

ISSN: 2543-6821 (online)

Journal homepage: <http://ceej.wne.uw.edu.pl>

Yasuhito Tanaka

Budget Deficit in a Growing Economy and Impossibility of Fiscal Collapse: A Continuous Time Analysis

To cite this article

Tanaka Y. (2024). Budget Deficit in a Growing Economy and Impossibility of Fiscal Collapse: A Continuous Time Analysis. Central European Economic Journal, 11(58), 305-319.

DOI: 10.2478/ceej-2024-0020

 To link to this article: <https://doi.org/10.2478/ceej-2024-0020>



Yasuhito Tanaka

Faculty of Economics, Doshisha University, Kamigyo-ku, Kyoto, Japan

corresponding author: yatanaka@mail.doshisha.ac.jp

Budget Deficit in a Growing Economy and Impossibility of Fiscal Collapse: A Continuous Time Analysis

Abstract

Using a continuous time dynamic model of growing economy we will show the following results. 1) When people derive utility from their money holding (or government bond holding) along with their consumption, a budget deficit is essential to achieve and maintain full employment under stable prices or inflation in a growing economy. 2) If we take into account that government spending due to budget deficits increases financial assets held by the private sector, and then consumption will occur from assets in addition to consumption from income, even when the interest rate on government bonds is higher than the real economic growth rate, the ratio of government debt to GDP can not diverge and the divergence is naturally prevented by mild inflation. The required inflation rate is such that the interest rate of the government bonds is smaller than the weighted average of the rate of return on capital and the nominal growth rate. Since the interest rate of the government bonds is usually considered smaller than the rate of return on capital, this is not a very demanding requirement. Thus, we need not worry at all about the accumulation of government debt or about the divergence of the debt to GDP ratio, which is often taken as an indicator of fiscal collapse.

Keywords

budget deficit | growing economy | infinitely living consumers | continuous time model | impossibility of fiscal collapse

JEL Codes

E12, E24

1. Introduction

The purpose of this paper is to prove the following results using a relatively simple mathematical model.

1. When people derive utility from their money holding (or government bond holding) along with their consumption, a budget deficit is essential to achieve and maintain full employment under stable prices or inflation in a growing economy.

2. With the following items (a) and (b) in mind, even when the interest rate on government bonds is higher than the real economic growth rate, the ratio of government debt to GDP does not diverge and the divergence is naturally prevented by mild inflation. The required inflation rate is such that the interest rate of the government bonds is smaller than the weighted average of the rate of return on capital and the nominal growth rate. Since the interest rate of government bonds is usually considered smaller than the rate of return on capital, this is not a very demanding requirement.

(a) Government spending due to budget deficits increases government bonds or base money held by banks and increases financial assets held by the private sector.

(b) As financial assets increase, consumption will occur from assets in addition to consumption from income.

If the GDP ratio of government debt diverges and becomes infinitely large, then the GDP ratio of private financial assets also becomes infinitely large, and the GDP ratio of consumption from assets also becomes infinitely large. However, since consumption is a part of GDP, such a situation cannot occur and a contradiction arises. In such a case, an increase in consumption demand would cause a rise in prices, which would prevent the debt-to-GDP ratio from diverging. This is a naturally occurring phenomenon, not caused by some policy.

Thus, we need not worry at all about the accumulation of government debt or about the divergence of the debt-to-GDP ratio, which is often taken as an indicator of fiscal collapse.

If current account deficits continue and net external debt accumulates, it will eventually have to be repaid, which will be a major obstacle to the country's economy. However, as long as the government debt remains domestic, there is no problem at all.

In Section 2, we explain the methodology of this paper and present a literature review.

Section 3 presents this paper's model and analyses of the behaviour of consumers, firms, and market equilibrium. We will prove the necessity of a budget deficit for full employment under constant prices. We show also that larger budget deficits cause inflation, or we need a larger budget deficit to realise full employment under inflation. The policy of increasing money through budget deficits in line with the rate of economic growth as indicated by the conclusions of this section is consistent with the Monetarist $k\%$ rule.

Section 4 presents the explicit values of the savings and that of money holding in the steady state.

In Section 5, a case with interest-producing government bonds instead of money will be considered. It will be shown that the debt-to-GDP ratio should be constant in a steady-state growth path, and the large propensity to consume leads to a small debt-to-GDP ratio.

In Section 6, we will prove that divergence of the debt-to-GDP ratio to infinity cannot occur even if the interest rate of the government bond is larger than the real growth rate, that is, fiscal collapse is impossible. In that case, inflation raises the nominal growth rate and prevents the debt-to-GDP ratio from diverging to infinity. The required inflation rate is such that the interest rate of the government bonds is smaller than the weighted average of the rate of return on capital and the nominal growth rate. Hyper-inflation will not occur. It is the interest on government bonds that causes divergence. Without interest, divergence does not occur.

Section 7 is the concluding section.

2. Methodology and literature review

A macroeconomic model that includes microeconomic foundations for consumers' behaviour and firms' behaviour will be used. Consumers are assumed to live infinitely, and utility maximisation over an infinite time is considered. People's utility depends on

their holding of money or government bonds as well as their consumption. A continuous time dynamic model will be used.

Economic growth is not based on the assumption that new generations will be born one after another, but rather on the assumption that the population of the same generation will increase. This can be interpreted as economic growth due to technological progress that increases labour productivity. As a result, the size of the economy continuously increases, and the accumulated financial assets must be discounted by the growth rate.

In the above discussion, the permanence of the state or mankind is assumed. Under this assumption, both government debt and private financial assets will continue to accumulate forever. If the destruction of the nation or the extinction of the human race are foreseen, people will try to use up all of their assets by then, so consumption will increase further, full employment can be maintained even with budget surpluses, and both private financial assets and government debt will gradually decline and disappear on the day of destruction.

In Sections 3 and 4, we consider budget deficit due to money issuance, not government bonds. In Sections 5 and 6, we examine the case of budget deficit by government bonds. Please see Oguri (2011) for the relationship between government bonds and money.

There is little literature written from the viewpoint of not considering the accumulation of government debt to be a problem, except for those who belong to MMT (Modern Money Theory, for example, Wray (2015), Mitchell, Wray and Watts (2019), Kelton (2020) or its periphery. Since MMT members do not like to use mathematical models, such papers are even scarcer. In this section, we would like to briefly mention some of the literature that we consulted in writing this paper.

Most of the discussions about the debt-to-GDP ratio use a simple calculation about primary budget balances, the interest rate, and the growth rate. In his 2022 and 2023 articles, Blanchard introduces the following problem: "When does the level of debt become unsafe? To answer this question, we need a definition of 'unsafe'. I propose the following: Debt becomes unsafe when there is a non-negligible risk that, under existing and likely future policies, the ratio of debt to GDP will steadily increase, leading to default at some point. The natural way to proceed is then straightforward. The dynamics of the debt ratio

depend on the evolution of three variables: primary budget balances (that is, spending net of interest payments minus revenues); the real interest rate (the nominal rate minus the rate of inflation); and the real rate of economic growth.” (Blanchard, 2002/3)

However, in a steady state under full employment with or without inflation, the necessary budget deficit for full employment is determined by several parameters of the economy. The larger budget deficit raises the inflation rate, and the large (or small) propensity to consume leads to the small (or large) budget deficit required for full employment under constant price or inflation. Therefore, the large propensity to consume leads to a small debt-to-GDP ratio.

While this paper uses a continuous time model that assumes people live infinitely, we have also analysed problems related to budget deficits using overlapping generation models. In doing so, we referred to Diamond (1965), J. Tanaka (2010, 2011a, 2011b, 2013) and Otaki (2007, 2009, 2015). As for economic growth, this paper uses an exogenous growth model, but we have also used an endogenous growth model due to investment by firms with reference to Grossman and Yanagawa (1993) and Maebayashi and J. Tanaka (2021).

There are papers, for example, Weil (1987, 1989), that use continuous time models which assume people live infinitely. However, in this research, the budget constraint in the discrete time model is considered implicitly in our mind with reference to Tachibana (2006), and the budget constraint in the continuous time model is derived by making the time interval of the discrete time model infinitesimal.

We follow Lerner’s functional finance theory (Lerner, 1944). He did not consider whether the government should run surpluses or deficits to be meaningful in and of itself. He believes that fiscal policy should be used to realise full employment avoiding inflation. For more on Lerner’s functional finance theory, see Forstater (1999).

Our model is a kind of neoclassical model, but its spirit may be post-Keynesian in that it does not abandon the goal of full employment out of a dislike of budget deficits or the accumulation of government debt. Lopez-Gallardo (2000) is a study of budget deficits and full employment from a post-Keynesian standpoint. It is inspired by Minsky (1986) and related to Mosler (1997-1998), Wray (1998) and Kregel (1998). Mosler, Wray and Kregel (M-W-K), as a policy for full

employment, proposed the following, as described in Lopez-Gallardo ((2000), p. 550):

“Let the government assume the role of employer of last resort at a given wage rate, so that anybody willing to work at that rate will get a job from the government. Government expenditure will thus expand, but will not entail any complication because “the purchasing ability of the government is limited only by what is available for sale in exchange for dollars” (Mosler, 1997-1998, p.169), while this availability, we are told, is elastic below full employment. Now, government expenditure will grow probably over and above tax receipts, and a budget deficit will ensue. However, M-W-K demonstrate, with explanations rich in theoretical, historical, and institutional details, that the government can simply create enough new money, or otherwise sell securities, to finance the deficit with an unchanging rate of interest.”

Lopez-Gallardo (2000) discusses various problems with this proposal, which are not of interest to this paper.

Another description from a post-Keynesian standpoint, according to Sawyer (2020), is that “Kalecki (1944) argued that there would be the need for permanent budget deficits in the face of intentions to save exceeding intentions to invest.”

This paper is also an example of an analysis, using a simple mathematical model, of the following statement by J. M. Keynes:

“Unemployment develops, that is to say, because people want the moon; — men cannot be employed when the object of desire (i.e. money) is something which cannot be produced and the demand for which cannot be readily choked off. There is no remedy but to persuade the public that green cheese is practically the same thing and to have a green cheese factory (i.e. a central bank) under public control.” (Keynes (1936), Chap. 17)

The goal of macroeconomic policy is, motto-wise, “full employment without inflation”. It is not right to be concerned about fiscal surpluses or deficits, since they are merely the means to that goal and the result of that goal. Whether or not to repay government debt with taxes should be determined based on how it will affect prices and employment. Repayment is not a natural assumption, and whether it is repaid or not has no value in itself.

3. Holding of money and budget deficit in a growing economy

3.1. Consumers' behaviour

Utility function

We consider an exogenous growth model in which consumers infinitely live and hold money for the reason of liquidity and so on.

The consumer's utility over an infinite time is

$$\int_{t=0}^{\infty} e^{-\delta t} u\left(c_t, \frac{m_t}{p_t}\right) dt. \quad (1)$$

The utility function is

$$u\left(c_t, \frac{m_t}{p_t}\right) = \alpha \ln c_t + (1 - \alpha) \ln \frac{m_t}{p_t}. \quad (2)$$

c_t is the real value of the consumption by a consumer, and p_t is the price of the good. m_t is the nominal value of the money holding of the consumer. Therefore, m_t/p_t is the real money holding. The consumer's utility depends on the consumption and the real money holding. $\delta > 0$ is the discount rate. α is the propensity to consume of the consumers. $0 < \alpha < 1$.

Budget constraint

The budget constraint for the consumer is

$$\dot{b}_t = (1 - \tau)w_t l_t - p_t c_t - r_t m_t + (r_t - n)b_t. \quad (3)$$

b_t is the per capita savings of the consumer, and $\dot{b}_t$ is the time derivative of b_t . Generally, the time derivative of a variable x is denoted by $\dot{x}$. w_t is the wage rate, l_t is an indicator of whether the consumer is employed or not. 1 if employed, 0 if not. The meanings of this equation are as follows.

1. $\dot{b}_t$ is the change in the per capita value of the savings.
2. $(1 - \tau)w_t l_t - p_t c_t$ is the difference between the per capita disposable labour income and consumption.
3. $b_t - m_t$ is the portion of the per capita savings that is invested in productive capital, which generates interest (return) r_t .
4. We assume that people live infinitely and no new generation will be born, however, the current generation's

population will grow. Hence the savings must be discounted by the growth rate as expressed by $-nb_t$.

Utility maximisation

The present value Hamiltonian is written as follows.

$$H_t = e^{-\delta t} u\left(c_t, \frac{m_t}{p_t}\right) + \lambda_t [(1 - \tau)w_t l_t - p_t c_t - r_t m_t + (r_t - n)b_t]. \quad (4)$$

λ_t is the Lagrange multiplier. The first order conditions are

$$\frac{\partial H_t}{\partial c_t} = e^{-\delta t} \frac{\alpha}{c_t} - \lambda_t p_t = 0, \quad (5)$$

and

$$\frac{\partial H_t}{\partial m_t} = e^{-\delta t} \frac{1 - \alpha}{m_t} - \lambda_t r_t = 0. \quad (6)$$

The costate equation is

$$\frac{\partial H_t}{\partial b_t} = (r_t - n)\lambda_t = -\dot{\lambda}_t. \quad (7)$$

By (5) and (6), we get

$$c_t = \frac{\alpha}{p_t} [(1 - \tau)w_t l_t + (r_t - n)b_t - \dot{b}_t],$$

and

$$m_t = \frac{1 - \alpha}{r_t} [(1 - \tau)w_t l_t + (r_t - n)b_t - \dot{b}_t].$$

They mean

$$\frac{e^{-\delta t}}{\lambda_t} = (1 - \tau)w_t l_t + (r_t - n)b_t - \dot{b}_t.$$

Thus,

$$\lambda_t = \frac{\alpha}{p_t c_t} e^{-\delta t} = \frac{1 - \alpha}{r_t m_t} e^{-\delta t}.$$

Differentiating this with respect to t ,

$$\dot{\lambda}_t = -\delta \frac{1-\alpha}{r_t m_t} e^{-\delta t} = -\delta \lambda_t.$$

Then, from (7) we find

$$r_t = n + \delta. \quad (9)$$

This is the equilibrium interest rate (rate of return).

Steady state

Let us consider a steady state. Under constant prices, w_t , c_t , m_t and b_t are constant in the steady state. They are the wage rate, and the per capita values of real consumption, nominal money holding and nominal savings. Then,

$$\dot{b}_t = 0.$$

On the other hand, under inflation at a constant rate π , c_t is constant, but w_t , m_t and b_t increases at the rate of π . Then,

$$\dot{b}_t = b_t \pi.$$

Denote the labour supply or the employment under full employment by L_t^f . Also, we denote

$$B_t = b_t L_t^f, C_t = c_t L_t^f, M_t = m_t L_t^f.$$

The real value of the capital is

$$K_t = \frac{B_t - M_t}{p_t}.$$

Denote the real capital per labour under full employment by

$$k_t = \frac{K_t}{L_t^f}.$$

In the steady state under full employment,

$$\dot{k}_t = 0.$$

3.2. Firms' behaviour

Let y_t be the output, K_t be the capital, and L_t be the employment of a firm. Then, the production function is written as follows.

$$y_t = F(K_t, L_t) = L_t f(k_t) = L_t F(k_t, 1).$$

We assume the constant returns to scale property for the production function. We normalise so that the number of firms is one. Each firm maximises its profit. The profit of a firm is

$$p_t y_t - p_t r_t K_t - w_t L_t = p_t L_t f(k_t) - p_t r_t K_t - w_t L_t.$$

The first order conditions for profit maximisation are

$$p_t r_t = p_t \frac{\partial F}{\partial K_t} = p_t f'(k_t),$$

and

$$w_t = p_t \frac{\partial F}{\partial L_t} = p_t [f(k_t) - f'(k_t) k_t].$$

$\frac{\partial F}{\partial K_t}$ and $\frac{\partial F}{\partial L_t}$ are the marginal productivity of capital and that of labour. From them, we have

$$w_t L_t = p_t [f(k_t) - f'(k_t) k_t] L_t,$$

and

$$p_t r_t K_t = p_t f'(k_t) K_t = p_t f'(k_t) k_t L_t$$

Then, we obtain

$$p_y y_t = w_t L_t + p_t r_t K_t.$$

This is the total nominal supply of the good.

The real value of the capital increases at the rate of n . Then,

$$\dot{K}_t = n K_t.$$

$\dot{K}_t$ is the time derivative of K_t . This increase in capital is the investment. We assume full employment. Therefore, $l_t = 1$ for all people.

3.3. Market equilibrium

We consider two cases, with and without inflation.

Without inflation

Denote the constant price by $\bar{p}$. In the steady state under full employment and constant price, the total consumption demand is

$$\bar{p}C_t = L_t^f \bar{p}c_t = \alpha[(1-\tau)w_t + (r_t - n)b_t] L_t^f = \alpha[(1-\tau)w_t L_t^f + (r_t - n)B_t].$$

The total money holding is

$$M_t = L_t^f m_t = \frac{1-\alpha}{r_t} [(1-\tau)w_t + (r_t - n)b_t] L_t^f = \frac{1-\alpha}{r_t} [(1-\tau)w_t L_t^f + (r_t - n)B_t]. \quad (10)$$

Let G_t be the nominal value of the fiscal expenditure. The total nominal demand is

$$G_t + \alpha[(1-\tau)w_t L_t^f + (r_t - n)B_t] + \bar{p}nK_t.$$

$\bar{p}nK_t$ is the nominal investment. The market clearing condition is

$$G_t + \alpha[(1-\tau)w_t L_t^f + (r_t - n)B_t] + \bar{p}nK_t = w_t L_t^f + \bar{p}r_t K_t. \quad (11)$$

From this we obtain (see Appendix 1)

$$G_t - \tau w_t L_t^f = nM_t. \quad (12)$$

So long as $0 < \alpha < 1$ and $n > 0$, this is positive. It is the budget deficit. Therefore, we have shown the following result.

Proposition 1

If consumers' utility depends on holding of money, a budget deficit is necessary for economic growth under full employment and constant prices.

Suppose that M_t is given. If, prior to that time, full employment was achieved, the budget deficit shown in (12) is necessary and sufficient for continuous full employment without inflation.

The policy of increasing money through budget deficits in line with the rate of economic growth as indicated by (12) is consistent with the Monetarist (Friedman's) k% rule (Halton, 2023).

Under inflation at a constant rate of π

In this case,

$$\dot{b}_t = \pi b_t.$$

Therefore, (10), (11), (A-1) in Appendix 1 and (12) are rewritten as

$$M_t = \frac{1-\alpha}{r_t} [(1-\tau)w_t L_t^f + (r_t - n)B_t - \pi B_t],$$

$$G_t + \alpha[(1-\tau)w_t L_t^f + (r_t - n)B_t - \pi B_t]$$

$$+ p_t n K_t = w_t L_t^f + p_t r_t K_t,$$

$$G_t - \tau w_t L_t^f + \alpha[(1-\tau)w_t L_t^f + (r_t - n)B_t - \pi B_t] + (n - r_t)B_t - nM_t + r_t M_t = (1-\tau)w_t L_t^f,$$

and

$$G_t - \tau w_t L_t^f = nM_t + \pi B_t.$$

This is greater than the value in (12). It means the following results.

Proposition 2

1. A budget deficit larger than its value under full employment and constant prices in (12) leads to inflation.

Or,

2. To achieve full employment under inflation, a budget deficit larger than that under constant prices in (12) is required.

4. Explicit values of the savings and money holding in the steady state

Without inflation

From (9) in the steady state, the value of the interest rate is

$$r_t = n + \delta.$$

Denote this value with $\tilde{r}$. Then, the capital-labour ratio, k_t , which is constant in the steady state, satisfies

$$f'(k_t) = \tilde{r}. \quad (13)$$

Denote this value of k_t with $\tilde{k}$. Then, the steady-state value of the capital is

$$\tilde{K}_t = \tilde{k} L_t^f,$$

also, we have

$$\tilde{K}_t = \frac{B_t - M_t}{\bar{p}}. \quad (14)$$

The steady-state value of the nominal wage rate is

$$w_t = \bar{p}[f(\tilde{k}) - f'(\tilde{k})\tilde{k}].$$

The steady-state value of the money holding is

$$M_t = (1 - \alpha) \frac{1}{\tilde{r}} [(1 - \tau)w_t L_t^f + (\tilde{r} - n)B_t].$$

It is rewritten as

$$M_t = (1 - \alpha) \frac{1}{\tilde{r}} (1 - \tau)w_t L_t^f + (1 - \alpha) \frac{\tilde{r} - n}{\tilde{r}} B_t. \quad (15)$$

From (14),

$$B_t - M_t = \bar{p}\tilde{K}_t \quad (16)$$

By (15) and (16), we obtain

$$B_t = \frac{(1 - \alpha) \frac{1}{\tilde{r}} (1 - \tau)w_t L_t^f + \bar{p}\tilde{K}_t}{1 - (1 - \alpha) \frac{\tilde{r} - n}{\tilde{r}}}. \quad (17)$$

This is the explicit solution for the value of the savings. By similar calculations, we get

$$\begin{aligned} M_t &= \frac{(1 - \alpha) \frac{1}{\tilde{r}} (1 - \tau)w_t L_t^f + \bar{p}\tilde{K}_t}{1 - (1 - \alpha) \frac{\tilde{r} - n}{\tilde{r}}} - \bar{p}\tilde{K}_t \quad (18) \\ &= \frac{(1 - \alpha) \frac{1}{\tilde{r}} (1 - \tau)w_t L_t^f + (1 - \alpha) \frac{\tilde{r} - n}{\tilde{r}} \bar{p}\tilde{K}_t}{1 - (1 - \alpha) \frac{\tilde{r} - n}{\tilde{r}}} \\ &= (1 - \alpha) \frac{(1 - \tau)w_t L_t^f + (\tilde{r} - n)\bar{p}\tilde{K}_t}{\tilde{r} - (1 - \alpha)(\tilde{r} - n)} \\ &= (1 - \alpha) \frac{(1 - \tau)w_t L_t^f + (\tilde{r} - n)\bar{p}\tilde{K}_t}{\alpha\tilde{r} + (1 - \alpha)n}. \end{aligned}$$

This is the explicit solution for the value of the money holding.

Under inflation at a constant rate of π

Under inflation at a constant rate of π ,

$$M_t = (1 - \alpha) \frac{1}{\tilde{r}} [(1 - \tau)w_t L_t^f + (\tilde{r} - n)B_t - \pi B_t].$$

Therefore, we get

$$B_t = \frac{(1 - \alpha) \frac{1}{\tilde{r}} (1 - \tau)w_t L_t^f + p_t \tilde{K}_t}{1 - (1 - \alpha) \frac{\tilde{r} - n - \pi}{\tilde{r}}}$$

and

$$\begin{aligned} M_t &= (1 - \alpha) \frac{(1 - \tau)w_t L_t^f + (\tilde{r} - n - \pi)p_t \tilde{K}_t}{\tilde{r} - (1 - \alpha)(\tilde{r} - n - \pi)} \\ &= (1 - \alpha) \frac{(1 - \tau)w_t L_t^f + (\tilde{r} - n - \pi)p_t \tilde{K}_t}{\alpha\tilde{r} + (1 - \alpha)(n + \pi)}. \end{aligned}$$

5. Government bond holding in the steady state

In this section, we consider the case where financial assets are held not in money but in interest-producing government bonds that have almost the same liquidity as money.

Budget constraint and utility maximisation

Let $i < \tilde{r}$ be the interest rate of the government bonds. It is usually smaller than the rate of return on capital for the risk premium. The budget constraint for the consumer is¹

$$\dot{b}_t = (1 - \tau) w_t l_t - p_t c_t - (r_t - i) m_t + (r_t - n) b_t.$$

m_t is the interest income from the government bond holding. Except for that, it is the same as (3).

The consumption and the government bond holding are

$$c_t = \frac{\alpha}{p_t} [(1 - \tau)w_t l_t + (r_t - n)b_t - \dot{b}_t],$$

and

$$m_t = \left(\frac{1 - \alpha}{r_t - i} \right) [(1 - \tau)w_t l_t + (r_t - n)b_t - \dot{b}_t]. \quad (21)$$

¹ We do not consider taxation of interest on government bonds, but it could be included. An interest tax would reduce the likelihood of a divergence of the debt-to-GDP ratio.

Without inflation

The total bond holding is

$$M_t = m_t L_t^f = \frac{1-\alpha}{r_t-i} [(1-\tau)w_t + (r_t-n)b_t] L_t^f = \frac{1-\alpha}{r_t-i} [(1-\tau)w_t L_t^f + (r_t-n)B_t]. \quad (22)$$

B_t is the total savings. From this

$$(r_t-i)M_t = (1-\alpha)[(1-\tau)w_t L_t^f + (r_t-n)B_t].$$

Then, (12) is rewritten as (please see Appendix 1)

$$G_t - \tau w_t L_t^f + iM_t = nM_t. \quad (23)$$

The left-hand side is the budget deficit including interest payments on the government bonds. Therefore, the same conclusion as that without interest on the government bonds is obtained.

In the steady-state growth path, GDP increases at the rate of n . The government debt M_t also increases at the rate of n . Therefore, when the economy is in a steady-state growth path, the debt-to-GDP ratio is constant, and the larger the value of α (propensity to consume) is, the smaller the debt-to-GDP ratio is.

Let $r_t = \tilde{r}$. Then, (22) is rewritten as

$$M_t = (1-\alpha) \frac{1}{\tilde{r}-i} (1-\tau)w_t L_t^f + (1-\alpha) \frac{\tilde{r}-n}{\tilde{r}-i} B_t$$

Then, (17) and (18) are rewritten as

$$B_t = \frac{(1-\alpha) \frac{1}{\tilde{r}-i} (1-\tau)w_t L_t^f + \tilde{p}\tilde{K}_t}{1 - (1-\alpha) \frac{\tilde{r}-n}{\tilde{r}-i}},$$

and

$$M_t = (1-\alpha) \frac{(1-\tau)w_t L_t^f + (\tilde{r}-n)\tilde{p}\tilde{K}_t}{\alpha\tilde{r} + (1-\alpha)n - i}.$$

This value has the same dimension as GDP.

Since

$$\tilde{r} = n + \delta,$$

the denominator of M_t is

$$\alpha\tilde{r} + (1-\alpha)n - i = n - i + \alpha\delta.$$

This must be positive. It means

$$i < n + \alpha\delta.$$

Under inflation at a constant rate of π

Under inflation, (22) is rewritten as

$$M_t = \frac{1-\alpha}{r_t-i} [(1-\tau)w_t L_t^f + (r_t-n)B_t - \pi B_t].$$

Then, (23) is rewritten as

$$G_t - \tau w_t L_t^f + iM_t = nM_t + \pi B_t. \quad (24)$$

(19) and (20) are rewritten as

$$B_t = \frac{(1-\alpha) \frac{1}{\tilde{r}-i} (1-\tau)w_t L_t^f + p_t \tilde{K}_t}{1 - (1-\alpha) \frac{\tilde{r}-n-\pi}{\tilde{r}-i}},$$

and

$$M_t = (1-\alpha) \frac{(1-\tau)w_t L_t^f + (\tilde{r}-n-\pi)p_t \tilde{K}_t}{\alpha\tilde{r} + (1-\alpha)(n+\pi) - i}.$$

This value has the same dimension as GDP. The denominator of M_t is

$$\alpha\tilde{r} + (1-\alpha)(n+\pi) - i = n - i + \alpha\delta + (1-\alpha)\pi.$$

This must be positive. It means

$$i < n + \alpha\delta + (1-\alpha)\pi. \quad (25)$$

This is the same as (30) in the next section, which is the condition for convergence of the debt-to-GDP ratio.

6. Impossibility of divergence of the debt-to-GDP ratio

Similarly to the previous section, financial assets are held not in money but in interest-producing government bonds. By (14),

$$\tilde{k} = \frac{b_t - m_t}{\bar{p}}.$$

It is constant. Then, under constant prices,

$$\dot{b}_t = \dot{m}_t.$$

On the other hand, under inflation at a constant rate of π , since

$$\tilde{k} = \frac{b_t - m_t}{p_t},$$

$$\dot{b}_t - \dot{m}_t = \tilde{k}\dot{p}_t = p_t\tilde{k}\pi = (b_t - m_t)\pi. \quad (26)$$

Suppose that $l_t=1$ (full employment), and that $r_t=\tilde{r}$. But b_t and m_t are not necessarily the equilibrium steady-state values, and they may change over time. The money holding of a consumer is

$$m_t = \frac{1-\alpha}{\tilde{r}-i} [(1-\tau)w_t l_t + (\tilde{r}-n)b_t - \dot{b}_t].$$

Without inflation

Without inflation,

$$\dot{m}_t = \left(\frac{1-\alpha}{\tilde{r}-i} \right) [(\tilde{r}-n)\dot{b}_t - \frac{\partial}{\partial t} \dot{b}_t].$$

In this case w_t is constant. This equation implies

$$\frac{\partial}{\partial t} \dot{b}_t = (\tilde{r}-n)\dot{b}_t - \left(\frac{\tilde{r}-i}{1-\alpha} \right) \dot{m}_t.$$

Since $\dot{b}_t = \dot{m}_t$,

$$\frac{\partial}{\partial t} \dot{m}_t = (\tilde{r}-n)\dot{m}_t - \left(\frac{\tilde{r}-i}{1-\alpha} \right) \dot{m}_t.$$

From (9), $\tilde{r}-n=\delta$. Therefore,

$$\frac{\partial}{\partial t} \dot{m}_t = \left(\frac{i-n-\alpha\delta}{1-\alpha} \right) \dot{m}_t. \quad (27)$$

Note that m_t is the per capita value of the bond holding. The total bond holding is $L_t^f m_t$, and L_t^f increases at the rate of n . Thus, m_t has the same dimension as the debt-to-GDP ratio². We get the following proposition.

Proposition 3

(27) means that when $i > n + \alpha\delta$, m_t increases over time if it is positive. Then, the debt-to-GDP ratio diverges. When $i < n + \alpha\delta$, m_t decreases over time if it is positive. Then, m_t converges to zero, and the debt-to-GDP ratio converges to a finite value.

Under inflation at a constant rate of π

Since GDP increases at the rate of n under constant prices, m_t , the per capita government debt holding, has the same dimension as the debt-to-GDP ratio. However, when inflation continues at a constant rate, the per capita government debt holding divided by the price level has the same dimension as the debt-to-GDP ratio. We have

$$\frac{\partial}{\partial t} \left(\frac{m_t}{p_t} \right) = \frac{1}{p_t} \left(\dot{m}_t - \frac{m_t}{p_t} \dot{p}_t \right) = \frac{1}{p_t} (\dot{m}_t - m_t \pi),$$

and

$$\begin{aligned} \frac{\partial}{\partial t} \left[\frac{\partial}{\partial t} \left(\frac{m_t}{p_t} \right) \right] &= \frac{1}{p_t} \left(\frac{\partial}{\partial t} \dot{m}_t - 2\dot{m}_t \pi + m_t \pi^2 \right) \\ &= \frac{1}{p_t} \left(\frac{\partial}{\partial t} \dot{m}_t - \dot{m}_t \pi \right) - \frac{\partial}{\partial t} \left(\frac{m_t}{p_t} \right) \pi. \end{aligned} \quad (28)$$

Note that π is assumed to be constant. By some calculations with $\tilde{r}-n=\delta$, we obtain (Appendix 2)

² The debt-to-GDP ratio is the total bond holding divided by GDP, which is equal to the per capita government bond holding divided by the per capita GDP. If the population grows, the per capita GDP is constant, so the per capita government bond holding and the debt-to-GDP ratio are of the same dimension.

$$\frac{\partial}{\partial t} \left[\frac{\partial}{\partial t} \left(\frac{m_t}{p_t} \right) \right] = \left(\frac{i - n - \alpha\delta - (1 - \alpha)\pi}{1 - \alpha} \right) \frac{\partial}{\partial t} \left(\frac{m_t}{p_t} \right).$$

We get the following proposition.

Proposition 4

(29) means that when

$$i > n + \alpha\delta + (1 - \alpha)\pi,$$

$\frac{\partial}{\partial t} \left(\frac{m_t}{p_t} \right)$ increases over time if it is positive. Then, the debt-to-GDP ratio diverges under inflation at the rate of π . On the other hand, when

$$i < n + \alpha\delta + (1 - \alpha)\pi, \quad (30)$$

$\frac{\partial}{\partial t} \left(\frac{m_t}{p_t} \right)$ decreases over time if it is positive. Then, $\frac{\partial}{\partial t} \left(\frac{m_t}{p_t} \right)$ converges to zero, and the debt-to-GDP ratio converges to a finite value. (30) is the same as (25) in the previous section.

Since $\tilde{r} = n + \delta$, (30) implies

$$i < \alpha\tilde{r} + (1 - \alpha)(n + \pi). \quad (31)$$

The meaning of (31) is that the interest rate of the government bonds should be smaller than the weighted average of the rate of return on capital $\tilde{r}$ and the nominal growth rate $n + \pi$.

Impossibility of fiscal collapse and prevention of debt-to-GDP ratio divergence by mild inflation

These discussions mean that in (23) when $i > n + \alpha\delta$, M_t diverges to infinity relative to $G_t - \tau w_t L_t^f$ under constant prices. However, then (23) cannot hold. An increase in M_t will increase interest income and thereby consumption. As the debt-to-GDP ratio increases, this increase in consumption is expected to lead to excess demand, which in turn will cause prices to rise. Eventually, we have

$$i < n + \alpha\delta + (1 - \alpha)\pi,$$

and the debt-to-GDP ratio converges to a finite value. These discussions mean the following proposition.

Proposition 5

The divergence of the debt-to-GDP ratio to infinity cannot occur. It is prevented by inflation. The inflation rate does not have to be very large. It is sufficient to have a value such that (30) or (31) is satisfied. There is no danger of the so-called *hyper-inflation*, and as stated in the introduction, this is a naturally occurring phenomenon, not caused by policy. It is the interest on government bonds that causes inflation here; without interest on the government bonds, there would be no inflation.

7. Concluding remarks

7.1. Policy implications

We have made it clear that under the assumption that people derive utility from their holding of money or government bonds, budget deficits are essential for an economy to grow at constant prices or under continuous inflation. However, it has also been proven that the government debt-to-GDP ratio, which many economists and journalists are concerned about as a criterion for fiscal collapse or as a measure of fiscal health, cannot diverge. Therefore, macroeconomic policies should be managed in such a way as to prevent the economy from falling into a recession while avoiding as high a rate of inflation as possible, and not to be preoccupied with the fiscal balance of payments.

In this paper, we used a continuous time dynamic model to study the role of budget deficits in a growing economy in which consumers hold money or government bonds for liquidity and live forever. We have shown that the debt-to-GDP ratio cannot diverge even when the budget deficit is financed by interest-producing government bond issuance.

In Japan and other countries, there is often concern about the accumulation of government debt and the increase in the debt-to-GDP ratio. However, the goal of macroeconomic policy is to achieve near-full employment while avoiding inflation as much as possible, not to control government debt. This paper has shown that rising debt-to-GDP ratios are not a concern.

7.2. Further remarks

There is a persistent belief among many that government debt should eventually be repaid by taxpayers, and that new government spending through the issuance of bonds would place a burden on future generations. However, we believe this may not be the case. Regardless of who purchases the bonds, government spending increases the financial assets of those who receive the spending, while taxation decreases their financial assets. The difference is the budget deficit, and if the budget deficit continues, the financial assets held by the public will continue to increase along with government debt. What problems will the accumulated financial assets cause? It is thought that people's consumption depends not only on their income but also on their accumulated assets, including interest on government bonds. Therefore, if the financial assets held by the public increase along with government debt, consumption will increase under full employment, leading to higher prices. Although the government debt-to-GDP ratio tends to be more problematic than the total amount of government debt, if prices rise and nominal GDP increases, the government debt-to-GDP ratio will decline. The analysis in this paper focused on this point. As mentioned in the introduction, the net external debt accumulated by the current account deficit must eventually be repaid, and if it becomes too large, it will become an obstacle to the economy, but as long as the government debt remains domestic, there is no problem.

In many countries around the world, and especially in Japan, the question of financial resources is always raised when the government implements any kind of policy. Raising taxes reduces people's disposable income, which in turn reduces consumption and worsens the economy. When the economy is strong and full employment has been achieved, new government spending becomes an inflationary factor, and tax increases may be necessary to control it. However, even when the economy is not doing well, the issue of financial resources is discussed. This is because the redemption of government bonds is a prerequisite. Government spending increases demand, but taxes reduce demand by reducing disposable income. Is it not the government's duty to strike a reasonable balance between the two and implement the necessary public policies? Even if government policies result in budget deficits and accumulated government debt, is there not a problem if full employment is maintained at stable prices, i.e., constant prices or mild inflation?

Pensions and other social security benefits should be promoted, and if technically feasible, the construction of airports and new high-speed railroads should also be promoted. The technology and productive capacity of the country and its people are the constraints on the economy, not the financial resources.

References

- Blanchard, O. (2022). *Deciding When Debt Becomes Unsafe*. International Monetary Fund. <https://www.imf.org/en/Publications/fandd/issues/2022/03/Deciding-when-debt-becomes-unsafe-Blanchard#:~:text=When%20does%20the%20level%20of,to%20default%20at%20some%20point>.
- Blanchard, O. (2023). *Fiscal Policy under Low Interest Rates*. The MIT Press, Cambridge, Massachusetts, United States.
- Diamond, P. (1965). National Debt in a Neoclassical Growth Model. *American Economic Review*. 55, 1126–1150.
- Forstater, M. (1999). Functional Finance and Full Employment: Lessons from Lerner for Today? *Working Paper No. 272*. Levy Economics Institute, Bard College, Annandale-on-Hudson, New York, United States.
- Grossman, G., & Yanagawa, N. (1993). Asset Bubbles And Endogenous Growth. *Journal of Monetary Economics*. 31, 3–19. [https://doi.org/10.1016/0304-3932\(93\)90014-7](https://doi.org/10.1016/0304-3932(93)90014-7).
- Halton, C. (2023). *K-Percent Rule: What It is, How It Works*. <https://www.investopedia.com/terms/k/k-percent-rule.asp#:~:text=The%20K-Percent%20Rule%20was%20a%20proposal%20by%20economist,growth%20of%20gross%20domestic%20product%20%28GDP%29%20each%20year>.
- Kalecki, M. (1944). The White Paper on Employment Policy. *Bulletin of the Oxford University Institute of Statistics*. 6, 137–144.
- Kelton, S. (2020). *The Deficit Myth: Modern Monetary Theory and the Birth of the People's Economy*. Public Affairs, New York, United States.
- Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money*. Macmillan, London, U.K.
- Kregel, J. (1998). Price Stability & Full Employment as Complements in a New Europe. *Paper presented at*

the Post Keynesian Workshop. University of Tennessee, Knoxville.

Lerner, A. P. (1944). *The Economics of Control: Principles of Welfare Economics*. Macmillan, London, U.K.

Lopez-Gallardo, J. (2000). Budget Deficit and Full Employment. *Journal of Post Keynesian Economics*. 22, 549–563. <https://doi.org/10.1080/01603477.2000.11490258>.

Maebayashi, N., & Tanaka, J. (2021). Limited Asset Market Participation and Fiscal Sustainability. *The Society for Economic Studies The University of Kitakyushu Working Paper Series No. 2020-4*. https://www.kitakyu-u.ac.jp/economy/uploads/2020_4.pdf

Mitchell, W., Wray, L. R., & Watts, M. (2019). *Macroeconomics*. Red Globe Press, London, U.K.

Minsky, H. (1986). *Stabilizing an Unstable Economy*. New Haven: Yale University Press.

Mosler, W. (1997–1998). Full Employment and Price Stability. *Journal of Post Keynesian Economics*. 20, 167–182. <https://doi.org/10.1080/01603477.1997.11490146>.

Oguri, S. (2011). On the Nature of Government Bank Notes (in Japanese). *Hikone Ronso*. 2011 Winter, 118–133.

Otaki, M. (2007). The Dynamically Extended Keynesian Cross and the Welfare-Improving Fiscal Policy. *Economics Letters*. 96, 23–29. <https://doi.org/10.1016/j.econlet.2006.12.005>.

Otaki, M. (2009). A Welfare Economics Foundation for the Full-Employment Policy. *Economics Letters*. 102, 1–3. <https://doi.org/10.1016/j.econlet.2008.08.003>.

Otaki, M. (2015). *Keynesian Economics and Price Theory: Re-orientation of a Theory of Monetary Economy*. Springer Japan, Tokyo, Japan.

Sawyer, M. (2020). Kalecki on Budget Deficits and the Possibilities for Full Employment. *Review of Political Economy*. 32, 548–562. <https://doi.org/10.1080/09538259.2020.1831203>.

Tachibana, T. (2006). *Maximum Principle (in Japanese)*, mimeo. https://www.le.chiba-u.jp/~ttachi/doc/Derive_Hamiltonian_110625.pdf.

Tanaka, J. (2010). *Keynes keizaigaku no kiso (in Japanese)*. Kyushu Daigaku Shuppankai, Kitakyushu, Japan.

Tanaka, J. (2011a). Macroeconomic Theory Based on “Non-Walrasian Approach” Chapter 2 IS-LM Model and Its Application (in Japanese). *The Society for Economic Studies The University of Kitakyushu Working Paper Series No. 2011-2*. https://www.kitakyu-u.ac.jp/economy/uploads/2011_01.pdf.

Tanaka, J. (2011b). Macroeconomic Theory Based on “Non-Walrasian Approach” Chapter 3 AD-AS Model and Its Application (in Japanese). *The Society for Economic Studies The University of Kitakyushu Working Paper Series No. 2011-4*. https://www.kitakyu-u.ac.jp/economy/uploads/2011_03.pdf.

Tanaka, J. (2013). Welfare Analysis of Fiscal Policies in a Fixed Price Overlapping Generations Model. *The Society for Economic Studies The University of Kitakyushu Working Paper Series No. 2012-11*. https://www.kitakyu-u.ac.jp/economy/uploads/2012_11.pdf.

Weil, P. (1987). Permanent Budget Deficits and Inflation. *Journal of Monetary Economics*. 20, 393–410. [https://doi.org/10.1016/0304-3932\(87\)90022-5](https://doi.org/10.1016/0304-3932(87)90022-5).

Weil, P. (1989). Overlapping Families of Infinitely-Lived Agents. *Journal of Public Economics*. 38, 183–198. [https://doi.org/10.1016/0047-2727\(89\)90024-8](https://doi.org/10.1016/0047-2727(89)90024-8).

Wray, L. R. (1998). Zero Unemployment and Stable Prices. *Journal of Economic Issues*. 32, 539–545. <https://doi.org/10.1080/00213624.1998.11506062>.

Wray, L. R. (2015). *Modern Money Theory: A Primer on Macroeconomics for Sovereign Monetary Systems* (2nd ed.). Palgrave Macmillan, New York, United States.

Appendices

1. Derivations of (12) and (23)

Arranging (11),

$$G_t - \tau w_t L_t^f + \alpha[(1 - \tau)w_t L_t^f + (r_t - n)B_t] + \bar{p}nK_t = (1 - \tau)w_t L_t^f + \bar{p}r_t K_t.$$

Further, by $\bar{p}K_t = B_t - M_t$, we get

$$G_t - \tau w_t L_t^f + \alpha[(1 - \tau)w_t L_t^f + (r_t - n)B_t] + n(B_t - M_t) = (1 - \tau)w_t L_t^f + r_t(B_t - M_t).$$

Then,

$$G_t - \tau w_t L_t^f + \alpha[(1 - \tau)w_t L_t^f + (r_t - n)B_t] + (n - r_t)B_t - nM_t + r_t M_t = (1 - \tau)w_t L_t^f. \quad (\text{A-1})$$

From (10),

$$r_t M_t = (1 - \alpha)[(1 - \tau)w_t L_t^f + (r_t - n)B_t].$$

Substituting this into (A-1), we get

$$G_t - \tau w_t L_t^f + [(1 - \tau)w_t L_t^f + (r_t - n)B_t] - (r_t - n)B_t - nM_t = (1 - \tau)w_t L_t^f.$$

Then, we obtain

$$G_t - \tau w_t L_t^f = nM_t.$$

In the case of government bond holding we have

$$r_t M_t = (1 - \alpha)[(1 - \tau)w_t L_t^f + (r_t - n)B_t] + iM_t.$$

Therefore, we get

$$G_t - \tau w_t L_t^f + iM_t = nM_t.$$

2. Derivation of (29)

From (21),

$$\dot{m}_t = \left(\frac{1 - \alpha}{\tilde{r} - i} \right) \left[(1 - \tau)w_t \pi + (\tilde{r} - n)\dot{b}_t - \frac{\partial}{\partial t} \dot{b}_t \right].$$

In this case w_t increases at the rate of π . This equation means

$$\frac{\partial}{\partial t} \dot{b}_t = (1 - \tau)w_t \pi + (\tilde{r} - n)\dot{b}_t - \left(\frac{\tilde{r} - i}{1 - \alpha} \right) \dot{m}_t.$$

Since, from (26) $\dot{b}_t - \dot{m}_t = (b_t - m_t)\pi$, we obtain

$$\frac{\partial}{\partial t} \dot{b}_t - \frac{\partial}{\partial t} \dot{m}_t = (\dot{b}_t - \dot{m}_t)\pi.$$

This means

$$\frac{\partial}{\partial t} \dot{m}_t = \frac{\partial}{\partial t} \dot{b}_t - (\dot{b}_t - \dot{m}_t)\pi.$$

Therefore,

$$\frac{\partial}{\partial t} \dot{m}_t = (1 - \tau)w_t\pi + (\tilde{r} - n)\dot{b}_t - \left(\frac{\tilde{r} - i}{1 - \alpha}\right)\dot{m}_t - (\dot{b}_t - \dot{m}_t)\pi.$$

Again by (21),

$$(1 - \tau)w_t = \left(\frac{\tilde{r} - i}{1 - \alpha}\right)m_t - (\tilde{r} - n)b_t + \dot{b}_t.$$

From this,

$$\begin{aligned} \frac{\partial}{\partial t} \dot{m}_t &= \left[\left(\frac{\tilde{r} - i}{1 - \alpha}\right)m_t - (\tilde{r} - n)b_t + \dot{b}_t\right] \pi + (\tilde{r} - n)\dot{b}_t - \left(\frac{\tilde{r} - i}{1 - \alpha}\right)\dot{m}_t - (\dot{b}_t - \dot{m}_t)\pi \\ &= \left[\left(\frac{\tilde{r} - i}{1 - \alpha}\right)m_t - (\tilde{r} - n)b_t\right] \pi + (\tilde{r} - n)\dot{b}_t - \left(\frac{\tilde{r} - i}{1 - \alpha}\right)\dot{m}_t + \dot{m}_t\pi. \end{aligned}$$

By $\dot{b}_t - \dot{m}_t = (b_t - m_t)\pi$, $\dot{b}_t - b_t\pi = \dot{m}_t - m_t\pi$.

Thus,

$$\frac{\partial}{\partial t} \dot{m}_t = \left[\left(\frac{\tilde{r} - i}{1 - \alpha}\right)m_t - (\tilde{r} - n)m_t\right] \pi + (\tilde{r} - n)\dot{m}_t - \left(\frac{\tilde{r} - i}{1 - \alpha}\right)\dot{m}_t + \dot{m}_t\pi,$$

Therefore,

$$\begin{aligned} \frac{\partial}{\partial t} \dot{m}_t - \dot{m}_t\pi &= \left[\left(\frac{\tilde{r} - i}{1 - \alpha}\right)m_t - (\tilde{r} - n)m_t\right] \pi + (\tilde{r} - n)\dot{m}_t - \left(\frac{\tilde{r} - i}{1 - \alpha}\right)\dot{m}_t = \left(\tilde{r} - n - \frac{\tilde{r} - i}{1 - \alpha}\right)(\dot{m}_t - m_t\pi) \\ &= \left(\tilde{r} - n - \frac{\tilde{r} - i}{1 - \alpha}\right)\frac{\partial}{\partial t}\left(\frac{m_t}{p_t}\right)p_t. \end{aligned}$$

Then, by (28)

$$\frac{\partial}{\partial t} \left[\frac{\partial}{\partial t} \left(\frac{m_t}{p_t}\right)\right] = \left(\tilde{r} - n - \frac{\tilde{r} - i}{1 - \alpha}\right)\frac{\partial}{\partial t} \left(\frac{m_t}{p_t}\right) - \frac{\partial}{\partial t} \left(\frac{m_t}{p_t}\right)\pi = \left(\tilde{r} - n - \frac{\tilde{r} - i}{1 - \alpha} - \pi\right)\frac{\partial}{\partial t} \left(\frac{m_t}{p_t}\right).$$

Since $\tilde{r} - n = \delta$,

$$\frac{\partial}{\partial t} \left[\frac{\partial}{\partial t} \left(\frac{m_t}{p_t}\right)\right] = \left(\frac{i - n - \alpha\delta - (1 - \alpha)\pi}{1 - \alpha}\right)\frac{\partial}{\partial t} \left(\frac{m_t}{p_t}\right).$$