

OPTIMIZING BUDGET DEFICIT IN MULTI-CONSTRUCTION PROJECTS USING SEQUENTIAL QUADRATIC PROGRAMMING

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Abstract

Construction companies frequently struggle with poor cash flow management of their projects. Financial terms, including retainage, advance payments, and interest rates, significantly impact the project's cash flow. This study investigates the financial aspects of projects with close beginnings. In addition, considers how to deal with financial deficits by suggesting a multi-project scheduling optimization model to minimize maximum negative cash flow while maintaining maximum profit. Sequential quadratic programming algorithms (SQP) generate workable schedules that optimally use available resources. Several scenarios have been used to test and examine the model. The outcomes indicate a decrease in negative cash flow for company 1 from (-1,852,096) to (-1,817,485) and for company 2 from (-484,524) to (-459,769) in scenario 1. Furthermore, a decrease in negative cash flow to (-1,661,660) alongside a profit of (776,593) for company 1 and a decrease to (-434,970) alongside a profit of (141,228) for company 2 in scenario 2. On the other hand, a decrease in negative cash flow to (-1,698,992) alongside a profit of (786,243) for company 1 and a decrease to (-370,815) alongside a profit of (209,363) for company 2 in scenario 3.

Keywords:

Optimizing;
Budget Deficit;
Multi-Construction Projects;
Sequential Quadratic
Programming;
Maximum Negative Cash Flow.

1 Introduction

Cash flow management is a complex but essential challenge for companies of all sizes and individuals because a lack of cash resources is one of the leading causes of bankruptcy in the construction industry. Controlling cash inflows and outflows is critical for any company seeking to complete the project on time and within budget. Financial burdens and stress can accumulate without proper planning and effective cash flow management, which can eventually lead to the failure of the whole entity. Conversely, the construction sector experiences the highest insolvency rates relative to other sectors [1]. Poor financial management is a leading cause of failure for many construction firms, primarily due to inadequate cash flow forecasting attention [2]. The time-cost trade-off challenge falls under the broader category of project scheduling issues, examining ways to adjust project activities to find an optimal balance between project duration and expense. For contractors working on actual projects, navigating the trade-off between project cost, completion time, and uncertainty is considerable for contractors [3]. Time-cost optimization methods, rooted in the critical path method (CPM), have seen widespread application in the construction sector for many years. A key benefit of employing CPM is its utility in helping contractors pinpoint the critical tasks necessary for the on-time completion of a project. Typically, time and cost trade-off analyses presume that shortening the project's duration is achievable exclusively through critical activities, with a set limit on the crash time available for each activity [4]. Optimization algorithms have emerged as a nontraditional tool in construction management applications to avoid the drawback of traditional statistical methods and, as such, have been employed in time cost trade-off analysis individually to obtain the optimum solution while considering cost constraints [5].

2 Literature review

The genetic algorithm (GA) approach has been employed in finance-based scheduling to create schedules based on the critical path method (CPM) that displays cash flow patterns meeting periodic financial requirements within predefined cash limits. These schedules display cash flow requirements for periodic financial needs within predetermined cash limits [6]. Presented a multi-objective optimization method to determine the most advantageous project tender offer for standard repetitive construction projects, considering various resource alternatives for all tasks. The formulation aims to minimize the project's duration, the price of the project/bid, and the maximum working capital while maximizing the project's net present value [7]. Focuses on improving cash management and profitability by minimizing negative cash flows in construction projects and adjusting the timing of construction activities according to the lowest cash flow availability. A genetic algorithm was presented to generate financial schedules that reduce negative cash flow and enhance profit, pinpointing the precise amount and timing of cash inflows and outflows at each period's end. [8]. Introduces a multi-objective optimization model, MAWA-TLBO, designed to address discrete time-cost trade-off issues by simultaneously optimizing the project's total duration and cost. A genetic algorithm is used to develop finance-focused schedules to reduce negative cash flow and optimize profit by determining the inflow or outflow amounts and their timing at the end of every period [9]. Presented a model to optimize project costs with limited resources and alternative production processes (APPs), providing optimal network diagrams, Gantt charts, histograms, and S-curves for project management. The project planner can optimize production while scheduling activities in discrete time units at a minimum cost [10]. Introduces a model designed to optimize the balance between time, cost, and financing by utilizing a hybrid algorithm, GALP, which merges genetic algorithms (GA) with linear programming (LP) for optimization. The effectiveness of this model is demonstrated by testing both small and large network scenarios in the project, ensuring optimal outcomes in financing costs and profitability. The tests also validate the efficiency and configuration of each model, establishing the feasibility of applying the proposed models to large network structures [11]. Introduces a viable approach to address the multi-objective optimization challenge in construction project management, focusing on project duration and cost. The method aims to minimize the likelihood of discrepancies between the project's optimized forecasts and its real-world outcomes [12]. Presented a model to schedule and fund a project using the approach that suggests minimizing financing by shifting activities with a lag to improve cash flow without extending project length, then analyzing long- and short-term loans. The model generates a modified cash flow with less contractor financing needed and feasible input and outflow patterns based on the optimal financing alternative, which aims to address cash shortages at the lowest cost. The model calculates the best advance payment to avoid borrowing [13]. Explores the financial considerations involved in managing multiple projects at once and examines how to handle financial constraints by introducing a multi-project scheduling optimization model aimed at maximizing profits and reducing the overall duration of projects. A genetic algorithm and finance-oriented scheduling techniques are used to create viable project timelines. These methods balance the financing needs of project activities and the funds available at any given time [14].

3 Scheduling the critical path method

Finance-based scheduling shifts activities without changing the project's completion date to achieve specific financial goals [15]. The Critical Path Method (CPM) was used to schedule project activities by determining when each activity would occur. This study focuses on an early start, early finish, late start, late finish, total float, and free float among the activity times, all scheduled according to CPM. The user of the proposed model in this paper is only required to enter the names of the activities, the dates on which they began, and their predecessors. For each activity, the model will calculate the CPM. Although the projects are carried out concurrently, the model user must specify the start date of the first activity in each project because the projects do not always begin on the same day. Each project will have a deadline, and changing the start dates of the activities will only fall within each activity's free float. As a result, the optimization model will determine each activity's start, end, and delayed start and finish dates to meet the contractor's financial goals [16].

$$\text{Activity delayed start} = \text{Activity start date} + X \quad (1)$$

Where: X= Number of days shifted within the free float calculated by the optimization algorithm.

4 Model development

Negative cash flow affects project financing expenses and profit. Scheduling professionals can create plans that optimize project profitability when cash flow is effectively managed. This aims to prevent a budget deficit without postponing project completion by minimizing negative cash flow. A cash flow management plan is necessary to profit as much as possible. Fig. 1 shows the build of this model.

SQP uses a quadratic programming subproblem to find the best solution for this subproblem to construct an optimal time for projects. This method is used to construct an inclusive model of different cash problems by three scenarios with the predetermined objective function to find optimal starting times for each scenario. The first scenario solves the problem of devising schedules corresponding to a minimum negative cash flow without reducing profit (without extending the project's time). The second scenario extends the project's time and reduces the negative cash flow problem while maintaining the maximum profit. The third scenario uses a short-term loan from the bank, recovering this loan at an interest rate determined according to the bank's policy as monthly payments until the project's end.

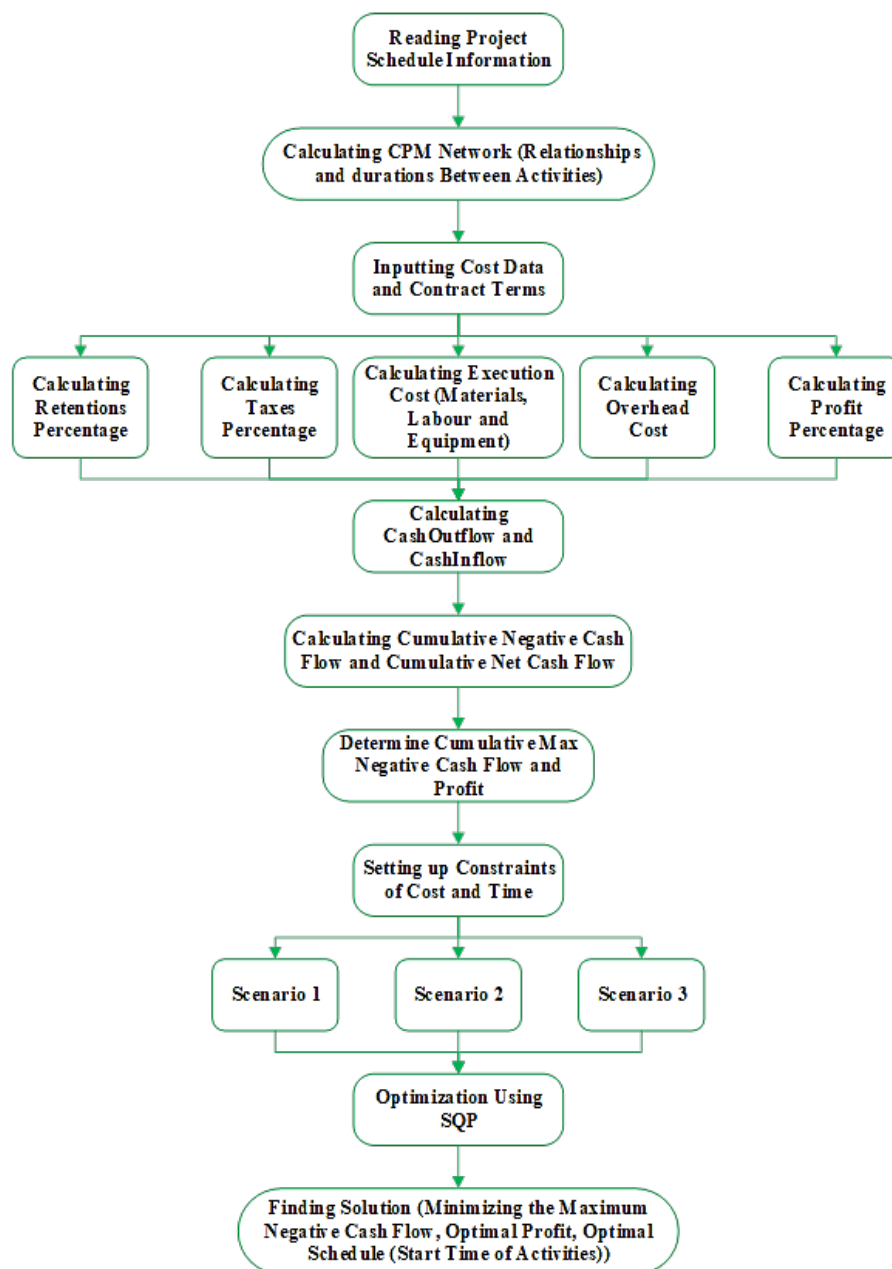


Fig. 1: Flow chart for model development

5 Sequential Quadratic Programming Concept

Since the late 1970s, Sequential Quadratic Programming (SQP) has been considered the most helpful method for finding solutions to nonlinearly constrained optimization problems. As known with most optimization methods, SQP is a conceptual method, not a single algorithm, from which many algorithms have evolved. Due to the solid theoretical and computational foundations, commercial and public domain SQP algorithms were developed and used to solve a huge number of vital practical problems. Large-scale versions have been recently tested with promising results. Despite all these problems being very important, the biggest SQP strength is its capability to find solutions for problems with nonlinear constraints. That is why it is claimed that (NLP) should at least include one function of nonlinear constraints [17].

The fundamental idea behind SQP is modeling NLP at any given solution, say x_k , by a quadratic programming subproblem and then using this solution to form a better approximation x_{k+1} . This process is performed repeatedly to form an approximation sequence that would hopefully converge to a solution x^* . The possible ideal way to understand the performance and theory of SQP is to see the method as the natural extension of both Newton as well as quasi-Newton methods to set the constrained optimization, keeping in mind that an appropriate choice of the quadratic subproblem is needed. Consequently, it is expected that the SQP methods would share the characteristics of Newton-like methods, such as rapid convergence when the iterates are close to the solution but possible erratic behavior that needs to be controlled while other the iterates are far away from a solution. In general, this correspondence is acceptable, but the presence of the constraint complicates the analysis and the SQP implementation [18].

5.1 SQP Advantages

The use of SQP advantages can be summarized [19]:

- 1) Very efficient for highly constrained problems.
- 2) Generally, requires fewer function evaluations (efficient for problems with expensive function evaluations).
- 3) Requires first derivatives but can work only with function values.
- 4) Can capitalize on a good initial point.
- 5) Good for a sequence of similar problems.
- 6) Infeasibility detection.

5.2 SQP mechanism

One of the most well-known and reliable methods for nonlinear continuous optimization is sequential quadratic programming (SQP). It finds a solution using gradient information, starting with a single point as input. An excellent initial solution is necessary for SQP to have a better chance of achieving an acceptable solution and avoiding the local optima. As with unconstrained optimization, this algorithm permits a close approximation of Newton's method for constrained optimization. A quasi-Newton update is applied to approximating the Hessian of the Lagrangian function at each iteration. This output is a Quadratic Programming (QP) subproblem, the solution of which is used to inform the iterative search for the optimum time [20]. The entire SQP algorithm can be summarized in seven steps [21].

- 1) Initialize the values of x (initial feasible solution) and initial Lagrange multipliers such that $c_i(x_0) \geq 0$ for $i = 1, 2, \dots, q$ and $\mu_0 \geq 0$. Also, stopping criteria for decreasing objective function were set, and the initial Hessian matrix was set to the identity matrix.
- 2) Compute the gradient g_k , the constraint matrices a_k , c_k and the Jacobian matrices (gradient of constraints with respect to different variables) A_{ek} and A_{ik} using the current feasible solution. Where g_k is a gradient of an objective function, A_{ek} and A_{ik} are the Jacobian of equality and inequality constraints.
- 3) Solve the QP problem for δx and compute Lagrange multipliers λ_{k+1} and $\hat{\mu}_{k+1}$. Where λ and $\hat{\mu}$ vectors of Lagrange multipliers.
- 4) Compute α_k represents a scalar obtained by a line search.
- 5) Set $x_{k+1} = x_k + \alpha_k \delta x$. Where x_k is the Current point, α_k is a scalar obtained by a line search, δx is the activation degree.
- 6) Compute new values of $\hat{\mu}_{k+1}$ after modifying $\delta x = \alpha_k \delta x$.

7) If the objective function's decrease is insignificant, output solution $x^* = x_{k+1}$ and stop. Otherwise, return to step the next step 2. Fig. 2 shows the steps for the optimal solution in the SQP algorithm.

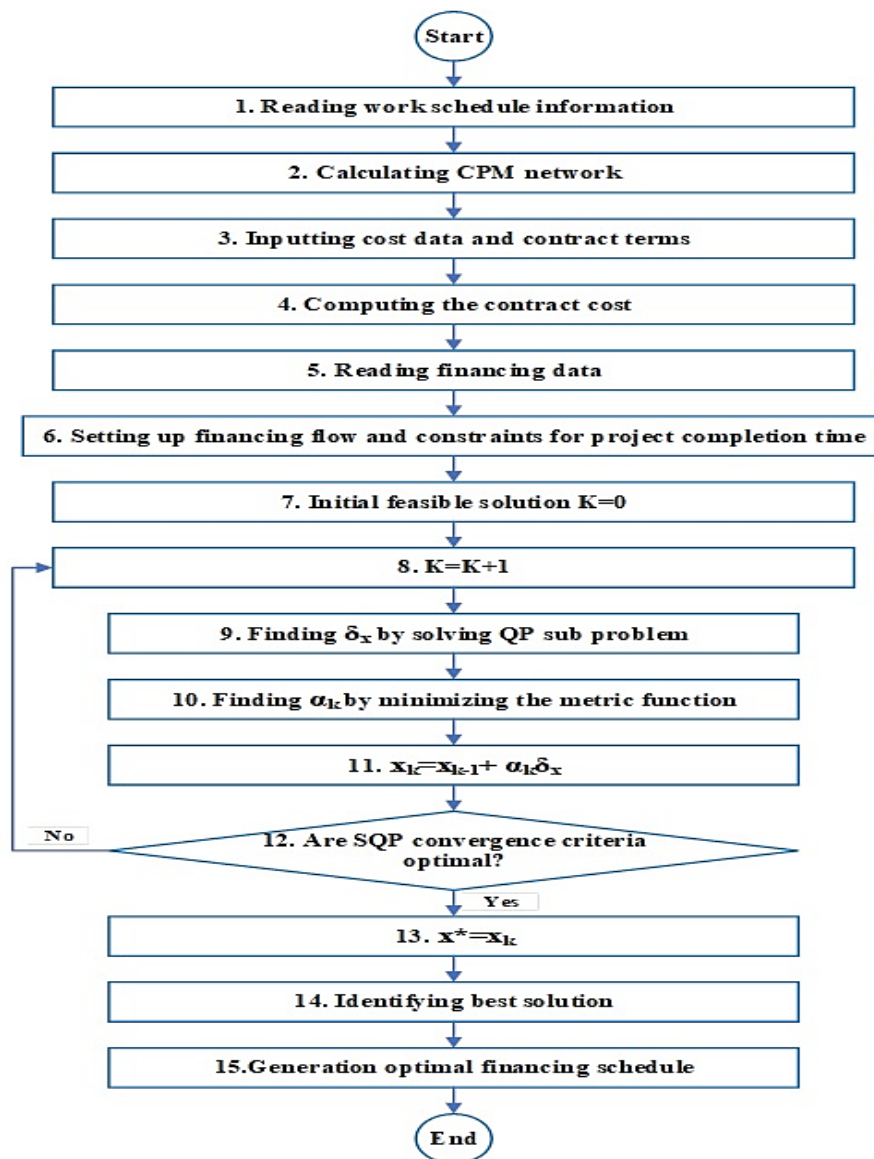


Fig. 2: SQP mechanism for the optimal solution

6 Data acquisition

An optimization model was developed using a sample of construction projects from both public and private sector companies. The basis of scenario analysis is several constraints, including project profit and completion time. The four projects that were finished in the time frame (2020–2021) provided the data required to create the models. Records kept by private and Ministry of Construction, Housing, Municipalities, and Public Works companies were used as sources of information. One contractor is in charge of the entire project, which entails the following tasks: replacing damaged items, removing damaged ones, and maintaining some damaged ones. Table 1 shows general information for the projects that included maintenance and rehabilitation work. Tables 2 and 3 show project costs extracted from the bill of quantities (BOQ). Tables 4 and 5 show each work clause, duration, direct cost (materials, labour), and the relationship between activities.

Table 1: Maintenance and rehabilitation projects in companies

Projects	Companies	Sector	Year
Numaniyah District Road (Kut - Baghdad) in Wasit Governorate	Hamorabi State Company for Constructing Contracts	Public	2020-2021
Al-Ahrar District Road (Kut - Baghdad) in Wasit Governorate		Public	2020-2021
Thaileb Bridge in Anbar Governorate	Romoz Al Tecnologia Company for Trading and Contracting Ltd	Private	2020-2021
Al-Fokanee Bridge in Anbar Governorate		Private	2020-2021

Table 2: BOQ and duration for activities in company 1

Activity ID	Activity Description	Original Duration (Day)	Amount of Activityx10 ³ IQD
Numaniyah road			
A1000	Mobilization	22	20,000
A1005	Remove Exist Asphalt	20	60,000
A1010	Embankment	50	700,000
A1020	Shoulder Stabilized	50	25,000
A1030	Box Calvert	30	15,500
A1040	Subbase Layer	50	425,000
A1050	Base Layer 15 Cm Thickness	40	825,000
A1060	Binder Layer 7 Cm Thickness	35	455,000
A1070	Wearing Layer 5 Cm Thickness	30	384,000
A1090	Guard Rail	30	19,300
A1100	Road Marking CL	20	32,800
A1110	Road Marking EL	20	88,350
A1120	Road Signs	20	69,920
A1130	Road Lighting	30	70,000
Total			3,189,870
Al-ahrar road			
A1000	Mobilization	22	20,000
A1005	Remove Exist Asphalt	20	60,000
A1010	Embankment	50	700,000
A1020	Shoulder Stabilized	50	25,000
A1030	Box Calvert	40	23,500
A1040	Subbase Layer	50	425,000
A1050	Base Layer 15 Cm Thickness	40	825,000
A1060	Binder Layer 7 Cm Thickness	35	455,000
A1070	Wearing Layer 5 Cm Thickness	30	384,000
A1090	Guard Rail	30	19,300
A1100	Road Marking CL	20	32,800
A1110	Road Marking EL	20	88,350
A1120	Road Signs	20	69,920
A1130	Road Lighting	30	70,000
Total			3,197,870

Table 3: BOQ and duration for activities in company 2

Activity ID	Activity Description	Original Duration (Day)	Amount of Activity x 10 ³ IQD
Thaileb bridge			
A1000	Alternative Road and Signs	19	34,751
A1010	Demolishing and Removing the Damaged Bridge Parts	28	391,473
A1020	Repairing Cracks in the Existing Bridge Parts	10	28,455
A1030	Columns	25	42,152
A1040	Cross Head Beams	25	32,411
A1050	Bearing Pads and Expansion Joint Seals Works	14	15,000
A1060	Precast Concrete Girders	50	180,452
A1070	Deck Slab and Diaphragms	27	123,452
A1080	Walls of drain water channel	15	27,815
A1090	Cover of Water Drain	7	8,741
A1100	Galvanized Steel Bridge Hand Railings	8	15,472
A1110	Paving Using Surface Layer	10	40,125
A1120	Painting for Bridge	5	14,822
A1130	Install new Expansion Joints	5	15,482
A1140	Bridge Gully Drain	5	3,751
A1150	Paint for Road Marking	8	19,822
A1160	Cleaning The Site Area	5	8,000
Total			1,002,176

Al-Fokanee bridge			
A1000	Site Preparation	10	30,000
A1010	Demolishing and Removing Destroyed Parts	18	25,000
A1020	Repairing Works	13	23,510
A1030	Concrete Pipes Culvert Works	25	28,500
A1040	Casting the Sulphate Resistant Plain	30	40,153
A1050	Casting Reinforced Concrete Pads	15	12,552
A1060	Installing Bearing Pads of Girders	10	15,422
A1070	Installing the Concrete Girders	15	100,252
A1080	Casting Diaphragm	10	25,488
A1090	Spreading Clay Layers	10	38,000
A1100	Fascia panels installing	15	13,411
A1110	Spreading Sub-base Layers	10	48,500
A1120	Sub-base Layer " Class C" for Shoulders of the Road	10	10,000
A1130	Casting the Deck Slab and Sidewalks and Approach Slab	30	86,222
A1140	Base Asphalt Layer (Stabilizer)	8	85,152
A1150	Binder Layer	8	48,200
A1160	Surface Layer	7	35,122
A1170	Painting The Bridge Parts	5	15,224
A1180	Bridge Gully Drain	5	3,500
A1190	Install new Expansion Joints	8	23,500
A1200	Galvanized Guardrail Works	10	80,000
A1210	Walkway and Road Marking Works	5	12,900
Total			800,608

Table 4: Work Items with essential data for company 1

Activity ID	Duration (Day)	Predecessor	Direct cost x10 ³ IQD	Lead	ESD	EFD	LSD	LFD	TF	FF
Numaniyah road										
A1000	22	-	15,600	0	1	22	1	22	0	0
A1005	20	A1000	46,800	0	23	42	53	72	30	30
A1010	50	A1000	546,000	0	23	72	23	72	0	0
A1020	50	A1010	19,500	-30	43	92	43	92	0	0
A1030	30	A1000	12,090	0	23	52	43	72	20	20
A1040	50	A1005,A1010,A1030	331,500	-20	73	122	73	122	0	0
A1050	40	A1040	643,500	-15	108	147	108	147	0	0
A1060	35	A1050	354,900	-10	138	172	138	172	0	0
A1070	30	A1060	299,520	-10	163	192	163	192	0	0
A1090	30	A1040	15,054	0	123	152	143	172	20	0
A1100	20	A1070,A1120	25,584	0	193	212	193	212	0	0
A1110	20	A1070,A1120	68,913	0	193	212	193	212	0	0
A1120	20	A1090,A1130	54,538	0	153	172	173	192	20	20
A1130	30	A1040	54,600	0	123	152	143	172	20	0
Al-ahrar road										
A1000	22	-	15,600	0	62	83	62	83	0	0
A1005	20	A1000	46,800	0	84	103	114	133	30	30
A1010	50	A1000	546,000	0	84	133	84	133	0	0
A1020	50	A1010	19,500	-30	104	153	104	153	0	0
A1030	40	A1000	18,330	0	84	123	94	133	10	10
A1040	50	A1005,A1010,A1030	331,500	-20	134	183	134	183	0	0
A1050	40	A1040	643,500	-15	169	208	169	208	0	0
A1060	35	A1050	354,900	-10	199	233	199	233	0	0
A1070	30	A1060	299,520	-10	224	253	224	253	0	0
A1090	30	A1040	15,054	0	184	213	204	233	20	0
A1100	20	A1070,A1120	25,584	0	254	273	254	273	0	0
A1110	20	A1070,A1120	68,913	0	254	273	254	273	0	0
A1120	20	A1090,A1130	54,538	0	214	233	234	253	20	20
A1130	30	A1040	54,600	0	184	213	204	233	20	0

Table 5: Work Items with essential data for company 2

Activity ID	Duration (Day)	Predecessor	Direct cost x10 ³ IQD	Lag	ESD	EFD	LSD	LFD	TF	FF
Thaileb bridge										
A1000	19	-	27,801	0	1	19	10	28	9	9
A1010	28	-	313,178	0	1	28	1	28	0	0
A1020	10	A1000,A1010	22,764	0	29	38	133	142	104	104
A1030	25	A1000,A1010	33,722	0	29	53	29	53	0	0
A1040	25	A1030	25,929	0	54	78	54	78	0	0

A1050	14	A1040	12,000	0	79	92	79	92	0	0
A1060	50	A1050	144,362	0	93	142	93	142	0	0
A1070	27	A1060,A1020	98,762	0	143	169	143	169	0	0
A1080	15	A1070	22,252	0	170	184	170	184	0	0
A1090	7	A1080	6,993	0	185	191	188	194	3	3
A1100	8	A1080	12,378	0	185	192	187	194	2	2
A1110	10	A1080	32,100	0	185	194	185	194	0	0
A1120	5	A1100	11,858	0	193	197	203	207	10	10
A1130	5	A1090,A1100,A1110	12,386	0	195	199	195	199	0	0
A1140	5	A1130	3,001	0	200	204	203	207	3	3
A1150	8	A1130	15,858	0	200	207	200	207	0	0
A1160	5	A1120,A1140,A1150	6,400	0	208	212	208	212	0	0
Al-Fokanee bridge										
A1000	10	-	24,000	0	32	41	32	41	0	0
A1010	18	A1000	20,000	0	42	59	42	59	0	0
A1020	13	A1010	18,808	0	60	72	77	89	17	17
A1030	25	A1010	22,800	0	60	84	65	89	5	5
A1040	30	A1010	32,122	0	60	89	60	89	0	0
A1050	15	A1020,A1030,A1040	10,042	0	90	104	90	104	0	0
A1060	10	A1050	12,338	0	105	114	105	114	0	0
A1070	15	A1060	80,202	0	115	129	115	129	0	0
A1080	10	A1070	20,390	0	130	139	135	144	5	5
A1090	10	A1070	30,400	0	130	139	155	164	25	0
A1100	15	A1070	10,729	0	130	144	130	144	0	0
A1110	10	A1090	38,800	0	140	149	165	174	25	25
A1120	10	A1110	8,000	0	150	159	193	202	43	38
A1130	30	A1080,A1100	68,978	0	145	174	145	174	0	0
A1140	8	A1110,A1130	68,122	0	175	182	175	182	0	0
A1150	8	A1140	38,560	0	183	190	183	190	0	0
A1160	7	A1150	28,098	0	191	197	191	197	0	0
A1170	5	A1120,A1160	12,179	0	198	202	203	207	5	5
A1180	5	A1160	2,800	0	198	202	203	207	5	5
A1190	8	A1160	18,800	0	198	205	200	207	2	2
A1200	10	A1160	64,000	0	198	207	198	207	0	0
A1210	5	A1170,A1180,A1190,A1200	10,320	0	208	212	208	212	0	0

7 Calculation of cash flows

To calculate cash flows initially, the contractor inputs the cost data and contract conditions, as shown in Table 6. These data vary from company to company, depending on the contractor's execution costs percentage before the contract assignment. Then, the project cash flow forecast is developed and integrated with the project financing flow in the next process. Table 7 shows the maximum negative cash flow is (-1,852,096) at the end of the 6th month, and the total profit is (811,243) in company 1. Finally, Table 8 shows the maximum negative cash flow is (-484,524) at the end of the 2nd month, and the total profit is (234,362) in company 2. The Sequential Quadratic Programming (SQP) algorithm is employed to identify optimal schedules that reduce the maximum negative cash flow and enhance project profitability. An illustrative calculation of cash flows for company 1 follows:

1) Total receipts: Total progress payment to the contractor subtracted from the discounts of retention and taxes.

2) Total cost: The total costs incurred by the contractor in each month (materials, labour, overhead).

3) Net cash flow (N_t) each month.

N_t in month 2 = Total receipts in month 2 + Total cost in month 2

N_t in month 2 = 108,462 - 405,216 = -296,754

4) Cumulative net cash flow (CN_t) each month.

CN_t in month 2 = N_t in month 1 + N_t in month 2

CN_t in month 2 = -105,084 - 296,754 = -401,838

5) Cumulative cash flow (F_t) each month.

F_t in month 2 = CN_t in month 1 + Total cost in month 2

F_t in month 2 = -105,084 - 405,216 = -510,300

Table 6: Project cost data and contract conditions

Data type	Item	Amount	
		Company 1	Company 2
Cost data	Materials (expressed in percentage) from each item in BOQ	42%	43%
	Labour (expressed in percentage) from each item in BOQ	36%	37%
	Overhead (expressed in percentage) from each item in BOQ	6%	7%
	Profit (expressed in percentage) from each item in BOQ	12.7%	13%
Contract conditions	Retention (expressed in percentage) from contract cost (10% from progress payment until 5% from contract cost)	5%	0%
	Taxes (expressed in percentage) from progress payment (bill to owner)	3.3%	0%
	Number of months between submitting pay requests	1 month	1 month
	Lag in responding to payment requests (months)	1 month	1 month
	Lag to make the final payment and return the retained money (months)	0 month	0 month
	Delay penalty (expressed in percentage)	cost/day= (contract cost/ contract duration) x 11.5%	cost/day= (contract cost) x 0.5%

Table 7: Cash flow computation depending on schedule in C1*

Month	Bill to Owner	Retention	Taxes	Total Receipts	Execution Cost	Overhead	Total Cost	Net Cash Flow N_t	Cumulative Net Cash Flow CN_t	Cumulative Cash Flow F_t
1					-97,578	-7,506	-105,084	-105,084	-105,084	-105,084
2	125,100	12,510	4,128	108,462	-376,272	-28,944	-405,216	-296,754	-401,838	-510,300
3	482,400	48,240	15,919	418,241	-354,851	-27,296	-382,148	36,093	-365,745	-783,986
4	454,938	45,494	15,013	394,431	-759,720	-58,440	-818,160	-423,729	-789,474	-1,183,905
5	974,000	97,400	32,142	844,458	-942,206	-72,477	-1,014,683	-170,225	-959,699	-1,804,157
6	1,207,956	115,743	39,863	1,052,350	-828,654	-63,743	-892,397	159,953	-799,746	-1,852,096
7	1,062,377		35,058	1,027,319	-890,594	-68,507	-959,101	68,218	-731,528	-1,758,847
8	1,141,787		37,679	1,104,108	-488,306	-37,562	-525,868	578,240	-153,289	-1,257,396
9	626,034		20,659	605,375	-220,633	-16,972	-237,605	367,770	214,482	-390,893
10	282,863		9,334	273,529	-23,623	-1,817	-25,440	248,089	462,568	189,040
11	30,285		1,000	29,285				29,285	811,243	462,568
Total	6,387,740	319,387	210,795	5,857,558	-4,982,437	-383,264	-5,365,702			

* Concerns company 1

Table 8: Cash flow computation depending on schedule in C2*

Month	Bill to Owner	Retention	Taxes	Total Receipts	Execution Cost	Overhead	Total Cost	Net Cash Flow N_t	Cumulative Net Cash Flow CN_t	Cumulative Cash Flow F_t
1					-340,979	-29,836	-370,815	-370,815	-370,815	-370,815
2	426,224			426,224	-104,561	-9,149	-113,710	312,514	-58,301	-484,524
3	130,701			130,701	-103,085	-9,020	-112,105	18,596	-39,705	-170,406
4	128,856			128,856	-123,335	-10,792	-134,127	-5,271	-44,975	-173,831
5	154,168			154,168	-255,692	-22,373	-278,065	-123,897	-168,871	-323,040
6	319,615			319,615	-195,888	-17,140	-213,028	106,587	-62,285	-381,900
7	244,860			244,860	-305,317	-26,715	-332,032	-87,172	-149,457	-394,317
8	381,646			381,646	-13,369	-1,170	-14,539	367,107	217,649	-163,996
9	16,713			16,713					234,362	217,649
Total	1,802,783			1,802,783	-1,442,226	-126,195	-1,568,421			

* Concerns company 2

** There are no retentions and taxes because UNDP Iraq funds them.

8 Optimization model

The primary goal is to create a tool designed to assist contractors in reducing negative cash flow while maintaining profit. To accomplish this objective, a detailed model encompassing a variety of cash-related issues was developed alongside three different scenarios. The Sequential Quadratic

Programming (SQP) method was programmed in MATLAB, which was then utilized to determine the optimal schedule for addressing the cash problem.

8.1 Apply scenario 1

Utilizing the SQP technique to maximize profits and minimize negative cash flow addresses the issue by generating schedules that reduce negative cash flow. Below are the objective function and constraints specific to this scenario:

$$\text{Minimize: } F_t \quad (2)$$

$$E_{st} \leq A_{st} \leq L_{st} \quad (3)$$

Where: F_t : maximum cumulative cash flow, E_{st} : is the early start time for activity according to schedule, A_{st} : is the activity start data in the project, L_{st} : is the late start time for activity according to time schedule. Critical path activities are started early to generate schedules, and non-critical activities are started at the optimal start times while maintaining a link between them. These schedules generate corresponding cash requirements profiles. The SQP procedure then looks for a schedule that minimizes negative cumulative cash flow while keeping maximum profit. Tables 9 and 10 show the optimized cash flow calculation by applying scenario 1 in two companies. Also, Tables 11 and 12 show constraints in the two companies. On the other hand, the noncritical activities' start times have changed to reach the optimum schedule with minimum negative cash flow and optimum profit, as shown in Tables 13 and 14 which compare the original and optimal start times for the activities in the two companies. Fig. 3 and 4 compare the two companies' initial negative cumulative cash flow and the optimized negative cumulative cash flow.

Table 9: Cash flow optimization computation for the S1* in C1*

Month	Bill to Owner	Retention	Taxes	Total Receipts	Execution Cost	Overhead	Total Cost	Net Cash Flow N_t	Cumulative Net Cash Flow CN_t	Cumulative Cash Flow F_t
1					-81,523	-6,271	-87,794	-87,794	-87,794	-87,794
2	104,517	10,452	3,449	90,616	-392,327	-30,179	-422,506	-331,890	-419,684	-510,300
3	502,983	50,298	16,598	436,087	-350,220	-26,940	-377,160	58,927	-360,758	-796,844
4	449,000	44,900	14,817	389,283	-761,144	-58,550	-819,693	-430,410	-791,168	-1,180,451
5	975,825	97,583	32,202	846,040	-910,144	-70,011	-980,155	-134,115	-925,283	-1,771,323
6	1,166,852	116,154	38,506	1,012,192	-828,474	-63,729	-892,202	119,990	-805,294	-1,817,485
7	1,062,146		35,051	1,027,095	-916,943	-70,534	-987,477	39,618	-765,676	-1,792,771
8	1,175,568		38,794	1,136,774	-497,406	-38,262	-535,668	601,106	-164,570	-1,301,345
9	637,701		21,044	616,657	-220,633	-16,972	-237,605	379,052	214,482	-402,175
10	282,863		9,334	273,529	-23,623	-1,816	-25,442	248,087	462,568	189,040
11	30,285		1,000	29,285				29,285	811,243	462,568
Total	6,387,740	319,387	210,795	5,857,558	-4,982,437	-383,264	-5,365,702			

* Concerns scenario 1 in company 1

Table 10: Cash flow optimization computation for the S1* in C2*

Month	Bill to Owner	Retention	Taxes	Total Receipts	Execution Cost	Overhead	Total Cost	Net Cash Flow N_t	Cumulative Net Cash Flow CN_t	Cumulative Cash Flow F_t
1					-340,979	-29,836	-370,815	-370,815	-370,815	-370,815
2	426,224			426,224	-81,797	-7,157	-88,954	337,270	-33,545	-459,769
3	102,246			102,246	-103,085	-9,020	-112,105	-9,859	-43,404	-145,650
4	128,856			128,856	-146,099	-12,784	-158,883	-30,027	-73,430	-202,286
5	182,623			182,623	-220,772	-19,318	-240,090	-57,467	-130,896	-313,519
6	275,965			275,965	-230,808	-20,196	-251,004	24,961	-105,935	-381,900
7	288,510			288,510	-305,317	-26,715	-332,032	-43,522	-149,457	-437,967
8	381,646			381,646	-13,369	-1,169	-14,538	367,108	217,649	-163,997
9	16,713			16,713				16,713	234,362	217,649
Total	1,802,783			1,802,783	-1,442,226	-126,195	-1,568,421			

* Concerns scenario 1 in company 2

Table 11: Constraints S1 in C1

Activity	Variable	Project	Constraints
A1000	x(1)	1	$x(1) = 1$
A1005	x(2)	1	$23 \leq x(2) \leq 53$
A1010	x(3)	1	$x(3) = 23$
A1020	x(4)	1	$x(4) = 43$
A1030	x(5)	1	$23 \leq x(5) \leq 43$
A1040	x(6)	1	$x(6) = 73$
A1050	x(7)	1	$x(7) = 108$
A1060	x(8)	1	$x(8) = 138$
A1070	x(9)	1	$x(9) = 163$
A1090	x(10)	1	$123 \leq x(10) \leq 143$
A1100	x(11)	1	$x(11) = 193$
A1110	x(12)	1	$x(12) = 193$
A1120	x(13)	1	$153 \leq x(13) \leq 173$
A1130	x(14)	1	$123 \leq x(14) \leq 143$
A1000	x(15)	2	$x(15) = 62$
A1005	x(16)	2	$84 \leq x(16) \leq 114$
A1010	x(17)	2	$x(17) = 84$
A1020	x(18)	2	$x(18) = 104$
A1030	x(19)	2	$84 \leq x(19) \leq 94$
A1040	x(20)	2	$x(20) = 134$
A1050	x(21)	2	$x(21) = 169$
A1060	x(22)	2	$x(22) = 199$
A1070	x(23)	2	$x(23) = 224$
A1090	x(24)	2	$184 \leq x(24) \leq 204$
A1100	x(25)	2	$x(25) = 254$
A1110	x(26)	2	$x(26) = 254$
A1120	x(27)	2	$214 \leq x(27) \leq 234$
A1130	x(28)	2	$184 \leq x(28) \leq 204$

Table 12: Constraints S1 in C2

Activity	Variable	Project	Constraints
A1000	x(1)	1	$1 \leq x(1) \leq 10$
A1010	x(2)	1	$x(2) = 1$
A1020	x(3)	1	$29 \leq x(3) \leq 133$
A1030	x(4)	1	$x(4) = 29$
A1040	x(5)	1	$x(5) = 54$
A1050	x(6)	1	$x(6) = 79$
A1060	x(7)	1	$x(7) = 93$
A1070	x(8)	1	$x(8) = 143$
A1080	x(9)	1	$x(9) = 170$
A1090	x(10)	1	$185 \leq x(10) \leq 188$
A1100	x(11)	1	$185 \leq x(11) \leq 187$
A1110	x(12)	1	$x(12) = 185$
A1120	x(13)	1	$193 \leq x(13) \leq 203$
A1130	x(14)	1	$x(14) = 195$
A1140	x(15)	1	$200 \leq x(15) \leq 203$
A1150	x(16)	1	$x(16) = 200$
A1160	x(17)	1	$x(17) = 208$
A1000	x(18)	2	$x(18) = 32$
A1010	x(19)	2	$x(19) = 42$
A1020	x(20)	2	$60 \leq x(20) \leq 77$
A1030	x(21)	2	$60 \leq x(21) \leq 65$
A1040	x(22)	2	$x(22) = 60$
A1050	x(23)	2	$x(23) = 90$
A1060	x(24)	2	$x(24) = 105$
A1070	x(25)	2	$x(25) = 115$
A1080	x(26)	2	$130 \leq x(26) \leq 135$
A1090	x(27)	2	$130 \leq x(27) \leq 155$
A1100	x(28)	2	$x(28) = 130$
A1110	x(29)	2	$140 \leq x(29) \leq 165$
A1120	x(30)	2	$150 \leq x(30) \leq 193$
A1130	x(31)	2	$x(31) = 145$
A1140	x(32)	2	$x(32) = 175$
A1150	x(33)	2	$x(33) = 183$
A1160	x(34)	2	$x(34) = 191$
A1170	x(35)	2	$198 \leq x(35) \leq 203$

A1180	x(36)	2	$198 \leq x(36) \leq 203$
A1190	x(37)	2	$198 \leq x(37) \leq 200$
A1200	x(38)	2	$x(38) = 198$
A1210	x(39)	2	$x(39) = 208$

Table 13: Original and optimal start time for S1 in C1

Activity	Original start time (Day)	Optimal start time using SQP
Numaniyah road		
A1000	1	1
A1005	23	30
A1010	23	23
A1020	43	43
A1030	23	28
A1040	73	73
A1050	108	108
A1060	138	138
A1070	163	163
A1090	123	128
A1100	193	193
A1110	193	193
A1120	153	172
A1130	123	141
Al-ahrar road		
A1000	62	62
A1005	84	85
A1010	84	84
A1020	104	104
A1030	84	91
A1040	134	134
A1050	169	169
A1060	199	199
A1070	224	224
A1090	184	184
A1100	254	254
A1110	254	254
A1120	214	218
A1130	184	189
Max Negative Cash Flow	-1,852,096	-1,817,485
Profit	811,243	811,243

Table14: Original and optimal start time for S1 in C2

Activity	Original start time (Day)	Optimal start time using SQP
Thaileb bridge		
A1000	1	9
A1010	1	1
A1020	29	109
A1030	29	29
A1040	54	54
A1050	79	79
A1060	93	93
A1070	143	143
A1080	170	170
A1090	185	186
A1100	185	187
A1110	185	185
A1120	193	201
A1130	195	195
A1140	200	202
A1150	200	200

A1160	208	208
Al-Fokanee bridge		
A1000	32	32
A1010	42	42
A1020	60	72
A1030	60	63
A1040	60	60
A1050	90	90
A1060	105	105
A1070	115	115
A1080	130	132
A1090	130	137
A1100	130	130
A1110	140	165
A1120	150	158
A1130	145	145
A1140	175	175
A1150	183	183
A1160	191	191
A1170	198	203
A1180	198	199
A1190	198	200
A1200	198	198
A1210	208	208
Max Negative Cash Flow	-484,524	-459,769
Profit	234,362	234,362

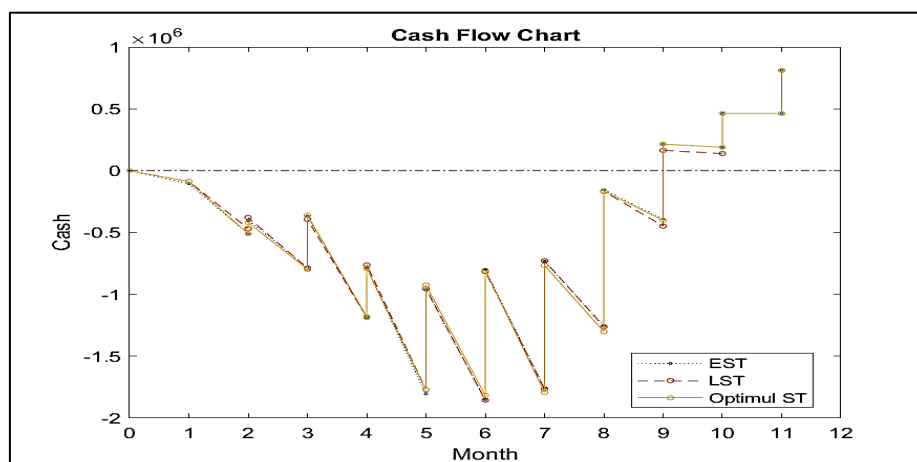


Fig. 3: Original and optimized cash flow chart S1 in C2

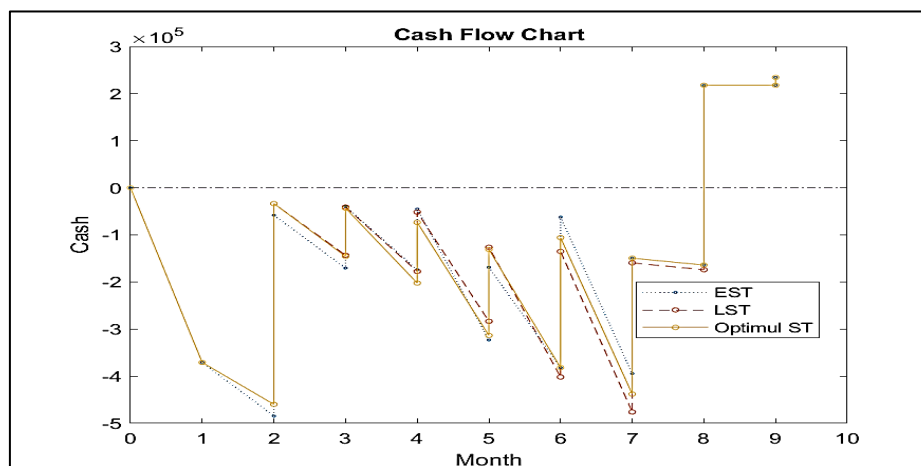


Fig. 4: Original and optimized cash flow chart S1 in C2

8.2 Apply scenario 2

(Maximizing Profit) by extending the project and reducing the negative cash flow problem while maintaining the maximum profit (Assuming the projects are extended for 30 days). Below are the objective function and constraints specific to this scenario:

Maximize: G (4)

$E_{st} \leq A_{st} \leq M$ (5)

Where: G : is the maximum profit, E_{st} : is the early start time for activity according to schedule, A_{st} : is the activity start data in the project and M : is the latest start time for activity according to the maximum delay in start time for the last activity. Optimal start times for activities are used to generate new time schedules while maintaining dependency between activities. It is possible to see how much cash each schedule requires. Afterward, the SQP procedure looks for a schedule that generates the maximum profit while generating the minimum cumulatively negative cash flow possible. Tables 15 and 16 show the optimized cash flow calculation by applying scenario 2 in two companies. Also, Tables 17 and 18 show constraints in the two companies. On the other hand, the activities' start time has changed to reach the optimum schedule with minimum negative cash flow and optimum profit, as shown in Tables 19 and 20, which compare the original and optimal start time for the activities in the two companies. Fig. 5 and 6 compare the two companies' initial negative cumulative cash flow and the optimized negative cumulative cash flow.

Table 15: Cash flow optimization computation for the S2* in C1*

Month	Bill to Owner	Retention	Taxes	Total Receipts	Execution Cost	Overhead	Total Cost	Net Cash Flow N_t	Cumulative Net Cash Flow CN_t	Cumulative Cash Flow F_t
1					-48,360	-3,720	-52,080	-52,080	-52,080	-52,080
2	62,000	6,200	2,046	53,754	-344,929	-26,533	-371,462	-317,708	-369,788	-423,542
3	442,217	44,222	14,593	383,402	-345,761	-26,597	-372,358	11,044	-358,744	-742,146
4	443,283	44,328	14,628	384,327	-612,154	-47,089	-659,243	-274,916	-633,660	-1,017,987
5	784,813	78,481	25,899	680,433	-915,281	-70,406	-985,688	-305,255	-938,915	-1,619,348
6	1,173,438	117,344	38,723	1,017,371	-661,396	-50,877	-712,272	305,099	-633,817	-1,651,187
7	847,943	28,812	27,982	791,149	-954,425	-73,417	-1,027,843	-236,694	-870,511	-1,661,660
8	1,223,622		40,380	1,183,242	-639,188	-49,168	-688,356	494,886	-375,624	-1,558,867
9	819,472		27,043	792,429	-390,071	-30,005	-420,077	372,352	-3,272	-795,701
10	500,091		16,503	483,588	-70,872	-5,452	-76,323	407,265	403,992	-79,596
11	90,861		2,998	87,863				87,863	776,593	403,992
Total	6,387,740	319,387	210,795	5,857,558	-4,982,437	-383,264	-5,365,702			

* Concerns scenario 2 in company 1

Table 16: Cash flow optimization computation for the S2 in C2

Month	Bill to Owner	Retention	Taxes	Total Receipts	Execution Cost	Overhead	Total Cost	Net Cash Flow N_t	Cumulative Net Cash Flow CN_t	Cumulative Cash Flow F_t
1					-327,810	-28,683	-356,493	-356,493	-356,493	-356,493
2	409,763			409,763	-72,162	-6,314	-78,476	331,287	-25,207	-434,970
3	90,203			90,203	-93,469	-8,179	-101,648	-11,445	-36,653	-126,855
4	116,837			116,837	-121,275	-10,612	-131,887	-15,050	-51,702	-168,539
5	151,594			151,594	-191,891	-16,791	-208,682	-57,088	-108,791	-260,384
6	239,864			239,864	-245,525	-21,483	-267,008	-27,144	-135,934	-375,799
7	306,906			306,906	-240,869	-21,076	-261,945	44,961	-90,974	-397,879
8	301,086			301,086	-149,225	-13,057	-162,282	138,804	47,825	-253,261
9	186,530			186,530				186,530	141,228	47,825
Total	1,802,783			1,802,783	-1,442,226	-126,195	-1,568,421			

* Concerns scenario 2 in company 2

Table 17: Constraints S2 in C1

Activity	Variable	Project	Constraints
A1000	$x(1)$	1	$x(1) = -1$
A1005	$x(2)$	1	$-x(2) \leq -23, x(1) - x(2) \leq -22$

A1010	x(3)	1	- x(3) <= -23, x(1)- x(3) <= -22
A1020	x(4)	1	- x(4) <= -43, x(3)- x(4) <= -20
A1030	x(5)	1	- x(5) <= -23, x(1)- x(5) <= -22
A1040	x(6)	1	- x(6) <= -73, x(2)- x(6) <= -20, x(4)- x(6) <= -30, x(5)- x(6) <= -30
A1050	x(7)	1	- x(7) <= -108, x(6)- x(7) <= -35
A1060	x(8)	1	- x(8) <= -138, x(7)- x(8) <= -30
A1070	x(9)	1	- x(9) <= -163, x(8)- x(9) <= -25
A1090	x(10)	1	- x(10) <= -123, x(6)- x(10) <= -50
A1100	x(11)	1	x(11) <= 223, - x(11) <= -193, x(9)- x(11) <= -30, x(13)- x(11) <= -20
A1110	x(12)	1	x(12) <= 223, - x(12) <= -193, x(9)- x(12) <= -30, x(13)- x(12) <= -20
A1120	x(13)	1	x(13) <= -153, x(10)- x(13) <= -30, x(14)- x(13) <= -30
A1130	x(14)	1	- x(14) <= -123, x(6)- x(14) <= -50
A1000	x(15)	2	x(15) = -62
A1005	x(16)	2	- x(16) <= -84, x(15)- x(16) <= -22
A1010	x(17)	2	- x(17) <= -84, x(15)- x(17) <= -22
A1020	x(18)	2	- x(18) <= -104, x(17)- x(18) <= -20
A1030	x(19)	2	- x(19) <= -84, x(15)- x(19) <= -22
A1040	x(20)	2	- x(20) <= -134, x(16)- x(20) <= -20, x(18)- x(20) <= -30, x(19)- x(20) <= -40
A1050	x(21)	2	- x(21) <= -169, x(20)- x(21) <= -35
A1060	x(22)	2	- x(22) <= -199, x(21)- x(22) <= -30
A1070	x(23)	2	- x(23) <= -224, x(22)- x(23) <= -25
A1090	x(24)	2	- x(24) <= -184, x(20)- x(24) <= -50
A1100	x(25)	2	x(25) <= 284, - x(25) <= -254, x(23)- x(25) <= -30, x(27)- x(25) <= -20
A1110	x(26)	2	x(26) <= 284, - x(26) <= -254, x(23)- x(26) <= -30, x(27)- x(26) <= -20
A1120	x(27)	2	- x(27) <= -214, x(24)- x(27) <= -30, x(28)- x(27) <= -30
A1130	x(28)	2	- x(28) <= -184, x(20)- x(28) <= -50

Table 18: Constraints S2 in C2

Activity	Variable	Project	Constraints
A1000	x(1)	1	- x(1) <= -1
A1010	x(2)	1	x(2) = -1
A1020	x(3)	1	- x(3) <= -29, x(1)- x(3) <= -19, x(2)- x(3) <= -28
A1030	x(4)	1	- x(4) <= -29, x(1)- x(4) <= -19, x(2)- x(4) <= -28
A1040	x(5)	1	- x(5) <= -54, x(4)- x(5) <= -25
A1050	x(6)	1	- x(6) <= -79, x(5)- x(6) <= -25
A1060	x(7)	1	- x(7) <= -93, x(6)- x(7) <= -14
A1070	x(8)	1	- x(8) <= -143, x(7)- x(8) <= -50, x(3)- x(8) <= -10
A1080	x(9)	1	- x(9) <= -170, x(8)- x(9) <= -27
A1090	x(10)	1	- x(10) <= -185, x(9)- x(10) <= -15
A1100	x(11)	1	- x(11) <= -185, x(9)- x(11) <= -15
A1110	x(12)	1	- x(12) <= -185, x(9)- x(12) <= -15
A1120	x(13)	1	- x(13) <= -193, x(11)- x(13) <= -8
A1130	x(14)	1	- x(14) <= -195, x(10)- x(14) <= -7, x(11)- x(14) <= -8, x(12)- x(14) <= -10
A1140	x(15)	1	- x(15) <= -200, x(14)- x(15) <= -5
A1150	x(16)	1	- x(16) <= -200, x(14)- x(16) <= -5
A1160	x(17)	1	x(17) <= 238, - x(17) <= -208, x(13)- x(17) <= -5, x(15)- x(17) <= -5, x(16)- x(17) <= -8
A1000	x(18)	2	x(18) = -32
A1010	x(19)	2	- x(19) <= -42, x(18)- x(19) <= -10
A1020	x(20)	2	- x(20) <= -60, x(19)- x(20) <= -18
A1030	x(21)	2	- x(21) <= -60, x(19)- x(21) <= -18
A1040	x(22)	2	- x(22) <= -60, x(19)- x(22) <= -18
A1050	x(23)	2	- x(23) <= -90, x(20)- x(23) <= -13, x(21)- x(23) <= -25, x(22)- x(23) <= -30
A1060	x(24)	2	- x(24) <= -105, x(23)- x(24) <= -15
A1070	x(25)	2	- x(25) <= -115, x(24)- x(25) <= -10
A1080	x(26)	2	- x(26) <= -130, x(25)- x(26) <= -15
A1090	x(27)	2	- x(27) <= -130, x(25)- x(27) <= -15
A1100	x(28)	2	- x(28) <= -130, x(25)- x(28) <= -15
A1110	x(29)	2	- x(29) <= -140, x(27)- x(29) <= -10
A1120	x(30)	2	- x(30) <= -150, x(29)- x(30) <= -10
A1130	x(31)	2	- x(31) <= -145, x(26)- x(31) <= -10, x(28)- x(31) <= -15
A1140	x(32)	2	- x(32) <= -175, x(29)- x(32) <= -10, x(31)- x(32) <= -30
A1150	x(33)	2	- x(33) <= -183, x(32)- x(33) <= -8
A1160	x(34)	2	- x(34) <= -191, x(33)- x(34) <= -8
A1170	x(35)	2	- x(35) <= -198, x(30)- x(35) <= -10, x(34)- x(35) <= -7
A1180	x(36)	2	- x(36) <= -198, x(34)- x(36) <= -7
A1190	x(37)	2	- x(37) <= -198, x(34)- x(37) <= -7

A1200	$x(38)$	2	$-x(38) \leq -198, x(34) - x(38) \leq -7,$
A1210	$x(39)$	2	$x(39) \leq 238, -x(39) \leq -208, x(35) - x(39) \leq -5, x(36) - x(39) \leq -5,$ $x(37) - x(39) \leq -8, x(38) - x(39) \leq -10$

Table 19: Original and optimal start time for S2 in C1

Activity	Original start time (Day)	Optimal start time using SQP
Numaniyah road		
A1000	1	1
A1005	23	56
A1010	23	26
A1020	43	46
A1030	23	46
A1040	73	76
A1050	108	111
A1060,	138	148
A1070	163	173
A1090	123	153
A1100	193	203
A1110	193	203
A1120	153	183
A1130	123	153
Al-ahrar road		
A1000	62	62
A1005	84	124
A1010	84	94
A1020	104	114
A1030	84	104
A1040	134	144
A1050	169	179
A1060	199	209
A1070	224	234
A1090	184	214
A1100	254	264
A1110	254	264
A1120	214	244
A1130	184	214
Max Negative Cash Flow	-1,852,096	-1,661,660
Profit	811,243	776,593

Table 20: Original and optimal start time for S2 in C2

Activity	Original start time (Day)	Optimal start time using SQP
Thaileb bridge		
A1000	1	19
A1010	1	1
A1020	29	93
A1030	29	38
A1040	54	63
A1050	79	88
A1060	93	102
A1070	143	152
A1080	170	179
A1090	185	197
A1100	185	196
A1110	185	194
A1120	193	209
A1130	195	204
A1140	200	212
A1150	200	209
A1160	208	217

Al-Fokanee bridge		
A1000	32	32
A1010	42	53
A1020	60	71
A1030	60	77
A1040	60	72
A1050	90	102
A1060	105	117
A1070	115	127
A1080	130	144
A1090	130	150
A1100	130	142
A1110	140	162
A1120	150	176
A1130	145	157
A1140	175	187
A1150	183	195
A1160	191	203
A1170	198	210
A1180	198	215
A1190	198	210
A1200	198	210
A1210	208	220
Max Negative Cash Flow	-484,524	-434,970
Profit	234,362	141,228

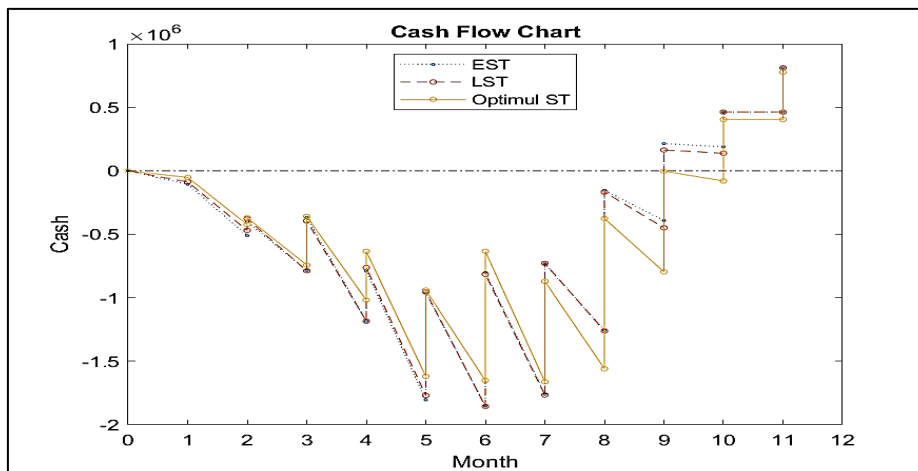


Fig. 5: Original and optimized cash flow chart S2 in C1

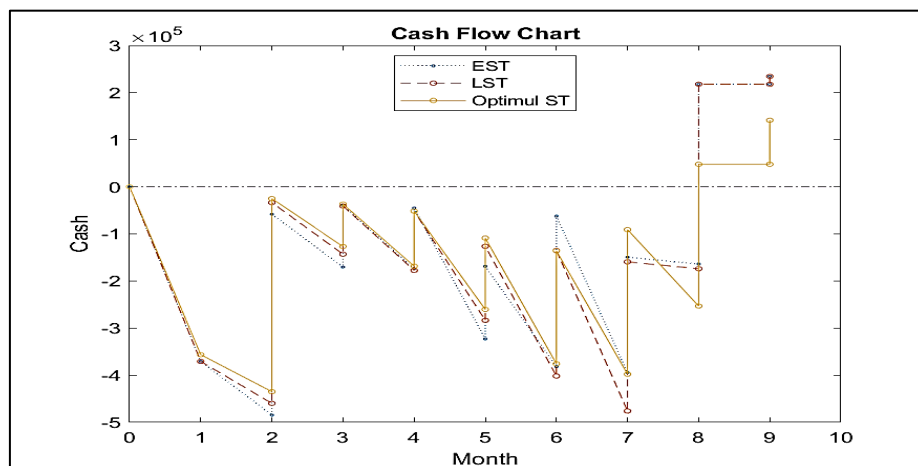


Fig. 6: Original and optimized cash flow chart S2 in C2

8.3 Apply scenario 3

The third scenario is used to find a solution for the problem of devising schedules that correspond to minimum negative cash flow by the imposition of using a short-term loan from the bank and recovering this loan at an interest rate determined according to the bank's policy as monthly payments until the end of the project. This procedure reduces the profits of the project. (Assuming the loan's value is 250000×10^3 with an interest rate of 10%). Below are the objective function and constraints specific to this scenario:

$$\text{Minimize: } I_t \quad (6)$$

$$E_{st} \leq A_{st} \leq L_{st} \quad (7)$$

Where: I_t is the maximum interest rate, E_{st} is the early start time for activity according to schedule, A_{st} is the activity start data in the project and L_{st} is the late start time for activity according to time schedule. Critical path activities are started early to generate schedules and non-critical activities are started at optimal start times while maintaining a link between them. These schedules generate corresponding cash requirements profiles. The SQP procedure then looks for a schedule that minimizes negative cumulative cash flow. Tables 21 and 22 show the optimized cash flow calculation by applying scenario 3 in two companies. Also, Tables 23 and 24 show constraints in the two companies. On the other hand, the noncritical activities' start times have changed to reach the optimum schedule with minimum negative cash flow and optimum profit, as shown in Tables 25 and 26 which compare the original and optimal start times for the activities in the two companies. Fig. 7 and 8 compare the two companies' initial negative cumulative cash flow and the optimized negative cumulative cash flow.

Table 21: Cash flow optimization computation for the S3* in C1*

Month	Bill to Owner	Retention	Taxes	Total Receipts	Execution Cost	Overhead	Total Cost	Net Cash Flow N_t	Cumulative Net Cash Flow CN_t	Cumulative Cash Flow F_t
1					-83,460	-6,420	-89,880	-89,880	160,120	-89,880
2	107,000	10,700	3,531	92,769	-412,248	-29,596	-441,844	-349,075	-188,955	-281,724
3	493,267	49,327	16,278	427,662	-375,835	-26,795	-402,630	25,032	-163,923	-591,585
4	446,583	44,658	14,737	387,188	-798,920	-59,340	-858,260	-471,072	-634,995	-1,022,183
5	989,000	98,900	32,637	857,463	-937,404	-69,993	-1,007,396	-149,933	-784,928	-1,642,391
6	1,166,543	115,802	38,496	1,012,245	-850,738	-63,326	-914,064	98,181	-686,747	-1,698,992
7	1,055,433		34,829	1,020,604	-941,778	-70,329	-1,012,107	8,497	-678,250	-1,698,853
8	1,172,151		38,681	1,133,470	-508,484	-36,999	-545,482	587,988	-90,262	-1,223,732
9	616,646		20,349	596,297	-269,948	-18,650	-288,598	307,699	217,436	-378,860
10	310,831		10,258	300,573	-51,122	-1,817	-52,940	247,633	465,068	164,495
11	30,286		999	29,287	-27,500		-27,500	1,787	786,243	437,568
Total	6,387,740	319,387	210,795	5,857,558	-5,257,437	-383,264	-5,640,701			

* Concerns scenario 3 in company 1

Table 22: Cash flow optimization computation for the S3* C2*

Month	Bill to Owner	Retention	Taxes	Total Receipts	Execution Cost	Overhead	Total Cost	Net Cash Flow N_t	Cumulative Net Cash Flow CN_t	Cumulative Cash Flow F_t
1					-340,979	-29,836	-370,815	-370,815	-120,815	-370,815
2	426,224			426,224	-138,936	-9,149	-148,085	278,139	157,324	-268,899
3	130,701			130,701	-137,460	-9,020	-146,480	-15,779	141,545	10,844
4	128,856			128,856	-157,710	-10,792	-168,502	-39,646	101,900	-26,956
5	154,168			154,168	-290,067	-22,373	-312,440	-158,272	-56,371	-210,540
6	319,615			319,615	-230,263	-17,140	-247,403	72,212	15,840	-303,775
7	244,860			244,860	-339,692	-26,715	-366,407	-121,547	-105,707	-350,567
8	381,646			381,646	-47,744	-1,170	-48,914	332,732	227,018	-154,628
9	16,713			16,713	-34,375		-34,375	-17,662	209,363	192,643
Total	1,802,783			1,802,783	-1,717,226	-126,195	-1,843,421			

* Concerns scenario 3 in company 2

Table 23: Constraints S3 in C1

Activity	Variable	Project	Constraints
A1000	x(1)	1	$x(1) = 1$
A1005	x(2)	1	$23 \leq x(2) \leq 53$
A1010	x(3)	1	$x(3) = 23$
A1020	x(4)	1	$x(4) = 43$
A1030	x(5)	1	$23 \leq x(5) \leq 43$
A1040	x(6)	1	$x(6) = 73$
A1050	x(7)	1	$x(7) = 108$
A1060	x(8)	1	$x(8) = 138$
A1070	x(9)	1	$x(9) = 163$
A1090	x(10)	1	$123 \leq x(10) \leq 143$
A1100	x(11)	1	$x(11) = 193$
A1110	x(12)	1	$x(12) = 193$
A1120	x(13)	1	$153 \leq x(13) \leq 173$
A1130	x(14)	1	$123 \leq x(14) \leq 143$
A1000	x(15)	2	$x(15) = 62$
A1005	x(16)	2	$84 \leq x(16) \leq 114$
A1010	x(17)	2	$x(17) = 84$
A1020	x(18)	2	$x(18) = 104$
A1030	x(19)	2	$84 \leq x(19) \leq 94$
A1040	x(20)	2	$x(20) = 134$
A1050	x(21)	2	$x(21) = 169$
A1060	x(22)	2	$x(22) = 199$
A1070	x(23)	2	$x(23) = 224$
A1090	x(24)	2	$184 \leq x(24) \leq 204$
A1100	x(25)	2	$x(25) = 254$
A1110	x(26)	2	$x(26) = 254$
A1120	x(27)	2	$214 \leq x(27) \leq 234$
A1130	x(28)	2	$184 \leq x(28) \leq 204$

Table 24: Constraints S3 in C2

Activity	Variable	Project	Constraints
A1000	x(1)	1	$1 \leq x(1) \leq 10$
A1010	x(2)	1	$x(2) = 1$
A1020	x(3)	1	$29 \leq x(3) \leq 133$
A1030	x(4)	1	$x(4) = 29$
A1040	x(5)	1	$x(5) = 54$
A1050	x(6)	1	$x(6) = 79$
A1060	x(7)	1	$x(7) = 93$
A1070	x(8)	1	$x(8) = 143$
A1080	x(9)	1	$x(9) = 170$
A1090	x(10)	1	$185 \leq x(10) \leq 188$
A1100	x(11)	1	$185 \leq x(11) \leq 187$
A1110	x(12)	1	$x(12) = 185$
A1120	x(13)	1	$193 \leq x(13) \leq 203$
A1130	x(14)	1	$x(14) = 195$
A1140	x(15)	1	$200 \leq x(15) \leq 203$
A1150	x(16)	1	$x(16) = 200$
A1160	x(17)	1	$x(17) = 208$
A1000	x(18)	2	$x(18) = 32$
A1010	x(19)	2	$x(19) = 42$
A1020	x(20)	2	$60 \leq x(20) \leq 77$
A1030	x(21)	2	$60 \leq x(21) \leq 65$
A1040	x(22)	2	$x(22) = 60$
A1050	x(23)	2	$x(23) = 90$
A1060	x(24)	2	$x(24) = 105$
A1070	x(25)	2	$x(25) = 115$
A1080	x(26)	2	$130 \leq x(26) \leq 135$
A1090	x(27)	2	$130 \leq x(27) \leq 155$
A1100	x(28)	2	$x(28) = 130$
A1110	x(29)	2	$140 \leq x(29) \leq 165$
A1120	x(30)	2	$150 \leq x(30) \leq 193$
A1130	x(31)	2	$x(31) = 145$
A1140	x(32)	2	$x(32) = 175$
A1150	x(33)	2	$x(33) = 183$
A1160	x(34)	2	$x(34) = 191$
A1170	x(35)	2	$198 \leq x(35) \leq 203$
A1180	x(36)	2	$198 \leq x(36) \leq 203$
A1190	x(37)	2	$198 \leq x(37) \leq 200$

A1200	x(38)	2	x(38) = 198
A1210	x(39)	2	x(39) = 208

Table 25: Original and optimal start time for S3 in C1

Activity	Original start time (Day)	Optimal start time using SQP
Numaniyah road		
A1000	1	1
A1005	23	28
A1010	23	23
A1020	43	43
A1030	23	43
A1040	73	73
A1050	108	108
A1060	138	138
A1070	163	163
A1090	123	123
A1100	193	193
A1110	193	193
A1120	153	173
A1130	123	141
Al-ahrar road		
A1000	62	62
A1005	84	92
A1010	84	84
A1020	104	104
A1030	84	85
A1040	134	134
A1050	169	169
A1060	199	199
A1070	224	224
A1090	184	202
A1100	254	254
A1110	254	254
A1120	214	227
A1130	184	187
Max Negative Cash Flow	-1,852,096	-1,698,922
Profit	811,243	786,243

Table 26: Original and optimal start time for S3 in C2

Activity	Original start time (Day)	Optimal start time using SQP
Thaileb bridge		
A1000	1	1
A1010	1	1
A1020	29	29
A1030	29	29
A1040	54	54
A1050	79	79
A1060	93	93
A1070	143	143
A1080	170	170
A1090	185	185
A1100	185	185
A1110	185	185
A1120	193	193
A1130	195	195
A1140	200	200
A1150	200	200
A1160	208	208
Al-Fokanee bridge		

A1000	32	32
A1010	42	42
A1020	60	60
A1030	60	60
A1040	60	60
A1050	90	90
A1060	105	105
A1070	115	115
A1080	130	130
A1090	130	130
A1100	130	130
A1110	140	140
A1120	150	150
A1130	145	145
A1140	175	175
A1150	183	183
A1160	191	191
A1170	198	198
A1180	198	198
A1190	198	198
A1200	198	198
A1210	208	208
Max Negative Cash Flow	-484,524	-370,815
Profit	234,362	209,363

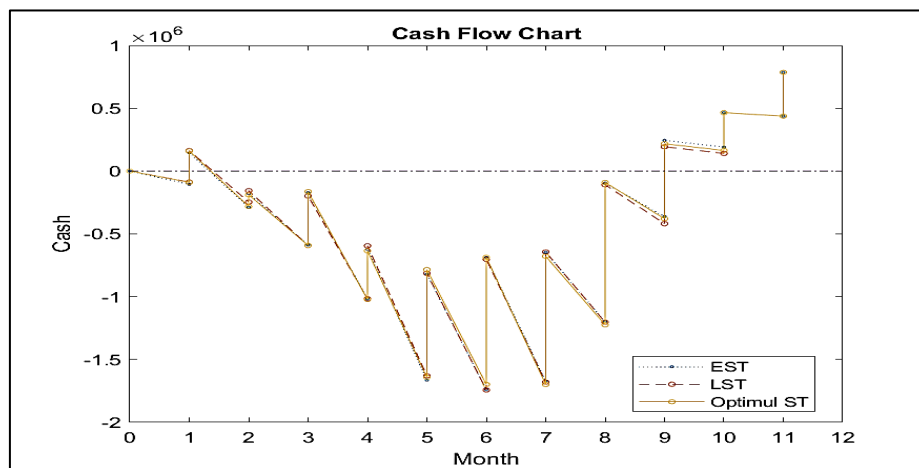


Fig. 7: Original and optimized cash flow chart S3 in C1

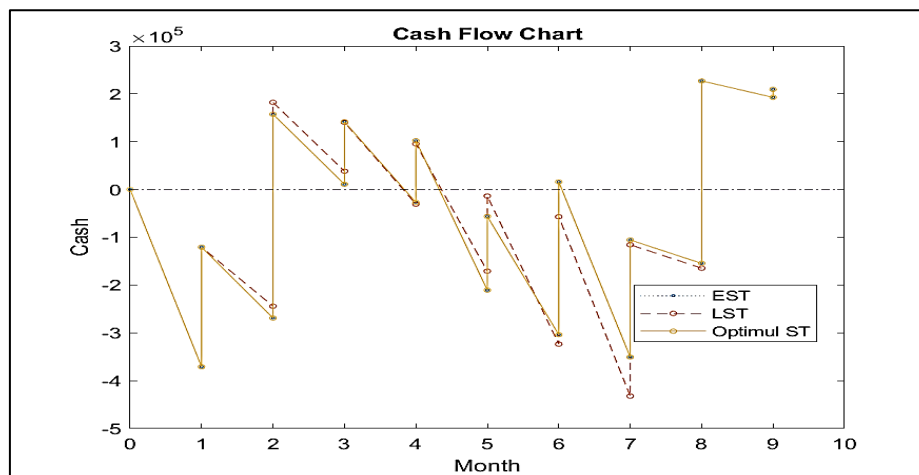


Fig. 8 Original and optimized cash flow chart S3 in C2

8 Conclusions

This study uses SQP to optimize budget deficit in multi-construction projects for companies and conducts periodic financial auditing on behalf of the contractor. As a result of the work carried out in this research, the following conclusions can be withdrawn:

- 1) The model's flexibility is shown in its ability to mitigate financial strain by rescheduling activities without extending completion time. Therefore, the proposed model determines the best scheduling strategy to meet contractors' financial requirements related to schedule problems.
- 2) The model allows the user to examine various project execution scenarios (and their impact on time and total cost) to minimize negative cash flow without extending the time of the project, extending the time of the project and using short-term loans. These scenarios determine the ideal combination of monthly financing inflow and outflow schedules.
- 3) When applying scenario 1, which refers to not extending the time of the project, the schedule becomes risky due to the loss of flexibility of noncritical activities and the use of their float. Therefore, a delay in any activity can cause a delay in the project as a whole.
- 4) One of the inevitable aspects of any construction project is the variation orders. This model does not account for any costs incurred due to changes in scope or variation orders.
- 5) Where the standard bid documents give the right to contractors' commitment to providing financial capability and financial resources as a condition of obtaining the contract, failing to pay the contractors on time for progress payments will leave them in a precarious financial position, especially in the absence of advance payment.

Abbreviations

CPM	Critical Path Method
ESD	Early Start Day
EFD	Early Finish Day
EST	Early Start Time
EFT	Early Finish Time
FF	Free Float
GA	Genetic Algorithm
GALP	Genetic Algorithms and Linear Programming
IQD	Iraq Dinar
LSD	Late Start Day
LSD	Late Finish Day
LST	Late Start Time
LFT	Late Finish Time
LP	Linear Programming
NLP	Nonlinear Optimization Problems
QP	Quadratic Problems
UNDP	United Nations Development Programme
SQP	Sequential Quadratic Programming
TF	Total Float

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