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COMPARATIVE STUDY OF STRESSES DUE TO FORCED MOUNTING OF A SHAFT LOCATED ON FOUR BEARINGS FOR THREE CASES: COAXIAL BEARINGS, RIGID NON-COAXIAL BEARINGS AND ELASTIC NON-COAXIAL BEARINGS

Cornel MARIN
Valahia University of Targoviste
E-mail: marin.cor@yahoo.com

Abstract: *The paper presents a comparative study of the stresses that occur for three cases of shaft mounting in a speed reducer: the maximum stresses in a continuous beam located on four coaxial rigid supports, the maximum stresses in the same beam located on four coaxial rigid supports and the maximum stresses in the same beam on four non-coaxial rigid and elastic supports. In the first case of shaft mounting, it is mounted on four bearings with non-coaxial and rigid support, in the second case, the shaft is mounted on the four coaxial and rigid bearings, and in the third case the shaft is mounted on four bearings with non-coaxial rigid and elastic end bearings, the intermediate bearings being mounted in the support by means of elastic elements. The statically indeterminate problem is solved in this paper using the load function method ψ from Strength of Materials and the numerical calculation is done for a particular numerical case, using the step function Φ(x-a) from the professional program MATHCAD. The solution method allows the numerical calculation of reactions, the drawing of internal stress diagrams and the calculation of maximum stresses in the dangerous section.*

Keywords: *Stresses in the forced assembly of a shaft, load function, step function Mathcad*

1. INTRODUCTION

The calculation model for the study of stresses and deformations corresponding to the forced mounting of a shaft on four bearings with non-coaxial bearings is the continuous beam located on four rigid supports [1], [2], [3]:

Case 1 - the shaft is forced mounted on four bearings with non-coaxial and rigid bearings having the intermediate support offset both in the vertical plane (w2=0.002 mm; w3=0.001mm) and in the horizontal plane (v2=0.001 mm; v3=0.001 mm) ;

Case 2 - the shaft is mounted on four bearings with bearings with coaxial and rigid support;

Case 3 - the shaft is forced mounted on four bearings with bearings with non-coaxial and elastic support, the intermediate bearings being mounted in the support by means of elastic elements; the intermediate support are offset by the same values as in case 1: in the vertical plane (w2=0.002 mm; w3=0.001mm); in the horizontal plane (v2=0.001 mm; v3=0.001 mm).

For calculating reactions, drawing bending moment diagrams and calculating maximum stresses in dangerous sections, the loading function ψ is used [1], [2], [3]. Using the professional calculation program MATHCAD, this method allows both calculating reactions and drawing stress and strain diagrams using the step function Φ(x-a) in the MATHCAD program for the given numerical values of the parameters as well as calculating maximum stresses in the dangerous section.

2. CALCULATION MODEL FOR CASE 1

The calculation model used for the first case of assembly of the shaft forcedly mounted on four non-coaxial and rigid bearings with the intermediate support offset both in the vertical plane (w2=0.002 mm; w3=0.001mm) and in the horizontal plane (v2=0.001 mm; v3=0.001 mm), is a continuous beam located on four rigid point supports that are not at the same level with the axis of the bar with the intermediate supports offset in the vertical plane, having the bending stiffness EI constant along its entire length as in figure 1.

For the general case, the continuous beam is located on four supports located between them at distances b2, b3 and b4, two consoles having lengths a and c and is loaded with different concentrated forces P, distributed forces q and force couples N that are located at distances d, e, f and g from the end of the bar. To determine the reactions from the four bearings V1, V2, V3 and V4, the two equilibrium equations from Statics and two equations from Mechanics of deformable bodies obtained from writing the equation of the three arrows for the triplets of supports 1-2-3, respectively 2-3-4 [1], [2], [3] are used:

$$\begin{cases} \sum F_{zs} \downarrow = V_1 + V_2 + V_3 + V_4 \\ \sum \overleftarrow{M}_{4s} = V_1 \cdot (b_2 + b_3 + b_4) + V_2 \cdot (b_3 + b_4) + V_3 \cdot b_4 \end{cases} \tag{1}$$

$$\begin{cases} EI \cdot \left[w_1 \cdot b_3 - w_2 \cdot (b_2 + b_3) + w_3 \cdot b_2 \right] = \\ = \Psi_1 \cdot b_3 - \Psi_2 \cdot (b_2 + b_3) + \Psi_3 \cdot b_2 \\ EI \cdot \left[w_2 \cdot b_4 - w_3 \cdot (b_3 + b_4) + w_4 \cdot b_3 \right] = \\ = \Psi_2 \cdot b_4 - \Psi_3 \cdot (b_3 + b_4) + \Psi_4 \cdot b_3 \end{cases} \tag{2}$$

where:

- $\sum F_{zs} \downarrow$ is the sum of the external forces along the axis direction *Oz;*
- $\sum \bar{M}_{4s}$ the sum of the moments of the forces and the external couples along an axis parallel to Oy passing through the right support (support 4);
- $\Psi_1,\ \Psi_2\ \Psi_3$ *şi* Ψ_4 are the load functions corresponding to the supports *1, 2, 3 şi 4*.

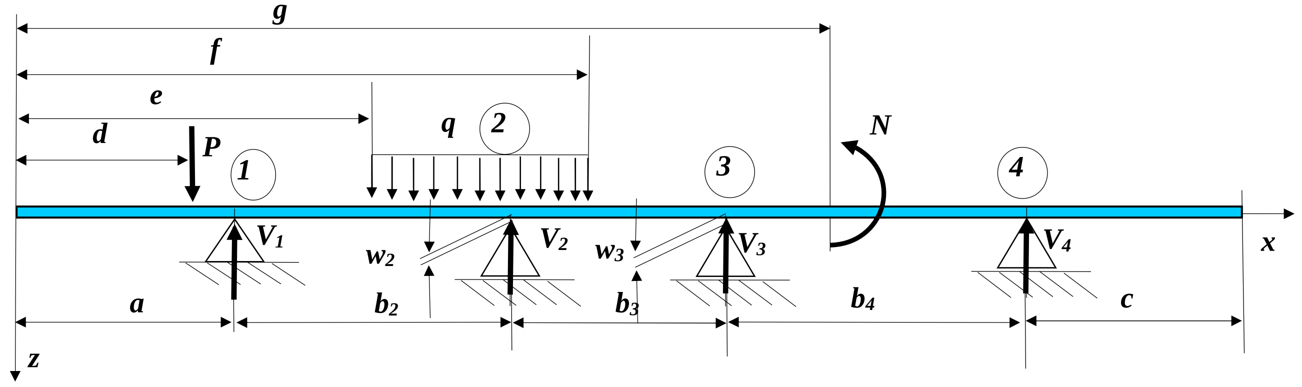

Figure 1

The values of deflections in rigid supports are taken into account: $w_1= w_4=0,\ w_2 \neq 0;\ w_3 \neq 0$ and the load functions from the supports are expressed ($\Psi_1,\ \Psi_2\ \Psi_3$ *şi* Ψ_4) as sums of the load functions of external loads ($\Psi_{1s},\ \Psi_{2s}\ \Psi_{3s}$ *şi* Ψ_{4s}) and the charge functions corresponding to the unknown reactions:

$$\begin{cases} \Psi_1 = \Psi_{1s}\,; \qquad \Psi_2 = \Psi_{2s} - \dfrac{V_1 b_2^3}{6}\;; \\ \Psi_3 = \Psi_{3s} - \dfrac{V_1 (b_2+b_3)^3}{6} - \dfrac{V_2 b_3^3}{6}; \\ \Psi_4 = \Psi_{4s} - \dfrac{V_1 (b_2+b_3+b_4)^3}{6} - \dfrac{V_2 (b_3+b_4)^3}{6} - \dfrac{V_3 b_4^3}{6}. \end{cases} \tag{3}$$

Substituting relations (3) into equations (2) we obtain two equivalent equations having the reactions as unknowns V_1, V_2 şi V_3:

$$\begin{cases} \Psi_{1s} b_3 - \left(\Psi_{2s} - \dfrac{V_1 b_2^3}{6}\right)(b_2+b_3) + \left(\Psi_{3s} - \dfrac{V_1(b_2+b_3)^3}{6} - \dfrac{V_2 b_3^3}{6}\right) b_2 = \\ = EI\left[-w_2\cdot(b_2+b_3)+w_3\cdot b_2\right] \\ \left(\Psi_{2s} - \dfrac{V_1 b_2^3}{6}\right) b_4 + \left(\Psi_{3s} - \dfrac{V_1(b_2+b_3)^3}{6} - \dfrac{V_2 b_3^3}{6}\right)(b_3+b_4) + \\ + \left(\Psi_{4s} - \dfrac{V_1(b_2+b_3+b_4)^3}{6} - \dfrac{V_2(b_3+b_4)^3}{6} - \dfrac{V_3 b_4^3}{6}\right) b_2 = \\ = EI\left[w_2\cdot b_4 - w_3\cdot(b_3+b_4)\right] \end{cases} \tag{4}$$

If we introduce the notations in equations (4):

$$\begin{cases} A_{2s} = \Psi_{1s}\cdot b_3 - \Psi_{2s}\cdot(b_2+b_3) + \Psi_{3s}\cdot b_2 \\ A_{3s} = \Psi_{2s}\cdot b_4 - \Psi_{3s}\cdot(b_3+b_4) + \Psi_{4s}\cdot b_3 \end{cases}; \qquad \begin{cases} B_2 = E\cdot I\cdot\left[-w_2\cdot(b_2+b_3)+w_3\cdot b_2\right] \\ B_3 = E\cdot I\cdot\left[w_2\cdot b_4 - w_3\cdot(b_3+b_4)\right] \end{cases} \tag{5}$$

these become:

$$\begin{cases} \dfrac{V_1 b_2^3 (b_2+b_3)}{6} - \dfrac{V_1(b_2+b_3)^3 b_2}{6} - \dfrac{V_2 b_2 b_3^3}{6} = B_2 - A_{2s} \\ -\dfrac{V_1 b_2^3}{6} b_4 + \dfrac{V_1(b_2+b_3)^3(b_3+b_4)}{6} + \dfrac{V_2 b_3^3(b_3+b_4)}{6} - \\ -\dfrac{V_1(b_2+b_3+b_4)^3 b_3}{6} - \dfrac{V_2(b_3+b_4)^3 b_3}{6} - \dfrac{V_3 b_4^3 b_3}{6} = B_3 - A_{3s} \end{cases} \tag{6}$$

Solving the system formed by equations (1) and (6) we obtain the following calculation formulas for the four reactions:

$$\begin{cases} V_1 = \dfrac{\dfrac{3(b_3+b_4)(A_{2S}-B_2)}{b_2 b_3} - \dfrac{3(A_{3S}-B_3)}{2b_4} + \dfrac{b_3 b_4}{4}\sum \bar{M}_{4S}}{b_2\left(0{,}75\cdot b_3^2 + b_2 b_3 + b_2 b_4 + b_3 b_4\right)} \\ V_2 = \dfrac{6(A_{2S}-B_2)}{b_2 b_3^3} - \dfrac{V_1}{b_3^2}(b_2+b_3)(2b_2+b_3) \\ V_3 = \dfrac{1}{b_4}\left[\sum \bar{M}_{4S} - (b_3+b_4)V_2 - (b_2+b_3+b_4)V_1\right] \\ V_4 = \sum F_{zs} - V_1 - V_2 - V_3 \end{cases} \tag{7}$$

3. CALCULATION MODEL FOR CASE 2

The calculation model used for the second case of mounting the shaft on four bearings with bearings having the four coaxial supports is a particular case of the model from Case 1 in which the values of the vertical and horizontal misalignments are zero:

$w_2 = w_3 = 0; \; v_2 = v_3 = 0$

4. CALCULATION MODEL FOR CASE 3

The calculation model used for the third case of forced mounting of the shaft on four bearings with bearings as non-coaxial, rigid and elastic supports, having the intermediate support offset by the same values as in case 1: in the vertical plane (w2=0.002 mm; w3=0.001mm) and in the horizontal plane (v2=0.001 mm; v3=0.001 mm) is a continuous beam located on four rigid and elastic point supports, having the intermediate supports offset in the vertical plane and the bending stiffness EI constant along its entire length as in figure 2.

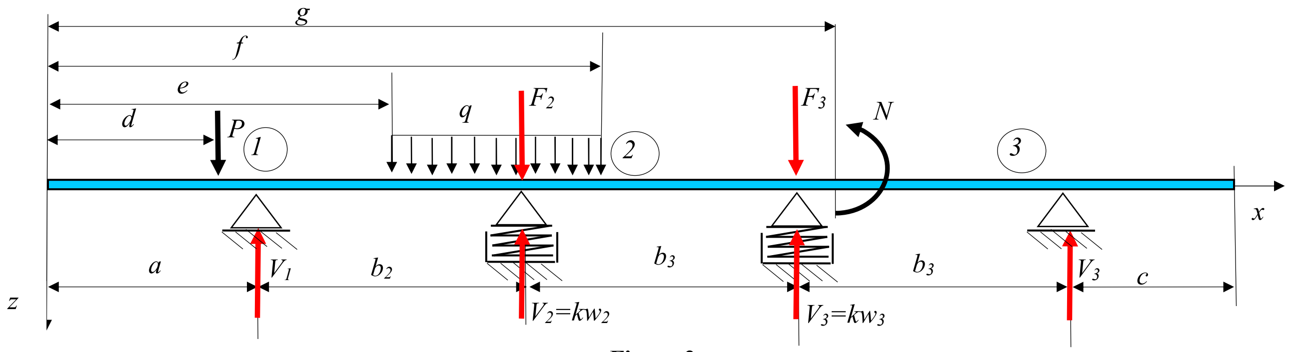

Figure 2

For the general case, the continuous beam is located on four supports located at distances b2, b3 and b4, two consoles having lengths a and c and is loaded with different concentrated forces P, distributed forces q and force couples N which are located at distances d, e, f and g from the end of the beam.

To determine the reactions from the four bearings V1, V2 , V3 and V4, the two equilibrium equations from Statics and two equations from Mechanics of deformable bodies obtained from writing the equation of the three arrows for the triplets of supports 1-2-3 , respectively 2-3-4 [1], [2]:

$$\begin{cases}\sum F_{zs}\downarrow = V_1+V_2+V_3+V_4\\ \sum \bar{M}_{4s} = V_1\cdot(b_2+b_3+b_4)+V_2\cdot(b_3+b_4)+V_3\cdot b_4\end{cases} \qquad (8)$$

$$\begin{cases}EI\cdot\left[w_1\cdot b_3 - w_2\cdot(b_2+b_3)+w_3\cdot b_2\right]=\\ =\Psi_1\cdot b_3-\Psi_2\cdot(b_2+b_3)+\Psi_3\cdot b_2\\ EI\cdot\left[w_2\cdot b_4 - w_3\cdot(b_3+b_4)+w_4\cdot b_3\right]=\\ =\Psi_2\cdot b_4-\Psi_3\cdot(b_3+b_4)+\Psi_4\cdot b_3\end{cases} \qquad (9)$$

în care:

- is the sum of the external forces along the axis direction Oz;
- the sum of the moments of the forces and the external couples along an axis parallel to Oy passing through the right support (support 4);
- Ψ1, Ψ2, Ψ3 and Ψ4 are the load functions corresponding to the supports 1, 2, 3 și 4.
- the deflections w2 and w3 in the intermediate supports are proportional to the corresponding reactions V2 and V3:

$$w_2=\frac{V_2}{k};\quad w_3=\frac{V_3}{k} \qquad (10)$$

- additional forces F2 and F3 occur due to forced assembly due to misalignments in the vertical plane:

$$F2=k\cdot w2=2000\ N;\quad F3=k\cdot w3=1000\ N; \qquad (11)$$

The load functions from the supports (Ψ1, Ψ2, Ψ3 and Ψ4) are expressed as sums of the load functions of the external loads ((Ψ1s, Ψ2s, Ψ3s and Ψ4s) and the load functions corresponding to the unknown reactions.:

$$\begin{cases}\Psi_1=\Psi_{1s};\qquad \Psi_2=\Psi_{2s}-\dfrac{V_1 b_2^3}{6};\\ \Psi_3=\Psi_{3s}-\dfrac{V_1(b_2+b_3)^3}{6}-\dfrac{V_2 b_3^3}{6};\\ \Psi_4=\Psi_{4s}-\dfrac{V_1(b_2+b_3+b_4)^3}{6}-\dfrac{V_2(b_3+b_4)^3}{6}-\dfrac{V_3 b_4^3}{6}.\end{cases} \qquad (12)$$

Substituting relations (3) into equations (2) we obtain two equivalent equations having as unknowns the reactions V1, V2 and V3:

$$\begin{cases}\Psi_{1s} b_3-\left(\Psi_{2s}-\dfrac{V_1 b_2^3}{6}\right)(b_2+b_3)+\left(\Psi_{3s}-\dfrac{V_1(b_2+b_3)^3}{6}-\dfrac{V_2 b_3^3}{6}\right)b_2=\\ =EI\left[-\dfrac{V_2}{k}\cdot(b_2+b_3)+\dfrac{V_3}{k}\cdot b_2\right]\\ \left(\Psi_{2s}-\dfrac{V_1 b_2^3}{6}\right)b_4+\left(\Psi_{3s}-\dfrac{V_1(b_2+b_3)^3}{6}-\dfrac{V_2 b_3^3}{6}\right)(b_3+b_4)+\\ +\left(\Psi_{4s}-\dfrac{V_1(b_2+b_3+b_4)^3}{6}-\dfrac{V_2(b_3+b_4)^3}{6}-\dfrac{V_3 b_4^3}{6}\right)b_2=\\ =EI\left[\dfrac{V_2}{k}\cdot b_4-\dfrac{V_3}{k}\cdot(b_3+b_4)\right]\end{cases} \qquad (13)$$

If we introduce the notations in equations (4):

$$\begin{cases} A_{2s} = \Psi_{1s} \cdot b_3 - \Psi_{2s} \cdot (b_2 + b_3) + \Psi_{3s} \cdot b_2 \\ A_{3s} = \Psi_{2s} \cdot b_4 - \Psi_{3s} \cdot (b_3 + b_4) + \Psi_{4s} \cdot b_3 \end{cases} \quad (14)$$

these becomes:

$$\begin{cases} \frac{V_1 b_2^3 (b_2 + b_3)}{6} - \frac{V_1 (b_2 + b_3)^3 b_2}{6} - \frac{V_2 b_2 b_3^3}{6} = \\ = EI\left[-\frac{V_2}{k} \cdot (b_2 + b_3) + \frac{V_3}{k} \cdot b_2 \right] - A_{2s} \\ -\frac{V_1 b_2^3}{6} b_4 + \frac{V_1 (b_2 + b_3)^3 (b_3 + b_4)}{6} + \frac{V_2 b_3^3 (b_3 + b_4)}{6} - \\ -\frac{V_1 (b_2 + b_3 + b_4)^3 b_3}{6} - \frac{V_2 (b_3 + b_4)^3 b_3}{6} - \frac{V_3 b_4^3 b_3}{6} = \\ = EI\left[\frac{V_2}{k} \cdot b_4 - \frac{V_3}{k} \cdot (b_3 + b_4) \right] - A_{3s} \end{cases}$$

(15)

The system formed by equations (8) and (15) is solved in MATHCAD, having as unknowns the reactions V1, V2, V3, and V4, which is written in matrix form:

(16)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ (b_2 + b_3 + b_4) & (b_3 + b_4) & b_4 & 0 \\ \frac{b_2 (b_2 + b_3)(-2 b_2 b_3 - b_3^2)}{6} & -\frac{b_2 b_3^3}{6} + \alpha \cdot (b_2 + b_3) & -\alpha \cdot b_2 & 0 \\ -\frac{b_2^3 b_4}{6} + \frac{(b_2 + b_3)^3 (b_3 + b_4)}{6} - \frac{(b_2 + b_3 + b_4)^3 b_3}{6} & \frac{b_3^3 (b_3 + b_4)}{6} - \frac{(b_3 + b_4)^3 b_3}{6} - \alpha \cdot b_4 & -\frac{b_4^3 b_3}{6} + \alpha \cdot (b_3 + b_4) & 0 \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{Bmatrix} = \begin{Bmatrix} \sum F_{zs} \\ \sum \overleftarrow{M}_{4s} \\ -A_{2s} \\ -A_{3s} \end{Bmatrix}$$

5. NUMERICAL CALCULATION OF MAXIMUM STRESSES FOR CASE 1

On the input main shaft of the gear speed reducer in figure 3, three cylindrical gears with inclined teeth are mounted as in figure 4. The shaft receives movement from a belt whell D1=75 mm assembled on the cone at a speed n1= 1500 rpm; the electric motor has a power P1=15 kW which it transmits to three working machines with the powers: P2=4 kW; P3=5 kW; P4=6 kW and the speeds n1, n2 and n3 respectively through the three whell gears D2=250, D3=220 and D4=200 mm located at distances e1=25, e2=55, e3=85, e4=115 mm from the shaft end in accordance with figure 4. Gears have teeth inclined at an angle β=20^0. The four bearings with bearings A, B, C and D are located between them at distances a=40, b2=30, b3=30, b4=30 mm, in accordance with figure 4. The four supports used for mounting the bearings in the gearbox housing are not coaxial: the bearing supports B and C being offset from the line of supports A and D: in the vertical plane by w2= 0.002 mm and w3= 0.001 mm and in the horizontal plane they are offset respectively by v2= 0.001 mm v3= 0.001.

The following will be calculated/represented:

1. the total torques Mt1 transmitted through the main shaft from the engine to the three working machines (Mt2 , Mt3 , Mt4);

2. the external forces acting on the shaft as well as the reactions from the four bearings;

3. diagrams of axial, torsional, shear, bending stresses in the main shaft;

4. maximum equivalent bending moment Mimax in the dangerous section;

5. maximum equivalent stress corresponding to bending and torsional stresses in the dangerous section, according to the Von MISSES theory.

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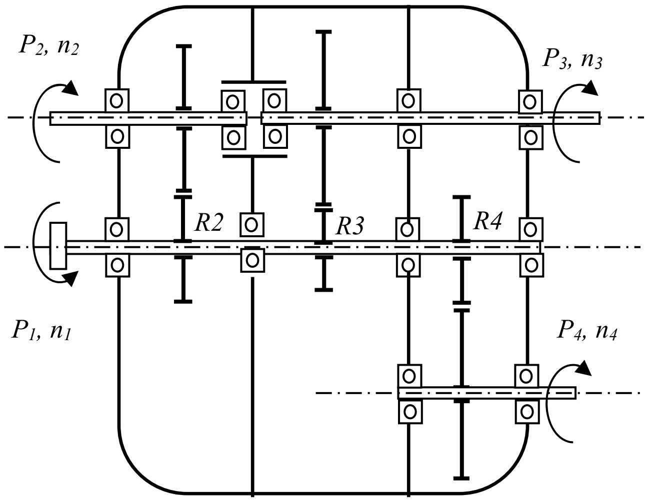

Figure 3

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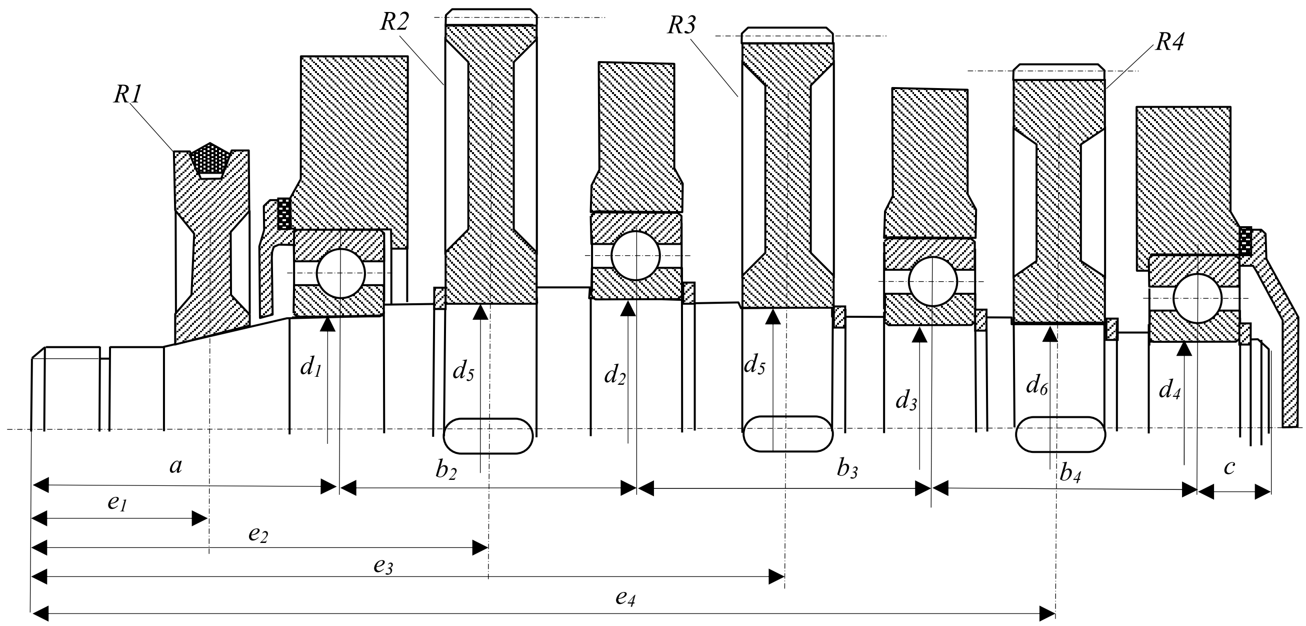

Figure 4

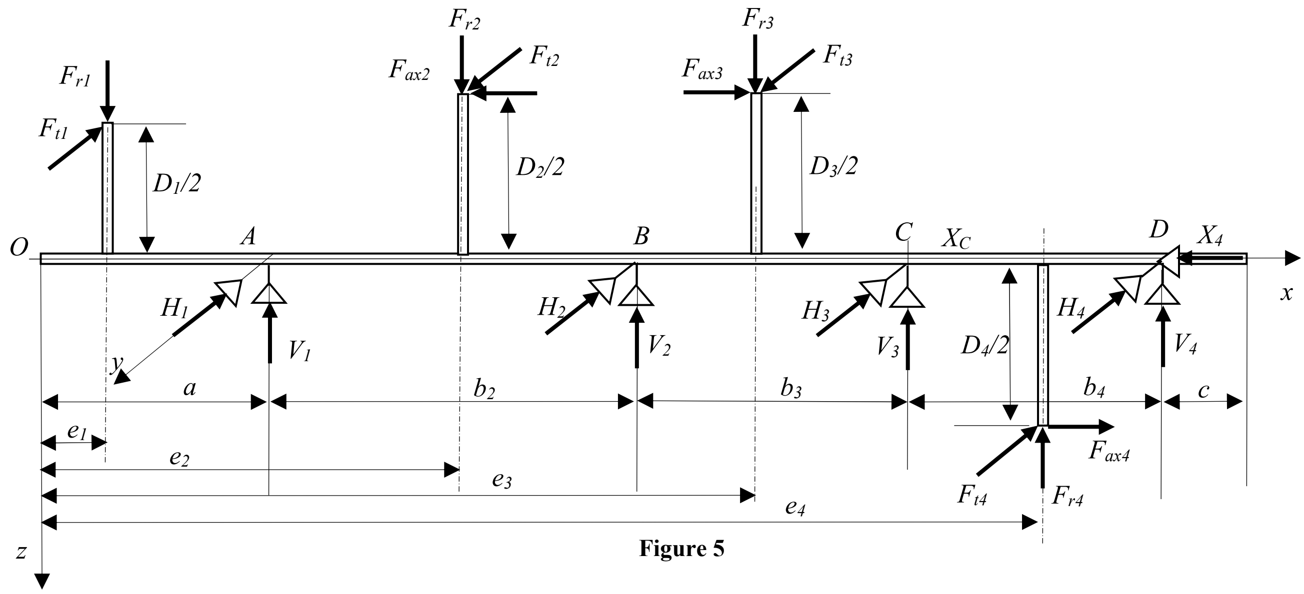

Figure 5

5.1. Calculation of torques and forces in the belt wheel and gears

The shaft loading scheme in both the vertical plane Oxz and the horizontal plane Oxy is shown in figure 5. The reactions from the four bearings are:

- in the vertical plane V1, V2, V3 and V4,
- in the horizontal plane: H1, H2, H3 and H4;
- in the axial direction X4

The calculation relations for the torques, tangential, radial and axial forces in the three gears and the tangential and radial forces in the belt wheel are:

$$M_{t1} = \frac{30000}{\pi}\frac{P_0}{n};\quad M_{t2} = \frac{30000}{\pi}\frac{P_2}{n};M_{t3} = \frac{30000}{\pi}\frac{P_3}{n};$$

$$F_{t1} = \frac{2M_{t1}}{D_1};\quad F_{r1} = 0{,}5 \cdot F_{t1};$$

$$F_{t2} = \frac{2M_{t2}}{D_2};\quad F_{r2} = F_{t2}\frac{tg\alpha_n}{\cos\beta};\quad F_{ax2} = F_{t2} \cdot tg\beta$$

$$F_{t3} = \frac{2M_{t3}}{D_3};\quad F_{r3} = F_{t3}\frac{tg\alpha_n}{\cos\beta};\ F_{ax3} = F_{t3} \cdot tg\beta$$

$$F_{t4} = \frac{2M_{t4}}{D_4};\quad F_{r4} = F_{t4}\frac{tg\alpha_n}{\cos\beta};\quad F_{ax4} = F_{t4} \cdot tg\beta \tag{16}$$

The numerical results obtained in MATHCAD for the torsional torques, tangential force in the belt wheel, the tangential, radial and axial forces of the three gears are:

- torsional torques:

Mt1 = 95492.966   Mt2 = 25464.791

$Mt3 = 31830.989 \qquad Mt4 = 38197.186$ Nmm

-tangential forces:

$Ft1 = 1091.348 \quad Ft2 = 203.718 \quad Ft3 = 289.373$

$Ft4 = 381.972$ N

- radial forces:

$Fr1 = 545.674 \quad Fr2 = 75.291 \quad Fr3 = 106.948$

$Fr4 = 141.171$ N

- axial forces:

$Fax2 = 35.921 \quad Fax3 = 51.024 \quad Fax4 = 67.352$ N

5.2 Reactions in the bearings

The numerical results obtained in MATHCAD for the reactions in the vertical plane in the bearings A, B, C and D were obtained using relations (7) and have the values:

$V1 = 11997.123 \qquad V2 = -25271.408$

$V3 = 16926.459 \qquad V4 = -3065.432$ N

The numerical results obtained in MATHCAD for the reactions in the horizontal plane in the bearings A, B, C and D were obtained using relations (7) and have the values:

$H1 = 4982.648 \qquad H2 = -10455.329$

$H3 = 2895.363 \qquad H4 = 1597.089$ N

5.3. Diagrams of axial, torsional, shear, bending stresses in the main shaft;

The diagrams of axial, torsional, shear and bending stresses in the main shaft are presented in Figure 6, figure 7 and Figure 8.

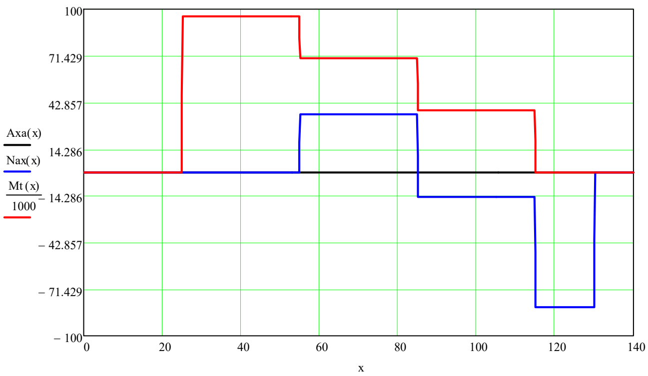

Figure 6. Torsional couple Mtx and axial internal forces Nx diagrams

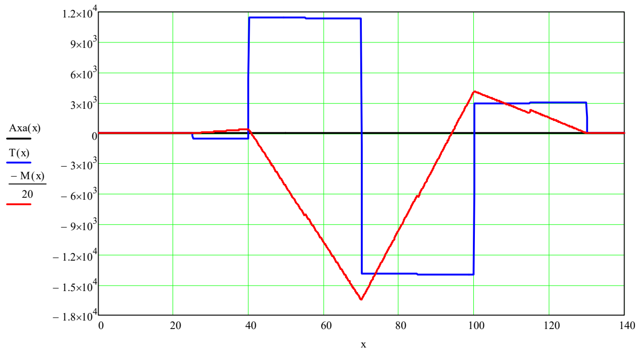

Figure 7. Shear forces Tz and bending couple Miy diagrams in the vertical plane Oxz

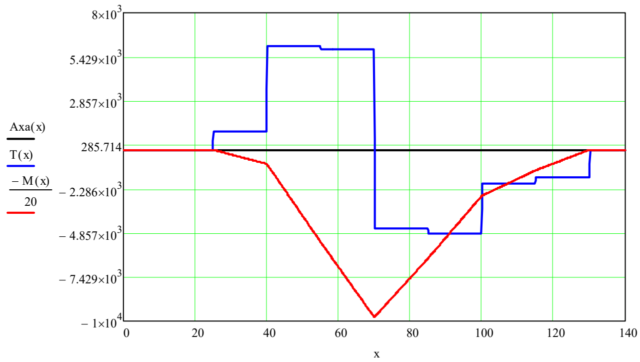

Figure 8. Shear forces Ty and bending couple Miz diagrams in the horizontal plane Oxz

5.4. The maximum equivalent bending moment Mimax in the dangerous section

From the analysis of the diagrams in Fig. 7 and 8, the dangerous section is the one corresponding to the bearing B located at the distance x=70 mm (Fig.5):

- the maximum value of the moment Miymax is:

$Miymax := M(70) \qquad Miymax = 329738.864$ Nmm

- the maximum value of the moment Mizmax is:

$Mizmax := M(70) \qquad Mizmax = 195534.345$ Nmm

The maximum bending moment in the dangerous section B is:

$$Mimax := \sqrt{Miymax^2 + Mizmax^2}$$

$Mimax = 383355.446$ Nmm

The maximum torsional moment in dangerous section B is:

$Mt1 - Mt2 = 70028.175$ Nmm

6.5. Maximum equivalent stress in the dangerous section

Flexural modulus of section B:

$$Wy := \frac{\pi \cdot d2^3}{32} \qquad Wy = 12271.846 \ mm^3$$

Torsional modulus of section B:

$$Wp := \frac{\pi \cdot d2^3}{16} \qquad Wp = 24543.693 \ mm^3$$

Maximum bending stress in section B:

$$\sigma max := \frac{Mimax}{Wy} \qquad \sigma max = 31.239 \ MPa$$

Maximum torsional stress in section B:

$$\tau max := \frac{Mt1 - Mt2}{Wp} \qquad \tau max = 2.853 \ MPa$$

The maximum equivalent stress in the dangerous section B according to the Von MISSES resistance theory is:

$$\sigma ech5 := \sqrt{\sigma max^2 + 3\tau max^3} \qquad \sigma ech5 = 32.335 \ MPa$$

6. NUMERICAL CALCULATION OF MAXIMUM STRESSES IN CASE 2

The input data for Case 2 are the same as those for Case 1 (figure 4). The difference between Case 2 and Case 1 is that the four bearings A, B C and D are located face to face mounted in the gearbox housing in four coaxial supports, this being a particular case of Case 1 where w2=w3=0 and v2=v3=0.

6.1. Calculation of torques and forces in the belt whell and gear wheels

The shaft loading scheme in both the vertical plane Oxz and the horizontal plane Oxy is presented in figure 5. The reactions in the four bearings are:

- in the vertical plane V1, V2, V3 and V4,
- in the horizontal plane: H1, H2, H3 and H4;
- in the axial direction X4

For the tangential forces in the belt wheel, the tangential, radial and axial forces of the two gears, the same numerical results are obtained as in Case 1.

6.2 Reactions in the bearings

The numerical results obtained for the reactions in the vertical plane in the bearings A, B, C and D were obtained using relations (7) and have the values:

$$V1 = 1088.815 \quad V2 = -727.716 \quad N$$

$$V3 = 563.998 \quad V4 = -338.355 \quad N$$

The numerical results obtained for the reactions in the horizontal plane in the bearings A, B, C and D were obtained using same relations (7) and have the values:

$$H1 = 2255.571 \quad H2 = -7728.252 \quad N$$

$$H3 = 5622.440 \quad H4 = -1129.988 \quad N$$

6.3. Diagrams of axial, torsional, shear, bending stresses in the main shaft

The diagrams of axial, torsional, shear and bending stresses in the main shaft are presented in:

- Fig. 6 –Torsional couple Mtx and axial internal forces Nx diagrams
- Fig. 9 – Shear forces Tz and bending couple Miy diagrams in the vertical plane Oxz
- Fig. 10 – Shear forces Ty and bending couple Miz diagrams in the horizontal plane Oxy.

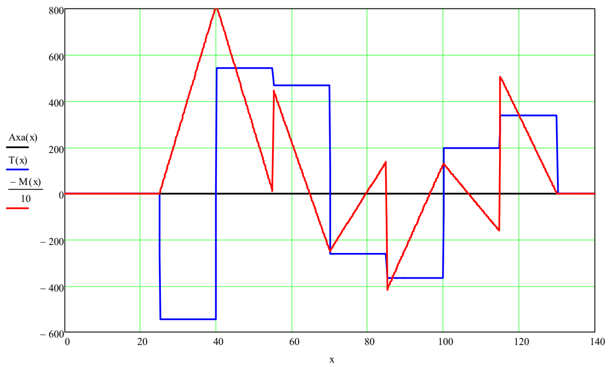

Figure 9. Shear forces Tz and bending couple Miy diagrams in the vertical plane Oxz

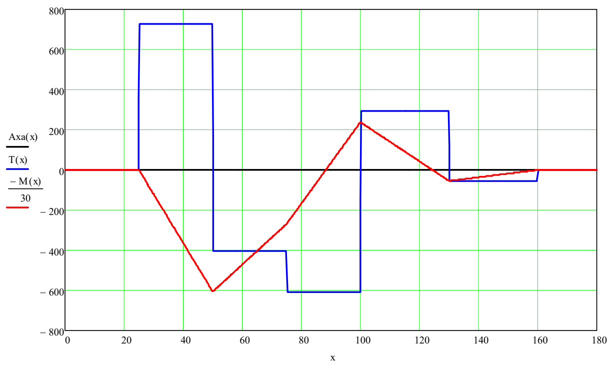

Figure 10. Shear forces Ty and bending couple Miz diagrams in the horizontal plane Oxz

6.4. The maximum equivalent bending moment Mimax in the dangerous section

From the analysis of the diagrams in Fig. 9 and 10, the dangerous section is the one corresponding to the bearing B. The maximum value of the moment Miymax is:

$$Miymax2 := M(70) \qquad Miymax2 = 2489.629 \ Nmm$$

The maximum value of the moment Mizmax is:

$$Mizmax2 := M(70) \qquad Mizmax2 = 113722.036 \ Nmm$$

The maximum bending moment in the dangerous section B is:

$$Mimax2 := \sqrt{Miymax2^2 + Mizmax2^2}$$

$Mimax2 = 113749.284$ Nmm

The maximum torsional moment in the dangerous section B is:

$Mt1 - Mt2 = 70028.175$ Nmm

6.5. Maximum equivalent stresses in the dangerous section

Flexural modulus of section B:

$$Wy := \frac{\pi \cdot d2^3}{32} \qquad Wy = 12271.846 \text{ mm}^3$$

Torsional modulus of section B:

$$Wp := \frac{\pi \cdot d2^3}{16} \qquad Wp = 24543.693 \text{ mm}^3$$

Maximum bending stress in section B:

$$\sigma max := \frac{Mimax2}{Wy} \qquad \sigma max = 9.269 \text{ MPa}$$

Maximum torsional stress in section B:

$$\tau max := \frac{Mt1 - Mt2}{Wp} \qquad \tau max = 2.853 \text{ MPa}$$

The maximum equivalent stress in the dangerous section B according to the Von MISSES resistance theory is:

$$\sigma ech5 := \sqrt{\sigma max^2 + 3\tau max^3}$$

$\sigma ech5 = 12.474$ MPa

7. NUMERICAL CALCULATION OF MAXIMUM STRESSES IN CASE 3

The input data for Case 3 are the same as those for Case 1 (figure 4). The difference between Case 3 and Case 1 is that the two bearings B and C are mounted in the gearbox housing by means of elastic elements, according to figure 11. The main shaft is force-mounted on four bearings with bearings having non-coaxial and elastic supports; the intermediate supports are offset by the same values as in case 1: in the vertical plane (w2=0.002 mm; w3=0.001mm); in the horizontal plane (v2=0.001 mm; v3=0.001 mm).

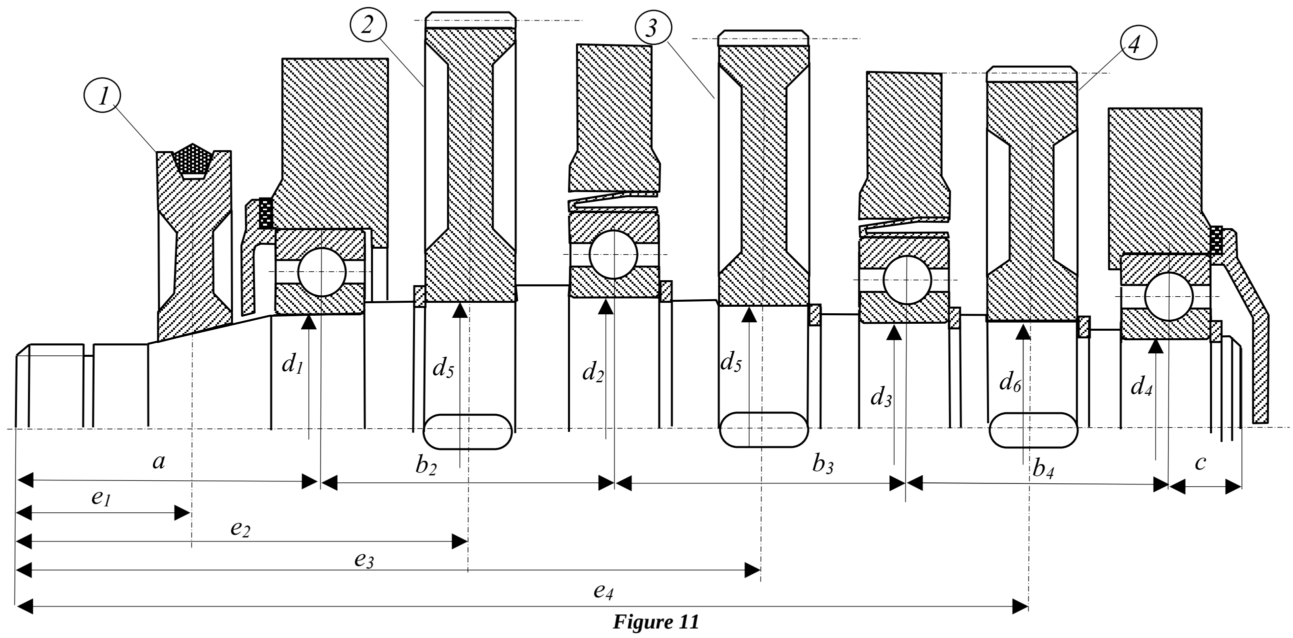

Figure 11

7.1. Calculation of torques and forces in the belt wheel and gears

The shaft loading scheme in both the vertical plane Oxz and the horizontal plane Oxy is shown in figure 5.

The reactions in the four bearings are:

- in the vertical plane V1, V2, V3 and V4,
- in the horizontal plane: H1, H2, H3 and H4;
- in the axial direction X4

The numerical results obtained from the belt wheel, the tangential, radial and axial forces in the three gears are:

-tangential forces:

$Ft1 = 1091.348$ $\quad Ft2 = 203.718$

$Ft3 = 289.373$ $\quad Ft4 = 381.972$ N

- radial forces:

$Fr1 = 545.674$ $\quad Fr2 = 75.291$

$Fr3 = 106.948$ $\quad Fr4 = 141.171$ N

- axial forces:

$Fax2 = 35.921$ $Fax3 = 51.024$ $Fax4 = 67.352$ N

7.2 Reactions in the bearings

The numerical results obtained for the reactions in the vertical plane in the bearings A, B, C and D were obtained using relations (7) and have the values:

$V1 = -441.458$ $V2 = -451.767$ N

$V3 = -397.080$ $V4 = -1122.953$ N

The numerical results obtained for the reactions in the horizontal plane in the bearings A, B, C and D were obtained using same relations (7) and have the values:

$H1 = -1449.352$ $H2 = -671.339$ N

$H3 = -376.617$ $H4 = -482.922$ N

7.3. Diagrams of axial, torsional, shear, bending stresses in the main shaft;

The diagrams of axial, torsional, shear and bending stresses in the main shaft are presented in:

 Fig. 6 –Torsional couple Mtx and axial internal forces Nx diagrams

 Fig. 12 – Shear forces Tz and bending couple Miy diagrams in the vertical plane Oxz

 Fig. 13 – Shear forces Ty and bending couple Miz diagrams in the horizontal plane Oxy.

7.4. Maximum equivalent bending moment Mimax in the dangerous section

From the analysis of the diagrams in Figure 9 and Figure 10, the dangerous section is the one corresponding to bearing B .

The maximum value of the moment Miymax is:

$Miymax := M(70)$ $Miymax = -43418.570$ Nmm

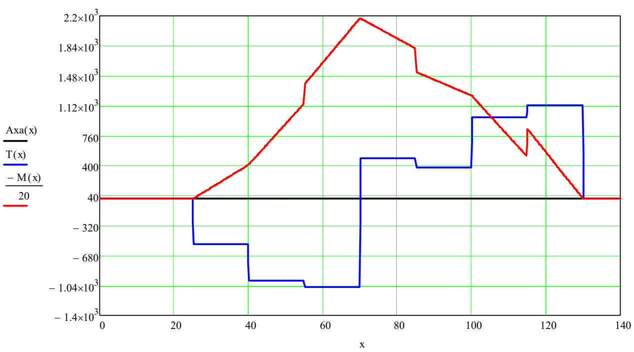

Figure 12. Shear forces Tz and bending couple Miy diagrams in the vertical plane Oxz

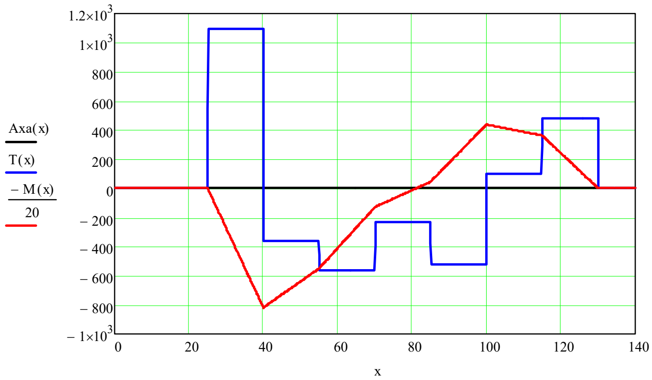

Figure 13. Shear forces Ty and bending couple Miz diagrams in the horizontal plane Oxz

The maximum value of the moment Mizmax is::

$Mizmax := M(70)$ $Mizmax = 2574.346$ Nmm

The maximum bending moment in the dangerous section B is:

$$Mimax := \sqrt{Miymax^2 + Mizmax^2}$$

$Mimax = 43494.821$ Nmm

The maximum torsional moment in the dangerous section B is:

$Mt1 - Mt2 = 70028.175$ Nmm

6.5. Maximum equivalent stresses in the dangerous section

Flexural modulus of section B:

$$Wy := \frac{\pi \cdot d2^3}{32} \qquad Wy = 12271.846 \ mm^3$$

Torsional modulus of section B:

$$Wp := \frac{\pi \cdot d2^3}{16} \qquad Wp = 24543.693 \ mm^3$$

Maximum bending stress in section B:

$$\sigma max := \frac{Mimax}{Wy} \qquad \sigma max = 3.544 \ MPa$$

Maximum torsional stress in section B:

$$\tau max := \frac{Mt1 - Mt2}{Wp} \qquad \tau max = 2.853 \ MPa$$

The maximum equivalent stress in the dangerous section B according to the Von MISSES resistance theory is:

$$\sigma ech5 := \sqrt{\sigma max^2 + 3\tau max^3}$$

$\sigma ech5 = 9.069$ MPa

8. CONCLUSIONS

The comparative analysis of the results obtained for the three shaft assembly cases is based on the centralized data in Table 1 for the bearing reactions, the maximum bending moment and the equivalent stress according to the Von Misses strength theory:

Table 1

Shaft mounting case	V1 (N)	V2 (N)	V3 (N)	V4 (N)	H1 (N)	H2 (N)	H3 (N)	H4 (N)	M_{imax} (Nmm)	σ_{5max} (MPa)
Case 1 Rigid non-coaxial bearings	11.997	-25.271,4	16.926,5	-3.065,4	4.982,6	-10.455,3	2.895,4	1.597	383.355,4	32,33
Case 2 Rigid coaxial bearings	1088,8	-727,7	564	-338,3	2.255,6	-7.728,2	5.622,4	-1.130	113.749,3	12,47
Case 3 Elastic non-coaxial bearings	-441.5	-451,8	-397	-1.123	1.449,3	-671,3	-376,6	-482,9	43.494,8	9,07

Analyzing the results in table 1, the following particularly interesting conclusions can be drawn:

1. The highest values of the reactions in the bearings are obtained for Case 1, for the shaft mounted forcibly in rigid non-coaxial support;
2. The value of the maximum bending moment in the dangerous section (bearing B, x=70 mm), is also obtained for Case 1, the shaft mounted forcibly in non-coaxial support, and is approximately three times higher than in Case 2;
3. The value of the torsional moment is preserved, not being affected by the coaxiality deviations
4. The maximum normal bending stress σmax in the dangerous section (bearing B, x=70 mm), is also obtained for Case 1, the shaft mounted forcibly in non-coaxial support, and is approximately 2.5 times higher than in Case 2;
5. The maximum tangential torsional stress τmax in the dangerous section (bearing B, x=70 mm), is the same 2,85 MPa for the three mounting cases, not being affected by the shaft mounting mode;
6. The lowest values of the bearing reactions are obtained for Case 3, for the shaft mounted forcibly in non-coaxial support with elastic supports;
7. The minimum bending moment value in the dangerous section (bearing B, x=70 mm), is obtained for the Case 3 and is approximately 8.8 times lower than in Case 1;
8. The maximum normal bending stress σmax in the dangerous section (bearing B, x=70 mm), is obtained for Case 3 and is approximately 3.5 times lower than in Case 1.

The final conclusion that emerges is that the coaxiality deviations of the bearing mounting support (Case 1) significantly influence the additional shaft loading and a proposed solution for their elimination would be the mounting Case 3 proposed in this paper.

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