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COALITIONAL CONTROL ALGORITHM FOR A VEHICLE PLATOONING APPLICATION – A SELFISH APPROACH

BY

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Abstract. Nowadays, the public roads are getting more congested than ever, with the increase of vehicle numbers. As such, the necessity of improving the traffic flow, while maintaining the passengers comfort is paramount. In this context, a vehicle platoon, defined as a group of different vehicles, which travel with the same velocity and maintain a prescribed inter-vehicle distance, is considered by the control community as an accepted control solution for efficient traveling. This work provides a selfish coalitional control algorithm suitable for a vehicle platooning application, which is a particular case of a networked system composed of multiple sub-systems (or vehicles), which share information via a communication network. The main idea behind the coalitional control is to design a dynamic communication network, in which the communication links between the agents (which control each vehicle) are enabled or disabled, according to the selected communication topology. Moreover, a selfish approach is envisioned, in which each agent is allowed to switch between the communication topologies and to choose its own, most beneficial one, while ensuring the optimality of the entire vehicle platoon. The control algorithm was tested in simulation, on a five vehicle,

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heterogeneous platoon, and the results show the efficiency of the proposed algorithm.

Keywords: Coalitional control, networked systems, communication architecture, switching communication topologies, vehicle platoon.

1. Introduction

Traffic management on public roads aims to provide suitable solutions for problems such as: traffic safety, fuel consumption or air pollution, all caused by the increased number of vehicles (Fakhfakh *et al.*, 2020). Vehicle platooning is an application designed for the Intelligent Transportation Systems (ITS), consisting in coordinating a group of vehicles, to travel with the same velocity, imposed by the group leader (hereafter called leader vehicle), while maintaining a safe inter-vehicle distance between the platoon's follower vehicles (Maiti *et al.*, 2017). According to the review work provided in (Li *et al.*, 2022), the basic operations for a connected and automated vehicle (CAV) platoon are: i) stabilizing, (or keeping the platoon stable), by maintaining the desired inter-vehicle distance and constant velocity, ii) merging, in which different vehicles or short platoons are combined in a long platoon, and iii) splitting, where a long platoon is separated into different vehicles or short platoons. Moreover, the impacts of communication delay on vehicle platoon string stability is analyzed in the review work given in (Zhang *et al.*, 2023). Within this review work, stability-related concepts for platoons are provided, such as: i) local stability, which is defined for each follower vehicle from the platoon, as the capability to maintain the same velocity or acceleration as its preceding vehicle, ii) string stability, which is defined as preventing the propagation of errors or disturbances towards the end of the platoon, and iii) traffic flow stability, which is defined as the effect on traffic flow and speed, introduced by traffic flow disturbances caused by entrances or exits from ramps.

Although vehicle platooning is not a novel approach used for safe travelling on highways, in recent years there is an increased interest within the control community, to develop control strategies suitable for a vehicle platooning application. In (Wang *et al.*, 2024), a fixed-time integral terminal sliding mode control algorithm for a vehicle platoon with longitudinal dynamics is presented. The control law is specifically designed to ensure both vehicle and string stability. In (Quyang *et al.*, 2021), a distributed controller with L_2 string stability designed for a vehicle platoon is proposed. Moreover, two communication topologies and a switching mechanism for changing the platoon leader are tested. In normal driving, the platoon follower vehicles receive information from their predecessor, whereas, in case of an emergency, all the communication is received from the first vehicle, i.e., the platoon leader. In (Zou *et al.*, 2019), an adaptive control law for controlling autonomous vehicle platoons is given. Both velocity and position

control are achieved using a self-tuning algorithm, in which the control gain varies. Thus, the gain from the adaptive control law has an initial value, which is modified in order to achieve zero tracking error. In (Xu *et al.*, 2019), a distributed platoon formation control for a homogeneous vehicle platoon with both lateral and longitudinal dynamics is presented. The algorithm combines a planning control level, which uses a multi-objective flocking algorithm and lateral stability constraints to compute the reference trajectory, which is sent to the lower control level that employs a distributed model predictive control (DMPC) algorithm, to ensure zero tracking error. In (Ploeg *et al.*, 2014), a cooperative adaptive cruise control (CACC) algorithm, using an H_∞ optimal controller is presented. The idea is to specifically design the controller to ensure the string stability for the vehicle platoon, in both simulation and experimental tests. In (Chanfreut *et al.*, 2022), a robust DMPC algorithm for reference tracking designed in the coalitional control framework is given. Using the coalitional formulation, the idea is to allow dynamic clusters of sub-systems, by switching between different communication topologies. The reference tracking DMPC algorithm is designed using the robust tube-based MPC methodology, and is tested on a 12 truck system, resulting in a maximum of 2048 communication topologies, each one defined by certain communication links between different sub-systems being enabled or disabled. In (Masero *et al.*, 2023), a robust, tube-based coalitional MPC with plug-and-play formulation is provided. The latter allows the control algorithm to be reconfigured online, when sub-systems are added or removed from the network, and was tested using a 4 truck system, in which a fifth truck was added during the simulation.

In this work, a coalitional control (CC) algorithm suitable for a vehicle platooning application is proposed. As previously mentioned, the core idea in coalitional control is to selectively cluster sets of sub-systems, which are connected with a common communication network. This means that, although all sub-systems from the network are connected to the communication network, i.e., there exists a communication link between every sub-system, not all the communication links are enabled. For the coalitional control strategy, a communication topology is specified by the communication links activated between sub-systems. There are two extreme topologies, namely, the decentralized topology, in which no communication links are activated between sub-systems, and the centralized topology, in which all communication links are activated between all sub-systems (Fele *et al.*, 2017). A vehicle platoon traveling in a longitudinal motion is a particular case of a networked system, where each sub-system is a vehicle, which must exchange information in order to fulfil the desired control specifications. As such, a coalition between vehicles, defined by a specific communication topology, means that only those vehicles within the cluster (or coalition) exchange relevant information (Maxim *et al.*, 2024). The proposed selfish approach for the coalitional control algorithm is designed to

allow each platoon vehicle to choose its own most beneficial communication topology. It results that each vehicle will periodically evaluate the performance cost criteria corresponding to each available topology, and afterwards will select the communication topology with the minimum cost.

With respect to previous works from the literature, the main contributions of this paper are as follows:

1. For each communication topology, a gain feedback controller is computed by optimizing an iterative cost function, using a large set of initial data points.
2. An algorithm for switching between available communication topologies is designed. Within the procedure, the communication topology is chosen by each agent, by periodically evaluating the performance cost functions for each communication topology.
3. Each agent selfishly chooses between the available communication topologies, and selects the communication topology, that has the minimum cost.
4. The heterogeneous platoon model is simplified, and reconfigured in a compact structure, to ensure the parametrization of the algorithm, regardless on the vehicle position in the platoon.

The remainder of this paper is structured as follows: in Section 2 the proposed coalitional control algorithm is provided, whereas the vehicle platoon model is presented in Section 3. The simulation results and discussion are given in Section 4, followed in Section 5 by the conclusions and future work ideas.

2. Coalitional Control (CC) methodology

In this section, the proposed coalitional control strategy is given. Let us introduce a networked system composed of N sub-systems coupled through the states, connected through a uni-directional communication network. Each sub-system $i = \overline{1, N}$ is described using a discrete-time, state-space model as follows:

$$\begin{cases} x_i(k+1) = A_i x_i(k) + B_{i,i} u_i(k) + B_{i,i-1} x_{i-1}(k) \\ y_i(k) = C_i x_i(k) \end{cases} \quad (1)$$

where $x_i \in \mathbb{R}$ is the state variable, $u_i \in \mathbb{R}$ is the input variable and $y_i \in \mathbb{R}$ is the output variable for sub-system $i = \overline{1, N}$; k is the discrete-time index and $x_{i-1} \in \mathbb{R}$ is the state variable corresponding to sub-system $i-1$, which represents the coupling signal between sub-systems; matrices A_i , B_i , $B_{i,i-1}$, C_i have adequate dimensions.

As previously mentioned, a coalitional control framework is defined by a dynamic communication network, which allows to design various communication topologies. A communication topology consists of different statuses for the communication links that allow the information flow between the

sub-systems. If an uni-directional communication link between sub-systems i and $i-1$ is enabled, it means that sub-system i will receive relevant information from sub-system $i-1$ (i.e., the state variable $x_{i-1}(k)$). Using this coalitional control framework, an optimal state feedback gain matrix K is computed for each communication topology. The zero values from the K matrix represent disabled links, whereas the non-zero values correspond to enabled communication links.

In this work, the coalitional control was specifically designed using a vehicle platooning application. Note that, a vehicle platoon is a particular case of a networked system, in which the coupling is only between consecutive sub-systems (or vehicles) and the communication flow is uni-directional, i.e., vehicle i , $i = \overline{2, N-1}$ sends information to vehicle $i+1$, and receives information from vehicle $i-1$. Note that, the last vehicle with index $i = N$ is only receiving information from its predecessor, but does not have a successor vehicle to send information. The first vehicle from the platoon, denoted with index $i = 1$ is called the leader vehicle and it imposes the platoon travel velocity, whereas the other platoon vehicles, denoted with indices $i = \overline{2, N}$ are called followers, and need to maintain a desired inter-vehicle distance.

Moreover, four different communication topologies are designed (illustrated in Fig. 1), and evaluated using a $N=5$ vehicle platooning application. Hence, the default communication topology is the decentralized topology, denoted with C_0 , in which no information exchange is allowed.

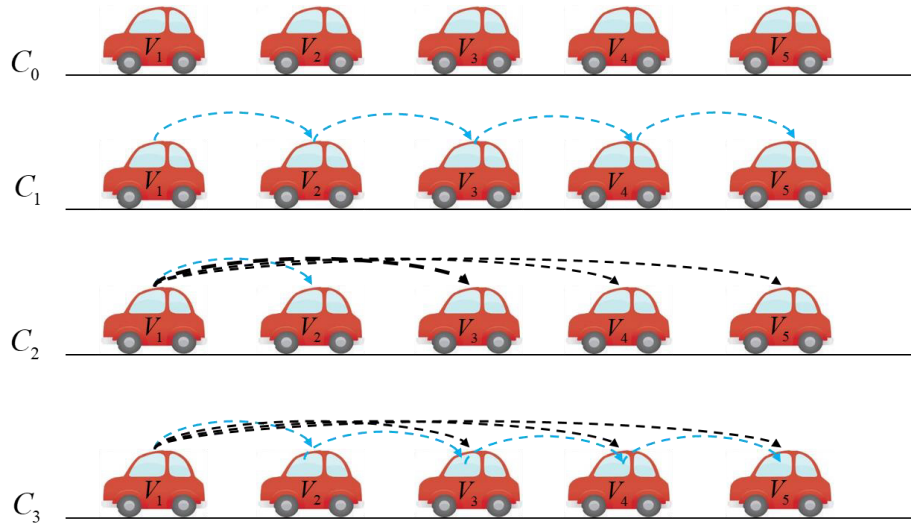


Fig. 1 – Coalitional control with four communication topologies (blue dashed arrow – information flow between consecutive vehicles, black dashed arrow – information flow from the leader vehicle).

Within the communication topology C_1 , the information is exchanged only between consecutive vehicles, and is depicted in Fig. 1 with blue dashed arrow. In the communication topology C_2 , the information exchange is uni-directional from the leader vehicle, towards each follower vehicle, and is depicted in Fig. 1 with black dashed arrow. Finally, the communication topology C_3 is a combination between topologies C_1 and C_2 , in which a follower vehicle receives relevant information from both the leader (black dashed arrow) and its predecessor (blue dashed arrow).

It is noteworthy to mention that the proposed uni-directional communication topologies are well known in the literature (Zheng *et al.*, 2017) and have the following names: C_1 is called predecessor-following topology, C_2 is called leader-following topology, whereas C_3 is called predecessor-leader following topology.

In addition to these main communication topologies which were evaluated independently, a switching mechanism is designed, which allows each platoon vehicle to select a desired communication topology, based on a minimization of a cost index criteria. This evaluation is performed periodically during the simulation, involves all the platoon vehicles and is individually executed by each vehicle. This results in a selfish coalitional control for the platoon, because, the vehicles do not reach a common agreement for a communication topology. Instead, each vehicle selfishly choses its own most beneficial topology, and enables the corresponding communication links, according to the selected topology.

2.1. Computation of the state feedback matrix

In this section, the method to compute the state feedback gain controller corresponding to each communication topology is provided.

- Communication topology C_0

Let us consider a vehicle platoon with $N=5$ vehicles. For each platoon vehicle with index i , $i = \overline{1, N}$, the following optimization problem is solved:

$$J_i(K_{i,i}) = \sum_{r_1 \in R_{ref}} \sum_{j=1}^M \left(C_i x_i(k+j)^T C_i x_i(k+j) \right) + \left(\Delta u_i(k+j)^T R_i \Delta u_i(k+j) \right), \quad (2)$$

s.t. (1),

with $u_i(k) = K_{i,i} x_i(k)$.

where r_1 is the velocity reference trajectory for the leader vehicle, R_{ref} is a set that contains several predefined reference trajectories r_1 , M is the size of the

optimization window, $\Delta u_i(k+j) = u_i(k+j) - u_i(k+j-1)$ is the increment of the control action, and R_i is the input weight matrix.

Note that, for the decentralized topology C_0 , the resulting matrix K is diagonal, and each element from the main diagonal $K_{i,i}$ is separately computed by solving the optimization problem (2). The idea is to increase the size of the platoon, one vehicle at the time, and to compute for each follower vehicle, the corresponding gain controller, considering that the controllers for all the preceding vehicles are already available. This is necessary due to the coupling between vehicles, i.e., in (1), vehicle $i=2$ receives information from vehicle $i=1$, which means that to compute the controller gain matrix $K_{2,2}$ corresponding to vehicle $i=2$, all the information pertaining to vehicle $i=1$ must be known.

Remark 1. Note that, when computing the feedback gain matrix for subsystem $i=1$ by solving the optimization problem (2), the velocity reference trajectory r_1 is used in model (1) as a coupling term for the leader vehicle, i.e., for $i=1$, $x_{i-1}(k) = r_1(k)$. This is done, because the leader vehicle, as the first vehicle in the platoon, does not have a vehicle placed in front, that it must follow, but we can envision a ‘virtual’ vehicle which travels with the velocity r_1 , which also must be followed by the platoon leader. Moreover, the same rationale goes for the follower vehicles $i = \overline{2, N}$, which do not explicitly use the velocity reference trajectory r_1 , when computing the optimal feedback gain matrix $K_{i,i}$, but remain nonetheless state coupled with their preceding vehicle, and furthermore the entire platoon is connected in an in-chain architecture.

Remark 2. The optimization problem (2) is iterative, and at each iteration within the minimization, a new reference trajectory $r_1 \in R_{ref}$ is used. The reference set is selected by the user, and must contain a sufficient number of different velocity reference trajectories, that cover the entire operating range. In this manner, the resulting K matrix is computed without the bias introduced by a certain operation point, which is more favorable for the platoon operation.

- Communication topology C_1

For each platoon vehicle with index i , $i = \overline{2, N}$, the following optimization problem is solved:

$$J_i(K_{i,i}, K_{i-1,i}) = \sum_{r_1 \in R_{ref}} \sum_{j=1}^M \left(C_i x_i(k+j)^T C_i x_i(k+j) \right) + \left(\Delta u_i(k+j)^T R_i \Delta u_i(k+j) \right), \quad (3)$$

s.t. (1),

with $u_i(k) = K_{i,i} x_i(k) + K_{i,i-1} x_{i-1}(k)$.

Note that, the difference between problem (2) and problem (3) is the

manner in which the control action is computed. The only exception is the first vehicle, which does not receive information from its predecessor, resulting that for $i=1$, problem (3) becomes problem (2). For the follower vehicles, $i = \overline{2, N}$, when computing the unknown gains $K_{i,i}$, $K_{i,i-1}$, it is assumed that all the information corresponding to the preceding vehicles is known and available, i.e., $K_{1,1}$ was already computed for the leader vehicle and is considered a known variable.

- Communication topology C_2

For each platoon vehicle with index i , $i = \overline{2, N}$, the following optimization problem is solved:

$$J_i(K_{i,i}, K_{i,1}) = \sum_{r_1 \in R_{ref}} \sum_{j=1}^M \left(C_i x_i(k+j)^T C_i x_i(k+j) \right) + \left(\Delta u_i(k+j)^T R_i \Delta u_i(k+j) \right), \quad (4)$$

s.t. (1),

with $u_i(k) = K_{i,i} x_i(k) + K_{i,1} x_1(k)$.

For this topology, all follower vehicles receive information only from the leader.

- Communication topology C_3

For each platoon vehicle with index i , $i = \overline{2, N}$, the following optimization problem is solved:

$$J_i(K_{i,i}, K_{i,1}, K_{i,i-1}) = \sum_{r_1 \in R_{ref}} \sum_{j=1}^M \left(C_i x_i(k+j)^T C_i x_i(k+j) \right) + \left(\Delta u_i(k+j)^T R_i \Delta u_i(k+j) \right), \quad (5)$$

s.t. (1),

with $u_i(k) = K_{i,i} x_i(k) + K_{i,1} x_1(k) + K_{i,i-1} x_{i-1}(k)$.

For this topology, all follower vehicles receive information from both the leader and the preceding vehicle.

Remark 3. For each communication topology, in order to compute the feedback gain matrix K , the corresponding optimization problem is solved for each follower vehicle, using the same reference set R_{ref} , predefined by the user in the initialization phase of the methodology. However, it is important to notice that the computational time required to compute the K matrix for a certain topology, increases with the number of platoon vehicles (i.e., for a platoon with $N=5$ vehicles, to compute the feedback gain matrix K , corresponding to topology C_0 , problem (1) is repeated five times, for each vehicle separately, as the size of the platoon is gradually increased). This is because, in the proposed

selfish coalitional control methodology, for each communication topology, the feedback gain matrix K is computed incrementally, one vehicle at the time, i.e., with its corresponding $K_{i,i}$ values, starting from the leader, towards the last follower.

2.2. Methodology to selfishly select the communication topology

In this section, the methodology to automatically select a communication topology for each platoon vehicle is provided. Usually, in the general coalitional control framework, at certain time instants, the switching between available communication topologies occurs. This is done by evaluating the existing communication topologies, minimizing an overall cost function (for the entire system), and selecting the communication topology which has the smallest cost (Chanfreut *et al.*, 2019). In this work, the selfish characteristic of the proposed coalitional control algorithm is revealed in the manner used to select the communication topology. The idea is to allow each vehicle from the platoon, to select its own most beneficial communication topology. This is achieved by locally minimizing a topology cost function, from each vehicle's perspective. To this end, the following topology evaluation cost function is designed:

$$V_{C_j}(C_j, x_i(k)) = \sum_{l=0}^T x_i(k+l)^T \bar{Q}_i x_i(k+l) + u_i(k+l)^T \bar{R}_i u_i(k+l) + \alpha_i |C_j|, \quad (6)$$

where $C_j, j=0,3$ is the topology evaluated, $\bar{Q}_i, \bar{R}_i$ are weight matrices for the states and inputs variables, respectively, T is the window size, α_i is the weight of the communication cost and $|C_j|$ denotes the number of communication links which are enabled in the topology.

To further clarify the procedure used to selfishly select the communication topology, Algorithm 1 is provided.

Algorithm 1
For each sampling time k
If $\frac{(k-1)}{T} \in \mathbb{N}$
For each vehicle $i = \overline{1, N}$
- Compute V_{C_0} using (6), for topology C_0 , where $u_i = K_{i,i} x_i$
- Compute V_{C_1} using (6), for topology C_1 , where $u_1 = K_{1,1} x_1, u_i(k) = K_{i,i} x_i(k) + K_{i,i-1} x_{i-1}(k), i = \overline{2, N}$
- Compute V_{C_2} using (6), for topology C_2 , where $u_1 = K_{1,1} x_1, u_i(k) = K_{i,i} x_i(k) + K_{i,1} x_1(k), i = \overline{2, N}$

- Compute V_{C_3} using (6), for topology C_3 ,
where $\begin{cases} u_1 = K_{1,1}x_1, & u_2(k) = K_{2,2}x_2(k) + K_{2,1}x_1(k), \\ u_i(k) = K_{i,i}x_i(k) + K_{i,1}x_1(k) + K_{i,i-1}x_{i-1}(k), & i = \overline{3, N} \end{cases}$
- Find the minimum value between $V_{C_0}, V_{C_1}, V_{C_2}, V_{C_3}$ and select the communication topology corresponding to the minimum cost.
end for
else
- Use the topologies previously selected.
end if
end for

Remark 4. In Algorithm 1, each communication topology $C_j, j = \overline{0, 3}$ is separately evaluated by each platoon vehicle, using the cost function (6). This is performed periodically, at regular time intervals of T sampling periods, over a T - sized time window ($\text{each}(k-1)/T \in \mathbb{N}$, where $\mathbb{N}$ is the natural numbers set), starting from the current state $x_i(k)$. Thus, each vehicle will rank the communication topologies according to the V_{C_j} values, and will use the communication topology with the smallest cost. Once the communication topology is selected, this will remain unchanged for the next T sampling periods.

Remark 5. Within the selfish coalitional control algorithm, it is possible for each vehicle to choose a different communication topology than the communication topology selected by its neighbors. If this is the case, it remains in the responsibility of each agent (controller) to open the necessary communication links according to the selected communication topology, in order to receive the coupling information, i.e., the state trajectory.

Remark 6. In the proposed algorithm, only the available communication topologies $C_j, j = \overline{0, 3}$ are evaluated. This means that platoon maneuvers such as lane change, joining the platoon or splitting from the platoon are not implemented and are subject to future research.

3. Vehicle platoon description

In this section, details regarding the vehicle platooning application used to test the proposed selfish coalitional control are provided. In this work, a heterogeneous vehicle platoon consisting of $N=5$ mobile robots is used. The identification procedure which was performed in order to determine the models for each mobile robot was taken from (Maxim *et al.*, 2023), and was extended to a heterogeneous platoon. The model for each mobile robot was identified using

the System Identification Toolbox from Matlab, based on experimental data, in which, the input signal is in the range $[0 \dots 6]$ V and represents the voltage applied to the DC rear servomotor that ensures the longitudinal motion of the mobile robot. The output signal used in the identification is in the range $[0 \dots 0.8]$ m/s and represents the velocity of the mobile robot. In Fig. 2, the experimental data used in the identification is depicted, whereas in Fig. 3, for simplicity, only the identification validation for the platoon leader is provided. The models were identified with a best fit of 77.35%, 75.84%, 77.11%, 76.83% and 77.07%, for mobile robots V_1 , V_2 , V_3 , V_4 and V_5 , respectively.

The identified model for each platoon mobile robot $i = \overline{1,5}$ is a first order transfer function described as follows:

$$G_i(s) = \frac{K_i}{\tau_i s + 1} \quad (7)$$

where K_i is the gain and τ_i is the time constant.

Thus, the platoon leader mobile robot with index $i = 1$ is modelled by:

$$V_1 : G_1(s) = \frac{K_1}{\tau_1 s + 1} = \frac{0.09828}{0.1074s + 1}, \quad (8)$$

whereas the follower mobile robots with indices $i = \overline{2,5}$ have the following models:

$$\begin{aligned} V_2 : G_2(s) &= \frac{K_2}{\tau_2 s + 1} = \frac{0.1007}{0.1011s + 1}; & V_3 : G_3(s) &= \frac{K_3}{\tau_3 s + 1} = \frac{0.1017}{0.1077s + 1}; \\ V_4 : G_4(s) &= \frac{K_4}{\tau_4 s + 1} = \frac{0.1039}{0.0994s + 1}; & V_5 : G_5(s) &= \frac{K_5}{\tau_5 s + 1} = \frac{0.1078}{0.1052s + 1}. \end{aligned} \quad (9)$$

Furthermore, in Fig. 4, the step responses for the open loop transfer functions given in (8)-(9), corresponding to each mobile robot $i = \overline{1,5}$ are provided. Note that, according to the open loop dynamics, the mobile robot with the highest amplification gain, is placed at the end of the platoon, i.e., V_5 , while the mobile robot with the smallest amplification gain is placed as the platoon leader, i.e., V_1 .

Using the first order transfer functions models (8)-(9) identified for each mobile robot, a vehicle platooning application is envisioned, in which each vehicle is a mobile robot. As previously mentioned, in a vehicle platoon, there are different control objectives, depending on the vehicle position within the formation. Hence, a platoon leader, as the first vehicle, imposes the travel velocity for the entire platoon, whereas the follower vehicles need to maintain the formation, i.e., keep a safe inter-vehicle distance.

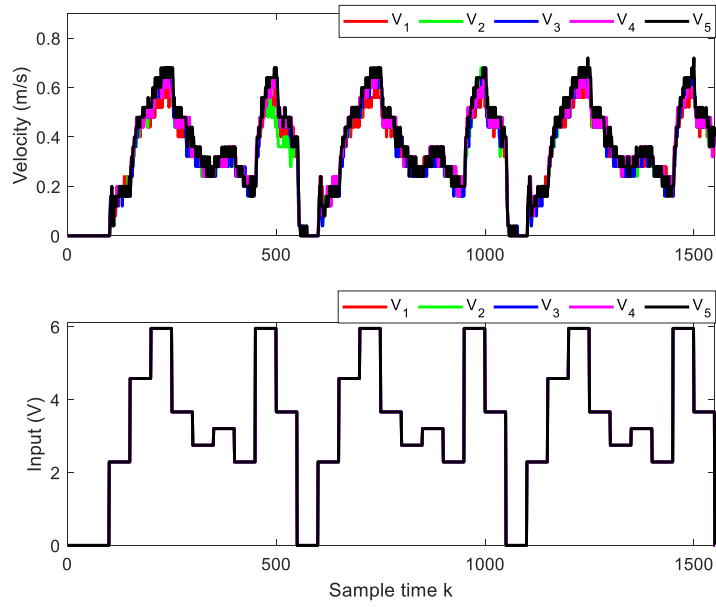


Fig. 2 – Experimental data used in the identification procedure.

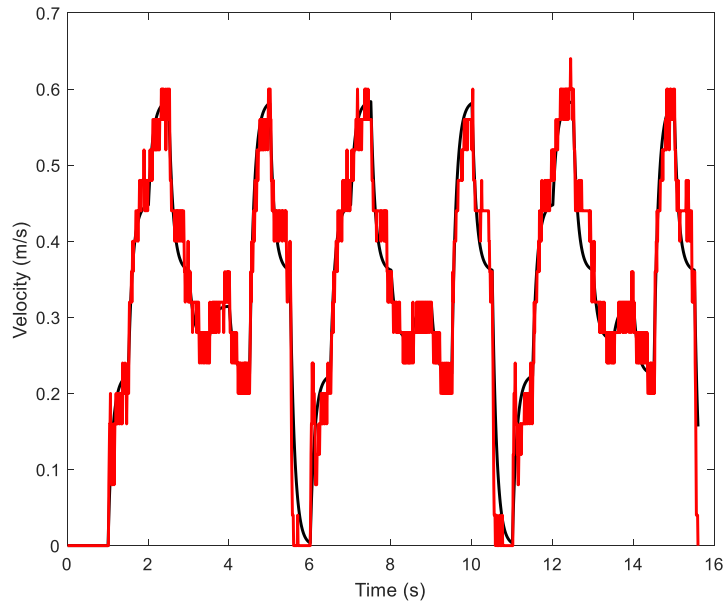


Fig. 3 – Measured (red line) and simulated model (black line) output for V_1 .

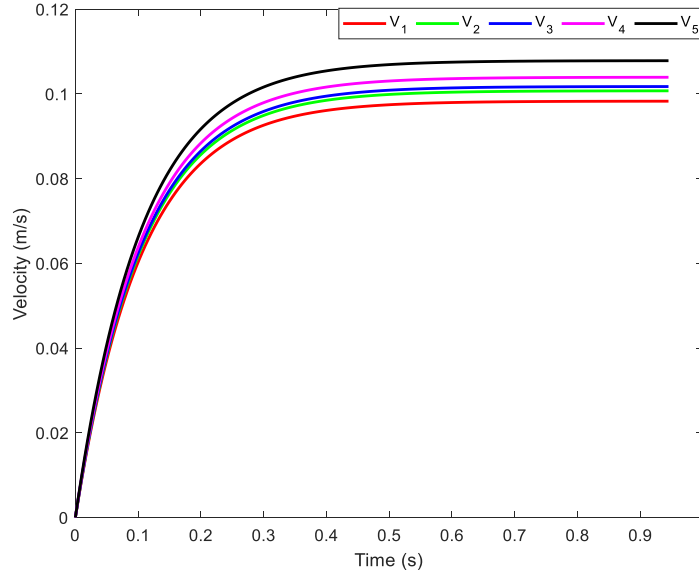


Fig. 4 – Open loop step response dynamics for the platoon mobile robots.

Let us consider a vehicle platoon, in the state-space formulation, in which each vehicle has the following continuous time, model description:

$$\begin{aligned} \begin{bmatrix} \dot{e}_{p_i} \\ \dot{v}_i \end{bmatrix} &= \underbrace{\begin{bmatrix} 0 & -1 \\ 0 & -\frac{1}{\tau_i} \end{bmatrix}}_{A_i} \underbrace{\begin{bmatrix} e_{p_i} \\ v_i \end{bmatrix}}_{x_i} + \underbrace{\begin{bmatrix} 0 \\ \frac{K_i}{\tau_i} \end{bmatrix}}_{B_{i,i}} u_i + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B_{i,i-1}} v_{i-1} \\ y_i &= \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C_i} x_i \end{aligned} \quad (10)$$

where the state vector $x_i = [e_{p_i} \quad v_i]^T$, $i = \overline{1,5}$, contains the position or headway error, computed as the difference between the inter-vehicle distance and the imposed distance reference r_i , $i = \overline{2,5}$, i.e., $e_{p_i} = p_{i-1} - p_i - r_i$, and the velocity v_i , respectively; the derivative of the headway error is computed using only the difference between the velocities of two consecutive vehicles $\dot{e}_{p_i} = v_{i-1} - v_i$ since the imposed headway reference is a constant value, selected for safety considerations; K_i , τ_i are the parameters of the first order transfer functions, with the numerical values given in (8)-(9); v_{i-1} is the velocity of the preceding vehicle; u_i is voltage applied on the servomotors that ensures the mobile robot's movement.

Note that, according to (10), each platoon vehicle is modelled equally. However, since the leader vehicle does not have a physical predecessor, in this work, we consider a ‘virtual’ vehicle that is placed in front of the platoon leader, and that imposes the velocity reference trajectory $r_1 \in R_{set}$. This means that the coupling signal v_{i-1} , $i=1$, does not come from the exterior, but is the velocity profile for the leader $v_{i-1} = r_1$, $i=1$. For the follower vehicles, the coupling signal is received via the communication network or measured with the onboard sensors, if the communication topology does not include enable links with the predecessor vehicle.

4. Simulation results and discussion

In this section, the simulation results obtained with the selfish coalitional control methodology are provided. As depicted in Fig. 1, there are four independent communication topologies or coalitions, each one with a corresponding optimal feedback gain matrix. Depending on the values of the feedback gain matrix, the implicit status of the communication links is revealed, i.e., a zero value means that the corresponding link is closed, and no information is exchanged.

The structure of the feedback gain matrix corresponding to each communication topology is a block matrix, in which each element $K_{i,i}$, $i = \overline{1,5}$ is a row vector, with the length imposed by the number of states.

- Communication topology C_0 (default case, without active communication links):

$$K = \begin{bmatrix} K_{1,1} & O & O & O & O \\ O & K_{2,2} & O & O & O \\ O & O & K_{3,3} & O & O \\ O & O & O & K_{4,4} & O \\ O & O & O & O & K_{5,5} \end{bmatrix} \quad (11)$$

where $O = [0 \ 0]$ is a zero row vector, and

$$\begin{aligned} K_{1,1} &= 1000 \times [1.440 \ -0.070], \\ K_{2,2} &= 1000 \times [6.423 \ -0.181], \quad K_{3,3} = 1000 \times [7.944 \ -0.174], \\ K_{4,4} &= 10000 \times [1.116 \ -0.140], \quad K_{5,5} = 10000 \times [1.552 \ -0.012]. \end{aligned} \quad (12)$$

- Communication topology C_1 (active communication links between consecutive vehicles):

$$K = \begin{bmatrix} K_{1,1} & O & O & O & O \\ K_{2,1} & K_{2,2} & O & O & O \\ O & K_{3,2} & K_{3,3} & O & O \\ O & O & K_{4,3} & K_{4,4} & O \\ O & O & O & K_{5,4} & K_{5,5} \end{bmatrix} \quad (13)$$

with

$$\begin{aligned} K_{1,1} &= 1000 \times [1.440 \quad -0.070], \\ K_{2,2} &= 1000 \times [6.458 \quad -0.197], \quad K_{2,1} = [35.611 \quad 43.352], \\ K_{3,3} &= 10000 \times [1.931 \quad -0.019], \quad K_{3,2} = 100 \times [0.005 \quad 1.156], \\ K_{4,4} &= 10000 \times [1.004 \quad -0.015], \quad K_{4,3} = 100 \times [0.006 \quad 1.204], \\ K_{5,5} &= 10000 \times [1.416 \quad -0.013], \quad K_{5,4} = 1000 \times [-1.119 \quad 0.132]. \end{aligned} \quad (14)$$

Note that the numerical values for the feedback gain matrices from the main diagonal, corresponding to the communication topology C_1 are different from the values given in (12), corresponding to the communication topology C_0 . This is the result of the proposed coalitional control methodology, in which the feedback gain matrix for each communication topology is computed independently, and incrementally, by increasing the length of the platoon.

- Communication topology C_2 (active communication links between the platoon's leader and each follower vehicles):

$$K = \begin{bmatrix} K_{1,1} & O & O & O & O \\ K_{2,1} & K_{2,2} & O & O & O \\ K_{3,1} & O & K_{3,3} & O & O \\ K_{4,1} & O & O & K_{4,4} & O \\ K_{5,1} & O & O & O & K_{5,5} \end{bmatrix} \quad (15)$$

with

$$\begin{aligned} K_{1,1} &= 1000 \times [1.440 \quad -0.070], \\ K_{2,2} &= 1000 \times [6.458 \quad -0.197], \quad K_{2,1} = [35.611 \quad 43.352], \\ K_{3,3} &= 10000 \times [1.931 \quad -0.019], \quad K_{3,1} = [1.008 \quad 1.167], \\ K_{4,4} &= 10000 \times [1.004 \quad -0.014], \quad K_{4,1} = [1.075 \quad 3.301], \\ K_{5,5} &= 10000 \times [1.552 \quad -0.007], \quad K_{5,1} = [1.000 \quad 1.025]. \end{aligned} \quad (16)$$

- Communication topology C_3 (a combination between C_1 and C_2 , with active communication links between a follower vehicle, its predecessor and the platoon leader):

$$K = \begin{bmatrix} K_{1,1} & O & O & O & O \\ K_{2,1} & K_{2,2} & O & O & O \\ K_{3,1} & K_{3,2} & K_{3,3} & O & O \\ K_{4,1} & O & K_{4,2} & K_{4,4} & O \\ K_{5,1} & O & O & K_{5,4} & K_{5,5} \end{bmatrix} \quad (17)$$

with

$$\begin{aligned} K_{1,1} &= 1000 \times [1.440 \quad -0.070], \\ K_{2,2} &= 1000 \times [6.458 \quad -0.197], K_{2,1} = [35.611 \quad 43.352], \\ K_{3,3} &= 1000 \times [7.945 \quad -0.146], \\ K_{3,1} &= [3.773 \quad 47.220], K_{3,2} = [1.883 \quad 37.697], \\ K_{4,4} &= 10000 \times [1.116 \quad -0.018], \\ K_{4,1} &= 100 \times [0.279 \quad 1.158], K_{4,3} = 100 \times [0.015 \quad 2.235], \\ K_{5,5} &= 10000 \times [1.552 \quad -0.012], \\ K_{5,1} &= [1.289 \quad 8.951], K_{5,4} = [0.948 \quad 10.245]. \end{aligned} \quad (18)$$

For each communication topology, the numerical values for the optimal feedback gain matrices, were obtained by solving the cost functions (2)-(5), with the following optimization parameters: $R_1 = 2$, $R_2 = 2$, $R_3 = 2$, $R_4 = 2$, and $R_5 = 1$ are the input weights, R_{ref} is the reference set, which contains 50 reference trajectories, as depicted in Fig. 5. Note that, the input weights parameters were empirically chosen, after multiple trials, so that each cost function reaches a feasible solution (2)-(5). Moreover, for each communication topology, the overall feedback gain matrix K was incrementally computed, one element at the time, starting with the leader vehicle. After the calculation of the feedback gain matrix $K_{1,1}$ corresponding to the leader vehicle, the platoon was increase with a follower vehicle. For the newly increased platoon, only the unknown feedback gain matrix corresponding to the follower was computed, considering that element $K_{1,1}$ for the leader is known. Following this reasoning, when computing the feedback gain matrix for the last vehicle from the platoon, the cost function is differently weighted, with a smaller input weight value.

Note that the numerical values of the optimal feedback gain matrices are influenced by the platoon model. As depicted in Fig. 2, the mobile robots are physically quite similar, in size and characteristics, which resulted a heterogeneous platoon, with similar specifications, as described by the numerical transfer functions models (8)-(9). However, the choice of the platoon model influences the proposed coalitional algorithm, in the sense that the numerical values for the main diagonal of the feedback gain matrices are alike. For other platoon models, the algorithm will result in different matrices, and without losing the generality of the algorithm, for larger platoon sizes, the K matrices must be redesigned accordingly.

The numerical values for the parameters from Algorithm 1 are: the sampling period $T_s = 0.01$ s, the window size $T = 50$, the weight matrices $\bar{Q}_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\bar{R}_i = 0$, $i = \overline{1,5}$, the weight of the communication cost $\alpha_i = 0$ for each communication topology. Note that if $\alpha_i \neq 0$, then the communication cost of each topology is also taken into consideration, when choosing the communication topology for each vehicle.

Each communication topology was independently tested in simulation, using one of the velocity reference profiles for the leader vehicle, given in set R_{ref} . The simulation results obtained with the communication topologies C_0, C_1, C_2, C_3 are depicted in Figs. 6, 7, 8 and 9, respectively. Moreover, the switching procedure given in Algorithm 1, which enables each vehicle to selfishly choose its own communication topology was also tested and the results are provided in Fig. 10, with the coalition selection corresponding to each platoon vehicle given in Fig. 11.

Note that, from the communication standard perspective, all four proposed communication topologies can be organized using dedicated short-range communications (DSRC) technology (Lazar *et al.*, 2023).

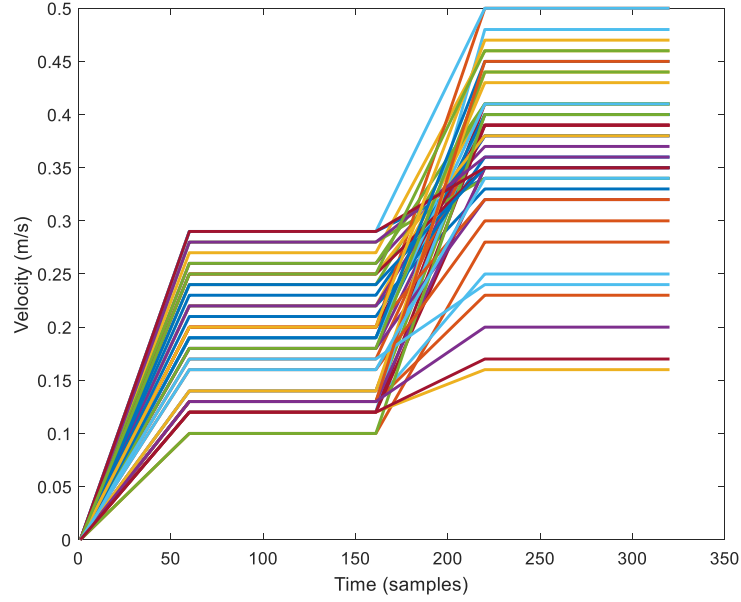


Fig. 5 – Velocity reference trajectories for the leader vehicle from set R_{ref} .

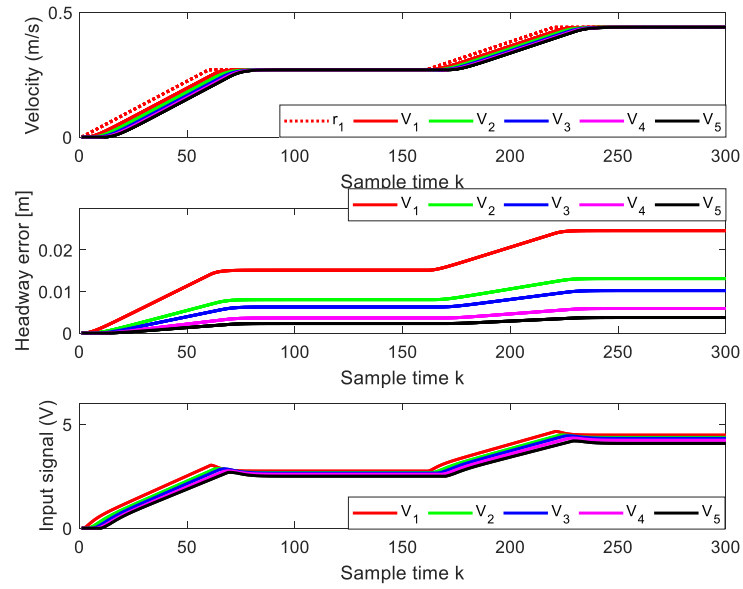
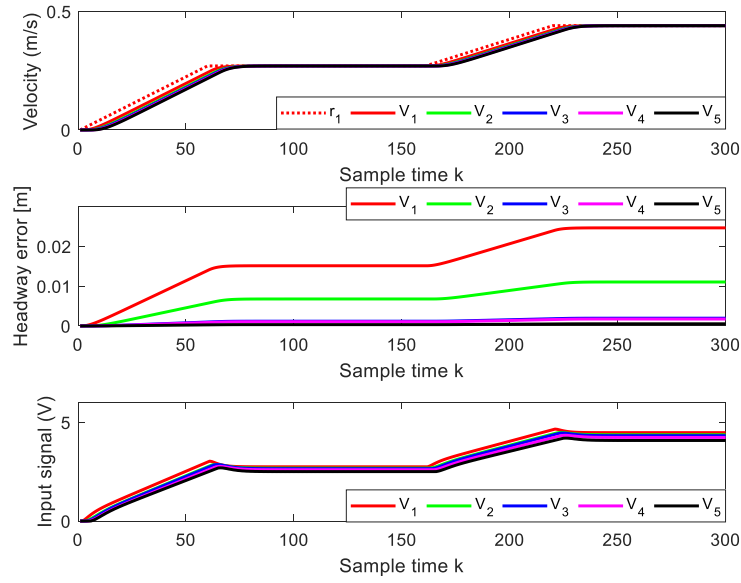
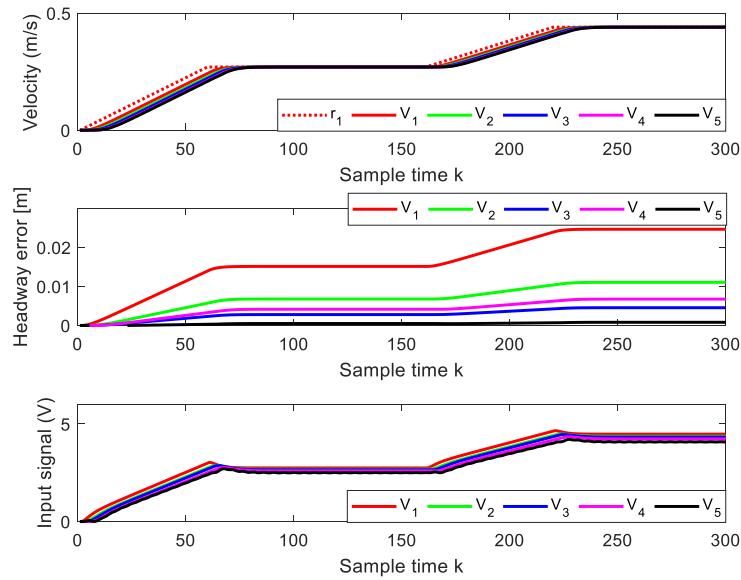


Fig. 6 – Simulation results obtained with the communication topology C_0 .

Fig. 7 – Simulation results obtained with the communication topology C_1 .Fig. 8 – Simulation results obtained with the communication topology C_2 .

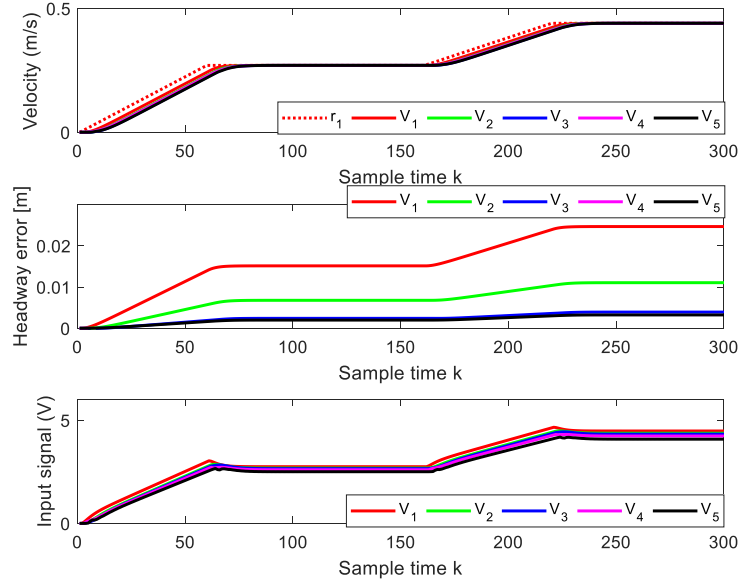


Fig. 9 – Simulation results obtained with the communication topology C_3 .

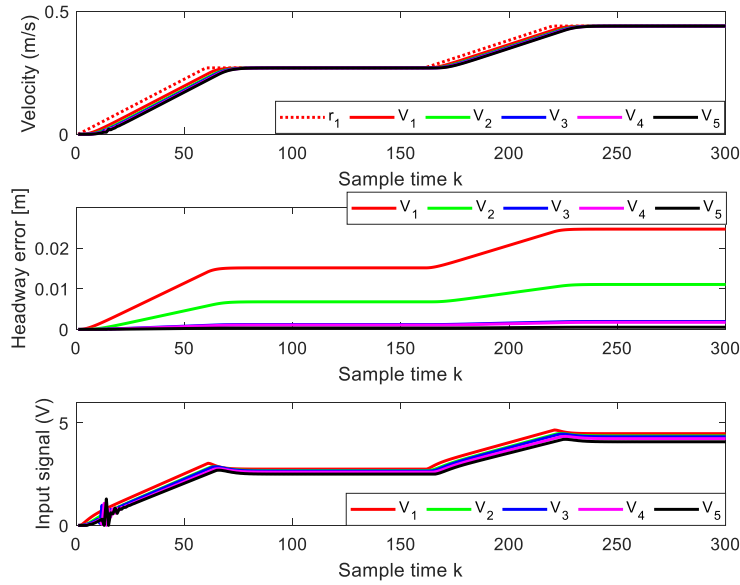


Fig. 10 – Simulation results obtained with the Algorithm 1 (selfish coalitional control methodology).

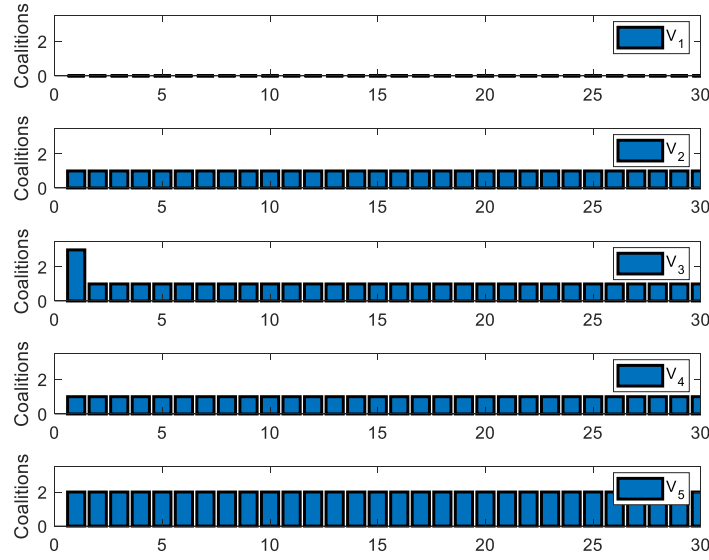


Fig. 11 – Coalition selection for each platoon vehicle using Algorithm 1.

For each simulation results, (given in Figs. 6, 7, 8, 9, 10), the upper subplot provides the velocity trajectories obtained for each platoon vehicle. The reference trajectory for the leader vehicle, is imposed by the ‘virtual’ vehicle placed in front of the platoon, and has the following profile: for the first 60 sample times, the velocity reference r_1 linearly increases from 0 to 0.3 m/s; after that, r_1 is maintained constant at the value 0.3 m/s for 100 sample times, followed by a constant increase to value 0.44 m/s, for the next 60 sample times; finally, r_1 is constant at the value of 0.44 m/s, until the end of the experiment. The headway errors for each platoon vehicle, including for the leader vehicle, which was computed with respect to the velocity reference r_1 are depicted in the middle subplot. The control efforts for each platoon vehicle are provided in the lower subplot. Note that, in Fig. 11, the coalition selection for each platoon vehicle, is depicted with a bar plot, which has a different value, at every $T=10T_s$ sample periods. However, only vehicle V_2 , changes the selected coalition, from C_3 which was active for the first $T=10T_s$ samples to coalition C_1 for the next sample periods. The remaining vehicles, selfishly select only a coalition, (or communication topology) for the entire simulation experiment, i.e., V_1 selects C_0 , V_2 selects C_1 , V_3 selects C_3 and C_1 , V_4 selects C_1 and V_5 selects C_2 , as depicted in Fig. 11.

In order to numerically evaluate the performances of each communication topology used separately, together with the performance achieved with the selfish switching procedure, the Mean Squared Error (MSE) for the velocity variable and mean value (MEAN) for the headway error and input signal indices were computed as follows:

$$\begin{aligned} MSE_{v_i} &= \frac{1}{M} \sum_{k=1}^M (r_i(k) - v_i(k)), \quad i = \overline{1,5}, \\ MEAN_{e_{p_i}} &= \frac{1}{M} \sum_{k=1}^M (e_{p_i}(k)), \quad i = \overline{1,5}, \\ MEAN_{u_i} &= \frac{1}{M} \sum_{k=1}^M (u_i(k)), \quad i = \overline{1,5}, \end{aligned} \quad (19)$$

where r_i is the imposed velocity reference trajectory from set R_{set} , whereas $r_i = v_{i-1}$, $i = \overline{2,5}$ for the follower vehicles, which need to maintain the velocity of their preceding vehicle. M denotes the size of the entire simulation, which in all the simulations is 320 sampling periods.

The numerical values for the performance indices given in (19) are provided in Tables 1, 2 and 3. The results show that, as expected, the leader vehicle has the same performance, regardless of the communication topology, because the control action is computed only using the feedback gain matrix $K_{1,1}$. Moreover, the proposed selfish coalitional control methodology ensures also the string stability of the platoon, i.e., the error values decrease towards the end of the platoon. This is visible in both Tables 1 and 2, where each follower vehicle has a smaller error value than its predecessor, and vehicle V_5 has the smallest MSE value per communication topology.

Table 1

MSE_{v_i} performance indices for the proposed communication topologies.

All values are multiplied with $\times 10^{-3}$

Topology/Vehicle	V ₁	V ₂	V ₃	V ₄	V ₅
C_0	0.1878	0.0602	0.0394	0.0169	0.0091
C_1	0.1878	0.0449	0.0045	0.0040	0.0020
C_2	0.1878	0.0449	0.0117	0.0206	0.0025
C_3	0.1878	0.0449	0.0096	0.0034	0.0075
<i>Algorithm 1</i>	0.1878	0.0449	0.0045	0.0042	0.0023

Table 2*MEAN_{e_{pi}} performance indices for the proposed communication topologies**All values are multiplied with $\times 10^{-3}$*

Topology/Vehicle	V ₁	V ₂	V ₃	V ₄	V ₅
C_0	0.3440	0.0945	0.0565	0.0187	0.0075
C_1	0.3440	0.0669	0.0021	0.0016	0.0002
C_2	0.3440	0.0669	0.0113	0.0246	0.0003
C_3	0.3440	0.0669	0.0083	0.0190	0.0057
Algorithm 1	0.3440	0.0669	0.0021	0.0016	0.0002

Table 3*MEAN_{u_i} performance indices for the proposed communication topologies*

Topology/Vehicle	V ₁	V ₂	V ₃	V ₄	V ₅
C_0	3.2644	3.1288	3.0683	2.9683	2.8507
C_1	3.2644	3.1352	3.1001	3.0120	2.9021
C_2	3.2644	3.1352	3.0922	2.9891	2.8793
C_3	3.2644	3.1352	3.0940	3.0289	2.9106
Algorithm 1	3.2644	3.1352	3.1001	3.0120	2.9021

The overall MSE performance indices, for the entire platoon, for each simulation test are the following:

$$MSE_v^{C_0} = \sum MSE_{v_i} = 0.3135 \times 10^{-3}, MSE_v^{C_1} = 0.2432 \times 10^{-3},$$

$$MSE_v^{C_2} = 0.2676 \times 10^{-3}, MSE_v^{C_3} = 0.2533, MSE_v^{Alg_1} = 0.2439 \times 10^{-3}.$$

The overall $MEAN_{e_p}$ and $MEAN_{u_i}$ indices have the following values numerical values:

$$MEAN_{e_p}^{C_0} = 0.5213 \times 10^{-3}, MEAN_{e_p}^{C_1} = 0.4148 \times 10^{-3},$$

$$MEAN_{e_p}^{C_2} = 0.4472 \times 10^{-3}, MEAN_{e_p}^{C_3} = 0.4439 \times 10^{-3}, MEAN_{e_p}^{Alg_1} = 0.4148.$$

Finally, the overall MEAN index values are:

$$MEAN_u^{C_0} = 15.2805, MEAN_u^{C_1} = 15.4138, MEAN_u^{C_2} = 15.3601,$$

$$MEAN_u^{C_3} = 15.4331, MEAN_u^{Alg_1} = 15.4138.$$

The overall performance evaluation of the entire platoon shows that the results obtained with the proposed selfish coalitional control algorithm given in Algorithm 1 have the same performance as the results obtained in the simulation test using only the communication topology C_1 for all platoon vehicles. This is

expected, since in Algorithm 1, when given the opportunity to choose their own communication topology, 3 out of 5 vehicles, selfishly and independently choose C_1 . Following this reasoning, using another velocity trajectory profile can result that in Algorithm 1, other coalitions will be selected by the vehicles, which will change the overall numerical performance evaluation. This investigation is the subject of the ongoing research related to the proposed selfish coalitional control methodology.

This numerical evaluation of the proposed selfish coalitional control methodology also reflects the results provided in Figs. 6-10, in which, visually, all communication topologies ensure the string stability of the platoon. Moreover, the hard input constraints imposed for the voltage applied on mobile robots servomotors, i.e., $0V < u_i \leq 6V$, are also fulfilled (see the lower subplot from the simulation results figures).

5. Conclusions

In this work, a vehicle platooning control solution, in the coalitional control framework was provided. Different from the standard coalitional control methodology, the proposed approach is selfish in both the controller design and the simulation validation. In the standard coalitional control, the coalitions (or communication topologies, with enabled and disabled communication links) are designed taking into account the overall system model, composed by aggregating all the sub-systems models. This results in a controller that is computed for the entire system. The selfish attitude towards the coalitional control promoted in this work, is shown when the optimal feedback gain corresponding to each communication topology is designed incrementally, from each sub-system perspective.

To further clarify, for the proposed vehicle platooning application, consisting of multiple sub-systems or vehicles in a chain architecture, the optimal feedback gain matrix was firstly computed for the leader vehicle. Next, the first follower was added in the platoon, and its gain matrix was also computed, considering that all the information for the leader is known. And so on, the platoon, was incrementally increased until the overall optimal gain feedback for the entire platoon, was obtained. Moreover, after the communication topologies were designed, i.e., the corresponding optimal state feedback gain matrices were computed, the control performance was separately evaluated using a five vehicle platoon, with heterogeneous vehicles models. Furthermore, the switching between the available communication topologies was also selfishly designed, so that, each vehicle has the opportunity to select its own communication topology, by minimizing a performance criteria applied for all the communication topologies.

Hence the main advantage of the proposed selfish coalitional solution is the flexibility of the selection of the communication topology implemented in the algorithm. In this work we design four main communication topologies, and in the standard (non-selfish) coalitional approach would result that the platoon can only chose and activate at one time, only one of these four communication topologies options. In the proposed selfish algorithm, each vehicle decides for oneself, which results that at one time, more communication topologies can be simultaneously chosen, thus increasing the control options.

The simulation results shown that when given the opportunity, each vehicle will chose its own most beneficial communication topology, which increases the overall platoon performance, that was achieved by using a single communication topology for each platoon vehicle.

Future work will focus on increasing the number of available communication topologies and will test the proposed selfish coalitional control in an experimental setup, consisting of a platoon of mobile robots.

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ALGORITM DE CONTROL COALIȚIONAL PENTRU O APLICAȚIE DE TIP PLUTON DE VEHICULE – O ABORDARE EGOISTĂ

(Rezumat)

În zilele noastre, drumurile publice devin mai aglomerate ca niciodată, odată cu creșterea numărului de vehicule. Ca atare, necesitatea îmbunătățirii fluxului de trafic, menținând în același timp confortul pasagerilor, este primordială. În acest context, un pluton de vehicule, definit ca un grup alcătuit din vehicule diferite, care se deplasează cu aceeași viteză și mențin o distanță prescrisă între vehicule, este considerat de comunitatea de control ca fiind o soluție de control acceptată pentru o deplasare eficientă. Această lucrare oferă un algoritm egoist de control coalițional, potrivit pentru o aplicație de tip pluton de vehicule, care este un caz particular al unui sistem în rețea compus din mai multe sub-sisteme (sau vehicule), care partajează informații printr-o rețea de comunicații. Ideea principală din spatele controlului coalițional este de a proiecta o rețea dinamică de comunicații, în care legăturile de comunicare dintre agenți (care controlează fiecare

vehicul) sunt activate sau dezactivate, în funcție de topologia de comunicare selectată. Mai mult, se consideră o abordare egoistă, în care fiecărui agent i se permite să comute între topologiile de comunicare și să își aleagă una proprie, i.e., cea mai benefică, asigurând în același timp optimalitatea întregului pluton de vehicule. Algoritmul de control a fost testat în simulare, pe un pluton eterogen de cinci vehicule, iar rezultatele arată eficiența algoritmului propus.