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PHYSICAL PRINCIPLES IN REVEALING THE WORKING MECHANISMS OF BRAIN. PART IV

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Abstract. The living brain is physically modeled as a universe, equivalent, in a way, to the physical universe we live in. While in the case of the physical universe the gravitation prevails, in the brain universe, if we may, the electricity prevails, but the mathematical description we provide is essentially the same in both cases. What is imperiously necessary in this approach is, first, a metric description of matter, then, of course, the physical interpretation of this description. These issues were treated, but only in their believed essentials, in the previous three instalments of the present work. The object of the present episode of the work is a classical space image of the matter. This image is worked out here having in the background the idea of a necessary physical connection between the matter of a universe and its fundamental physical structure. The matter can be characterized 'optically' as a Maxwell fish-eye, while the fundamental physical structure must be a dipole. Both these concepts are universal.

Keywords: neuron, continuum charge background, the holographic principle, Kepler's laws, memory of the universe, Maxwell fish-eye, the firing event.

1. Reiterating the Purpose of this Work, with a Synopsis

We have insisted in declaring, quite a few times by now, that with the present work (the series of articles published in this bulletin under the same title as the present one, to be cited as I, II and III, from now on) we carry out an analogy: *the brain is a universe*. This is held by us as essentially meaning that, from a physics' point of view the brain *must be described* as a universe. Taken face value, this statement may be considered an analogy; it has, obviously, discouragingly many physical aspects. However, in our opinion, such a step needs to be taken if we need to promote the physiology to a theoretical science with predictive capabilities. At present, the physiology, especially that referring to brain and heart, as a science, is by and large only a phenomenology, apparently reduced to the stage of cumulating facts, most of them disparate. It occurred to us that the reason for this condition stays exactly in the fact that the brain is considered just an organ like any other one, and there are no criteria of selection the fundamental physical facts in the construction of a guiding model for its functionality. Our idea of theoretical approach of the brain is conducted by taking heed of a special manner of doing physics in this world, within which the man lives and evolves. Thus, it will be not without merit, to exhibit the reason of that analogy, that appears to be the essential one for the concept. This aspect can cast a critical light on things, helping us to grasp the fundamental issues of a prospective physical theory of the brain. It derives from the following facts showing in what respect can the brain be considered as a universe, and *how do we understand to describe it* as such.

1.1. The Physical Characteristics of the Universe we Inhabit

We consider as an already firmly established – by theoretical physics, of course, and thereby scientific – truth that the quantization of light is a natural phenomenon of the world we inhabit. The truth of this statement is, indeed, to be expected based on, let us say, a kind of common-sense reasoning: *the light is the universal carrier of information in our world*. Factually speaking, this means that the light is present in all of the three essential instances involving the information we can extract from our experience and the manner we use it:

1 the light reaches us from afar, which speaks of the grand scale of the universe, *transfinite*, if it is to respect a mathematical nomenclature ([1]; see III, §1.1 for details);

2 the light determines a specific description of the *finite scale* of our world, forced upon us by the man's possibilities of perception, which are, in fact, the human abilities that establish the rules of natural philosophy;

3 the light determines the physics of the *infrafinite scale* of the world we inhabit, thereby imposing the experimental techniques helping in gathering the knowledge on this world. The techniques in question always involve microscopic processes of the world, which speaks, in fact, of an infrafinite scale of it.

In a word, the light is *the only* scale-transitive phenomenon of our world, and the *condition of quantization* referring to it – that is, the Planck's quantization procedure – is *the only law* that respects this property of scale transition. Let us further justify this statement.

For once, the legitimacy of such conclusion is positively secured, in our opinion, by the fact that the cosmic background radiation, for instance, the cosmic light of our existence, has definitely a Planck spectrum [2]. Indeed, this fact shows that the light obeys the *Wien displacement law*, which is an expression of the scale invariance in the physics of light ([3]; see also III, §3.2).We must recall that the Wien's displacement law *–the only physical criterion* against which any law of radiation was, and is, in fact, judged – shows that the experimental conclusions on the thermal radiation are *independent of the dimensions of the enclosure* used in studying experimentally this radiation [3], that is what we call *a Wien-Lummer enclosure* [4]. In other words, the universe around us can be considered, based on the NASA discoveries mentioned above (*loc*. *cit*. *ante*), as a Wien-Lummer enclosure, since the conclusions on light drawn based on a Wien-Lummer enclosure are valid at any scale of the world: finite, and its fictitious mind constructions, infrafinite and transfinite.

1.2. Brain as a Universe: Incentives of Consideration and Points of Analogy

Our first incentive in considering the brain as a universe is guided by an inverse reasoning, as it were: by the law of scale invariance, any Wien-Lummer enclosure can be considered as a universe. The skull containing a brain, obviously, makes an intuitively valid enclosure, even though it may not qualify as a Wien-Lummer one, for the light is conspicuously absent from the picture. However, there is another point to be taken in consideration with the concept of universe, and this is plainly suggested by physiology: the electric properties of the brain. The electricity is naturally produced in the brain, in fact one can tentatively say that considering the brain a universe, the electricity replaces the light from the physical universe of our existence. A few physiological research results encourage us into persisting in our idea. Let us survey them briefly.

First, and the foremost, we have the *concept of a dipole* that seems to have made, although implicitly, its entrance into physiology even by the end of 19th century, with the incipient concept of neuron [5], and was revived in later times under the pressure of facts (see [6]; also [7], and [8]; for a comprehensive documentation on physiology we recommend our source of an excellent basic

information on the subject [9]). For once, the concept of dipole in the neurophysiological practice asks for much more than the mathematical model usually provided by theoretical physics in order to represent the necessary fundamental physical structure in the brain matter [6]. The physics, as it is now, cannot provide an understanding of such a situation. Quoting:

> The dipole model provides *a useful way to localize* EEG *sources in the brain*, but only *when applied with an awareness of its underlying assumptions*. It has spurred the development of new, promising, related technologies, such as magnetoencephalography (MEG). This is a technique in which the magnetic fields generated by brain are mapped and the source is localized by solving the *magnetic inverse problem*. MEG *has the advantage that the magnetic fields it measures are less distorted by skull*, *scalp*, *and cerebrospinal fluid*, *than is the* EEG. This potentially provides electroencephalographers with more information than is contained in the EEG alone. However, the same assumptions and problems related to the EEG inverse problem apply to the MEG inverse problem. The dipole model in EEG is useful as a stepping stone to understanding functional neurophysiology, particularly at the neuronal and neuronal network levels. What *is most important*, however, *is not necessarily how well the model predicts physiology*. Rather, *it is determining when and why the model breaks down that furthers our understanding* ([6]; *emphasis added*, *a*/*n*).

In other words the neurologists usually associate shortcomings of the model with the geometrical and physical irregularities of the object of study: skull, scalp, cerebrospinal fluid, etc. All of these shortcomings, however, are suggested in a theoretical model provided by physics, and are simply limitations imposed on the reality due to the very limited mathematical possibilities of description. This is the reason why we chose to pay attention, from this moment on in this work, to what 'furthers our understanding' in neurology. That is, to "determining when and why the model breaks down", since this is what asks for assessing the model according to physiological necessities. However, there is a more important issue on which we have to direct our attention when characterizing the brain as a universe: the relation of this universe with the universe we inhabit. This relation is strictly determined by the essential condition of the creator of science serving in the description of a universe in general, that is, *the man*. It cannot be just mathematical, for it is exactingly determined by the fact that our senses mediate it, thus making out of the brain a first-hand depository of information they make possible.

Fact is that the dipole is the fundamental physical unit too, when it come to the description of the physical universe based on the very physical properties of light. Max Planck – who was the first to notice the law of quantization – invented the so-called resonator based on this physical structure, in order to account for the relation between light and matter. This invention, though, entices just as many shortcomings as its physiological counterpart, and basically of just about the same nature. This is why we undertook a study on the foundations of the very quantization procedure [10, 11] in order to determine what are the properties of the dipole and of the medium whose part it is. Briefly, the results of that study are the following: the dipoles as physical structures are dictated by the optical properties of the matter. Planck's procedure of *quantization of light* is also applicable to the *quantization of matter*, but the necessary properties of the light serving for the quantization of matter in the conditions of a Wien-Lummer enclosure, require dipoles of a magnetic nature. In this respect, we found out that in deciding what is 'light' and what is 'matter' in a universe in general, as a concept, the idea of propagation of signals constituting the equivalents of light comes first, which is critical in the case of brain. This fact may be considered as critically experienced by physiology [12].

Along this last line, the general conclusion of our study [10, 11], is that *when* propagation can be characterized by a path, both the rays, as paths of propagation of the specific signal of a universe, and the very medium in which this signal is propagated have the general characteristics imposed by an optical medium known as *Maxwell fish-eye*. This is perhaps the only physical medium admitting the dipole in an essential cosmological (remember, we conceive the brain as a universe!) position from the point of view of physics: the geodesics are always circles passing through two points, just like in the typical case of electric or magnetic line forces. Geometrically, this medium can be described as a Riemannian **sl**(2,R) manifold (for the concept see I, §2.3). Consequently, a first outcome of consideration of the brain as a universe would be that both the 'signals' in brain, and the 'brain matter', must satisfy this mathematical condition, which thus helps us in selection the fundamental signal, equivalent to light, in this universe. However, this mathematics calls for more physiological application senticed by the analogy presenting the brain as a universe. The conspicuous property of brain is that of memory, without which the man could not exist as such, let alone evolve in some way.

1.3. The Property of Memory of a Universe

Physiological studies of orientation in finite spaces led to the idea that the *Hippocampus* from the physical structure of brain, is the depository of the fundamental knowledge involving the space and time [13], in fact, tentatively speaking, of the knowledge in general [14]. This fact fostered

issues of a geometrical nature, involving the memory and space perception, waiting for a decisive solution. It turns out that this is a case where, actually, the physiology helps in physics' decisions of fundamental significance. For once, the Cartan's method of assigning coordinates to a place and carry them by memory in any other place (see III, Chapter 4) can be validated as a fundamental way of knowledge. On the other hand, it seems that the problem of memory can, even must, we should say, be settled holographically. Indeed, the idea of *holographic temporal recall* [15], can be carried over just naturally into the optical geometry of brain as a universe, in order to decide the mechanism of memory: the frequency – a central concept in Longuet-Higgins method for describing the temporal recall, devised on the basis of Dennis Gabor's discovery of holography – can be defined in an invariant geometrical way (see I, §3.1).

These, and the ones naturally correlated with them, are by and large the issues addresses within this part of our work, and the following one, having yet another taskin view, that occurred along the way of our study: *to provide selection criteria for a concept of universe in general*. We start with the fundamental concept of an optical medium, forcing the dipole, in a classical take of course, as a fundamental physical structure of the matter existing in a closed space, a category that proves also important, essential we should say, for physiology. However, it will be presented here only to show the points of intervention in order to built a right dipole model. The reason for this is that the model must describe a physical dipole that primarily serves for *depositing the information* communicated by senses to our brain. So, we find just natural that this instance of a dipole should serve in providing the criteria for a right model, and we follow closely the idea all along the present work.

The manner of extracting these criteria is dictated by the way we think the deposit in question is made in brain: *as a hologram*. This statement derives from the fact that by its very condition as a universe, the brain respects the wave-mechanical rules regarding the interpretation of matter, whereby the figments of our imagination must have an interpretation at any rate, in order to be used in guiding our experience (see I, §2.3).Speaking from the physics' point of view, we just need to *paraphrase* the definition of Charles Galton Darwin for the *concept of interpretation* [16] in order to make this concept into *a memory*. Paraphrasing, therefore, the words of that definition:

> It is almost impossible *to record the results of any of our senses except in terms of particles* – a firing, a deposit within place cell (see [13]) for the concept), etc – and this language must be *compatible with the language of waves*, which is used in the mathematical treatment. A necessary part of the discussion of any problem is therefore *the translation of the formal mathematical solution, which is in wave*

> *form, into terms of particles*. We *shall call this process the memory*, and only use the word in this technical sense… (see I, §2.3 for the original excerpt from Darwin's work just cited).

In other words, we cannot have an interpretation *but as a hologram*, since there is not a static universe in this world of our experience [11a], least of all the brain of our concept. Indeed, everything is in motion, a fact that was realized by man early, on the dawn of civilization. To wit: we have to recall the celebrated that '*tapantarhei*' of the old Heraclitus of Ephesus, which is exclusively taken as the possibility of a modern physical describing the universe. Anyway, once this solution of 'deposit' is settled for, the hard part of our problem is to decide the places to deposit the hologram in the universe of brain. In this respect, the physiology of the last quarter of the 20th century offer us a substantial help to physics. Namely, by the end of the last century, the ideas of imunology have been applied in order to construct a versatile model of *molecular deposit*, leading to a rational selection of the 'deposit places' in the brain ([17-21]; see also [22]). As usual, the physiological theory here is highly phenomenological, having not so much possibility of theoretical description. Should physics take this help, it needs to create the means to do it.

Therefore, what we have to do in this respect, and certainly in the long run, is to give also a theoretical physics' touch to this physiological theory. The limited expanse of the present part of the work, as well as necessities of further mathematical studies and documentation in physiology, surely will not allow us to complete the task in its entirety. So, we shall start concentrating here on the basic issues connected with the concept of memory.

2. The Concept of Dipole

Theoretically speaking, the concept of dipole is taken, in physiology, the way it is served by physics, with all the necessary cutlery, as it were (see, for instance, [23]). The conditions, though, in which the physical model is used, reflect both the physical necessity of producing solutions of technological use, and the physiological capabilities of characterization the medium where the model works. By describing the model as it is now, we *afortiori* have the places to intervene for improvements in the model or change in conception. However, let us assess the physical conditions of the existence of a fundamental physical structure in a universe, in general.

2.1. The Cosmological Oscillator

The principle of scale transition in physics promotes, as we already mentioned, the quantization into a position of essential physical law of the universe we inhabit [11]. In turn, the quantization procedure, whose epitome is the Max Planck's procedure of quantization of light, asks for a special physical structure of the matter in equilibrium with the light in that universe: *the electric dipole*. This physical structure allows for a state of equilibrium in the universe, which, speaking from physics' point of view, circumvents the problematic thermodynamical definition of the temperature for the radiation. Recall that the radiation is basically described as a field having a wave-like structure, while the definition of temperature requires a particle-like structure, involving, classically speaking, the kinetic energy of the particles of an ideal gas. Naturally, then, the light does not have the capability of supporting a statistic like the absolute temperature.

Therefore, classically speaking, the radiation does not have the possibility of a proper interpretation, and as such cannot benefit from a kinetical definition of the temperature like the ideal gas: the only way one can associate a temperature to it is by a thermodynamical equilibrium with matter, and this is realized in an enclosure of the Wien-Lummer type. In this circumstance, the electric dipole allows for an equilibrium between matter and radiation of a purelly electromagnetic nature, having, nevertheless, all of the virtues of a thermodynamical equilibrium. No doubt then – in view of the idea that the brain is a universe – the electric dipole eventually made its way into physiology, first of all with the physical details serving for the description of the heart [23]. However, here, the brain is the typical organ in need of a proper physical model of the dipole, not only electric, but perhaps of any other nature, since the brain qualifies as a universe by itself. Such a model must be adaptable to any conditions imposed by a universe in general. An example of such adaptation was presented in detail, physiologically of course, by Kenneth Catania, who has a suggestive description of such a dipole in the case of electric eels, used, naturally, for the necessities of sustaining the life on Earth (see [24], Fig. 7). We proceed to description of such an adjustable dipole, in an analytic way.

Ideally – from a geometrical point of view, of course – the definition of archetype classical dipole involves two electric point charges with their lines of force given by meridian circles – in a general view these must be taken as geodesic circles – of a family of intersecting spheres. A plane cross-section through the line joining the two charges of the dipole, containg the great circles of the intersecting spheres, is the suggestive image of such a dipole. Taking, in order to simplify our argument, the dipole axis as parallel to the y-axis of the coordinates necessary to our analytic-geometrical description, such an adaptable dipole would be pictured as a family of circles having the analytic equation of the form:

$$(x - u_0 - v_0 \tan\phi)^2 + y^2 = \frac{v_0^2}{\cos^2\phi} \tag{2.1.1}$$

The abscissa $x_0=u_0$, in this equation corresponds to the ordinates $y_0=\pm v_0$, so that all of the circles of this family pass through two fixed points from the plane (x,y), located symmetrically with respect to the axis of abscissae, and having the coordinates:

$$(u_0, v_0) \quad and \quad (u_0, -v_0) \tag{2.1.2}$$

If in each of these two positions there is a charge located, we have the image of the cross-section of a regular electric dipole. The whole physical picture of the field of such a dipole is obtained by a complete rotation of the plane of this analytic figure around the axis of the dipole. The following theory can be found in any basic course of analytic geometry.

Before proceeding any further, let us give first an interpretation of different parameters from the equation (2.1.1), u_0, v_0, and especially to the angle ϕ, destined to play an essential part in the theory. To this end, we need to take the most general family of plane circles in the form of a quadratic non-homogeneous form depending linearly on four parameters, a, b, c, d, which can be considered as coordinates of a circle as a geometrical entity by itself:

$$a(x^2 + y^2) + 2bx + 2cy + d = 0 \tag{2.1.3}$$

Obviously, we can talk of the circle $C(a, b, c, d)$, as of a point of coordinates (a, b, c, d). This equation can still be written in the form of a generic equation of the family

$$(ax + b)^2 + (ay + c)^2 = a^2R^2 \tag{2.1.4}$$

whence it becomes quite obvious that the radius R and the coordinates of the centers of a circles of family are defined by the new parameters according to equations

$$a^2R^2 \equiv b^2 + c^2 - ad, \quad ax_c + b = 0, \quad ay_c + c = 0 \tag{2.1.5}$$

The lower index 'c' in the last two equalities meaning that the coordinates x and y are those of the 'center'.

We are interested in calculating the angle between two circles of the family (2.1.1) at one of their intersection points from equation (2.1.2). This is the angle between their tangents at the point of intersection. Therefore, in order to perform this calculation, we need the slope of the tangent to a circle (2.1.3) in one of its points, and the simplest way to calculate it comes with the observation that for a circle, the tangent in any point is perpendicular to its radius through that point. This means that the slope of the tangent to the circle in a point is the negative of reciprocal of the slope of radius through that point. Using the equation of circle as in (2.1.4), this recipe gives right away:

$$\tan\theta = -\frac{ax+b}{ay+c} \tag{2.1.6}$$

where θ is the angle made by the tangent with the abscissa of the plane coordinate system. Obviously, in a common point of them, (x_0, y_0) say, the tangents of two intersecting circles C_1 and C_2 have the slopes

$$\tan\theta_1 = -\frac{a_1x_0+b_1}{a_1y_0+c_1}, \quad \tan\theta_2 = -\frac{a_2x_0+b_2}{a_2y_0+c_2} \tag{2.1.7}$$

with an evident correspondence between circles and parameters in the lower indices. Based on these, we can calculate the angle between the circles at their intersection point, as the difference between θ_1 and θ_2. However, it is best continuing to avail of equation (2.1.4), which in view of (2.1.6) gives directly the basic trigonometric functions of θ in the form:

$$\sin\theta = \pm\frac{ax+b}{aR}, \quad \cos\theta = \pm\frac{ay+c}{aR}$$

with R defined in equation (2.1.5). In order to settle our ideas based on simplest situations, we choose the plus sign in these expressions. Thus, in any point of intersection of two circles, the angle between them is given by:

$$\begin{aligned} \cos\chi &= \frac{b_1b_2+c_1c_2-\frac{1}{2}(d_1a_2+a_1d_2)}{a_1a_2R_1R_2}, \qquad \chi \equiv \theta_1-\theta_2 \\ \sin\chi &= \frac{(a_1c_2-a_2c_1)x_0+(a_2b_1-b_2a_1)y_0+(b_1c_2-b_2c_1)}{a_1a_2R_1R_2} \end{aligned} \tag{2.1.8}$$

where we have used the equation (2.1.3) in order to eliminate the coordinates of the point of intersection in the expression of *cosine*. Considering just the parameter ϕ as variable for the family of circles from equation (2.1.1), we have the identifications of circle coordinates:

$$a \equiv 1, \quad b \equiv -(u_0 + v_0 \tan\phi),$$
$$c \equiv 0, \quad d \equiv u_0^2 - v_0^2 + 2u_0 v_0 \tan\phi$$

Thus, the cosine from equation (2.1.8) can be calculated right away, with the result:

$$\cos\chi = \cos(\phi_1 - \phi_2) \quad \therefore \quad \chi = \phi_1 - \phi_2 (mod 2\pi) \tag{2.1.9}$$

where ϕ_1, ϕ_2 are the values of the parameter ϕ for the two circles C_1 and $C_2$of the family. In other words, the angle of two circles of the *one-parameter* family (2.1.1) is given by the difference of their parameters: with a proper choice of origin of its values, ϕ *measures the angle between the circles of the family*.

This simple geometry of the dipole admits a just as simple classical interpretation in terms of the concept of free particle. Recall, indeed, that classically, the concept of free particle has two differentiae: *a*) a special *trajectory*, namely *rectilinear*, and *b*) a special *equation of motion*, namely *linear in time*. Geometrically speaking the concept evolved in time, so that the trajectory of a free particle came to be generalized to *a geodesic*, in a general Riemannian geometry, while the equation of motion came to be generalized as the equation of the *affine parameter* of the geodesics. It is this last meaning of the concept of free particle, that allows us a classical implementation of the idea of interpretation, along the following lines.

Now, the electricity inside brain can be described by charge, in the same manner as the light in the classical physical optics: by the equation of a harmonic oscillator. Then, if the brain should be taken as a universe, the skull should be conceived as a Wien-Lummer enclosure. The fundamental structural unit of the brain matter must be a dipole which manifests itself electrically or magnetically, depending on the environment. The physiology confirms this conclusion plentifully, but what it cannot decide is the structure of the very dipole. Considering this dipole a classical one – two charges located, ideally speaking, in the same place – according to a Katz-type natural philosophy of charges ([25]; see II, §§4 *ff*) it should be a magnetic dipole, *i.e. a piece of matter with charges at the ends*, since we cannot possibly speak of a proper physical vacuum in the brain, and so much the more we cannot speak of vacuum in the brain as a universe. Each of these two end charges, though, must have partly the characteristic of a magnetic charge and partly the characteristic of an electric charge. This

'partly' is actually quantified by a *phase* that we have called the *split angle* of the charge (see II, §7) so that the charge can be quantified, in general, as the following equation shows:

$$q(\phi) = e \cdot \cos\phi + g \cdot \sin\phi \tag{2.1.10}$$

Here e is the magnitude of electric part, g is the magnitude of magnetic part of the charge, and ϕ its 'split angle'. In an abstract continuum of real numbers, this split of a charge can be considered as represented by a free particle:

$$\xi(\tau) = e + g \cdot \tau, \quad \xi(\tau) \stackrel{\text{def}}{=} \frac{q(\phi)}{\cos\phi}, \quad \tau \stackrel{\text{def}}{=} \tan\phi \tag{2.1.11}$$

Thus ξ can be considered as the coordinate of a free particle 'moving' uniformly in a 'time' τ, a circumstance which opens the possibility of an interpretation proper, as in the case above. In the language of geodesics, a circle of the family (2.1.1) is a geodesic of a Maxwell fish-eye optical medium [11]. The corresponding free particle is coordinated by the radius of a such a geodesic. The time of the motion of this coordinate is the coordinate of the centre of that geodesic [see equation (2.1.1) above]. In other words, if the charge settles everything in this universe, this is done in a specific mathematical way, which should be taken in consideration in any incidental interpretation.

This result is of a general mathematical nature, and is known as the *Arnold theorem* [26]. It can be formulated thus: if $q(\phi)$ is a solution of the second order differential equation

$$q''(\phi) + a \cdot q'(\phi) + b \cdot q(\phi) = 0 \tag{2.1.12}$$

then the two parameters defined by:

$$\tau \stackrel{\text{def}}{=} \frac{u_1(\phi)}{u_2(\phi)}, \quad \xi(\tau) \stackrel{\text{def}}{=} \frac{q(\phi)}{u_2(\phi)} \tag{2.1.13}$$

where (u_1,u_2) is a basis of two independent solutions of (2.1.12), are connected by the differential equation

$$\xi''(\tau) = 0 \tag{2.1.14}$$

Here a prime means derivative with respect to the variable indicated within parentheses. Physically, this theorem is taken as providing an equivalence between the harmonic oscillator and the free particle.

2.2. The Space of Quantization Procedure

The classical interpretation according to Arnold's theorem, makes out of the geodesic circles of spheres free particles. The *events* of the universe containing these free particles – the brain – are, by all appearances, *neuronal firing* events producing charges. These events can be seen as a kind of 'opposites' to the phenomena of producing the light in the physical universe of our experience: the production of light – a lightning phenomenon – means usually vanishing of charges. They occur with equal probability along the geodesics, which can be identified with the geodesics of spheres, but the problem is that of the Riemannian space hosting these geodesics: this geometrical structure should characterize the matter of the universe we are seeking for.

Of course, such a problem involves some theoretical requirements for the physical description of a light ray, or of its generic counterpart, conceptual component of a generic universe. Optically speaking – the optics being the only part of physics to be consulted most in this instance – such a description means knowledge of the equation of progression of the phenomenon of light along the ray: the propagation. At this juncture the optical properties of the medium supporting light are essential, and among the natural-philosophical prerequisites on such a medium, the *refraction phenomenon* is indispensable: propagation of light can be mathematically described by the *Euler equation* corresponding to the extremum of *optical path*. This path is defined as an integral (see, [27, 28], which are the works we follow closely):

$$I \overset{\text{def}}{=} \int n(x,y,z) \cdot \sqrt{\langle dx|dx\rangle} \tag{2.2.1}$$

where $(ds)^2 \equiv \langle dx|dx\rangle$ is the square of arclength of the corresponding *geometrical path*, *i.e.* of the path in the empty space hosting the optical medium of *refraction index n* ($\boldsymbol{x}$). This background space is assumed to be Euclidean. The variational problem associated with the integral from equation (2.2.1) – the Fermat's principle – provides the differential equation of propagation along the ray, where the *geometrical pathlength* is playing the part of a time of propagation. The differential equation in question is:

$$\frac{d}{ds}\left(n(\boldsymbol{x})\frac{d}{ds}|x\rangle\right) = \nabla n(\boldsymbol{x}) \tag{2.2.2}$$

In this kind of optical problems, the geometry is carried out in *coordinates of location* only. Thus, what is meant here by the symbol $|x\rangle$, as well as by $\boldsymbol{x}$, is a set of three coordinates in space, locating the positions in a Cartesian reference frame. Only, the Dirac's 'ket' notation suggests an algebraic realization of the vector as a 3×1 matrix, *i.e*. a matrix with three lines and a column. In the cases where the reference frame is unique in space, like in path optics, there is no difference in meaning between the two notations. Coming back to our discussion trend here regarding the classical optics, when algebraically expanded, the equation (2.2.2) gives

$$n(\boldsymbol{x})|x''\rangle + \{\nabla n(\boldsymbol{x}) \cdot |x'\rangle\}|x'\rangle = \nabla n(\boldsymbol{x}) \tag{2.2.3}$$

where the diacritical mark means derivative on *s*, the path length, and a dot between vectors means dot product, as usual.

Now, everything depends on the functional form of refraction index of the medium supporting the light. According to Planck's quantization procedure this optical medium needs to accommodate the fundamental material structure of *an electric dipole*. That is, we need to assume that in the Wien-Lummer enclosure of the skull type, enclosing the brain, the equivalent of a thermodynamical equilibrium creates a medium whose fundamental constitutive unit is a dipole. This condition comes down to assuming a special functional form of the refraction index of the medium. To wit, this special functional form is:

$$n(\boldsymbol{x}) = (1 + \langle x|x\rangle)^{-1} \quad \therefore \quad \nabla n(\boldsymbol{x}) = -2(1 + \langle x|x\rangle)^{-2}|x\rangle \tag{2.2.4}$$

where $\langle x|x\rangle$ is the sum of squares of the coordinates along the path of light, taken as scaled with a gauge length, as it were, in order to be considered pure numbers. Understandably, in view of the idea of the quantization of light (see [10], Chapter 2] the essential theme of the present work is justifying this choice for the refraction index of the optical medium. A 'choice' of the refraction index may seem as just a particular case of a medium, where the theoretical physics needs, in fact, the most general concept. However, let us not forget that the choice is only a mathematical fact, and the demand of generality is actually a matter of physics.

Start, for once, with the equation (2.2.3), which by using (2.2.4) can be rewritten as:

$$(1+\langle x|x\rangle)|x''\rangle - 2\langle x|x'\rangle|x'\rangle + 2|x\rangle = |0\rangle \tag{2.2.5}$$

Using the definition of the elementary arclength of the geometrical path, we have:

$$(ds)^2 = \langle dx|dx\rangle = \langle x'|x'\rangle(ds)^2 \quad \therefore \quad \langle x'|x'\rangle = 1 \tag{2.2.6}$$

so that, in a Euclidean background, we can get the relations

$$\langle x'|x''\rangle = 0, \quad \langle x''|x''\rangle + \langle x'|x'''\rangle = 0 \tag{2.2.7}$$

and use them in order to conclude on the equation (2.2.5). First, by differentiating (2.2.5) itself, we get:

$$(1+\langle x|x\rangle)|x'''\rangle - 2\langle x|x''\rangle|x'\rangle = |0\rangle \tag{2.2.8}$$

whence dot-multiplying this by $|x''\rangle$ and using the first relation from (2.2.7), gives

$$\langle x''|x'''\rangle = 0 \quad \therefore \quad \langle x''|x''\rangle = constant \tag{2.2.9}$$

Geometrically speaking, this last condition is referring to a curvature of the ray path through space: for this kind of continuum, the curvature of the path must be a constant. Returning then to equation (2.2.5) once again, however, this time only for dot-multiplying it by $|x''\rangle$ directly, and then using the first of equalities (2.2.7) and the result from the last of equations (2.2.9), gives the remarkable final equations to be used in adjusting the differential equation (2.2.8):

$$\frac{2\langle x|x''\rangle}{1+\langle x|x\rangle} = -\frac{1}{R^2}, \quad \langle x''|x''\rangle \equiv \frac{1}{R^2} \tag{2.2.10}$$

Here R is a non-dimensional constant, suggesting, again, the necessity of a gauge length to be introduced *via* the curvature of the geometrical path. Inserting this result into equation (2.2.8) produces the final equation of the propagation along the ray:

$$|x'''\rangle + \frac{1}{R^2}|x'\rangle = |0\rangle \tag{2.2.11}$$

This is the first occurrence, in the context of optics of material media, of this third order linear differential equation which, as we shall try to establish in the present work, is of a primary

importance in theoretical physics [see II, §8, equation (43)], as long as this one is viewed from the perspective of a natural philosophy involving the scale transitions.

While this task will be gradually fulfilled as we go all along with our work, for now we have an observation that needs to be made in order to properly guide the work itself. Namely, the *optical medium* described by the refraction index from equation (2.2.4) can be considered a Riemannian manifold which turns out to be of *finite volume* and *positive curvature*. Thus this optical medium satisfies the basic geometrical requirements for playing the part of a Wien-Lummer cavity. Indeed, the elementary optical path of this medium is conform-Euclidean, assuming that $\langle dx|dx\rangle$ is Euclidean. In view of the idea of physical generality, it is perhaps the moment to insist for a little while on the equation (2.2.1) from this geometrical point of view. Take the elementary path

$$(ds)^2 = \frac{\langle dx|dx\rangle}{(1+\langle x|x\rangle)^2} \tag{2.2.12}$$

as the Riemannian metric of this optical realm. This metric is, in the views adopted here, a conformal Euclidean metric, with the conformality factor being spherically symmetric. Using the spherical symmetry, we have

$$(ds)^2 = \left(\frac{dr}{1+r^2}\right)^2 + \left(\frac{r}{1+r^2}\right)^2 (d\Omega)^2, \quad (d\Omega)^2 \stackrel{\text{def}}{=} (d\theta)^2 + \sin^2\theta \cdot (d\varphi)^2 \tag{2.2.13}$$

where θ and φ are the spherical angles of colatitude and longitude, respectively and $r^2 \equiv \langle x|x\rangle$. This form of the metric does not tell us very much about the Riemannian geometry of the optical realm, but it can tell us a lot if we go over to a meaningful variable. So, let us assume that r takes values between two limits, as in the case of the radial coordinate in the classical Kepler problem. In this case it can be represented faithfully by formula:

$$r(\phi) = \tan\left(\frac{\phi}{2}\right) \tag{2.2.14}$$

so that the metric (2.2.13) can be written as

$$(ds)^2 = \frac{1}{4}\{(d\phi)^2 + \sin^2\phi \cdot (d\Omega)^2\} \tag{2.2.15}$$

This metric describes a realm spatially delimited by the magnitude $\tan\phi$, which can be, for instance, the magnitude of an *eccentricity vector*, in the case of a classical Kepler motion of the hyperbolic type, and with ϕ representing the arclength, ω say, of the geodesic on the unit sphere. Notice, however, a fact that seems more important for what follows here: namely that, in this case, (2.2.15) is still the metric of a unit sphere. Indeed, by introducing two complex variables:

$$z_1 = \cot\left(\frac{\phi}{2}\right)\cdot e^{i\omega}, \quad z_2 = -\tan\left(\frac{\phi}{2}\right)\cdot e^{i\omega} \tag{2.2.16}$$

where ω counts here as an arbitrary angle of longitude, the metric (2.2.15) can be reduced to the so-called *Lorentz form* [29]:

$$(ds)^2 = \frac{dz_1\cdot dz_2}{(z_1 - z_2)^2} \tag{2.2.17}$$

destined to play in important role in the theory of the memory, being connected with the *Schwarzian derivative*. However, more importantly, the metric (2.2.15) is the *space metric* of the so-called Schwarzschild solution for a sphere of incompressible fluid that can serve for the purposes of interpretation. It 'saved the day' so to speak in a delicate situation, on the occasion of Einstein-de Sitter debate, that instated the empty spacetime in physics, of which we shall discuss right away.

The metric (2.2.12) is the metric of the realm called *the Maxwell fish-eye*, indeed, as it was described long ago by Constantin Carathéodory in his exquisite mathematical researches regarding the geometrical optics [30]. With it, we have the first of the incentives of the choice of the functional form (2.2.4) for the refraction index. Indeed, the Maxwell fish-eye is a perfect optical device whereby all light rays have the *properties of the lines of force of an electric dipole*: circles passing through two fixed points representing the locations of the two component charges ([27, 28]; see §2.1 above). In a Maxwell fish-eye, all the light rays through any point in the space occupied by this medium, also pass through a point which is its inverse with respect to a given sphere. Should the necessity occur to operate an interpretation here, it obviously needs to be achieved by point particles having *charges*, and characterized by the Lorentz property: in order to acquire a charge of opposite sign, a position from such a medium needs to be *replicated by inversion* with respect to a certain, locally spherical surface.

Our contention is that this optical description is a characteristic of a universe in general: it is *a differentia of the concept of universe*. Then, the physiological discoveries regarding the applicability of the idea of dipole in the case of brain, constitute, again in our opinion based on the results of the previous section, further incentives for considering the brain a universe, and for describing it as such.

2.3. A Cosmological Geometry of Charge

A differentia of the concept of a universe in general, must be represented as a metric property that comes with the general relativity, when this doctrine is applied to cosmology: this is the only discipline that addressed the concept at a significant juncture. That addressing was stirred up by an important moment of our knowledge, known as *Einstein-de Sitterdebate* on the relativistic approach of the cosmological problem (see [31] and [32]; for the interesting– at least for us –details of this debate see [11b]). The whole moral of that historical debate – again, at least the way we take it, based on Katz's natural philosophy of charges, applied to cosmological situations – is that an empty universe is a virtual physical continuum of charges. The point is that, just like the light in the case of physical universe, the local charge can be considered a cosmological property.

It is perhaps worth insisting for a little while on the following issue that can be easily aroused by neurologists: is not the reference to general relativity stretching too much the attributions of physics regarding a brain theory? For, the reference to such very specific physics' issues, may appear to no avail in the case of brain, so why bother about it, in the first place!? We do not share this view, and for a good reason at that: the brain we are talking about here is a *universe existing in a uniform gravitational field*, that determined the life as it appears today. It seems to us impossible that such a circumstance does not have an effect on the structure of brain! While even a secret hope can be entertained – whose incentive will be expounded by us later on – that this cosmic incidental may be accountable for the creation of the *events of firing* in the brain, for now it suffices to say that the general relativity is *the only* physical theory addressing the problem of gravitation field cosmologically, and with sizable results at that. So we need to concentrate on it, indeed, and the Einstein-de Sitter debate seems the right moment to do that, inasmuch as it changed our way of thinking regarding the physical fields in physics, and anywhere else, for that matter. Consequently, let us insist upon this historical moment of knowledge, adopting a special point of view.

The debate started with Einstein's celebrated *Cosmological Considerations* [31]. Thereby, the author proposed a solution of the relativistic cosmological issue of the impossibility of prescribing boundary conditions for the metric tensor, in the case of field equations. According to his doctrine, the metric tensor should represent the gravitational field within matter. The proposal was a spatially

finite spherical universe, for which the boundary conditions are manifestly unnecessary. Einstein realized this proposal by considering the universe as an abstract four-dimensional quadratic manifold analogous to a sphere of constant radius *R*, after the model offered by special relativity:

$$\xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_4^2 = R^2 \tag{2.3.1}$$

projected on its three-dimensional manifold, ξ_4= constant for instance. Then, assuming that at the infrafinite scale the coordinates are differentials, and that the geometry is preserved for this abstract manifold, in the sense that the metric of this continuum can be written as a quadratic form having the same signature as the hypersphere:

$$(d\xi_1)^2 + (d\xi_2)^2 + (d\xi_3)^2 + (d\xi_4)^2 = (ds)^2 \tag{2.3.2}$$

the metric of the universe thus conceived must be

$$(ds)^2 \overset{\text{def}}{=} g^{ij} dx_i dx_j, \quad g^{ij} \equiv \delta^{ij} + \frac{x_i x_j}{R^2 - \sum_k x_k^2} \tag{2.3.3}$$

where $\xi_k \equiv x_k$ are the coordinates of a position in the chosen hyperplane of the hyperspace.

With a little stretching of the mathematical notions, this universe – known as the *static universe* of Einstein ever since – is actually a particular case of a hologram: being a projection, it surely qualifies as a 'photography'. With this observation, Willem de Sitter's objection can be better understood. Originally, it was referring to the fact that the prescriptions of relativity, which require a four-dimensional spacetime as an arena of reality, were simply forgotten in this case [32]. Assuming the idea of a hologram, one can see immediately this objection may be actually split in two subordinate ones. First, and mainly stressed by de Sitter himself, is that the manifest relativistic equivalence between space coordinates and time coordinate was lost in Einstein's construction: a three-dimensional universe cannot possibly describe but only a space filled with matter, and the time can only be a continuity parameter of a classical nature. As a matter of fact, with the metric (2.3.3) one forgets completely about time, for we only have a static universe, as mentioned above. So, de Sitter applies the Einstein's procedure to a five-dimensional abstract space, and projects this space onto a four-dimensional spacetime, thus considering the spacetime physical, according to the precepts of relativity, not the space alone, as Einstein did. Here the second subordinate objection surfaces, as

to the possible physical meaning of the assumed five-dimensional manifold. And, while we are on it, notice that neither (2.3.1) nor (2.3.2) may have a physical meaning, if not conceived as the manifolds of events, as in special relativity. The basic agreement of both Einstein and de Sitter is that this case cannot be accepted. However, there is a little subtlety in the relativistic equivalence of the time and space coordinates, that cannot be better illustrated than by the concept of a hologram, as we shall show right away.

Historically speaking, though, up to this point there was nothing to debate in what Einstein did. However, his general-relativistic doctrine obligated him to save the day, as it were, by a proposal that, according to his own declaration and subsequent work, he regretted all his life. Nevertheless, this proposal made history ever since, and with quite good reasons at that, we have to admit. It was the addition of a 'cosmological term' into the equations of gravitation field, initially proposed by him [33b] in order to give a kind of logical appearance to the lack of possibility of cosmological boundary conditions. Such an addendum was of a classical inspiration too: this is how one removes the Seeliger paradox connected with the classical Poisson equation describing the Newtonian gravitation. It consisted from using in the theory, instead of the fundamental tensor $\boldsymbol{G}$, used in the initial field equations, the modified tensor:

$$\boldsymbol{G} - \lambda \boldsymbol{g}, \quad G_{\mu\nu} \stackrel{\text{def}}{=} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \tag{2.3.4}$$

in order to discuss the cosmological problem. Here $\boldsymbol{R}$ is the Ricci curvature tensor of the abstract manifold, having the curvature R, and λ is the so-called *cosmological constant*.

This is the occasion that opened the 'Pandora's box' of the historical debate whose protagonists were Einstein and de Sitter: by the cosmological term added to $\boldsymbol{G}$ in equation (2.3.4), it was in fact assumed, implicitly, that *every universe*– therefore the concept, in general – must have a characteristic cosmological constant proportional to the curvature of its spacetime. Willem de Sitter proved that there are such universes, indeed, but *empty of matter*, according to Einstein's equations of gravitational field ($\boldsymbol{G}-\lambda\boldsymbol{g}=\boldsymbol{0}$), and therefore the Einsteinian doctrine that 'matter prevails over geometry' cannot be taken as valid: the metric tensor appears as just an incidental mathematical tool, not an essential physical ingredient, representing the physical field of gravitation, or something like that. It was a significant blow to the very Einstein doctrine, with important consequences for the future of physics. Some of these consequences will be presented here right away, but first let us pinpoint the issues at a closer look, through the idea of hologram.

Indeed, in order to do this job in utmost generality, let us use the previous idea –suggested by Einstein's projection method – of the universe as a hologram. Certainly, as we just said, by such a projection the universe qualifies as a 'photography', and the point of Willem de Sitter's objection to this method can then be associated with the fact that, a universe being three-dimensional, the projection must be a hologram, rather than just a simple photography. At rigor, it can be considered as a set of photographies, of the same object taken simultaneously at different 'phases', as it were, which is, in fact, the way a hologram is technically realized. To further elucidate this issue, while making our case along, notice that Einstein chose the universe ξ_4 = constant, when he would be completely free to choose any of the possibilities $\xi_k = constant$, as a matter of fact, with k taking the rest of the remaining values in turn. The correctness of the choice he made is, never the less, *apriori* assured by symmetry: a *sphere filled continuously with matter*, is the same any way you look at it! However, if the symmetry is somehow broken, for instance by an interpretation of matter with particles having no homogeneous numerical density inside the sphere, all of the four Einstein's projections are needed *simultaneously* in order to describe the resulting universe as a *bonafide* hologram.

This was the case signal ed by Willem de Sitter and, relativistic ally speaking, it arises no matter of the conditions of symmetry and continuity of matter, *i.e*. even in the ideal conditions of classical physics or relativity. However, in the case of relativity, the coordinates and the time must be equivalent from the point of view of any perspective of projection, so to speak. In his argument, de Sitter considered the Minkowski metric of the spacetime continuum. Then an Einstein projection at *constant time* would give the universe proper of classical mechanics, as a continuous set of *simultaneous points in Euclidean space*, if an incidental interpretation is in place. However, all the other three projections break the symmetry: they are, each one of them, *Lorentz three-dimensional worlds* different by the pair of space coordinates used in projection (see II, §3*ff* for an illustration of such a world, in the case of charges). So, if it is to consider the symmetry here, only these three last projections can be, in the last resort, considered as equivalent. Even in this case, the universe must then be, according to Einstein's recipe, necessarily described as a 'binary hologram', if we may say so: one of the 'photographies' is the regular Einsteinian static world at a given relativistic moment of time, the other is a Lorentzian three-dimensional world, which is a plane world, but 'in motion', as it were. Mention should be made, in connection to the historical case considered by us, that the parties involved in the Einstein-de Sitter debate did not consider but a single 'photography' instead of a hologram, and thus many conceptual details were left for the future to be revealed.

We stop here with these natural-philosophical considerations, for they are mostly issues of theoretical physics proper, to be deferred to appropriate works (see [11b] for details), and may become unnecessary for the problems of physiology. What we need to extract from this tale is the description of a universe in general, as a concept: it is, indeed, *a hologram*. And it is legal, we have to admit, to realize it as a single 'photography', as the protagonists of the historical debate did in fact. However, such a hologram must contain any number of 'phases' depending on the structure of matter contained in the universe to be described. For, the problem is, in fact, if we can *dispense with the idea of projection* and describe a hologram as a whole, in order to be able to discern 'the phases' in it, to be used in describing the universe as a physical structure, thus serving for prediction purposes. Recall, indeed, that in matters of brain, for instance, the prediction is essential and take specific forms, in need to be rationalized. The solution to this problem was virtually provided by Felix Klein even before Albert Einstein became… Einstein, but it was particularly obvious on the occasion of Einstein-de Sitter debate, for which Klein was a distinguished 'moderator', as it were. It is offered by the concept of *absolute or Cayleyan geometry*, to be presented in what follows from this section. We shall first discuss the case of an Einstein sphere, which can be taken as a guiding example. Then we shall extend the method of construction to the four-dimensional case, as a benefit of de Sitter's observations, to be used for a physical interpretation of the *a priori* conditions.

To start with, if we denote by X a point in the three-dimensional space, then a coordinate representation is given by a triple of numbers representing the point in the sense of Cartan (see III, Chapter 4). We can define them with reference to the residence space as follows: associate them arbitrarily, memorize them, and then carry them everywhere, thus realizing the position in any place *via* an adequate reference frame. A slight change in notation is in order here, in the sense that the lower indices will be assigned to points, rather than to coordinates:

$$X \stackrel{\text{def}}{=} (x, y, z) \tag{2.3.5}$$

Note that in this association, X is not necessarily taken as a vector: it is just *a triple of numbers*. We shall build the geometry based on the properties of the quadratic form from the left hand side of equation (2.3.1), however in the three-dimensional case:

$$(X, X) \equiv x^2 + y^2 + z^2 \tag{2.3.6}$$

The condition to have X as a real point is $(X, X) > 0$, even if this quantity is unspecified by the idea of a sphere in space, or something like that. The condition $(X, X) < 0$ defines, from a

geometrical point of view some purely imaginary points: there is not such a real thing as a negative sum of squares of real numbers. However, from a physical perspective, such points can be only partially imaginary, so to speak.

The physical interpretation of the resulting geometry depends on the condition $(X, X) = 0$, and this can always make sense in physics, pending a condition of quantization. For instance, it can represent the condition of equilibrium of forces within the static ensembles of particles serving for interpretation. Then stochastic processes can be defined in order to assimilate the fundamental physical quantities generating the three forces, with lengths, serving to realize the Cartan's program (see III, §4). These stochastic processes are to be defined, for instance, in terms of specific Lewis-Lutzky invariants ([34c], §4.3), and thus they should constitute the mathematical expression of the necessary memory in the global problem, analogous to the classical inertia. In such a case only the mass can be imaginary, but from the point of view of the equilibrium Newtonian forces – the static forces serving for interpretation –it can always be taken as imaginary.

In order to construct this *absolute geometry*, which we often have called here *Cayleyan* after its inventor's name ([35]; see for details and discussion [36]), we take the quadratic form (2.3.6) as a norm for the points in our space of points. It induces an internal multiplication of points by the *polarization* process:

$$(X_1, X_2) \stackrel{\text{def}}{=} x_1x_2 + y_1y_2 + z_1z_2$$

with an obvious correspondence between the indices of points and the indices of coordinates. This dot product helps us in characterizing a straight line in space – the essential concept necessary in constructing a metric. The straight line joining the points X_1 and X_2 is the locus of the points

$$X = \lambda X_1 + \mu X_2 \tag{2.3.7}$$

with λ and μ variable numbers representing some *homogeneous parameters*, coordinating the position of points along the straight line. This straight line intersects the absolute sphere $(X, X) = 0$ in two points, having the homogeneous parameters only partially determined – that is: up to an arbitrary factor – by the quadratic equation:

$$(X, X) \equiv \lambda^2(X_1, X_1) + 2\lambda\mu(X_1, X_2) + \mu^2(X_2, X_2) = 0$$

'Partially determined' means here 'up to an arbitrary factor'. For, indeed, we cannot determine from this equation, but only the ratios of these homogeneous coordinates – usually taken in analytical geometry as *non-homogeneous point coordinates* along the line – in the form of the roots of equation, *i.e.*

$$t \equiv \frac{\lambda}{\mu} = -\frac{(X_1, X_2)}{(X_1, X_1)} \pm \frac{\sqrt{(X_1, X_2)^2 - (X_1, X_1) \cdot (X_2, X_2)}}{(X_1, X_1)} \tag{2.3.8}$$

As it turns out, though, these two ratios are just enough for our purpose to build a metric of the space.

Let us explain the general philosophy of this construction: in geometry, a metric usually represents the *distance between two infinitesimally close* points, so that what we need first is to define a *distance* between points in general, and then go over to infinitesimally close points. Now, we do not have, within our experience, but the Euclidean distance: any other such quantity goes by analogy. The quantity that reduces to distance between two points in the Euclidean case turns out to be the *cross ratio* of four points on a straight line: *two of these points* are fixed and used as *a reference frame on the line*, while any other two points are giving the current pair of points between which we calculate the distance. The two points from equation (2.3.8) can be taken as the reference frame along the straight line defined by equation (2.3.7). Then the distance is given, up to a numerical factor, by the *logarithm of this cross ratio* (the so-called *Laguerre's formula*). Let us see how this philosophy really works.

Given the two points $X_{1,2}$ the straight line joining them contains all the points of the form $X = tX_1+X_2$. In order to define the distance between the two points, we can choose *arbitrarily* two other points, $X_{3,4}$ say, to play the part of the reference frame. Then the cross ratio of points on line is simply defined as the cross ratio of the corresponding non-homogeneous parameters t. So, we have:

$$(X_1, X_2; X_3, X_4) \stackrel{\text{def}}{=} \frac{t_1 - t_3}{t_1 - t_4} : \frac{t_2 - t_3}{t_2 - t_4} \tag{2.3.9}$$

The distance is simply *proportional* to the logarithm of this quantity, which is, obviously, an abstract number. It depends, of course, on the second pair of points, but this ambiguity can be substantially reduced if we refer the construction to the absolute of space. First, notice that according to equation (2.3.7), the parameter t has the values $t_2 = 0$ for the point X_2, and correspondingly, $t_1 = \infty$ for the point X_1. In view of this, the cross ratio (2.3.9) takes the simple form

$$(X_1, X_2; X_3, X_4) = \frac{t_4}{t_3} \tag{2.3.10}$$

The two points X_3 and X_4 must offer us the advantage of allowing a *standardization*, if we may say so, of this construction. Therefore, they must be chosen on the absolute: *every* pair of points in space has a corresponding pair of points on the absolute, these being the points where the corresponding straight line containing our points intersects the absolute. With this choice, the corresponding parameters $t_{3,4}$ are then given by the two ratios from the equation (2.3.8), so that equation (2.3.10) becomes

$$(X_1, X_2; X_3, X_4) = \frac{(X_1, X_2) + \sqrt{(X_1, X_2)^2 - (X_1, X_1) \cdot (X_2, X_2)}}{(X_1, X_2) - \sqrt{(X_1, X_2)^2 - (X_1, X_1) \cdot (X_2, X_2)}} \tag{2.3.11}$$

which is, indeed, sufficient for defining a metric by the Laguerre formula. This ratio, however, is complex of unit modulus so it cannot serve the intended purpose, which requires reality of the distance. The conclusion can be ascertained from the fact that the quantity under the sign of square root is always negative for real vectors in the Euclidean space. Nevertheless, according to Felix Klein, even with this cross-ratio, we can still construct a differential version of the distance by Cayley's method, *viz*. a metric of space [36]. Indeed, the distance according to Laguerre's formula is only *proportional* to the logarithm of the cross ratio, and therefore it involves an arbitrary constant. The logarithm of the cross ratio from equation (2.3.11) is a purely imaginary complex number, so that, if we choose the proportionality constant as a purely imaginary complex number the things are getting in order. Thus, the Laguerre distance given *via* the logarithm of the cross ratio (2.3.10) can be represented by the distance given via the logarithm of cross ratio (2.3.11), because the ratio of the two expressions involved in this equation is a purely imaginary complex number, and we are at liberty to choose an imaginary number as the constant defining the Laguerre distance.

With this philosophy, we can construct a differential version of the distance – a metric of space. Assuming that the two base points X_1 and X_2 are infinitesimally close $X_1=X$, $X_2=X+dX$, we can calculate the necessary quantities in equation (2.3.11) as

$$(X_1, X_2)^2 - (X_1, X_1) \cdot (X_2, X_2) = (X, dX)^2 - (X, X) \cdot (dX, dX)$$

Now, in the *real domain*, we can accept that the quantity $(X,dX)/(X,X)$ is an infinitesimal quantity of the first order, while $(dX,dX)/(X,X)$ is an infinitesimal quantity of the second order. Thus the cross ratio (2.3.11) can be expanded and, to first infinitesimal order of infinitesimals, it is

$$(X_1, X_2; X_3, X_4) = 1 + 2i\sqrt{\frac{(dX, dX)}{(X, X)} - \left(\frac{(X, dX)}{(X, X)}\right)^2}$$

The logarithm of this quantity is given, to the same first infinitesimal order, by the second term from the right hand side, which is, of course, a purely imaginary number, as we just said. Then, we can set things in order by Klein's recipe: multiply the logarithm with a purely imaginary constant quantity, iR say, in view of the fact that the metric *perse* is defined up to an arbitrary scale factor. Thus the Cayley-Klein – or absolute – metric of this geometry can finally be written in the form of a quadratic differential:

$$\left(\frac{ds}{R}\right)^2 = \frac{(dX, dX)}{(X, X)} - \left(\frac{(X, dX)}{(X, X)}\right)^2 \tag{2.3.12}$$

with R an arbitrary real quantity. This equation is a regularly considered form of the Cayley-Klein metric, with reference to any absolute of space. It turns out that this expression is also valid in *larger conditions of space definition*: *complex points, general definition of the absolute as a quadric in this space, etc*. Dan Barbilian, to mention a notable case, used it for the cases where (X,X) is a homogeneous polynomial of *arbitrary* degree – a *quantic*, in algebraic phraseology – thus generalizing the metric (2.3.12) even further [37].

However, as long as the absolute is a quadric – *i.e.* a general surface specified by an equation quadratic in the coordinates – using the properties of the dot and cross products of the real vectors in space, the metric (2.3.12) can be written in the form:

$$\left(\frac{ds}{R}\right)^2 = \frac{(X\wedge dX, X\wedge dX)}{(X, X)^2}, \quad X\wedge dX \stackrel{\text{def}}{=} (ydz - zdy, zdx - xdz, xdy - ydx) \tag{2.3.13}$$

The results obtained by projection are to be found by assuming that one of the coordinates – say z – is a constant, R say, so that, in such a case, we have

$$X\wedge dX \stackrel{\text{def}}{=} (-Rdy, Rdx, xdy - ydx) \tag{2.3.14}$$

If we apply this to the metric (2.3.13), we get the result:

$$\left(\frac{ds}{R}\right)^2 = R^2\frac{(dx)^2+(dy)^2}{(x^2+y^2+R^2)^2}+\frac{(xdy-ydx)^2}{(x^2+y^2+R^2)^2} \tag{2.3.15}$$

which, no question, coincides, with a result to be obtained from (2.3.3) up to a factor, for proper considerations, but reveals an interesting position of the metric of the *Maxwell fish-eye* (2.2.12) for the two-dimensional case. Before commenting on this result let us go over to one more dimension, as promised, in order to encompass also the case of charges.

So, let us now replicate the previous case for the three-dimensional space as carrier of the Cayley-Klein metric instead of the two-dimensional plane. The construction according to the methods of Cayleyan geometry guarantees that this will be a case of an Einstein universe, according to the taste of Willem de Sitter, and of the universe as a concept in general: no photography involved, just an incidental physical description. We just have to get started with the Einstein's quadratic form with four terms from equation (2.3.1), instead of that from equation (2.3.6):

$$X \overset{\text{def}}{=} (w,x,y,z), \quad (X,X) \equiv w^2+x^2+y^2+z^2 \tag{2.3.16}$$

Formally then, nothing changes from what has already been done this far, except the final formula (2.3.13): there is no possibility of a vectorial connection between the dot- and cross-products in the four-dimensional case, because the dimension of the cross-product space here is six, and therefore the cross-product cannot be assigned to a vector as in the three-dimensional case. However, an identity of the same sort as in the three-dimensional case is algebraically existent in this case, under the name of *Lagrange identity*, whose concise form in the current four-dimensional case is [38] (see [39], for a nice demonstration in the most general case): the sum of squares of the *six* 2×2 *different determinants* $S_{\alpha\beta}$, made out of the columns numberα and numberβ of the 2×4 table:

$$\begin{pmatrix} w_1 x_1 y_1 z_1 \\ w_2 x_2 y_2 z_2 \end{pmatrix}$$

can still be expressed by the difference between the product of the norms of the two lines of the table, as represented by the left hand side in equation (2.3.1), and the square of the dot-product of these lines, as induced by the norm thus defined, by a polarization procedure as in the three-dimensional case. To wit:

$$\sum S_{\alpha\beta}^2 = (X_1, X_1)\cdot(X_2, X_2) - (X_1, X_2)^2, \quad S_{\alpha\beta} \stackrel{\text{def}}{=} \begin{vmatrix} X_{1\alpha} & X_{1\beta} \\ X_{2\alpha} & X_{2\beta} \end{vmatrix} \tag{2.3.17}$$

Here, as in equation (2.3.5), we understand that the point X is specified by the four coordinates, as in equation (2.3.16). Within this convention the metric can be written as in equation (2.3.13), but with

$$X\wedge dX \stackrel{\text{def}}{=} (ydz - zdy, zdx - xdz, xdy - ydx, xdw - wdx, ydw - wdy, zdw - wdz) \tag{2.3.18}$$

Making w a constant: $w=\pm R$, as in equation (2.3.14), we get the Cayley-Klein metric in the form

$$\left(\frac{ds}{R}\right)^2 = R^2\frac{(dx)^2 + (dy)^2 + (dz)^2}{(R^2 + x^2 + y^2 + z^2)^2} + \frac{\sum(ydz - zdy)^2}{(R^2 + x^2 + y^2 + z^2)^2} \tag{2.3.19}$$

whose first term is:

$$\left(\frac{ds}{R}\right)^2 = R^2\frac{\langle dx|dx\rangle}{(R^2 + \langle x|x\rangle)^2} \tag{2.3.20}$$

and second term is the square of the differential area in space, up to a factor, of course:

$$\left(\frac{ds}{R}\right)^2 = \frac{R^2}{2}\frac{\mathrm{tr}(|x\rangle\langle dx| - |dx\rangle\langle x|)^2}{(R^2 + \langle x|x\rangle)^2} \tag{2.3.21}$$

Again, no doubt, the metric from equation (2.3.20) is, obviously, a *Maxwell fish-eye metric*, in the form once used by Constantin Carathéodory in the geometrical optics.

By the way of conclusion, one can see that the Einstein-de Sitter debate was, in fact, concentrated in representing a universe as an optical medium, according to the Planck's procedure of quantization. This conclusion is guaranteed by the inherent property of the agent carrying information in the physical universe – that is the light – which, if the dipole is the essential structure of matter in the universe, must be a differentia of the concept in general: every universe must be a Maxwell fish-eye in view of the relation between its 'matter' and its 'light'. It is these two last ingredients of the universe that need to be decided in a case by case study.

However, even in the matters of this decision there would seem to be a conceptual note, surpassing the phenomenology. Namely, the equation (2.3.1), therefore implicitly the norm (2.3.17) can be taken *apriori* as a constraint on charge, according to Katz's natural philosophy (see part II of the present work): a four-term quadratic form must represent a kind of 'norm' of the charge background, according to the precepts of that natural philosophy. Two associated charges of a dipole enter this norm. The association is realized by the phases, represented by their split angles in a holographic phenomenon. As we just said, this theory seems to surpass the phenomenology, thus being of a conceptual note: every universe should possess this property of connection between its 'matter' and its 'light'. The fact encouraging us along this path of thinking is, again, a fact of solid experience, part of the finite world we live in: the connection between light and charges as revealed by the electromagnetic nature of light. The universes naturally existing in this world must all satisfy this condition.

The bottom line of this description is that a universe can be described as a Wien-Lummer enclosure, and thereby the quantization should be in place, just as the quantization of light. Our contention is that the Einstein's abstract constraint (2.3.1) is an incidental normalization of the charge, necessary in the holographic construction of a universe having the metric (2.3.19). Any of the abstract variables ξ_μ, can be associated with any other one in a process of making a charge of a dipole. The other two remaining abstract variables make the other charge of the dipole. We have six such possibilities of construction of the dipoles. If one of the charges is fixed (the value R) then the metric is that of a Maxwell fish-eye, indeed. This statement is taken by us as a work load of the present development for the case of brain. However, before we start on this development, we need to reveal one more essential lesson of the Einstein-de Sitter debate.

3. Conclusions

There are sufficient incentives to consider the brain as a universe: the skull as an enclosure, the dipole as a fundamental physical structure, the electricity as a cosmic physical background, firing as an event, memory as a property of the universe. Each one of these incentives is object of a particular physics' study and, in fact, each one needs conceptual adjustments even within physics, in order to be considered usable for physiological necessities. The first to be considered is the connection between the existence of a universe and the structure of the fundamental physical structure of this universe is*the dipole*. Within today's mathematical possibilities of description of physical facts, we found that the connection can be explained by the fact that the matter of a universe has the structure

of a special *optical* medium: the Maxwell fish-eye. This is a medium that allows for the dipole as a unique fundamental physical structure.

However, the main concept to be adjusted in order to be aptly applied to the case of brain, appears to be the very concept of universe itself. We addressed this issue herefrom the very point of view of connection between its existence and its physical structure. To wit: it exists in an enclosure, regardles of space scale, and this existence imposes the dipole as a fundamental physical structure. These two facts are common for physics and physiology, and seem good candidates for defining a concept of universe in general. Everything else,then, comes along by the way of mathematics.

The mathematical description is guided by what we think is an obvious natural principle: the scale transition invariance. This principle certainly acts in physics. There are incentives to assume that it also acts in physiology, inasmuch as the man is a physical structure existing in the physical universe, and this existence is concurrently universal – a cosmic existence, so to speak – andlocal – a being struggling for life.

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