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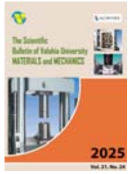

STUDY OF SOLICITATION STATES AT CIRCULAR AND ISOTROPIC PLATES

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Abstract: *In this paper we make a comparison between displacements and stresses of a circular and isotropic plate, which is solicitated at a pressure, using the analytical method and the same parameters obtained with finite element method. In this last case, the plate belong to the respectively recipient. From the two methods, we can conclusion that the results obtained both analytical and Cosmos Finite Element Method are appropriated.*

Keywords: *solicitation states, circular plates, isotropic materials*

1. INTRODUCTION

The study of solicitation states in circular and isotropic plates represents a fundamental topic in structural mechanics and material science. Circular plates are widely used in engineering applications such as mechanical components, aerospace structures, and civil engineering systems, where their ability to withstand complex loading conditions is critical. Because isotropic materials exhibit uniform properties in all directions, they provide an ideal framework for analyzing stress and deformation patterns under various boundary conditions and load distributions [1].

Understanding the stress distribution, bending moments, and deflections in such plates is essential for predicting structural performance and ensuring safety. Classical plate theory, combined with modern analytical and numerical methods, offers powerful tools to investigate these behaviors. In particular, the evaluation of radial and tangential stresses allows for a deeper insight into the mechanical response of plates subjected to uniform or concentrated loads [2].

This research aims to provide a comprehensive analysis of solicitation states in circular isotropic plates, highlighting both theoretical approaches and practical implications. By examining the interplay between geometry, material properties, and loading conditions, the study contributes to the broader understanding of plate mechanics and supports the development of more efficient structural designs.

The plane circular plates are structural elements with much smaller thicknesses than other dimensions, used to carry bending loads. They can be classified as membranes (which carry only tensile forces), thin plates (rigid, where the loads are transmitted by bending with shear force), and thick plates (where the calculations require three-dimensional elasticity). To simplify calculations for plates, simplifying assumptions are applied, such as the Kirchhoff-Love hypothesis and the midplane inextensibility hypothesis and others.

The researches to calculate the deformations and stresses states can be grouped in: analytical [3], [4], [5], numerical [3], [6] and phenomenal methods [3], [4].

The aggregate is charged with an uniform and distributed charge p = 1MPa. Because the axial-symmetrical load, the median plane will be deformed axial- symmetrically, also.

2. THEORETICAL CONSIDERATIONS

From the differential relationship for the axial-symmetrical bending of the circular plates, which are solicited transversal [3], it results:

$$\frac{d^3w}{dr^3}+\frac{1}{r}\frac{d^2w}{dr^2}-\frac{1}{r^2}\frac{dw}{dr}=\frac{T}{D} \quad (1)$$

We obtain the displacements:

$$w=\frac{p\cdot r^4}{64D}-C_1\cdot\frac{r^2}{4}-C_2\cdot \ln r+C_3 \quad (2)$$

$$\varphi = -\frac{p \cdot r^3}{16D} + C_1 \cdot \frac{r^2}{2} + \frac{C_2}{2} \tag{3}$$

In these accounts:

w represents the deflection;
T – the shearing load;
φ - the slope angle; r - the radius;
C_1, C_2, C_3 - the integrating constants;
D – the bending rigidity of the plate, which is:

$$D = \frac{E \cdot h^3}{12 \cdot \left(1 - \nu^2\right)}, \tag{4}$$

where: E is the modulus of longitudinal elasticity of the circular plate material; ν - the coefficient of the transversal working (the Young 's coefficient).

We note the external radius of the plate with R, like in figure 1. The constants are determined when we put the boundary conditions:

- on the outline, for $r = R, \varphi = 0, w = 0$;

- in the center, for $r = 0$, $\varphi = 0$.

The equations (2) and (3), with these values, become:

$$w = \frac{p}{64 \cdot D} \cdot \left(R^2 - r^2\right)^2 \tag{5}$$

$$\varphi = \frac{p \cdot r}{16 \cdot D} \cdot \left(R^2 - r^2\right) \tag{6}$$

The maximal deflection is in the center of the circular plate, at $r = 0$.

$$w_{max} = \frac{p \cdot R^4}{64 \cdot D} \tag{7}$$

In the studied case: $w = 0{,}952[mm]$.

We apply the following formulas, for calculate the stresses:

$$\sigma_{r\,max} = \frac{M_t}{\frac{h^2}{6}}; \tag{8}$$

$$\sigma_{t\,max} = \frac{M_r}{\frac{h^2}{6}} \tag{9}$$

where: M_r, M_t are the circumferential bending moments, respectively the radial bending moments.

In the center of the plate:

$$\sigma_r = \sigma_t = \frac{6}{h^2} \cdot M_t = 0{,}487 \cdot \frac{p \cdot R^2}{h^2} \tag{10}$$

In the studied case: $\sigma_r = 135{,}277\left[\frac{N}{mm^2}\right]$

On the outline of the plate:

$$\sigma_r = \frac{6}{h^2} \cdot M_t = \frac{3}{4} \cdot \frac{p \cdot R^2}{h^2} \tag{11}$$

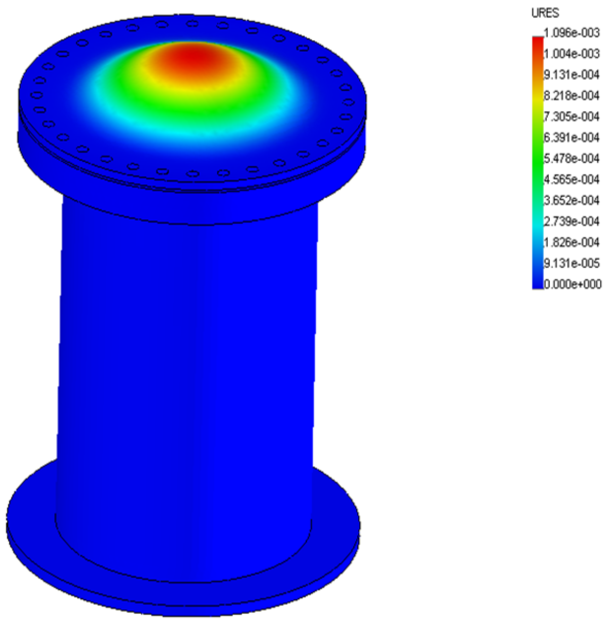

Figure 1. Results of displacements

$$\sigma_t = \frac{6}{h^2} \cdot M_r = 0{,}225 \cdot \frac{p \cdot R^2}{h^2} \quad (12)$$

In the studied cases:

$$\sigma_r = 208{,}(3)\left[\frac{N}{mm^2}\right]$$

$$\sigma_t = 62{,}5\left[\frac{N}{mm^2}\right]$$

The biggest value of stress is the value of σ_r on the outline.

If we put the dimensions of the plate in accordance with theory I of resistance, this formula becomes:

$$\sigma_a = \frac{3}{4} \cdot \frac{p \cdot R^2}{h^2},$$

and $h = 15{,}0099[mm]$.

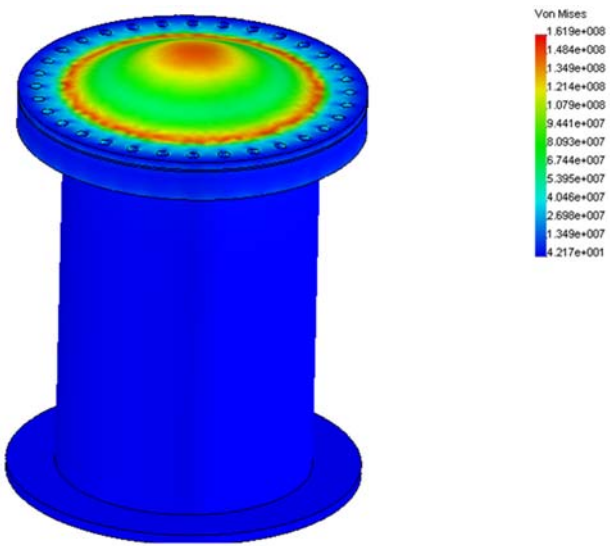

Figure 2. Results of stresses

3. OBTAINED RESULTS WITH FINITE ELEMENT METHOD

The program which is used is Cosmos Works 7.0.
The obtained results with the theoretical method and the obtained results with the virtual model are in absolute agreement.

For displacements we obtain figure 1 and for stresses we obtain figure 2.

4. CONCLUSIONS

From the two methods shown up, we can say that the results obtained both analytical and Cosmos Finite Element Method are appropriated, so given errors of the methods are small.

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