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MODELING OF ACTIVE BENDING BY THE METHOD NEWTON-RAPHSON AND THE MODIFIED NEWTON-RAPHSON

Roman HERDA[1,*], Miloš SLIVANSKÝ[1], Jaroslav SANDANUS[1]

[1] Department of Steel and Timber Structures, Slovak University of Technology, Bratislava, Slovakia,
[*] corresponding author: roman.herda@stuba.sk

Abstract

The paper deals with the modelling and analysis of a single layer timber gridshell produced by active bending. Two geometrically non-linear calculation methods, the Newton-Raphson method and the Modified Newton-Raphson method, are compared for the suitability of active bending modelling. The calculation is presented using the results of the timber lamella bending model. With this analysis, a more stable convergence of the calculation was achieved with the modified Newton-Raphson method than with the Newton-Raphson method. Based on the obtained results, the research was subsequently extended to the bending model of a single-layer network of timber beams, while the applied deformation method was the compression of supports. The suitability of using the modified Newton-Raphson method was verified on the model of a single-layer beam network, and the results were evaluated on five beam network models with different values of initial imperfection. During the processing of the parametric study, two methods of deforming the beam net in its supports were compared. The first method of shaping the beam mesh was the applied load using nodal forces. The second method of shaping the beam network was applied load using nodal displacements.

Keywords:

Active Bending;
Gridshell;
Timber;
Nonlinearity;
Newton-Raphson.

1 Introduction

Timber as a building material, thanks to the appropriate ratio of bending strength and Young's modulus, allows the use of elastic properties already in the construction process of the structure. A graphic comparison of materials based on Young´s modulus and bending strength is presented by Ashby [1] and Lienhard [2]. One of the ways to create a curved shape of the roof plane is to bend the load-bearing elements during construction to the required final geometry of the structure. This process has been used in practice for several shell structures (see Fig. 1) such as the Multihalle in Mannheim [3], The Savill Garden [4] The Weald and Downland [5].

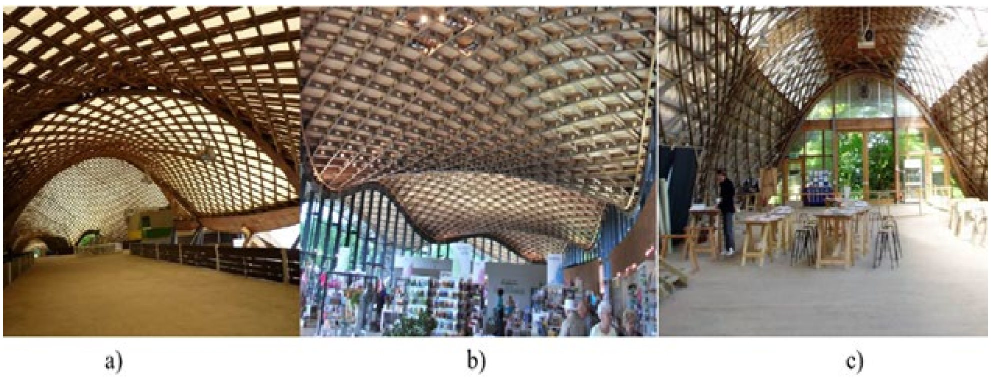

Fig. 1: a) Multihalle in Mannheim, b) The Savill Garden, c) Weald and Downland Gridshell [6].

Systematic elastic deformation is called active bending when the final shape of the structure is obtained as a result of bending the assembly of beams [7] (see Fig. 2). When designing structures using this method, it is essential to take into account the residual stresses introduced into the supporting elements during the assembly stage. Due to the occurrence of residual stresses, it is necessary to ensure a sufficient reserve in the resistance of the structure so that it can withstand the applied load throughout its lifetime. For structures made by the active bending method, the erection phase is one of the most critical phases in the life of the structure. Analysis of residual stress during active bending has been processed in researches Toussaint [8], Kuijvenhoven [9], V´Lier [10] and Gengnagel [11].

Fig. 2: The principle of creating a gridshell by the process of active bending [12].

When modelling and analysing the shaping of the structure using active bending, it is necessary to consider a geometrically non-linear calculation due to the occurrence of large deformations. Modern calculation software offers the possibility to analyse the structure geometrically non-linearly, but the stable convergence of the calculation depends directly on the type of non-linear method chosen, the number of increments and iterations, the complexity of the geometry, the method of bending the beams and the defined boundary conditions. For the non-linear analysis and form finding of membrane and shell constructions are in practice the most used The Newton-Raphson method, Modified Newton-Raphson method, Force Density method and Dynamic relaxation method [13], [14], [15].

A theoretical comparison of nonlinear methods has been processed in the researches Cuvilliers [16], Grančičová [17]. The article compares two nonlinear methods, the Newton-Raphson method and the Modified Newton-Raphson method.

2 Materials and methods

2.1 Theory of beam bending

The disadvantage of elastically bent members is the residual stress caused by the bending process. The higher the residual stress, the lower the stress reserve under the action of an external load. Assuming linear behaviour of the material, the residual stress σ_z at the edge of the section caused by bending around one axis can be defined according to Gengnagel [7] as follows:

$$\sigma_z = Et/(2R), \tag{1}$$

where:
t - thickness of the element,
R - radius of curvature,
E - Young´s modulus of elasticity.

It follows from relation (1) that in order to achieve the lowest possible residual stress, it is suitable for active bending to use a material with a low modulus of elasticity, a small thickness and a resulting large radius of curvature. From equation (1) according to Lienhard [2] it is possible to derive the relationship for determining the minimum radius of curvature R_{min}:

$$R_{min} = \frac{Et}{2\sigma Rd}. \qquad (2)$$

where:
σ_{Rd} - *limiting bending strength of the material.*

Using equation (2) and according to Collins and O´Regan [18] it is possible to determine the minimum radii of curvature for individual strength classes of timber members with the corresponding values of modulus of elasticity and characteristic value of flexural strength $f_{m,k}$ (see Fig. 3). Table 1 shows the values of the minimum radii of curvature for a 15 mm thick lamella and strength classes C16, C24, C30 and C40.

Table 1: Minimum radii of curvature of a 15 mm thick timber lamella.

	C16	C24	C20	C40
E_{0mean} [MPa]	8000	11000	12000	14000
$f_{m,k}$ [MPa]	16	24	30	40
R_{min} [m]	3.8	3.4	3	2.6

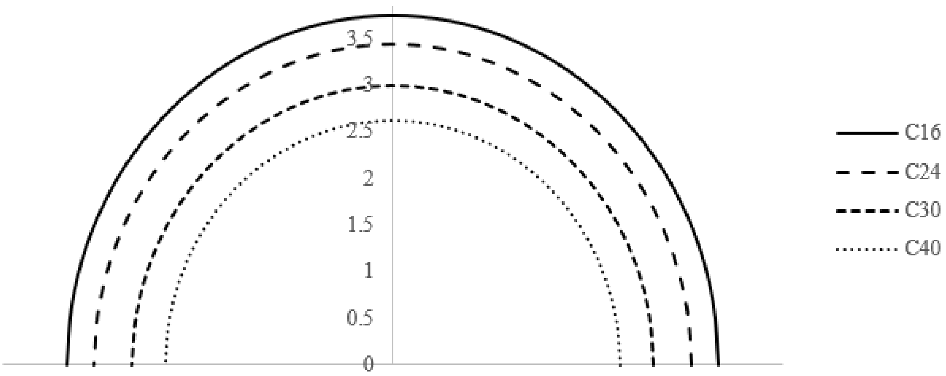

Fig. 3: Minimum radii of curvature of a 15 mm thick timber lamella.

2.2 The Newton-Raphson and the Modified Newton-Raphson

The Newton-Raphson method is described in Králik [19] as one of the fastest methods for solving nonlinear equations. This method is based on the assumption that the initial solution falls within the range of accepted solutions and that there is no divergence in the subsequent steps. Using this method requires a new stiffness matrix to be calculated at each iteration step.

The modified Newton-Raphson method is calculated using the same algorithm as the Newton-Raphson method. The main difference between the two methods is that the originally variable stiffness matrix is replaced by a constant approximation in the modified Newton-Raphson method [19]. Such a calculation requires a significantly higher number of defined iterations.

The suitability of both methods for simulating the active bending of the gridshell was verified on beam models in the SCIA Engineer software. The basic calculation model was asimply supported beam, to which an axial compressive load *P* was applied at the support points (see Fig. 4). The stability calculation defined the shape of the beam buckling with the corresponding equilibrium bifurcation coefficient and was further considered as an initial imperfection.

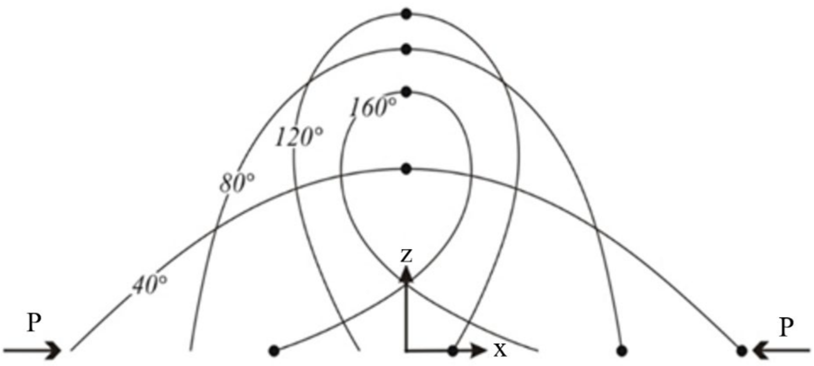

Fig. 4: Deformation curves of the beam caused by axial force [20].

3 Results and discussion

3.1 Comparison of geometrically nonlinear Newton-Raphson and modified Newton-Raphson methods

For the analysed beam, the material properties of a C30 timber lamella with a length of 3m and a cross section of 50x15mm were defined. During the calculation, the deformations during buckling of the beam loaded with increasing values of compressive force were monitored (see Fig. 5, Fig. 6, Fig. 7).

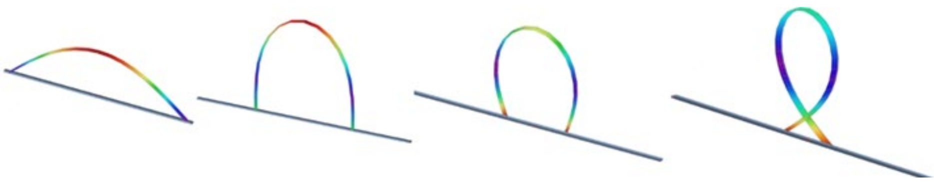

Fig. 5: Deformed shapes of the timber lamella from the increasing value of the applied compressive forces.

A comparison between the Newton-Raphson method and the Modified Newton-Raphson method is presented based on the dependence of the deformations U_z and U_x on the magnitude of the applied compressive load (see Fig. 6, Fig. 7). The deformation U_x is defined as the displacement of a support point in the direction of the load. The deformation U_z is defined as the vertical displacement of the point in the center of the beam.

The calculation using the Newton-Raphson method (see Fig. 6) was completed at a compressive load value of 0.153kN. As the load value was increased, the calculation diverged and the results indicated a general instability of the structure. The calculation did not reach stable convergence even when the number of iterations was increased.

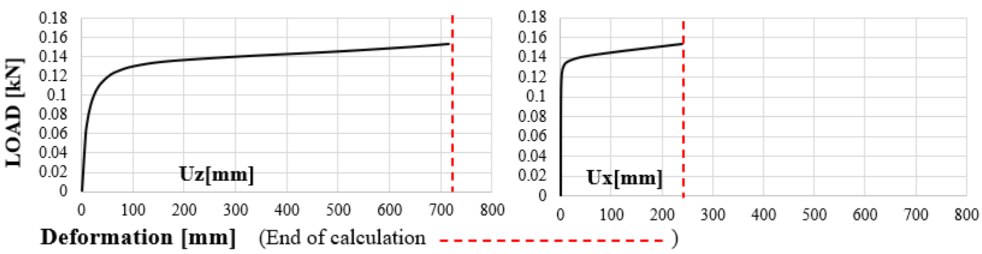

Fig. 6: Graphic representation of the dependence of Uz and Ux deformations on the magnitude of the applied compressive load using the Newton-Raphson method.

When using the modified Newton-Raphson method (see Fig. 7), a more stable convergence of the calculation was achieved even at higher values of the axial compressive force with corresponding beam deformations.

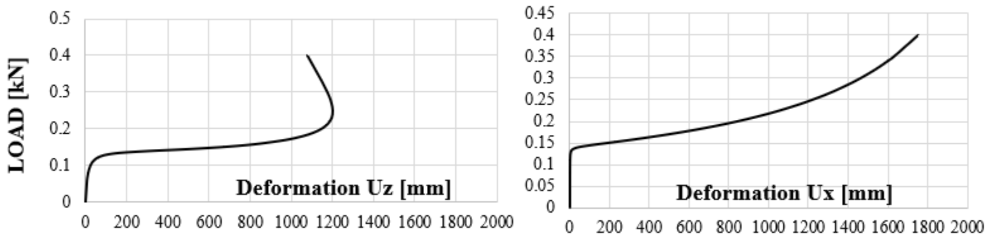

Fig. 7: Graphic representation of the dependence of Uz and Ux deformations on the magnitude of the applied compressive load using the Modified Newton-Raphson method.

The calculation showed that for the same load values, the same deformation values occur with both methods. However, by using the modified Newton-Raphson method, it is possible to analyse larger deformations of the structure due to the stable convergence of the calculation. Based on the results obtained, the study is extended to the case of bending a single-layer gridshell and analysis using the modified Newton-Raphson method.

3.2 Bending analysis of a single-layer gridshell

After proving the stable convergence of the calculation, the bending analysis of a single-layer gridshell with floor plan dimensions of 3mx3m and a mesh density of 0.5mx0.5m was carried out using the Modified Newton-Raphson method.

The defined material and cross-sections were the same as in the single lamella buckling study. Individual orthogonal beams were modelled as transversal and connected at the crossing points by short beams with defined values of rotational stiffness of 1E-05MNm/rad at both ends. The first step in the calculation was to define the boundary conditions of the beam system and introduce the initial load. The structure was supported in the centre of the beam network and the calculation was based on its own weight only. The deformed shape was taken and imported as a new initial imperfect state of the structure with modified boundary conditions (see Fig. 8).

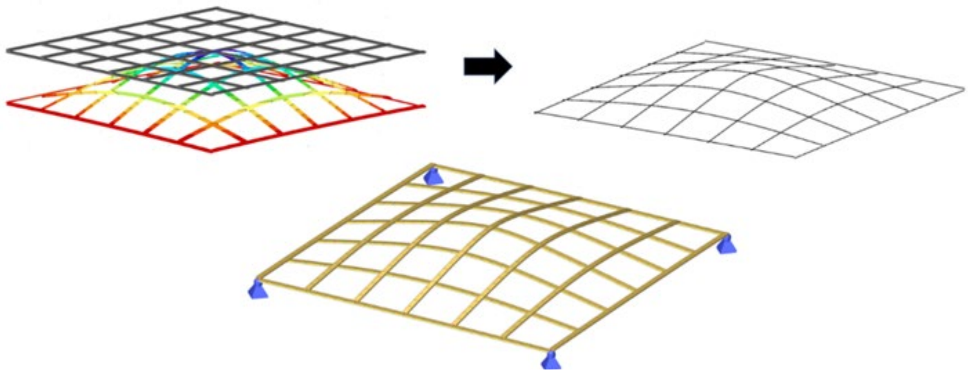

Fig. 8: Deformation of the beam net from its own weight and defining new boundary conditions.

The next step was to load the structure by introducing horizontal displacements into the supporting points towards the centre of gravity of the beam system, which introduced compressive stress into the structure, causing the beams to buckle and produce the resulting shape of the structure (see Fig. 9).

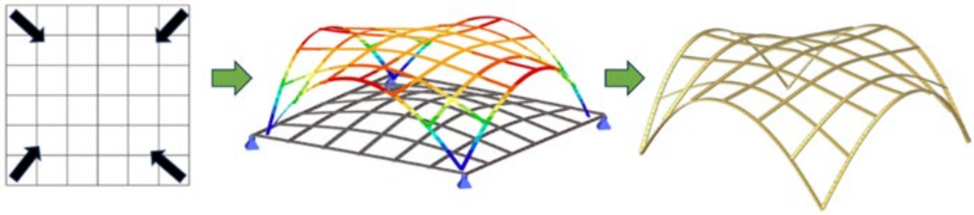

Fig. 9: Deformation of a single-layer wire mesh due to horizontal displacements.

Another way of loading that will cause the beam system to buckle is to introduce nodal forces at the same locations where the nodal displacements were previously defined. This type of loading requires the boundary conditions to be modified to allow displacements at the support points towards the centre of gravity of the beam net.

The study is devoted to comparing the methods of loading a single layer beam system using nodal displacements and nodal forces. The calculation was performed on 5 imperfect shapes of the beam net with different sizes of initial imperfection (see Fig. 10). The imperfect shapes were taken from the self-weight deformation shape of the planar mesh. A different weight multiplier was defined for each shape obtained to account for the different size of the imperfection.

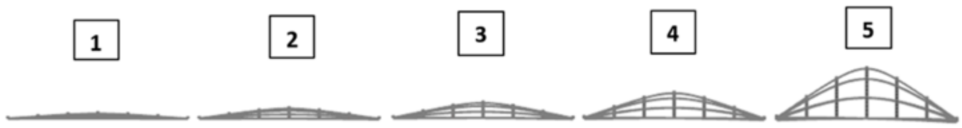

Fig. 10: Imperfect shapes of single-layer wire mesh - deformations due to self-weight.

Tables 2 and 3 show the resulting values of deformation, stress and internal forces. The displacement of U_{xy} in the tables is defined as the displacement of the corner of the object at the point of support towards the centre of gravity of the beam system. U_z is the maximum vertical deformation of the beam.

Table 2: Results from the computational model for loading a single-layer mesh with a constant value of displacement on imperfect shapes.

Imperfect shape	**σ_x [MPa]**	**U_{xy} [mm]**	**U_z [mm]**	**N [kN]**	**M_y [kNm]**
1. 60 [mm]	-28.4	60.9	312	-0.33	-0.04
2 120 [mm]	-27	60.9	311	-0.26	-0.04
3. 180 [mm]	-25	60.9	309	-0.18	-0.04
4. 300 [mm]	-25.3	60.9	304	-0.2	-0.04
5. 600 [mm]	-28.5	60.9	291	-0.3	-0.04

Due to the influence of the loading of the beam net with a constant displacement U_{xy} with a value of 60, 9 mm towards the centre of gravity of the structure, the value of the vertical deformation, which represents the uplift, takes a value of approximately 300 mm for all types of imperfections.

Table 3: Results from the computational model for loading a single-layer mesh with a constant value of nodal force -0,32 [kN] on imperfect shapes.

Imperfect shape	**σ_x [MPa]**	**U_{xy} [mm]**	**U_z [mm]**	**N [kN]**	**M_y [kNm]**
1. 60 [mm]	-29.3	60.9	332	-0.28	-0.05
2 120 [mm]	-35	90.2	398	-0.31	-0.06
3. 180 [mm]	-40.2	124	459	-0.32	-0.07
4. 300 [mm]	-49.2	200	561	-0.32	-0.09
5. 600 [mm]	-58.9	309	623	-0.33	-0.1

When the mesh shapes with different imperfections were loaded using constant values of node forces, the vertical deformation as well as the displacements in the corners of the object increased. For a 60 mm imperfect shape, the deformation of the corner of the object towards the centre of gravity of the beam net was 60.9 mm and the vertical deformation was 332 mm. For the initial shape with an imperfection of 600mm, the displacement U_{xy} in the corner of the object was 309 mm and the size of the vertical deformation of the beam was 623 mm. The final vertical coordinate of the mesh should be calculated as the sum of the imperfection and the vertical deformation.

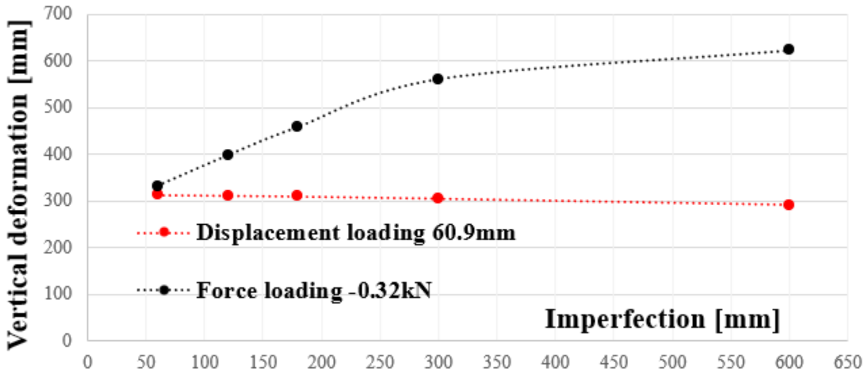

Fig. 11: Effect of the initial imperfection on the vertical deformation when the mesh is loaded with a constant displacement of the support and a constant value of the compressive force, respectively.

The graphical comparison above represents a parametric study (see Fig. 11), from which it can be observed that when the imperfect shape is loaded with a constant value of support displacement, an approximately constant value of vertical deformation occurs, and when the imperfect shape is loaded with a constant value of nodal forces, increasing values of vertical deformation occur. The same conclusion can be drawn for the deformation of U_{xy} (see Table 3).

3.3 Discussion

It is necessary to devote further research in to the issue of calculating active bending using the modified Newton-Raphson method in order to verify the obtained results based on experimental measurements. Further research will initially be based on a series of timber lamella bending experiments, and later a study will be carried out based on a gridshell bending experiment, which will achieve the final shape of the timber gridshell structure on a reduced scale.

Based on the obtained results, the goal of further research is to construct an experiment on a reduced scaled model according to Colabella [21] and Fernandes [22]. The resulting experimental model will be verified based on numerical calculation and evaluated according to [23].

The modified Newton-Raphson method demonstrated stable convergence, however, in modeling, it is necessary to pay increased attention to the rotational capacity at the point of contact of the orthogonal beams of the grid. This parameter has a significant impact on the final results and the stability of the convergence of the calculation. The magnitude of the value of the rotational capacity depends on the method of deformation of the beam network and on the specific type of connection that would be considered in the realization of the gridshell structure.

3 Conclusions

The static analysis of a gridshell made by active bending is complex. The design proceeds in a reverse manner from the resulting shape of the structure to the gridshell made on a flat base. Due to the non-linear relationship between load and response of the structure and the occurrence of large deformations during construction, the calculation is directly dependent on the geometrically non-linear method chosen. From the comparison of the non-linear Newton-Raphson and Modified Newton-Raphson methods on the example of the buckling of a beam subjected to compression, it can be seen that the modified Newton-Raphson method is more suitable for the analysis of active bending due to the possibility of obtaining larger deformations and stable convergence of the calculation.

From the parametric study of the deformation of a single-layer mesh with different imperfections, it can be concluded that loading the mesh with a constant displacement value produces approximately the same vertical deformation regardless of the increasing size of the initial imperfection. When loaded with a constant value of compressive force, increasing values of initial imperfections lead to increasing values of vertical deformations. The choice of erection method also has a significant effect on the resulting stress on the shell. The work analyzes the deformation of the shell by the compression of supports, but there are several methods, which are described in detail in [24] and [25].

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