t > t_2$ and $e_0 < E_6$, then $e^* = 0$. Where $t_2 = 2\sqrt{k} - \eta$, $E_4 = (a - c)(t + \eta)/4k$, $E_5 = (a - c)/(t + \eta)$, $E_6 = (a - c)/2\sqrt{k}$, $e^{C*} = [4ke_0 - (a - c)(t + \eta)]/[4k - (t + \eta)^2]$. The decision regions for enterprises under the centralised mode are shown in Figure 4.

Proof: The proof procedure is similar to decentralised decision-making, which we omit here.

Under the centralised decision-making mode of the supply chain, when$0 \le t \le t_2$, and $E_4 < e_0 < E_6$, $e^* = e^{C*}$; the optimal strategy for the supplier and retailer is (e^{C*}, p^{C*}), the market demand is Q^{C*}, and the supply chain's profit is π_{SC}^{C*}. Where:

$$e^{C*}=\frac{4ke_0-(a-c)(t+\eta)}{8k-(t+\eta)^2},\ p^{C*}=\frac{2k[a+c+(t-3\eta)e_0]-(t+\eta)at-(t+\eta)\eta c}{8k-(t+\eta)^2}$$

$$Q^{C*}=\frac{2k[a-c+(t+\eta)e_0]}{4k-(t+\eta)^2},\pi_{SC}^{C*}=\frac{[a-c+(t+\eta)e_0]^2k^2}{4k-(t+\eta)^2}$$

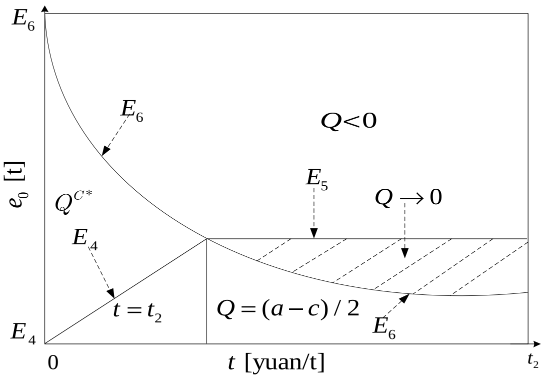

Fig. 4. Decision regions of enterprises under the centralised mode

Theorem 3: (1) a) If $0<e_0<(a-c)/2\sqrt{k}$, $0<t<2\sqrt{k}-\eta$, then $\partial e^{C*}/\partial t<0$; b) if $(a-c)/2\sqrt{k}<e_0<(a-c)/(t+\eta)$, $0<t<t_b$, then $\partial e^{C*}/\partial t<0$, indicating a negative correlation between the optimal inventory carbon emission level of used clothing and government carbon taxes. (2) If $(a-c)/2\sqrt{k}<e_0<(a-c)/(t+\eta)$, $t_b<t<2\sqrt{k}-\eta$, then $\partial e^{C*}/\partial\eta<0$, indicating that the optimal inventory carbon emission level of used clothing is positively correlated with government carbon taxes. (3) a) If $0<e_0<(a-c)/2\sqrt{k}$, $0<\eta<2\sqrt{k}-t$, then $\partial e^{C*}/\partial\eta<0$; b) if $(a-c)/2\sqrt{k}<e_0<(a-c)/(t+\eta)$, $0<\eta<\eta_a$, then $\partial e^{D*}/\partial\eta<0$, indicating a positive correlation between the optimal inventory carbon emission level of used clothing and government carbon labelling regulations. (4) If $(a-c)/2\sqrt{k}<e_0<(a-c)/(t+\eta)$, $\eta_b<\eta<2\sqrt{k}-t$, then $\partial e^{C*}/\partial\eta<0$, indicating a positive correlation between the optimal inventory carbon emission level of used clothing and government carbon labelling regulations. Where:

$$t_b=\frac{-(a-c)\eta+4ke_0-2\sqrt{4k^2e_0^2-k(a-c)^2}}{a-c},\eta_b=\frac{-(a-c)t+4ke_0-2\sqrt{4k^2e_0^2-k(a-c)^2}}{a-c}$$

Proof: The proof procedure is similar to theorem 2, which we omit here.

Theorem 3 indicates that when the supplier is an environmentally friendly supplier, under centralised control decisions, the optimal inventory carbon emission level of used clothing decreases as the carbon tax and carbon labelling increase, and increases as the carbon tax and carbon labelling decrease. This result is consistent with the conclusion under decentralised decision-making in the supply chain. Similar to the decentralised decision-making scenario, carbon taxes and carbon regulations can only effectively promote enterprise emission reductions when they are below a certain threshold; otherwise, they fail to incentivise emission reductions. On the other hand, when the supplier is polluting, the optimal inventory carbon emission level of the polluting supplier increase as the intensity of carbon taxes and regulations increase, and decrease as the carbon taxes and regulations decrease.

Comparison and optimisation analysis of decentralised and centralised decision-making modes

Corollary 1: (1) If $0 < e_0 < (a-c)/2\sqrt{2k}$, $0 < t < 2\sqrt{2k} - \eta$ or $(a-c)/2\sqrt{2k} < e_0 < (a-c)/2\sqrt{k}$, $0 < t < t_a$, or $(a-c)/2\sqrt{k} < e_0 < (a-c)/(\eta+t)$, $0 < t < t_a$, we can obtain $\partial e^{D*}/\partial t < 0, \partial e^{C*}/\partial t < 0$. This indicates that the optimal inventory carbon emission level of used clothing for the supplier is negatively correlated with the carbon tax under both centralised and decentralised modes. Meanwhile, If $0 < e_0 < (a-c)/2\sqrt{2k}$, $0 < \eta < 2\sqrt{2k} - t$ or $(a-c)/2\sqrt{2k} < e_0 < (a-c)/2\sqrt{k}$, $0 < \eta < \eta_a$ or $(a-c)/2\sqrt{k} < e_0 < (a-c)/(\eta+t)$, $0 < \eta < \eta_a$, we obtain $\partial e^{D*}/\partial \eta < 0, \partial e^{C*}/\partial \eta < 0$. This indicates that the optimal inventory carbon emission level of used clothing for the supplier is negatively correlated with carbon labelling under both centralised and decentralised modes.

(2) If $(a-c)/2\sqrt{2k} < e_0 < (a-c)/2\sqrt{k}$, $t_a < t < 2\sqrt{2k} - \eta$ or $(a-c)/2\sqrt{2k} < e_0 < (a-c)/2\sqrt{k}$, $0 < \eta < 2\sqrt{2k} - t$, we can see that, in the centralised decision-making mode of the supply chain, the supplier's inventory carbon emission level of used clothing of used clothing is negatively correlated with both the carbon tax and carbon labelling; however, in the decentralised decision-making mode of the supply chain, the supplier's inventory carbon emission level of used clothing of used clothing is positively correlated with the carbon tax and carbon labelling.

(3) If $(a-c)/2\sqrt{k} < e_0 < (a-c)/(\eta+t)$, $t_a < t < t_b$ or $(a-c)/2\sqrt{k} < e_0 < (a-c)/(\eta+t)$, $\eta_a < \eta < \eta_b$, we can see that, in the centralised decision-making mode of the supply chain, the supplier's inventory carbon emission level of used clothing is positively correlated with both the carbon tax and carbon labelling, however, in the decentralised decision-making mode of the supply chain, the supplier's inventory carbon emission level of used clothing is negatively correlated with both the carbon tax and carbon labelling.

(4) If $(a-c)/2\sqrt{2k} < e_0 < (a-c)/(\eta+t)$, $t_a < t < 2\sqrt{2k} - \eta$ or $(a-c)/2\sqrt{2k} < e_0 < (a-c)/(\eta+t)$, $\eta_a < \eta < 2\sqrt{2k} - t$, the supplier's inventory carbon emission level of used clothing is positively correlated with both the carbon tax and carbon labelling under both centralised and decentralised decision-making modes in the supply chain.

Proof: When $8e_0^2 - (a-c)^2 > 0 \Rightarrow 8ke_0/(a-c) > (a-c)e_0$, if $0 < t < 2\sqrt{2k} - \eta$ or $0 < \eta < 2\sqrt{2k} - t$, we obtain $\partial e^{D*}/\partial t < 0$, $\partial e^{C*}/\partial t < 0$, $\partial e^{D*}/\partial \eta < 0$ and $\partial e^{C*}/\partial \eta < 0$.

When $(a-c)/2\sqrt{2k} < e_0 < (a-c)/2\sqrt{k}$, if $0 < \eta < t_a$or $0 < t < \eta_a$, we can get $\partial e^{D*}/\partial t < 0, \partial e^{C*}/\partial t < 0, \partial e^{D*}/\partial \eta < 0$ and $\partial e^{C*}/\partial \eta < 0$. When $\eta_a < t < 2\sqrt{2k} -$

η or $t_a < \eta < 2\sqrt{2k} - t$, we obtain $\partial e^{D*}/\partial \eta > 0$, $\partial e^{C*}/\partial t < 0$, $\partial e^{D*}/\partial \eta > 0$ and $\partial e^{C*}/\partial \eta < 0$.

When $(a-c)/2\sqrt{k} < e_0 < (a-c)/(\eta+t)$, if $0 < \eta < t_b$ or $0 < t < \eta_b$, we can get $\partial e^{D*}/\partial t < 0$, $\partial e^{C*}/\partial t < 0$, $\partial e^{D*}/\partial \eta < 0$ and $\partial e^{C*}/\partial \eta < 0$. If $\eta_b < t < \eta_a$ or $t_b < \eta < t_a$, then $\partial e^{D*}/\partial t < 0$, $\partial e^{C*}/\partial t > 0$, $\partial e^{D*}/\partial \eta < 0$ and $\partial e^{C*}/\partial \eta > 0$. If $\eta_a < t < 2\sqrt{2k} - \eta$ or $t_a < \eta < 2\sqrt{2k} - t$, then $\partial e^{D*}/\partial t > 0$, $\partial e^{C*}/\partial t > 0$, $\partial e^{D*}/\partial \eta > 0$ and $\partial e^{C*}/\partial \eta > 0$.

Corollary 1 indicates that when the supplier is environmentally friendly, the implementation of carbon tax and carbon labelling systems can promote carbon emission reduction regardless of whether centralised or decentralised decision-making is adopted. If the supplier is polluting, in a centralised decision-making mode, inventory carbon emissions are negatively correlated with carbon taxes and carbon labelling, while in a decentralised decision-making mode, inventory carbon emissions are positively correlated with carbon taxes and carbon labelling. Additionally, if the supplier is a polluting supplier and the carbon tax and regulations exceed a certain threshold, the implementation of carbon tax and carbon labelling systems will not be effective in reducing carbon emissions.

Corollary 1 also indicates that if the supplier is an environmentally friendly supplier, the implementation of carbon regulations and carbon labelling systems can achieve a reduction in carbon emissions. However, if the supplier is a polluting supplier, under centralised decision-making, adopting appropriate carbon taxes and regulatory constraints can lead to a reduction in the inventory emission of used clothing.

Corollary 2: Based on the above analysis, by comparing the optimal inventory carbon emission levels, demand, sales prices, and supply chain profits under the decentralised and centralised decision-making modes, the following conclusions can be drawn:

(1) If $8k - (t+\eta)^2 > 0$, then $e^{C*} < e^{D*}$, $Q^{C*} < Q^{D*}$ and $\pi_{SC}^{C*} > \pi_m^{D*} + \pi_r^{D*}$;

(2) If $4k - (t+\eta)^2 > 0$, then $p^{C*} < p^{D*}$; if $2\sqrt{k} < (\eta+t) < 2\sqrt{2k}$, then $p^{C*} > p^{D*}$.

Proof: From Corollary 1 and Corollary 2, it is known that $Q^{C*} - Q^{D*} > 0$, $e^{C*} - e^{D*} = -2k(\eta+t)[(\eta+t)e_0 - a + c]/\{[8k-(t+\eta)^2][4k-(t+\eta)^2]\} < 0$. Similarly, it can be deduced that $\pi_{SC}^{C*}/(\pi_m^{D*} + \pi_r^{D*})$>1. Thus, (1) is proven.

$p^{D*} - p^{C*} = \{2k[2k-(t+\eta)^2][a-c-(t+\eta)e_0]\}/\{[8k-(t+\eta)^2][4k-(t+\eta)^2]\} < 0$. It is satisfied that, when $2k-(t+\eta)^2 > 0$ then $p^{C*} < p^{D*}$; and when $2\sqrt{k} < (t+\eta) < 2\sqrt{2k}$, then $p^{C*} > p^{D*}$.

Corollary 2 indicates that in the decentralised decision-making mode, prices are not fixed compared to centralised decision-making. The optimal inventory carbon emission level of used clothing is greater in decentralised decision-making than in centralised decision-making. Moreover, the demand and profits in decentralised decision-making are smaller compared to centralised decision-making.

Contract design for supply chain coordination

Due to the problem of double marginalisation in the supply chain under decentralised decision-making, it is necessary to design a contract that achieves Pareto optimality for both parties. This means that the profits of both the decentralised supplier and retailer should not be less than the profits in the decentralised decision-making mode. In other words, the additional profit generated by transitioning the supply chain from a decentralised to a centralised mode should be allocated. Under this contract, the profits of the supplier and the retailer are as follows:

$$\pi_m = (w - c - te)(a - bp - \eta e\ ) - k(e_0 - e)^2 \tag{8}$$

$$\pi_r = (p - w)(a - bp - \eta e\ ) \tag{9}$$

Theorem 4: When the wholesale price given by the supplier to the retailer satisfies $w_{min} < w' < w_{max}$, the profits of the supplier and retailer are not less than those under the wholesale price contract. Compared to the wholesale price contract, the increase in profits for the supplier and retailer are $\Delta\pi'_m = (w' - w_{min})\ Q'$ and $\Delta\pi'_r = (w_{max} - w')\ Q'$, respectively. At this point, the total increase in profit $\Delta\pi_{SC}$ satisfies $\Delta\pi_{SC} = (w_{max} - w_{min})\ Q'$.

Proof: Since the profits of both the supplier and retailer are not less than the maximised profits under decentralised decision-making, we have $\pi'_m > \pi_m^D$ and $\pi'_r > \pi_r^D$. This implies: $(p' - w)Q' > \pi_r^D$ and $(w - c - tc')Q' - k(e_0 - e)^2 > \pi_m^D$, thus, we can obtain $w \leq (p'Q' - \pi_r^D)$ and $w \geq (\pi_m^D + k(e_0 - e)^2 + cQ' - t\ \Delta'Q')/Q'$, then we obtain $\Delta\pi_{SC} = (w_{max} - w_{min})\ Q'$.

The theorem shows that when $w' = w_{min}$, the supplier's profit does not increase, and all the additional profit $\Delta\pi_{SC}$ is obtained by the retailer. Conversely, when $w' = w_{max}$, the retailer's profit does not increase, and all the additional profit $\Delta\pi_{SC}$ is obtained by the supplier.

Since in decentralised decision-making, the wholesale price w is not solely determined by the supplier, its determination is related to the bargaining power of both the supplier and the retailer. Therefore, the Nash bargaining model is used to determine the value of w [27]. Suppose the bargaining power of the retailer is α, where $0 < \alpha < 1$. Then, the corresponding bargaining power of the supplier is $1 - \alpha$. According to the Nash bargaining model, the problem of finding the optimal w can be formulated as N, $N{:}\,Max = (\pi'_r - \pi_r^D)^\alpha(\pi'_r - \pi_m^D)^{(1-\alpha)}$, which yields the following expression:

$$N{:}\,Max = [(p' - w)Q' - \pi_r^D]^\alpha[(w - c - tc')Q' - k(e_0 - e')^2 - \pi_m^D]^{(1-\alpha)} \tag{10}$$

Solving equation (10) yields $w = (p' - \pi_r^D - \alpha\Delta\pi_{SC})/Q'$, which shows that the wholesale price is negatively correlated with the retailer's bargaining power. This means that the stronger the retailer's bargaining power, the lower the wholesale price set by the supplier. Conversely, if the retailer's bargaining power is weaker, the wholesale price will be higher.

Theorem 5: According to the Nash bargaining model, there exists an optimal wholesale price contract (p', w', c') that satisfies supply chain coordination, and w meets the condition of $w_{min} < w' < w_{max}$. Here, p', c' are the optimal choices for the supplier and retailer under centralised decision-making.

Proof: Take the first derivative of equation (10) with respect to w and set the first derivative equal to zero. This yields:

$$\begin{aligned}\frac{\partial N}{\partial w} = \big[&[(p' - w)Q' - \pi_r^D]^\alpha[(w - c - tc')Q' - k(e_0 - e')^2 - \pi_m^D]^{(-\alpha)}\big] \\ &\cdot\{\frac{-\alpha\big((w - c - tc')Q' - k(e_0 - e')^2 - \pi_m^D\big)}{(p' - w)Q' - \pi_r^D} + 1 - \alpha\}\end{aligned} \tag{11}$$

we obtain $w' = \{p' - \pi_r^D - \alpha[(p' - c - te')Q' - k(e_0 - e')^2 - \pi_m^D]\}/Q' = (p' - \pi_r^D - \alpha\Delta\pi_{SC})/Q'$. From this, we find that: $w' - w_{max} = -\alpha\Delta\pi_{SC}/Q' < 0$ and $w' - w_{min} =$

$(1-\alpha)\Delta\pi_{SC}/Q' > 0$. Therefore, it can be concluded that $w_{min} < w' < w_{max}$. Proof completed.

According to Theorems 4 and 5, the increases in profits for the supplier and retailer under the optimal contract are given by $\Delta\pi_r = \alpha\Delta\pi_{SC}$ and $\Delta\pi_m = (1-\alpha)\Delta\pi_{SC}$, respectively. This indicates that, under this contract, the profits of the supplier and retailer are influenced by their respective bargaining powers. This contract achieves the optimisation of the overall supply chain profit.

Results and discussion

Based on the assumptions, similar to references [27, 31, 32], this chapter sets $a = 100$, $c = 20$, $k = 16$, and uses numerical simulation methods to verify the above theorems and corollaries. Since the baseline of low carbon level of inventory play a key role in the impact on both decentralised and centralised decision-making, this chapter selects different baselines of low carbon level of inventory to examine the effects of carbon taxes and carbon labelling on carbon emissions levels, sales prices, sales volumes, supply chain profits, and the increase in supply chain profits under coordination contracts in decentralised and centralised decision-making modes. Specifically, $e_0 = 5$ and $e_0 = 11$.

The impact of carbon tax and carbon labelling on the inventory carbon emission level of used clothing

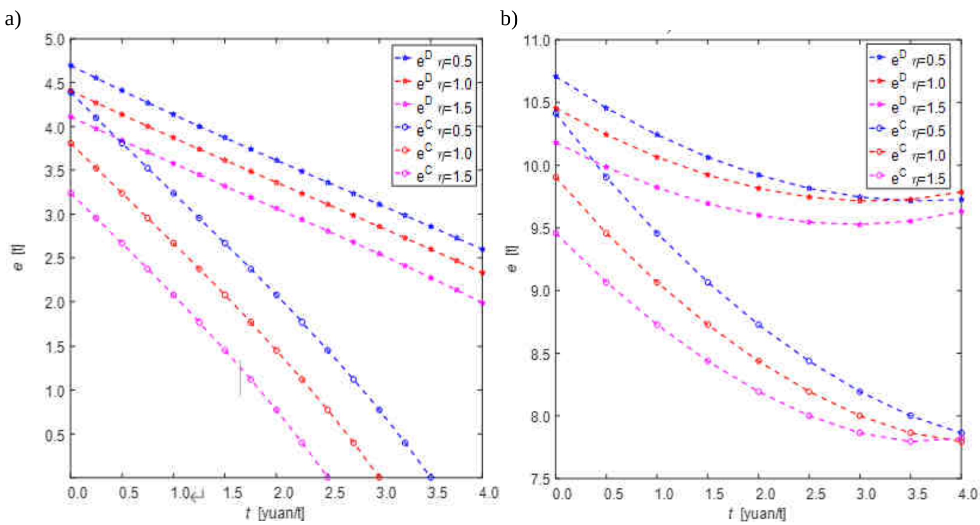

Fig. 5. Impact of changes t, η on the carbon emission level of used clothing e: a) $e_0 = 5$, b) $e_0 = 11$

Figure 5 illustrates the impact of carbon tax and carbon labelling on inventory carbon emissions level of used clothing. From Figure 5, we conclude that if the supplier is an environmentally friendly type, i.e., the used clothing inventory low carbon baseline level is relatively low ($e_0 = 5$): 1) in the decentralised decision-making mode, the inventory carbon emissions level of used clothing is more sensitive to government carbon regulations compared to the centralised decision-making mode. As a result, the inventory carbon emissions level of used clothing is higher in the centralised mode than in the decentralised mode; 2) as consumers become more aware of carbon labelling, the inventory carbon emissions level of used clothing decreases, indicating that carbon labelling effectively reduces the inventory emission of used clothing. If the supplier is a polluting type, i.e., the used clothing inventory low carbon baseline level is higher ($e_0 = 11$): 1) the inventory carbon emissions level of used clothing in the decentralised decision-making mode is greater than that in the centralised decision-making mode; 2) in the decentralised decision-making mode, the inventory carbon emissions level of used clothing initially decreases and then increases with the rise in carbon taxes, while in the centralised decision-making mode, the inventory carbon emissions level of used clothing is negatively correlated with carbon taxes; 3) when the tax rate remains constant, as consumer sensitivity to carbon labelling increases, carbon emissions gradually decrease in the centralised decision-making mode, whereas in the decentralised decision-making mode, carbon emissions initially decrease and then increase.

The impact of carbon taxes and carbon labels on demand

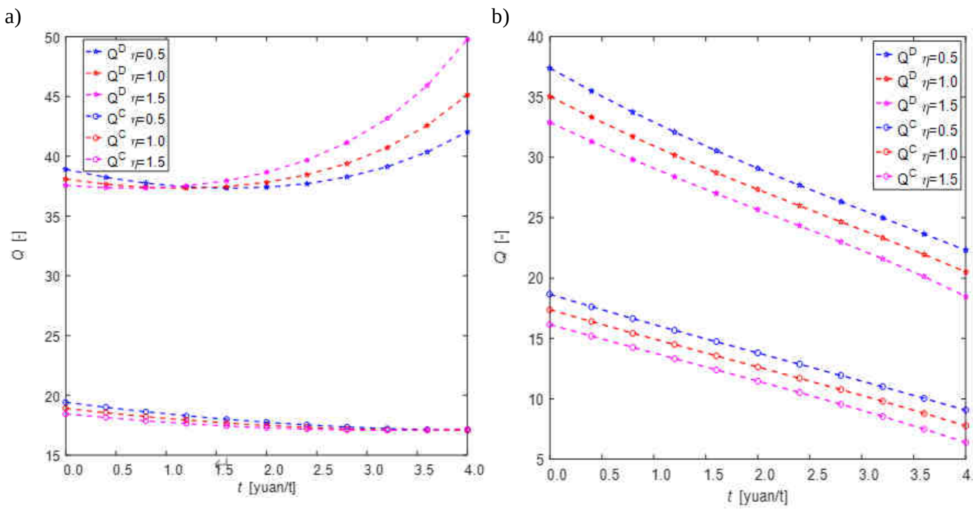

Fig. 6. Impact of changes t, η on the demand e: a) $e_0 = 5$, b) $e_0 = 11$

Figure 6 illustrates the impact of carbon tax and carbon labels on demand. From Figure 6, we conclude that if the supplier is environmentally friendly manufacturing, i.e. the used clothing inventory low carbon baseline level is relatively low ($e_0 = 5$): 1) the demand in decentralised mode is larger than that in centralised decision-making, and the demand in decentralised decision-making first decreases and then increases with the increase of carbon tax, and the demand in centralised decision-making is related to the carbon tax burden; 2) when the tax rate is fixed, with the increase of consumers' sensitivity to carbon labels, the demand under centralised decision-making first decreases and then increases, and the demand in decentralised mode is negatively correlated with carbon emissions. If the supplier is a non-environmentally friendly supply type, that is, the used clothing inventory low carbon baseline level is higher ($e_0 = 11$): 1) the demand of decentralised decision making is less than that of centralised decision making; 2) the demand is related to carbon tax burden in decentralised decision making and centralised decision making; 3) when the tax rate is fixed, with the increase of consumers’ sensitivity to carbon labelling, the demand is negatively correlated with carbon emissions in the centralised and decentralised decision-making modes.

The impact of carbon taxes and carbon labels on sales price

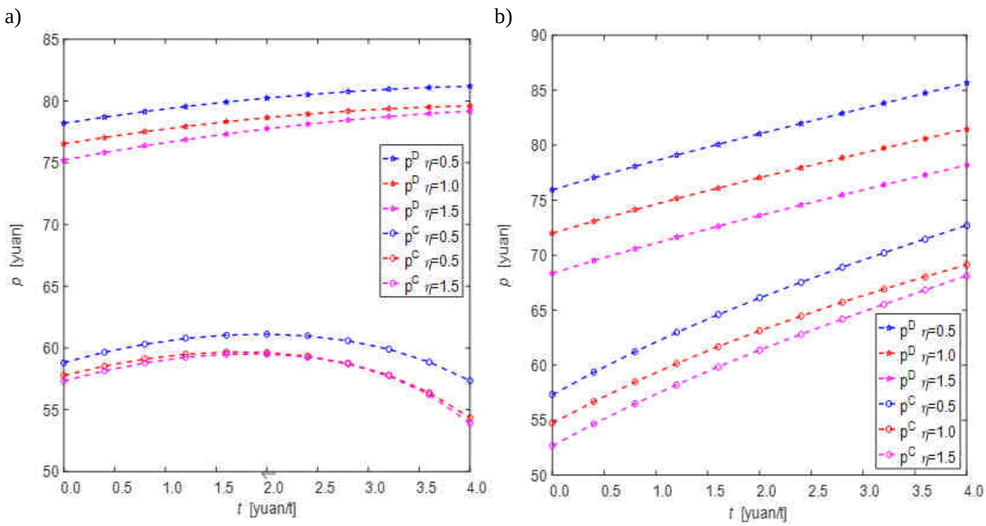

Fig. 7. Impact of changes t, η on the sales price e: a) $e_0 = 5$, b) $e_0 = 11$

Figure 7 illustrates the impact of carbon taxes and carbon labels on sales price. From Figure 7, we conclude that if the supplier is environmentally friendly, that is, the used clothing inventory low carbon baseline level is relatively low ($e_0 = 5$): 1) the sales price in decentralised decision-making mode is higher than that in centralised decision-making mode, and the sales price in centralised decision-making first increases and then decreases with the increase of carbon tax, and the sales price in decentralised decision-making is positively correlated with carbon tax; 2) when the tax rate is constant, with the increase of consumers’ sensitivity to carbon labelling, the sales price in the decentralised decision-making mode decreases with the increase of carbon labelling regulation, while the sales price in the centralised decision-making mode decreases first and then increases with the carbon labelling regulation. If the supplier is polluting, that is, the used clothing inventory low carbon baseline level is higher ($e_0 = 11$): 1) the sales price under decentralised decision-making is greater than that under centralised decision-making; 2) there is a positive correlation between sales price and carbon tax in decentralised decision-making and centralised decision-making; 3) when the tax rate is fixed, with the increase of consumers’ sensitivity to carbon labels, the sales price is positively correlated with carbon emissions in the centralised and decentralised decision-making models.

The impact of carbon tax and carbon labelling on profit

Figure 8 illustrates the impact of the carbon tax and carbon labelling on profit. From Figure 8, we conclude that if the supplier is of an environmentally friendly type, i.e., the used clothing inventory low carbon baseline level is relatively low ($e_0 = 5$): 1) the supply chain profit under decentralised decision-making is lower than that under centralised decision-making; 2) in the decentralised decision-making mode, the supply chain profit is negatively correlated with the carbon tax, while in the centralised decision-making mode, the supply chain profit initially decreases and then increases with the rise in the carbon tax; 3) when the tax rate remains constant, as consumer sensitivity to carbon labelling increases, the supply chain profit decreases in the decentralised decision-making mode. In the centralised decision-making mode, the supply chain profit initially decreases and then increases. If the supplier is of a non-environmentally friendly type, with a relatively high inventory low carbon baseline level ($e_0 = 11$): 1) the supply chain profit under the decentralised mode is lower than that under the centralised decision-making mode, and in both the decentralised and centralised decision-making modes, the supply chain profits are negatively correlated with the carbon tax; 2) when the tax rate remains constant, as consumer sensitivity to carbon labelling increases, the supply chain profits decrease in both the centralised and decentralised decision-making modes.

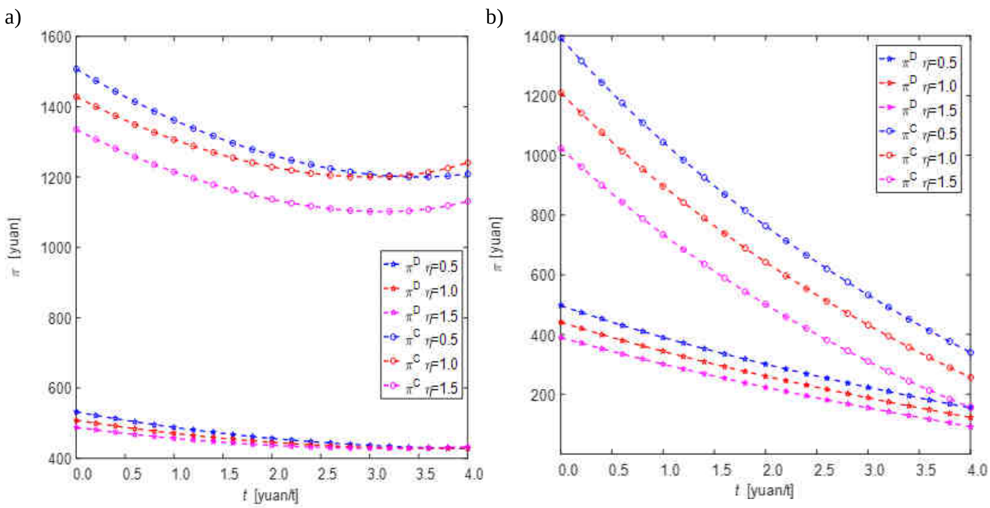

Fig. 8. Impact of changes t, η on the supply chain profit π e: a) $e_0 = 5$, b) $e_0 = 11$

The impact of carbon tax and carbon labelling on the coordination mechanism

Figure 9 illustrates the impacts of carbon tax and carbon labelling on the coordination mechanism. From Figure 9, we conclude that if the supplier is an environmentally friendly type, that is, when the used clothing inventory low carbon baseline level is relatively low ($e_0 = 5$): 1) the increase in the profit distribution of the supply chain initially decreases and then increases with the rise in carbon tax; 2) when the tax rate remains constant, the increase in the profit distribution of the supply chain initially decreases and then increases with the rise in the intensity of the carbon labelling. If the supplier is of a non-environmentally friendly type, with a relatively high inventory low carbon baseline level ($e_0 = 11$): 1) the increase in the profit distribution of the supply chain decreases with the rise in carbon tax; 2) when the tax rate remains constant, as consumer sensitivity to carbon labelling increases, the increase in the profit distribution of the supply chain decreases.

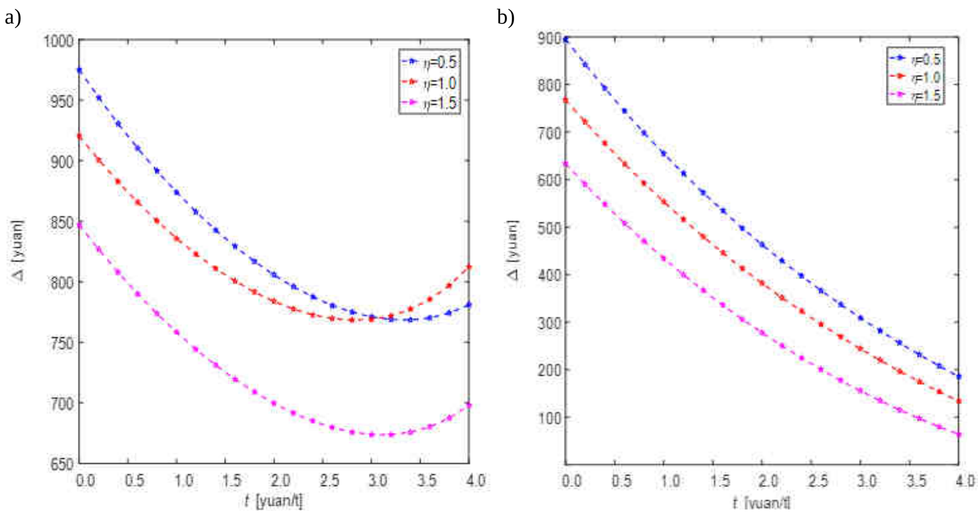

Fig. 9. Impact of changes t, η on the Coordination Mechanism e: a) $e_0 = 5$, b) $e_0 = 11$

Conclusion

This paper investigates a two-tier supply chain operating under carbon tax policies, where consumer preferences for carbon labelling influence demand. It examines the effects of carbon taxes and carbon labelling on suppliers' inventory carbon emissions and coordination decisions under both decentralised and centralised decision-making frameworks. The key findings are as follows:

- **Environmentally friendly suppliers**: When the supplier adopts environmentally friendly practices, the inventory carbon emission level of used clothing decreases as the carbon tax increases under both decision-making modes. Additionally, with a constant carbon tax, an increase in consumer perception of carbon labelling reduces the inventory carbon emission level of used clothing.
- **Polluting suppliers**: For suppliers with higher carbon emissions, the inventory carbon emission level of recycled clothing under decentralised decision-making initially decreases and then increases with a rise in the carbon tax. With a constant tax rate, as consumer perception of carbon labelling improves, the inventory carbon emission level first decreases and then increases. In centralised decision-making, carbon emissions are negatively correlated with the carbon tax. Moreover, as consumer perception of carbon labelling increases, the inventory carbon emission level of recycled clothing is lower.
- **Impact of initial conditions**: When suppliers have the same baseline of low carbon level of inventory, the effects of carbon tax and carbon regulation intensity on inventory carbon emission level of used clothing, demand, supply chain profits, and

coordination mechanisms are similar. However, their impact on sales prices is opposite.

- **Profit redistribution and coordination**: Since profits under decentralised decision-making are lower than those under centralised decision-making, implementing a Nash bargaining-based wholesale price contract can help coordinate the supply chain. By redistributing profit differences arising from the varying impacts of carbon tax and carbon labelling on supply chain revenues, both suppliers and retailers can achieve Pareto-optimal outcomes.

This study employs quantitative analysis to explore the decision-making processes associated with inventory carbon emissions under carbon tax and carbon labelling systems, uncovering key patterns of change. The findings provide valuable insights for government policy:

- For **non-environmentally friendly suppliers**, governments can enhance consumer awareness of environmental protection and provide subsidies for emission reduction efforts.
- For **environmentally friendly suppliers**, emphasis should be placed on fostering collaboration between suppliers and retailers to achieve minimal low-carbon inventory levels of used clothing.

However, this study has several limitations. For instance, it assumes a linear relationship between consumer demand and the inventory carbon emission level of used clothing. Future research could explore the impact of random consumer demand. Additionally, while this study considers single-cycle inventory carbon emission decisions, future work could extend to multi-cycle scenarios. Finally, the current focus is limited to carbon taxes and carbon labelling policies; future studies could examine the effects of other policy combinations, such as carbon trading, carbon allowances, and government subsidies, on inventory carbon emission decisions for used clothing suppliers.

Acknowledgements

The study is supported by a project funded by the Scientific Research Team Plan of Zhengzhou University of Aeronautics (23ZHTD01013), Management Discipline basic innovation ability promotion project of Zhengzhou University of Aeronautics (XKZX24014). We appreciate their support very much.

References

[1] Lang C, Zhang R. Second-hand clothing acquisition: The motivations and barriers to clothing swaps for Chinese consumers. Sustain Prod Consump. 2019;18:156-64. DOI: 10.1016/j.spc.2019.02.002.

[2] WRAP. Textiles. Available from: https://wrap.org.uk/resources/guide/textiles.

[3] Moon D. Promoting sustainable practices: Exploring secondhand clothing consumption patterns and reductions in greenhouse gas emissions in Japan. Sustain Prod Consump. 2024;45:294-305. DOI: 10.1016/j.spc.2024.01.007.

[4] Wu M, Al Mamun A, Yang Q, Masud M. Modeling the reuse intention and practices of secondhand clothing: Evidence from a developing nation. Hum Soc Sci Commun. 2023;10(1):1-12. DOI: 10.1057/s41599-023-02008-y.

[5] Hua G, Cheng TCE, Wang S. Managing carbon footprints in inventory management. Int J Prod Econ. 2011;132(2):178-85. DOI: 10.1016/j.ijpe.2011.03.024.

[6] Oreskes N. Metaphors of warfare and the lessons of history: time to revisit a carbon tax? Climatic Change. 2011;104(2):223-30. DOI: 10.1007/s10584-010-9887-5.

[7] Liu T, Wang Q, Su B. A review of carbon labeling: Standards, implementation, and impact. Renew Sust Energ Rev. 2016;53:68-79. DOI: 10.1016/j.rser.2015.08.050.
[8] Huang SP, Ma SZ, Pan Y, Li Y, Yuan YH, Tsai SB. An empirical study on how climate and environmental issues awareness affects low carbon use behaviour. Ecol Chem Eng S. 2020;27(1):55-66. DOI: 10.2478/eces-2020-0003.
[9] Zhang M, Tan J, Cheng ETC. Pricing of Chinese certified emissions reduction scheme in a supply chain under the cap-and-trade policy. Ecol Chem Eng S. 2024;31(1):31-48. DOI: 10.2478/eces-2024-0003.
[10] Liu W, Tsai S B, Wu CH, Wacławek M. Corporate environmental management and sustainable operation: theory and application. Ecol Chem Eng S. 2022;29(3):283-5. DOI: 10.2478/eces-2022-0020.
[11] Zhou MX, Li X. Low-carbon production control and resource allocation optimization. Int J Simul Model. 2022;21(2):352-363. DOI: 10.2507/IJSIMM21-2-CO9.
[12] Wang ZJ, Suo J. Optimization of flexible production logistics under low carbon constraint. Int J Simul Model. 2022;2:184-95. DOI: 10.2507/IJSIMM21-1-CO5.
[13] Zhao JX. An empirical study on emissions trading influence for the enterprise environmental responsibility. Ecol Chem Eng S. 2023;30(2):259-66. DOI: 10.2478/eces-2023-0027.
[14] Xu W, Xu S, Liu DY, Awaga AL, Rabia A, Zhang YY. Impact of fairness concerns on resource-sharing decisions: A comparative analysis using evolutionary game models in manufacturing enterprises. Adv Prod Eng Manage. 2024;19(2):223-38. DOI: 10.14743/apem2024.2.503.
[15] Feng Y, Liu Y, Chen Y. A robust multi-supplier multi-period inventory model with uncertain market demand and carbon emission constraint. Comput Ind Eng. 2022;165:107937. DOI: 10.1016/j.cie.2022.107937.
[16] Sebatjane M. Three-echelon circular economic production–inventory model for deteriorating items with imperfect quality and carbon emissions considerations under various emissions policies. Expert Syst Appl. 2024;252:124162. DOI: 10.1016/j.eswa.2024.124162.
[17] Sebatjane M. Sustainable inventory models under carbon emissions regulations: Taxonomy and literature review. Comput Oper Res. 2024;106865. DOI: 10.1016/j.cor.2024.106865.
[18] Wang Y, Shu J, Su J, Zhen L. Inventory placement with carbon cap-and-trade in guaranteed service supply chains. Transport Res E-Log. 2025;193:103813. DOI: 10.1016/j.tre.2024.103813.
[19] Young W, Hwang K, McDonald S, Oates CJ. Sustainable consumption: green consumer behaviour when purchasing products. Sustain Dev. 2010;18(1):20-31. DOI: 10.1002/sd.394.
[20] Zhao R, Zhong S. Carbon labelling influences on consumers' behaviour: A system dynamics approach. Ecol Indic. 2015;51:98-106. DOI: 10.1016/j.ecolind.2014.08.030.
[21] Fan T, Song Y, Cao H, Xia H. Optimal eco-labeling strategy with imperfectly informed consumers. Ind Manage Data Syst. 2019;119(6):1166-88. DOI: 10.1108/IMDS-06-2018-0256.
[22] Fischer C, Lyon TP. Competing environmental labels. J Econ Manage Strat. 2014;23(3):692-716. DOI: 10.1111/jems.12061.
[23] Jiang K, Zhang L, Zhang X, Wang Y. Sustainable implementation of the carbon-labeling policy with customer participation and government supervision. Comput Ind Eng. 2023;178:109100. DOI: 10.1016/j.cie.2023.109100.
[24] Yang M, Lin B. Differences in willingness to pay for various types of carbon-labeled products: A dynamic perspective. J Environ Manage. 2024;370:122936. DOI: 10.1016/j.jenvman.2024.122936
[25] Liu Z, Qian Q, Hu B, Li M. Cold chain inventory optimization model of three-level fresh products based on carbon labelling system. Environ Eng Manage J. 2023;22(5):813-24. DOI: 10.30638/eemj.2023.064.
[26] Wang Y, Yu Z, Jin M, Mao J. Decisions and coordination of retailer-led low-carbon supply chain under altruistic preference. Eur J Oper Res. 2021;293(3):910-25. DOI: 10.1016/j.ejor.2020.12.060.
[27] Gao H, Liu S, Xing D, Cao G. Optimization strategy of cooperation and emission reduction in supply chain under carbon tax policy. J Discret Math Sci C. 2018;21(4):825-35. DOI: 10.1080/09720529.2018.1479165.
[28] Wang S, Choi SH, Xiao J, Huang GQ. Low-carbon supply chain coordination through dual contracts considering pareto-efficiency. Int J Prod Res. 2024;62(18):6627-48. DOI: 10.1080/00207543.2023.2296976.
[29] Pu J, Chun W, Yang H. Low carbon strategy and coordination for a two-echelon automotive supply chain considering dual credit policy. Comput Ind Eng. 2024;192:110229. DOI: 10.1016/j.cie.2024.110229.
[30] Zhou X, Wei X, Lin J, Tian X, Lev B, Wang S. Supply chain management under carbon taxes: A review and bibliometric analysis. Omega-Int J Manage S. 2021;98:102295. DOI: 10.1016/j.omega.2020.102295.
[31] He P, Sun Y. The impacts of retailer's carbon tax on the price and carbon reduction decisions in supply chain. Comput Ind Eng. 2024;190:110034. DOI: 10.1016/j.cie.2024.110034.
[32] Cao G, Zhao X, Gao H, Tang M. A game theory analysis of intelligent transformation and sales mode choice of the logistics service provider[J]. Adv Prod Eng Manage. 2023;18(3):327-44. DOI: 10.14743/apem2023.3.476.