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STUDY OF THE INFLUENCE OF THE RELATIVE RIGIDITY OF AN ELASTIC HYBRID SYSTEM WITHOUT DAMPING, ON NATURAL SELF-PULSATION, AMPLITUDE AND DYNAMIC FACTOR DUE TO FORCED VIBRATION

Cornel MARIN

Valahia University of Targoviste, 13 Sinaia Alley, 130004, Targoviste, Romania

E-mail: marin.cor@yahoo.com

Abstract: *The paper aims to study the dynamic behavior of a hybrid elastic system without damping consisting of a beam in a cantilever supported on helical compression springs on which an electric motor is mounted. With the help of the MATHCAD professional program, the influence of the relative stiffness of the elastic bar subjected to bending and the springs subjected to compression was simulated, on the natural pulsation, the amplitude of the vibrations and the dynamic factor of the forced vibrations for two assembly variants of the electric motor*

Keywords: *forced vibration without damping, hybrid elastic system subjected to bending/compression.*

1. INTRODUCTION

The study of the dynamic behavior of an elastic mechanical system without damping consisting of a beam in a cantilever supported on two helical springs having different stiffnesses, an electric motor of mass m1 having a rotor of mass m_2 and the speed in normal operating mode n, can be done by numerical simulation using the MATHCAD professional program by changing the stiffness of the helical springs subjected to compression. To solve the vibration problem or to calculate natural self-pulsations, the method of influence coefficients [3], [6] is used, and for the calculation of reactions, the method of the load function Ψ [1], [2] is used. With the help of the calculation formula of the static arrow X and the equivalent stiffness K, the calculation formula of the dynamic vibration factor is obtained [5].

Seven spring stiffness values are simulated for two engine mounting cases:

– mounting the motor above the first spring (fig. 1);
– mounting the motor above the second spring (fig. 2)

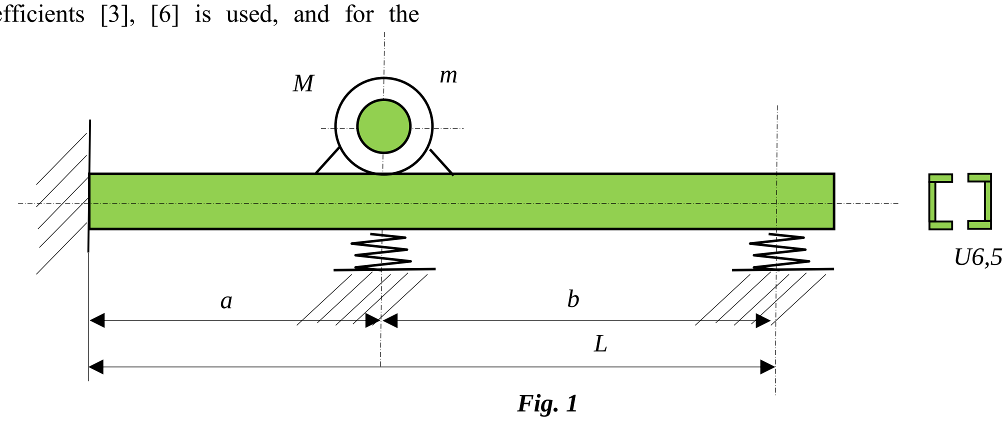

Fig. 1

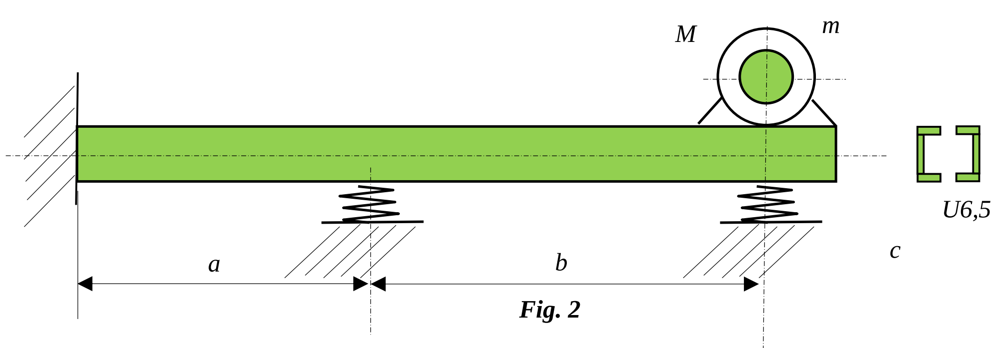

Fig. 2

2. METHOD OF INFLUENCE COEFFICIENTS

To study the vibrations of mechanical systems with n degrees of freedom formed by concentrated masses mounted on elastic beams subjected to bending, the method of influence coefficients is used, which consists in expressing the displacements x_1, x_2, x_3 ,... according to the inertial forces corresponding to the masses of the system [3], [6]:

$$\begin{cases} x_1 = \delta_{11} \cdot \Phi_1 + \delta_{12} \cdot \Phi_2 + \delta_{13} \cdot \Phi_3 + ... + \delta_{1n} \cdot \Phi_n \\ x_2 = \delta_{21} \cdot \Phi_1 + \delta_{22} \cdot \Phi_2 + \delta_{23} \cdot \Phi_3 + ... + \delta_{2n} \cdot \Phi_n \\ ... \\ x_n = \delta_{n1} \cdot \Phi_1 + \delta_{n2} \cdot \Phi_2 + \delta_{n3} \cdot \Phi_3 + ... + \delta_{nn} \cdot \Phi_n \end{cases} \quad (1)$$

where Φ_1, Φ_1, Φ_1, ... Φ_1 represent the inertial forces corresponding to the masses m_1, m_2, m_3 ,... m_n

$$\Phi_1 = -m_1 \cdot x_1; \quad \Phi_2 = -m_2 \cdot x_2; \ \Phi_3 = -m_3 \cdot x_3; \quad (2)$$

δ_{i1}, δ_{i2} ... δ_{in} - represent the influence coefficients (or the displacements of the sections corresponding to the masses m_1, m_2, m_3 ,... m_n , when a unitary force P=1 is applied in section i.

3. SETTING THE PROBLEM FOR THE FIRST ASSEMBLY CASE

An electric motor of mass m_1=10kg with the speed n=1500 rpm and the rotor of mass m_2=5kg with the eccentricity r=0.5mm, is mounted on a console of length L=1000mm embedded at one end, next to the first spring support (Fig. 3). The cantilever beam is elastically supported on two springs with stiffness k=500 N/mm at the a=600mm and b=400mm mounting distances. The console section consists of two U6.5 profiles (STAS 564-80) having the sectional moment of inertia Iy=57.5 cm^4. The mass of the cantilever is assumed to be neglected.

It will be determined:

1. the value of the natural pulsation p_1 and the amplitude X_1 of the transverse forced vibrations of the engine for the first mounting case (fig. 3);
2. the equivalent dynamic force P_{1d} due to forced vibrations of the mechanical system;
3. the variation of the natural self-pulsations p_1 of the elastic system for different values of the relative stiffness $\beta = E \cdot I_y / k \cdot L^3$.

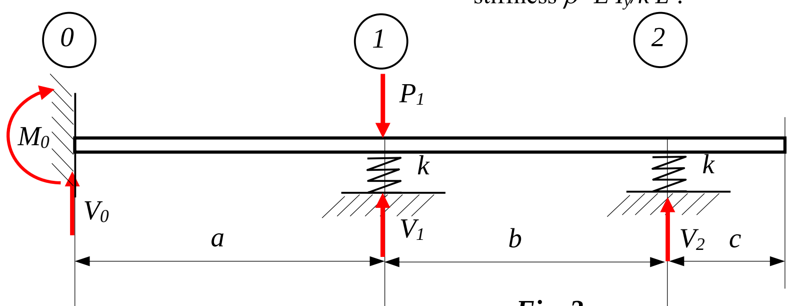

Fig. 3

To calculate the arrow w_1 corresponding to the position of the electric motor for the first assembly case (section 1) (Fig. 3) the reactions are determined using two equilibrium equations from STATICS [2]:

$$\begin{cases} V_0 + V_1 + V_2 = P_1 \\ M_0 - V_1 \cdot a - V_2 \cdot (a+b) + P_1 \cdot a = 0 \end{cases} \quad (3)$$

and two strain equations from STRENGTH OF MATERIALS written using the loading function Ψ[1]:

$$\begin{cases} E \cdot I_y \cdot \dfrac{V_1}{k} = -\dfrac{M_0 \cdot a^2}{2} - \dfrac{V_0 \cdot a^3}{6} \\ E \cdot I_y \cdot \dfrac{V_2}{k} = -\dfrac{M_0 \cdot (a+b)^2}{2} - \dfrac{V_0 \cdot (a+b)^3}{6} - \dfrac{V_1 \cdot b^3}{6} + \dfrac{P_1 \cdot b^3}{6} \end{cases} \quad (4)$$

If the system consisting of four equations with unknowns M_0, V_0, V_1 and V_2 is solved, the calculation formulas are obtained [5]:

$$\begin{cases} V_0 = P_1 \cdot \dfrac{6\alpha \cdot b\left[6\alpha + 3b(a+b)^2 - b^3\right]}{36\alpha^2 b + 12\alpha \cdot b\left[(a+b)^3 + a^3\right] + a^3 b^3 (3a+4b)} \\ M_0 = \dfrac{-V_0 \cdot \left[6\alpha \cdot (a+b) - a^3 \cdot b\right] + 6\alpha \cdot b \cdot P_1}{6\alpha - 3a^2 \cdot b} \end{cases}$$

$$\begin{cases} V_1 = P_1 - V_0 \dfrac{a+b}{b} - \dfrac{M_0}{b}; \\ V_2 = V_0 \dfrac{a}{b} + \dfrac{M_0}{b}. \end{cases} \quad (5)$$

where it was noted:

$$\alpha = \frac{E \cdot I_y}{k} \quad (6)$$

The arrow w_1 next to the motor is determined using the relationship written using the load function Ψ [1]:

$$w_1 = -\frac{V_0 \cdot a^3 + 3M_0 \cdot a^2}{6E \cdot I_y} \quad (7)$$

The equivalent stiffness K_1 of the elastic system is written as the ratio of the force P_1 to the arrow w_1:

$$K_1 = \frac{P_1}{w_1} \quad (8)$$

In the case of a system with one degree of freedom, the equations of the method of influence coefficients (1) are written:

$$x_1 = -\delta_{11} \cdot m_1 \cdot \ddot{x}_1 \quad \Rightarrow \quad \delta_{11} \cdot m_1 \cdot \ddot{x}_1 + x_1 = 0$$
$$where:\ \delta_{11} = \frac{w_1}{P_1} = \frac{1}{K_1} \qquad (9)$$

The characteristic equation allows to obtain the formula for the self-pulsation p_1:

$$r^2 + \frac{K_1}{m_1} = 0 \Rightarrow p_1 = \sqrt{\frac{K_1}{m_1}} \qquad (10)$$

The amplitude of the displacement forced vibrations X_1 is determined using the relation:

$$X_1 = \frac{m_2}{m_1} \frac{\omega^2}{p_1^2 - \omega^2},\quad where:\ \omega = \frac{2\pi \cdot n}{60} \qquad (11)$$

The equivalent dynamic force P_{1d} due to forced engine vibrations is calculated using the relation:

$$P_{1d} = \left(1 + \frac{X_1}{w_1}\right) \cdot P_1 \qquad (12)$$

By substituting the numerical values given in the formulas above, the following results are obtained:

$$\begin{cases} V_0 = 75{,}113 & N \\ M_0 = -38.633 & Nmm \\ V_1 = 8{,}799 & N \\ V_2 = 16{,}088 & N \\ \beta = 0{,}483 & \end{cases} \qquad \begin{cases} w_1 = 0{,}0176 & mm \\ K_1 = 5682{,}56 & N/mm \\ p_1 = 753{,}827 & rad/s \\ X_1 = 0{,}01135 & mm \\ P_{1d} = 164{,}485 & N \end{cases}$$

The dynamic behavior is simulated for seven values of spring stiffness k and relative stiffness $\beta = E \cdot Iy/k \cdot L^3$

The results from table 1 and the variation graph from figure 4 are obtained.

Table. 1

Spring stiffness	*Relative stiffness β*	*Natural pulsation (rad/s)*	*The amplitude of vibrations X1 (mm)*	*The equivalent dynamic factor*
k_6= 1,9· k =950 N/mm	0,254	867,702	0,008	1,637
k_5= 1,6 · k=800 N/mm	0,302	832,848	0,009	1,640
k_4=1,3 · k=650 N/mm	0,371	794,074	0,010	1,642
k=500 N/mm	0,483	753,827	0,011	1,645
k_3= 0,7· k= 350 N/mm	0,69	708,368	0,013	1,649
k_2=0,4· k= 200 N/mm	1,207	657,664	0,015	1,654
k_1=0,1· k= 50 N/mm	4,83	600,192	0,018	1,662

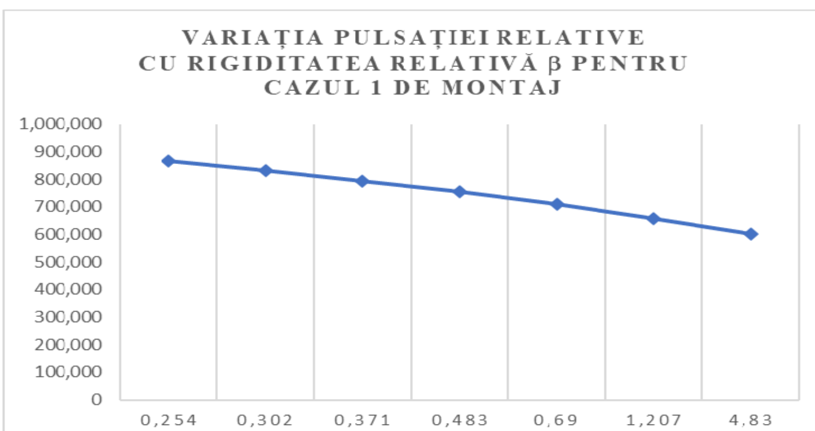

Fig. 4. Variation of natural self-pulsations with relative stiffness β for the first assembly case

4. SETTING THE PROBLEM FOR THE SECOND ASSEMBLY CASE

We consider the electric motor of mass m_1=10kg with the speed n=1500 rpm and the rotor of mass m_2=5kg with the eccentricity r=0.5mm, which is mounted on a console of length L=1000mm embedded at one end, in the right of the *second support spring* (Fig. 3).

The cantilever beam is elastically supported on two springs with stiffness k=500 N/mm at the distances a=600mm and b=400mm for left embed. The console section consists of two U6.5 profiles (STAS 564-80) having the moment of inertia Iy=57.5 cm^4. The mass of the cantilever is assumed to be neglected. It will be determined:

1. the value of the natural pulsation p_2 and the amplitude X_2 of the transverse forced vibrations of the engine for the second mounting case (fig. 5);
2. the equivalent dynamic force P_{2d} due to forced vibrations of the mechanical system;
3. the variation of the natural self-pulsations of the elastic system for different values of the relative stiffness $\beta = E \cdot I_y / k \cdot L^3$

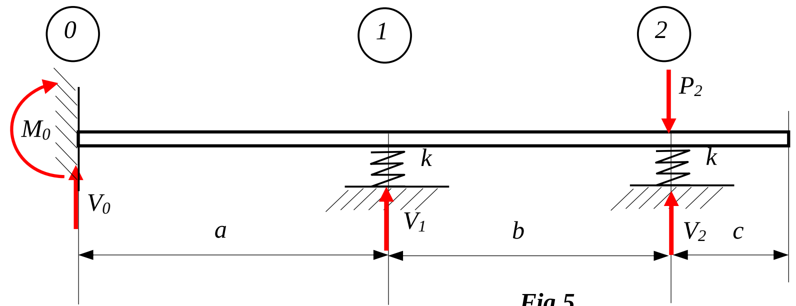

Fig.5

To calculate the arrow w_2 corresponding to the position of the electric motor for the first assembly case (Fig. 5) the reactions are determined using two equilibrium equations from STATICS [2]:

$$\begin{cases} V_0 + V_1 + V_2 = P_2 \\ M_0 - V_1 \cdot a - V_2 \cdot (a+b) + P_2 \cdot (a+b) = 0 \end{cases} \tag{13}$$

and two strain equations from STRENGTH OF MATERIALS written using the load function Ψ [1]:

$$\begin{cases} E \cdot I_y \cdot \dfrac{V_1}{k} = -\dfrac{M_0 \cdot a^2}{2} - \dfrac{V_0 \cdot a^3}{6} \\ E \cdot I_y \cdot \dfrac{V_2}{k} = -\dfrac{M_0 \cdot (a+b)^2}{2} - \dfrac{V_0 \cdot (a+b)^3}{6} - \dfrac{V_1 \cdot b^3}{6} \end{cases} \tag{14}$$

If the system consisting of four equations with unknowns M_0, V_0, V_1 and V_2 is solved, the calculation formulas are obtained [5]:

$$\begin{cases} V_0 = P_2 \cdot \dfrac{6\alpha \cdot b \cdot (6\alpha - 3a^2 \cdot b)}{36\alpha^2 b + 12\alpha \cdot b\left[(a+b)^3 + a^3\right] + a^3 b^3 (3a+4b)} \\ M_0 = -P_2 \cdot \dfrac{6\alpha \cdot b \cdot \left[6\alpha(a+b) - a^3 \cdot b\right]}{36\alpha^2 b + 12\alpha \cdot b\left[(a+b)^3 + a^3\right] + a^3 b^3 (3a+4b)} \end{cases}$$

$$\begin{cases} V_1 = -V_0 \dfrac{a+b}{b} - \dfrac{M_0}{b}; \\ V_2 = P_2 + V_0 \dfrac{a}{b} + \dfrac{M_0}{b}; \quad where\ \alpha = \dfrac{E \cdot Iy}{k}. \end{cases} \tag{15}$$

The arrow w_2 the section corresponding to the assembly of the motor is determined using the relationship written using the load function Ψ [1]:

$$w_2 = -\frac{V_0 \cdot (a+b)^3 + 3M_0 \cdot (a+b)^2 + V_1 \cdot b^3}{6E \cdot I_y} \tag{16}$$

The equivalent stiffness K_2 of the elastic system is written as the ratio of the force P_2 to the arrow w_2:

$$K_2 = \frac{P_2}{w_2} \tag{17}$$

In the case of a system with one degree of freedom, the equations of the method of influence coefficients (1) are written:

$$x_2 = -\delta_{22} \cdot m_2 \cdot \ddot{x}_2 \quad \Rightarrow \quad \delta_{22} \cdot m_2 \cdot \ddot{x}_2 + x_2 = 0$$

$$where: \ \delta_{22} = \frac{w_2}{P_2} = \frac{1}{K_2} \tag{18}$$

The characteristic equation allows to obtain the formula for the natural self-pulsation:

$$r^2 + \frac{K_2}{m_1} = 0 \Rightarrow p_2 = \sqrt{\frac{K_2}{m_1}} \tag{19}$$

The amplitude of the forced vibrations X_2 is determined using the relation:

$$X_2 = \frac{m_2}{m_1} \frac{\omega^2}{p_2^2 - \omega^2}, \ unde: \ \omega = \frac{2\pi \cdot n}{60} \tag{20}$$

The equivalent dynamic force P_{2d} due to forced engine vibrations is calculated using the relation

$$P_{2d} = \left(1 + \frac{X_2}{w_2}\right) \cdot P_2 \tag{21}$$

By substituting the numerical values given in the formulas above, the following results are obtained:

$$\begin{cases} V_0 = 45{,}917 & N \\ M_0 = -52.352 & Nmm \\ V_1 = 16{,}088 & N \\ V_2 = 37{,}995 & N \\ \beta = 0{,}483 \end{cases} \qquad \begin{cases} w_2 = 0{,}076 & mm \\ K_2 = 1315{,}96 & N/mm \\ p_2 = 362{,}761 & rad/s \\ X_2 = 0{,}0577 & mm \\ P_{2d} = 175{,}92 & N \end{cases}$$

The dynamic behavior is simulated for seven values of spring stiffness k and relative stiffness $\beta = E \cdot Iy / k \cdot L^3$

The results from table 2 and the variation graph from figure 6 are obtained:

Table 2

Spring stiffness	*Relative stiffness β*	*Natural pulsation (rad/s)*	*The amplitude of vibrations X2 (mm)*	*The equivalent dynamic factor*
k_6=2 k=950 N/mm	0,254	429,560	0,038	1,712
k_5=1,6 k=800 N/mm	0,302	408,547	0,043	1,723
k_4=1,3 k=650 N/mm	0,371	386,353	0,049	1,739
k=500 N/mm	0,483	362,761	0,0577	1,759
k_3=0,7 k= 350 N/mm	0,69	337,476	0,069	1,787
k_2=0,4 k= 200 N/mm	1,207	310,084	0,086	1,830
k_1=0,1 k= 50 N/mm	4,83	279,966	0,115	1,900

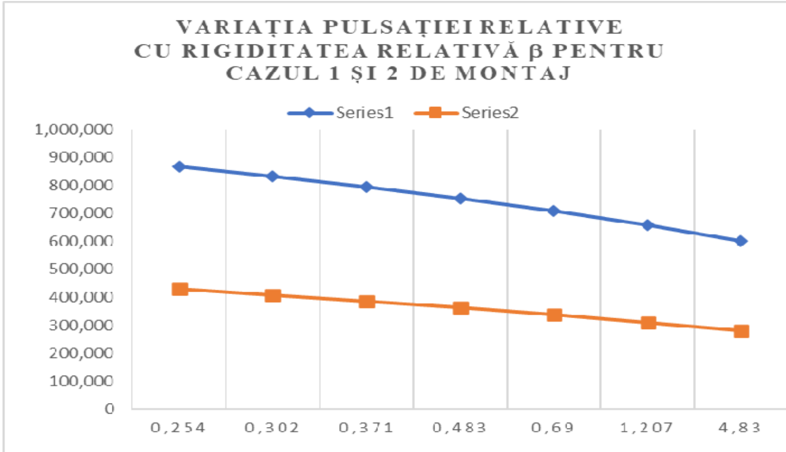

Fig. 6. Variation of natural self-pulsations with relative stiffness β for the second assembly case

5. CONCLUSIONS

From the analysis of the results presented in tables 1 and 2 with respect to figure 6, it can be seen that:
- the vibration amplitude X and the dynamic force amplification factor due to vibrations for the second mounting case is higher than in the first case;
- the values of the natural self-pulsation p_2 corresponding to the second mounting case are approximately equal to half of the values p1 corresponding to the first case;
- increasing the stiffness of the suspension springs of the beam leads to an increase in natural frequencies and a decrease in the amplitude of vibrations and the dynamic amplification factor.
- both for the first case and for the second mounting case, the vibrations are produced in the area of ante-resonance; there is no danger of operating at resonance (when the vibration amplitude and dynamic factor increase greatly).
- in such situations, when the own pulsation is lower than or equal to the motor pulsation, it is recommended to avoid operation in resonance mode, respectively in post-resonance mode
- if this is not possible, in order not to have a very high dynamic factor, a hydraulic damper or a dynamic absorber with damping is inserted [3].

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