

STUDY OF THE INFLUENCE OF THE DYNAMIC ABSORBER ON THE NATURAL PULSATION OF THE FORCED VIBRATION OF AN ELECTRIC MOTOR MOUNTED ON A CANTILEVER BEAM SUBJECTED TO BENDING AND SUPPORTED ON TWO HELICAL COMPRESSION SPRINGS

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Abstract: The paper aims to study the dynamic behavior of an elastic mechanical system without damping consisting of two masses m_1 and m_2 mounted on a cantilever beam subjected to bending and supported on two helical compression springs, or the influence of the dynamic mass absorber m_2 on the pulsation characteristics of the forced bending vibrations of the mass electric motor m_1 . Using the MATHCAD professional program, the influence of the mounting method of the dynamic absorber on the elastic system's own pulsations can be determined.

Keywords: undamped forced vibration, hybrid dynamic absorber from bending bars and helical compression springs.

1. INTRODUCTION

The study of the dynamic behavior of an elastic mechanical system without damping consisting of a beam in a cantilever supported on two helical springs having the same stiffness on which two concentrated masses m_1 and m_2 are mounted, is done by numerical simulation in MATHCAD, i.e. by changing the mounting position of the mass of the dynamic absorber m_2 (m_3). Using the calculation formulas of the static arrow and the method of influence coefficients [3], [6] for the calculation of natural pulsations, the vibrations of the system with two degrees of freedom are studied for four mounting cases of the engine and the dynamic absorber:

- near the two spring (Fig. 1.a and Fig. 2.a);
 - near the first spring and at the end of the beam (1.b);
 - near the second arch and at the end of the beam (2.b);
- The method of influence coefficients consists in expressing the dynamic displacements of a system with n degrees of freedom according to the inertial forces corresponding to the masses of the vibrating system [3], [6]:

$$\begin{cases} x_1 = \delta_{11} \cdot \Phi_1 + \delta_{12} \cdot \Phi_2 + \delta_{13} \cdot \Phi_3 + \dots + \delta_{1n} \cdot \Phi_n \\ x_2 = \delta_{21} \cdot \Phi_1 + \delta_{22} \cdot \Phi_2 + \delta_{23} \cdot \Phi_3 + \dots + \delta_{2n} \cdot \Phi_n \\ \dots \\ x_n = \delta_{n1} \cdot \Phi_1 + \delta_{n2} \cdot \Phi_2 + \delta_{n3} \cdot \Phi_3 + \dots + \delta_{nn} \cdot \Phi_n \end{cases} \quad (1)$$

where: $\Phi_1, \Phi_2, \Phi_3, \dots, \Phi_n$ represent the inertial forces corresponding to the masses $m_1, m_2, m_3, \dots, m_n$
 $\Phi_1 = -m_1 \cdot x_1; \Phi_2 = -m_2 \cdot x_2; \Phi_3 = -m_3 \cdot x_3; \dots \Phi_n = -m_n \cdot x_n;$

$\delta_{i1}, \delta_{i2} \dots \delta_{in}$ represent the influence coefficients (the displacements of the sections corresponding to the masses $m_1, m_2, m_3, \dots, m_n$, when a unitary force $P=1$ is applied in section i .

2. DEFINING THE PROBLEM

An electric motor of mass $m_1=10\text{kg}$ with speed $n=1500$ rpm and rotor mass $m_0=5\text{kg}$ with eccentricity $r=0.5\text{mm}$ is mounted on a cantilever beam of length $L=a+b+c$ embedded at one end, next to the first support spring (Fig.1.a). Due to the amplitudes of the forced vibrations a simple dynamic absorber without damping consisting of a mass m_2 is used which is mounted on the same console so that the amplitude of the vibrations of the electric motor decreases, the one-degree-of-freedom system becoming a two-degree-of-freedom system. The console of negligible mass is elastically supported on two springs at distances $a=600\text{mm}$ and $b=400\text{mm}$ ($c=300\text{mm}$), each having stiffness $k=500 \text{ N/mm}$. The console section consists of two U8 profiles (STAS 564-80) having the moment of inertia $I_y=57,5 \text{ cm}^4$. For the three assembly cases, the following will be determined:

1. the reactions $M_{01}, V_{01}, V_{11}, V_{21}$ and the values of the displacements δ_{11}, δ_{21} and δ_{31} for the first charging case (Fig. 3);
2. the reactions $M_{02}, V_{02}, V_{12}, V_{22}$ and the values of the displacements δ_{12}, δ_{22} and δ_{32} for the second charging case (Fig. 4);
3. the reactions $M_{03}, V_{03}, V_{13}, V_{23}$ and values of displacements δ_{13}, δ_{23} and δ_{33} for the third charging case (Fig. 5);
4. the values of the natural pulsations p_1 and p_2 of the vibrations of the system with two degrees of freedom and the distribution coefficients corresponding to the natural pulsations p_1, p_2 for

each of the three assembly cases and comparing the results with those obtained for the system with one

degree of freedom [7].

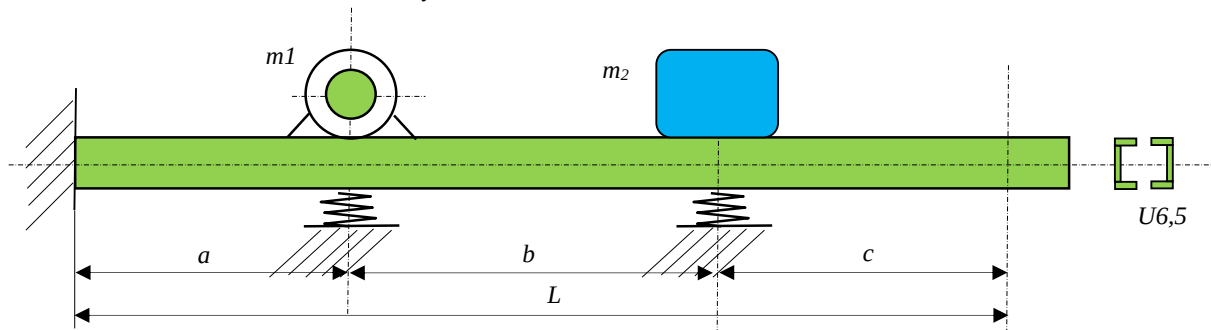


Fig. 1.a

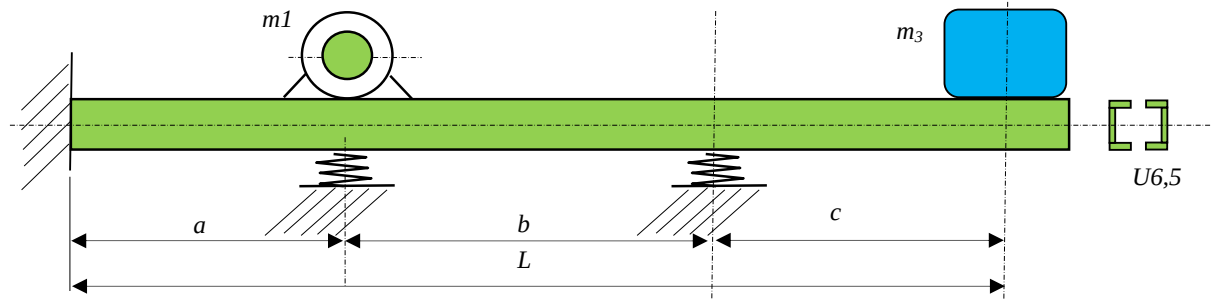


Fig. 1.b

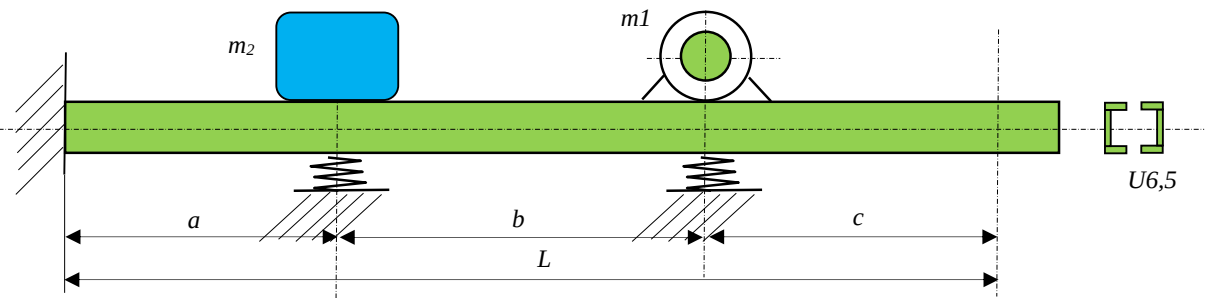


Fig. 2.a

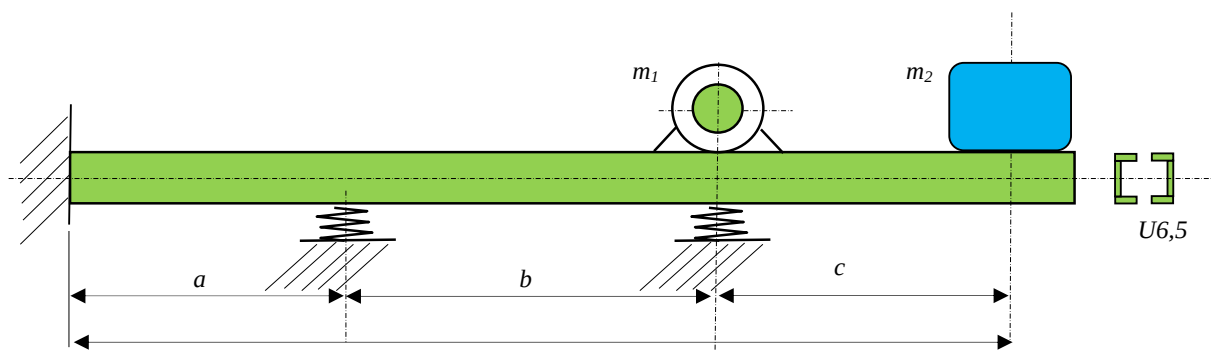


Fig. 2.b

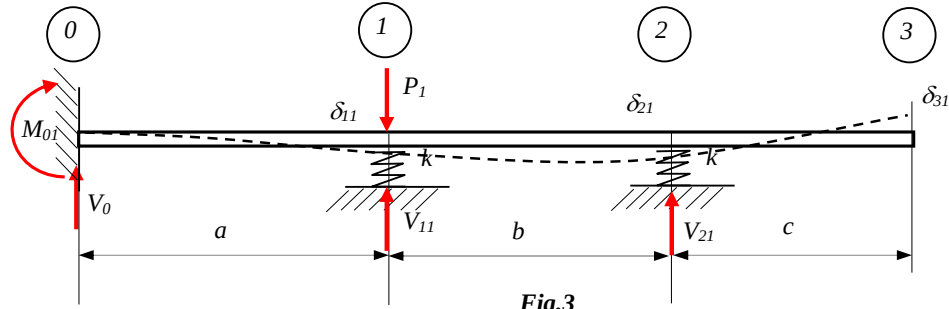


Fig.3

3. CALCULATION OF REACTIONS AND INFLUENCE COEFFICIENTS FOR THE FIRST ASSEMBLY CASE

The influence coefficients δ_{11} , δ_{21} and δ_{31} represent the arrows in the three sections 1, 2 and 3 corresponding to the loading with a unit force P_1 applied in section 1 (Fig. 3). In this case, the reactions M_{01} , V_{01} , V_{11} and V_{21} are calculated using the two equilibrium equations from Mechanics [1], [2]:

$$\begin{cases} V_{01} + V_{11} + V_{21} = P_1 \\ M_{01} - V_{11} \cdot a - V_{21} \cdot (a + b) + P_1 \cdot a = 0 \end{cases}$$

and the two strain equations - the arrows of sections 1 and 2 which are written using the loading function [1]:

$$\begin{cases} E \cdot I_y \cdot \frac{V_{11}}{k} = -\frac{M_{01} \cdot a^2}{2} - \frac{V_{01} \cdot a^3}{6} \\ E \cdot I_y \cdot \frac{V_{21}}{k} = -\frac{M_{01} \cdot (a + b)^2}{2} - \frac{V_{01} \cdot (a + b)^3}{6} - \frac{V_{11} \cdot b^3}{6} + \frac{P_1 \cdot b^3}{6} \end{cases} \quad (3)$$

Solving the system of four equations with four unknowns M_{01} , V_{01} , V_{11} and V_{21} yields the calculation formulas:

$$\begin{cases} V_{01} = P_1 \cdot \frac{6\alpha \cdot b [6\alpha + 3b(a + b)^2 - b^3]}{36\alpha^2 b + 12\alpha \cdot b [(a + b)^3 + a^3] + a^3 b^3 (3a + 4b)}; \\ M_{01} = \frac{-V_{01} \cdot [6\alpha \cdot (a + b) - a^3 \cdot b] + 6\alpha \cdot b \cdot P_1}{6\alpha - 3a^2 \cdot b}; \\ V_{11} = P_1 - V_{01} \frac{a + b}{b} - \frac{M_{01}}{b}; \\ V_{21} = V_{01} \frac{a}{b} + \frac{M_{01}}{b}; \text{ where } \alpha = \frac{E \cdot I_y}{k} \end{cases} \quad (4)$$

The corresponding displacements δ_{11} , δ_{21} and δ_{31} written using the loading function have the expressions:

$$\begin{cases} \delta_{11} = -\frac{V_{01} \cdot a^3 + 3M_{01} \cdot a^2}{6E \cdot I_y} \\ \delta_{21} = -\frac{V_{01} \cdot (a + b)^3 + 3M_{01} \cdot (a + b)^2 + V_{11} \cdot b^3 - P_1 \cdot b^3}{6E \cdot I_y}; \\ \delta_{31} = -\frac{V_{01} \cdot (a + b + c)^3 + 3M_{01} \cdot (a + b + c)^2 + V_{11} \cdot (b + c)^3 - P_1 \cdot (b + c)^3 + V_{21} \cdot c^3}{6E \cdot I_y}. \end{cases} \quad (5)$$

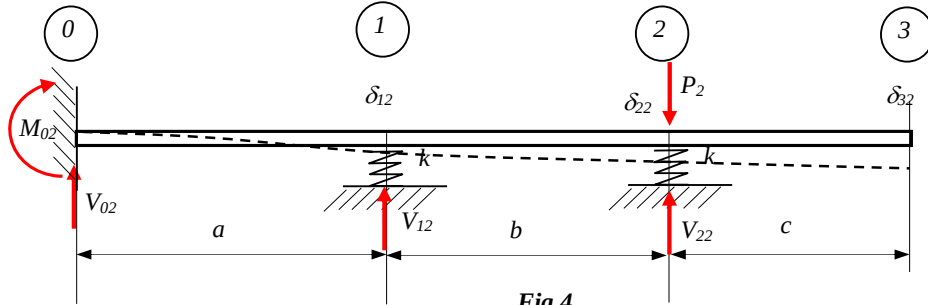


Fig.4

4. CALCULATION OF REACTIONS AND INFLUENCE COEFFICIENTS FOR THE SECOND ASSEMBLY CASE

The influence coefficients δ_{12} , δ_{22} and δ_{32} represent the arrows in the three sections 1, 2 and 3 corresponding to the loading with a unit force P_2 applied in section 2 (Fig. 4).

In this case, the reactions M_{02} , V_{02} , V_{12} and V_{22} are calculated using the two equilibrium equations from Mechanics [1], [2]:

$$\begin{cases} V_{02} + V_{12} + V_{22} = P_2 \\ M_{02} - V_{12} \cdot a - V_{22} \cdot (a + b) + P_2 \cdot (a + b) = 0 \end{cases}$$

and the two strain equations - the arrows of sections 1 and 2 which are written using the loading function [1]:

$$\begin{cases} E \cdot I_y \cdot \frac{V_{12}}{k} = -\frac{M_{02} \cdot a^2}{2} - \frac{V_{02} \cdot a^3}{6} \\ E \cdot I_y \cdot \frac{V_{22}}{k} = -\frac{M_{02} \cdot (a + b)^2}{2} - \frac{V_{02} \cdot (a + b)^3}{6} - \frac{V_{12} \cdot b^3}{6} \end{cases} \quad (6)$$

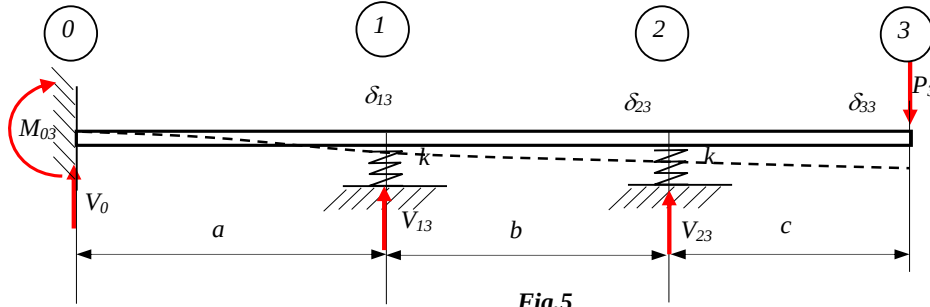
Solving the system of four equations with four unknowns M_{02} , V_{02} , V_{12} and V_{22} yields the calculation formulas:

(7)

$$\begin{cases} V_{02} = P_2 \cdot \frac{6\alpha \cdot b \cdot (6\alpha - 3a^2 \cdot b)}{36\alpha^2 b + 12\alpha \cdot b[(a+b)^3 + a^3] + a^3 b^3 (3a+4b)}; \\ M_{02} = -P_2 \cdot \frac{6\alpha \cdot b \cdot [6\alpha(a+b) - a^3 \cdot b]}{36\alpha^2 b + 12\alpha \cdot b[(a+b)^3 + a^3] + a^3 b^3 (3a+4b)}; \\ V_{12} = -V_{02} \frac{a+b}{b} - \frac{M_{02}}{b}; \\ V_{22} = P_2 + V_{02} \frac{a}{b} + \frac{M_{02}}{b}; \text{ where } \alpha = \frac{E \cdot I_y}{k}. \end{cases}$$

The corresponding displacements δ_{12} , δ_{22} and δ_{32} written using the loading function have the expressions:

$$\begin{cases} \delta_{12} = -\frac{V_{02} \cdot a^3 + 3M_{02} \cdot a^2}{6E \cdot I_y}; \\ \delta_{22} = -\frac{V_{02} \cdot (a+b)^3 + 3M_{02} \cdot (a+b)^2 + V_{12} \cdot b^3}{6E \cdot I_y}; \\ \delta_{32} = -\frac{V_{02} \cdot (a+b+c)^3 + 3M_{02} \cdot (a+b+c)^2 + V_{12} \cdot (b+c)^3 - P_2 \cdot c^3 + V_{22} \cdot c^3}{6E \cdot I_y} \end{cases} \quad (8)$$



5. CALCULATION OF REACTIONS AND INFLUENCE COEFFICIENTS FOR THE THIRD ASSEMBLY CASE

The influence coefficients δ_{13} , δ_{23} and δ_{33} represent the arrows in the three sections 1, 2 and 3 corresponding to the loading with a unit force P_3 applied in section 3 (Fig. 5). In this case, the reactions M_{03} , V_{03} , V_{13} and V_{23} are calculated using the two equilibrium equations from Mechanics [1], [2]:

$$\begin{cases} V_{02} + V_{12} + V_{22} = P_2 \\ M_{02} - V_{12} \cdot a - V_{22} \cdot (a+b) + P_2 \cdot (a+b+c) = 0 \end{cases}$$

and the two strain equations - the arrows of sections 1 and 2 which are written using the loading function [1]:

$$\begin{cases} E \cdot I_y \cdot \frac{V_{13}}{k} = -\frac{M_{03} \cdot a^2}{2} - \frac{V_{03} \cdot a^3}{6} \\ E \cdot I_y \cdot \frac{V_{23}}{k} = -\frac{M_{03} \cdot (a+b)^2}{2} - \frac{V_{03} \cdot (a+b)^3}{6} - \frac{V_{13} \cdot b^3}{6} \end{cases} \quad (9)$$

Solving the system of four equations with four unknowns M_{03} , V_{03} , V_{13} and V_{23} yields the calculation formulas:

$$\begin{cases} V_{03} = P_3 \cdot \frac{36\alpha^2 \cdot b - 18\alpha \cdot b \cdot c(a+b)^2 - 18\alpha \cdot a^2 \cdot b(b+c) + 3a^2 b^4 c}{36\alpha^2 b + 12\alpha \cdot b[(a+b)^3 + a^3] + a^3 b^3 (3a+4b)}; \\ M_{03} = -\frac{V_{03} \cdot [6\alpha \cdot (a+b) - a^3 \cdot b] - 6\alpha \cdot c \cdot P_3}{6\alpha - 3a^2 b}; \\ V_{13} = -P_3 \cdot \frac{c}{b} - V_{03} \frac{a+b}{b} - \frac{M_{03}}{b}; \\ V_{23} = P_3 \cdot \frac{b+c}{b} + V_{03} \frac{a}{b} + \frac{M_{03}}{b}; \text{ where } \alpha = \frac{E \cdot I_y}{k}. \end{cases} \quad (10)$$

The corresponding displacements δ_{13} , δ_{23} and δ_{33} written using the loading function have the expressions:

$$\begin{cases} \delta_{13} = -\frac{V_{03} \cdot a^3 + 3M_{03} \cdot a^2}{6E \cdot I_y}; \\ \delta_{23} = -\frac{V_{03} \cdot (a+b)^3 + 3M_{03} \cdot (a+b)^2 + V_{13} \cdot b^3}{6E \cdot I_y}; \\ \delta_{33} = -\frac{V_{03} \cdot (a+b+c)^3 + 3M_{03} \cdot (a+b+c)^2 + V_{13} \cdot (b+c)^3 + V_{23} \cdot c^3}{6E \cdot I_y} \end{cases} \quad (11)$$

6. CALCULATION OF NATURAL PULSATIONS AND EIGENMODES OF VIBRATION FOR THE FOUR ASSEMBLY CASES

6.a Calculation of natural pulsations for the first case of mounting m1-m2 (Fig. 1.a)

To solve problems using the MATCAD program, enter the parameters:

$$\beta = \frac{L^3 \cdot 10^{-3}}{E \cdot I_y} \quad \frac{m}{N} \quad (12)$$

$$\left\{ \alpha_{1i} = \delta_{1i} \frac{E \cdot I_y}{P \cdot L^3}; \alpha_{2i} = \delta_{2i} \frac{E \cdot I_y}{P \cdot L^3}; \alpha_{3i} = \delta_{3i} \frac{E \cdot I_y}{P \cdot L^3}; i = 1, 2, 3 \right.$$

Using the quantities defined above, the influence coefficients δ_{1i} , δ_{2i} , δ_{3i} are written as follows:

$$\begin{cases} \delta_{1i} = \alpha_{1i} \cdot \beta \cdot P \\ \delta_{2i} = \alpha_{2i} \cdot \beta \cdot P \\ \delta_{3i} = \alpha_{3i} \cdot \beta \cdot P \end{cases} \quad (13)$$

To calculate the natural self-pulsations of the system with two degrees of freedom for the first mounting case m1-m2, the method of influence coefficients is used, and the differential equations (1) are written as follows:

$$\begin{cases} x_1 = \delta_{11} \cdot \Phi_1 + \delta_{12} \cdot \Phi_2 \\ x_2 = \delta_{21} \cdot \Phi_1 + \delta_{22} \cdot \Phi_2 \end{cases}, \text{ unde: } \begin{cases} \Phi_1 = -m_1 \cdot \ddot{x}_1 \\ \Phi_2 = -m_2 \cdot \ddot{x}_2 \end{cases}$$

$$\Rightarrow \begin{cases} x_1 + \delta_{11} \cdot m_1 \ddot{x}_1 + \delta_{12} \cdot m_2 \ddot{x}_2 = 0 \\ x_2 + \delta_{21} \cdot m_1 \ddot{x}_1 + \delta_{22} \cdot m_2 \ddot{x}_2 = 0 \end{cases} \quad (14)$$

The system of differential equations (14) admits harmonic solutions of the form:

$$\begin{cases} x_1 = a_1 \cdot \cos(pt) \\ x_2 = a_2 \cdot \cos(pt) \end{cases} \Rightarrow \begin{cases} \ddot{x}_1 = -p^2 \cdot a_1 \cdot \cos(pt) \\ \ddot{x}_2 = -p^2 \cdot a_2 \cdot \cos(pt) \end{cases} \quad (15)$$

Substituting the solutions and their derivatives into the system of differential equations of the second order, the homogeneous system of equations is obtained with the vibration amplitudes a_1 and a_2 as unknowns:

$$\begin{cases} a_1(1 - p^2 \cdot \delta_{11} \cdot m_1) - a_2 \cdot p^2 \cdot \delta_{12} \cdot m_2 = 0 \\ -a_1 \cdot p^2 \cdot \delta_{21} \cdot m_1 + a_2(1 - p^2 \cdot \delta_{22} \cdot m_2) = 0 \end{cases} \quad (16)$$

The homogeneous system of equations with the vibration amplitudes a_1 and a_2 as unknowns admits non-zero solutions if its determinant is zero:

$$\begin{vmatrix} 1 - p^2 \cdot \alpha_{11} \cdot \beta \cdot P \cdot m_1 & -p^2 \cdot \alpha_{12} \cdot \beta \cdot P \cdot m_2 \\ -p^2 \cdot \alpha_{21} \cdot \beta \cdot P \cdot m_1 & 1 - p^2 \cdot \alpha_{22} \cdot \beta \cdot P \cdot m_2 \end{vmatrix} = 0 \quad (17)$$

The positive solutions of the equation of the fourth degree (17) represent the two natural self-pulsations of the system and have the expressions:

$$p_1 = \sqrt{\frac{1}{\beta \cdot P} \cdot \frac{\alpha_{11} \cdot m_1 + \alpha_{22} \cdot m_2 - \sqrt{(\alpha_{11} \cdot m_1 + \alpha_{22} \cdot m_2)^2 - 4m_1 \cdot m_2 (\alpha_{11} \cdot \alpha_{22} + \alpha_{12} \cdot \alpha_{21})}}{2m_1 \cdot m_2 (\alpha_{11} \cdot \alpha_{22} + \alpha_{12} \cdot \alpha_{21})}}$$

$$p_2 = \sqrt{\frac{1}{\beta \cdot P} \cdot \frac{\alpha_{11} \cdot m_1 + \alpha_{22} \cdot m_2 + \sqrt{(\alpha_{11} \cdot m_1 + \alpha_{22} \cdot m_2)^2 - 4m_1 \cdot m_2 (\alpha_{11} \cdot \alpha_{22} + \alpha_{12} \cdot \alpha_{21})}}{2m_1 \cdot m_2 (\alpha_{11} \cdot \alpha_{22} + \alpha_{12} \cdot \alpha_{21})}} \quad (18)$$

The eigenmodes of vibration are given by the values of the distribution coefficients obtained from the first homogeneous equation (16):

$$(1 - p^2 \cdot \alpha_{11} \cdot \beta \cdot P \cdot m_1) a_1 - p^2 \cdot \alpha_{12} \cdot \beta \cdot P \cdot m_2 \cdot a_2 = 0 \quad (19)$$

The distribution coefficients μ_{21} , μ_{22} corresponding to the self-pulsation p_1 are calculated with the formulas:

$$\begin{cases} \mu_{21} = \frac{a_{21}}{a_{11}} = \frac{1 - p_1^2 \cdot \alpha_{11} \cdot \beta \cdot P \cdot m_1}{p_1^2 \cdot \alpha_{12} \cdot \beta \cdot P \cdot m_2} \\ \mu_{22} = \frac{a_{22}}{a_{12}} = \frac{1 - p_2^2 \cdot \alpha_{11} \cdot \beta \cdot P \cdot m_1}{p_2^2 \cdot \alpha_{12} \cdot \beta \cdot P \cdot m_2} \end{cases} \quad (20)$$

6.b. Calculation of natural pulsations for the second mounting case m1-m3 (Fig. 1.b)

The two eigenpulsations of the system and the distribution coefficients for this case are calculated similarly and have the expressions:

$$p_1 = \sqrt{\frac{1}{\beta \cdot P} \cdot \frac{\alpha_{11} \cdot m_1 + \alpha_{33} \cdot m_3 - \sqrt{(\alpha_{11} \cdot m_1 + \alpha_{33} \cdot m_3)^2 - 4m_1 \cdot m_3 (\alpha_{11} \cdot \alpha_{33} + \alpha_{13} \cdot \alpha_{31})}}{2m_1 \cdot m_3 (\alpha_{11} \cdot \alpha_{33} + \alpha_{13} \cdot \alpha_{31})}}$$

$$p_2 = \sqrt{\frac{1}{\beta \cdot P} \cdot \frac{\alpha_{11} \cdot m_1 + \alpha_{33} \cdot m_3 + \sqrt{(\alpha_{11} \cdot m_1 + \alpha_{33} \cdot m_3)^2 - 4m_1 \cdot m_3 (\alpha_{11} \cdot \alpha_{33} + \alpha_{13} \cdot \alpha_{31})}}{2m_1 \cdot m_3 (\alpha_{11} \cdot \alpha_{33} + \alpha_{13} \cdot \alpha_{31})}} \quad (21)$$

6.c. Calculation of eigenpulsations for the third mounting case m2-m1 (Fig. 2.a)

The two natural pulsations of the system and the distribution coefficients for this case are identical to those obtained in paragraph 6.a

6.d. Calculation of natural pulsations for the fourth mounting case m2-m3 (Fig. 2.b)

The two eigenpulsations of the system and the distribution coefficients for this case are calculated similarly and have the expressions:

$$p_1 = \sqrt{\frac{1}{\beta \cdot P} \cdot \frac{\alpha_{22} \cdot m_2 + \alpha_{33} \cdot m_3 - \sqrt{(\alpha_{22} \cdot m_2 + \alpha_{33} \cdot m_3) - 4m_2 \cdot m_3 \cdot (\alpha_{22} \cdot \alpha_{33} + \alpha_{23} \cdot \alpha_{32})}}{2m_2 \cdot m_3 (\alpha_{22} \cdot \alpha_{33} + \alpha_{23} \cdot \alpha_{32})}}$$

$$p_2 = \sqrt{\frac{1}{\beta \cdot P} \cdot \frac{\alpha_{22} \cdot m_2 + \alpha_{33} \cdot m_3 + \sqrt{(\alpha_{22} \cdot m_2 + \alpha_{33} \cdot m_3) - 4m_2 \cdot m_3 \cdot (\alpha_{22} \cdot \alpha_{33} + \alpha_{23} \cdot \alpha_{32})}}{2m_2 \cdot m_3 (\alpha_{22} \cdot \alpha_{33} + \alpha_{23} \cdot \alpha_{32})}} \quad (22)$$

7. RESULTS OBTAINED

The results obtained for the static deflection w , the equivalent stiffness K , the displacement amplitude and the natural pulsation for the three mounting cases of the motors without dynamic vibration absorbers in

section 1 (Fig. 4), in section 2 (Fig. 5) and in section 3 (Fig.6), are given in table 1 [7].

The results obtained for pulsations and natural modes of vibration in the four mounting cases of masses m_1 , m_2 and m_3 are given in table 2:

Table 1

The mounting case of the electric motor	Arrow w (mm)	Stiffness K (N/mm)	Amplitude of vibrations X (mm)	Natural self-pulsation p (rad/s)	Dynamic factor
Case 1 (Fig.4)	0,0176	5682,256	0,01135	753,827	1,645
Case 2 (Fig.5)	0,076	1315,96	0,0577	362,761	1,759
Case 3 (Fig.6)	0,175	572,055	0,190	239,177	2,085

Table 2

The mounting case of the dynamic absorber	The natural pulsations p_1 (rad/s)	The natural pulsations p_2 (rad/s)	The mean pulsation $(p_1+p_2)/2$	The distribution coefficient $\mu_{21} (\mu_{31})$	The distribution coefficient $\mu_{22} (\mu_{32})$
Cazul 1 m_1 - m_2 (Fig.1.a)	332,887	1728,64	1030,763	2,258	-0,443
Cazul 2 m_1 - m_3 (Fig.1.b)	232,120	1211,98	722,05	3,946	-0,253
Cazul 3 m_2 - m_1 (Fig.2.a)	332,887	1728,64	1030,763	2,258	-0,443
Cazul 4 m_2 - m_3 (Fig.2.b)	201,287	1583,97	892,628	1,540	-0,650

8. CONCLUSIONS

From the analysis of the results obtained in table 1 and table 2, the following conclusions can be drawn:

- It is observed that with the increase of the mounting distance of the motor from the mounting, the static arrow w increases the vibration amplitude X and the dynamic factor and decreases the equivalent stiffness K and the inherent pulsation p ;
- the inherent pulsation of the system for the three mounting cases is lower than the motor pulsation ($\omega=157.08$ rad/s) and the system operates in pre-resonance mode, so there is no danger of operating at resonance;
- in case 1: by adding the dynamic mass absorber m_2 to the system with a mass degree of freedom m_1 (case 1), the natural pulsation p_1 is smaller, the natural pulsation p_2 is larger, and the average of the natural pulsations p_1 and p_2 is also higher than the natural pulsation p from table 1;
- in case 2: by adding the dynamic mass absorber m_3 to the system with a mass degree of freedom m_1 (case 2), the natural pulsation p_1 is smaller, the natural pulsation p_2 is larger, and the average of the natural pulsations p_1

and p_2 is approximately equal to the self-pulsation p from the table;

- in case 3: by adding the dynamic mass absorber m_1 to the system with a mass degree of freedom m_2 (case 3), the natural pulsation p_1 is smaller but very close ($p=362.761 / p_1=332.887$) and the natural pulsation p_2 is higher than the natural pulsation p from table 1;

- in case 4: by adding the dynamic mass absorber m_3 to the system with a mass degree of freedom m_2 (case 4), the natural pulsation p_1 is smaller but very close ($p=239.177 / p_1=201.287$), the natural pulsation p_2 is higher than the natural pulsation p from table 1;

- in practice, it is aimed that the own pulsations of the system with two degrees of freedom are as far as possible from the pulsations corresponding to the electric motors in order to avoid the phenomenon of resonance.

- in the case of using an electric motor with a speed of 2300 rpm, the excitation pulsation ($\omega=240.8$ rad/sec) is approximately equal to the own pulsation for case 3a ($p=239.177$ rad/sec), the motor having mass m_3 and it is mandatory to use the dynamic absorber mass m_1 (case 2) or the dynamic absorber mass m_2 (case 4).

Therefore, the presented calculation method and the MATHCAD simulation program allow the estimation of dangerous operating modes at resonance from the design phase and provide a solution to avoid operating in resonance mode when the vibration amplitudes and the dynamic factor have impermissibly high values.

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