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MULTIPLE ALGEBRAIC FUNCTION NONLINEAR SYSTEMS

BY

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Abstract. In order to obtain a more complex dynamic behaviour for a non-linear system several methods were used, including higher order design, system parameters variation and connecting analogue and discrete subsystems. The present paper analyses an approach to generate wide bandwidth signals by connecting three elementary algebraic building blocks to obtain a more complex non-linear function to be included in the analogue dynamic system. The resulting non-linear system attractor is presented, its Poincare section is analysed and statistical numerical simulations results are shown to highlight the possible application as noise generator.

Keywords: non-linear systems, analogue circuits, chaotic systems, noise generators.

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1. Introduction

Complex time evolution of chaotic systems present research interest for applications in true random number generation (Nejati, 2012; Callegari, 2007), noise generation (Endo, 2007), spread spectrum modulating systems (Sambas, 2015), and chaotic encryption modules (Andreatos, 2014). Most nonlinear systems presented in the literature use relatively simple algebraic building blocks. The analogue noise generator proposed in this paper is composed of two building blocks: one linear dynamic and the other nonlinear algebraic. The chosen sub-systems of the proposed nonlinear system are designed starting from an analog nonlinear system, first introduced in (Grigoraş and Grigoraş, 2015) exhibiting chaotic behavior. The main contribution of this research is the complex algebraic building block, composed of three simpler functions: saturation, sine and two's complement residue. For parameter choice, bifurcation diagrams were performed and examples are presented. Simulation results performing both dynamic and statistical analysis confirm the possible use of the designed four-order analog-discrete nonlinear system for noise generation.

2. Non-linear System Design

The proposed analogue system, intended to generate unpredictable non-periodic signals, is designed starting from a four-order dynamic prototype connected to an algebraic nonlinear building block.

The Lure topology used in the design of the proposed analog system is suggested in Fig. 1, being composed on a feedback loop with a linear dynamic system and a nonlinear algebraic function.

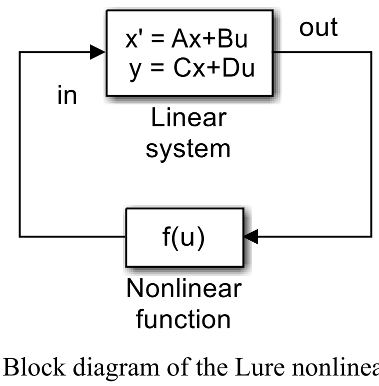

Fig. 1 – Block diagram of the Lure nonlinear system.

The linear part of the Lure feedback loop is designed starting from the state equations (1), first introduced in (Grigoraş and Grigoraş, 2017):

$$\begin{cases} \dfrac{dx}{dt} = -a \cdot x(t) + \omega_0 \cdot y(t) - z(t) \\ \dfrac{dy}{dt} = -\omega_0 \cdot x(t) + z(t) \\ \dfrac{dz}{dt} = -x(t) - y(t) - \omega_0 \cdot z(t) + In(t) \\ \dfrac{dv}{dt} = b \cdot x(t) - c \cdot v(t) + d \cdot In(t) \end{cases} . \tag{1}$$
$$Out(t) = v(t)$$

In equation (1) *x(t), y(t), z(t)* and *v(t)* are the state variables of the linear dynamic subsystem, *v(t)* is considered as its output signal, ω_0 is the free oscillating frequency, *a* parameter is its dampening factor, the *b*, *c* and *d* coefficients are internal coupling gains.

The resulting linear subsystem leads to the order four implementation block diagram, depicted in Fig. 2, highlighting the dependence of the dynamic behavior on the five constant gains implementing the state equations coefficients.

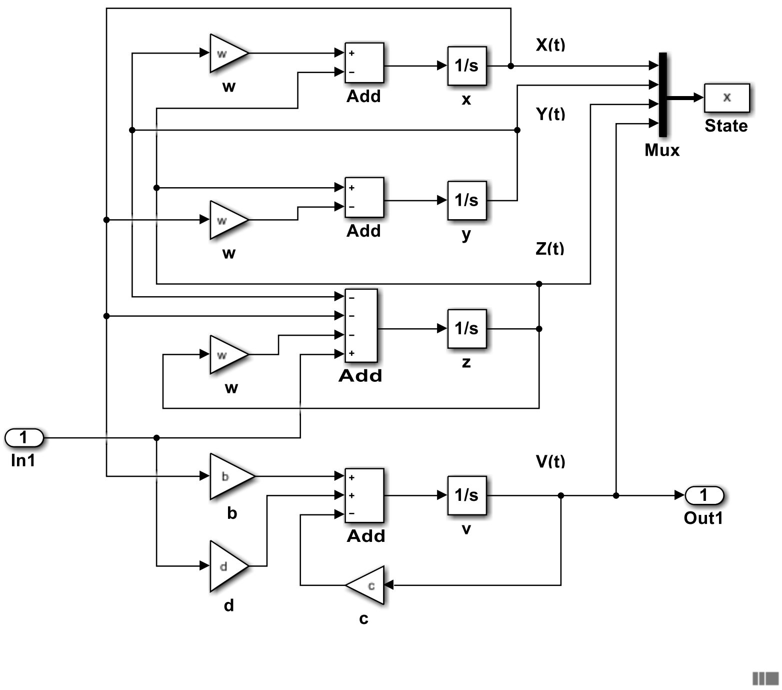

Fig. 2 – Block diagram of the linear subsystem.

The characteristic part in the design of the proposed system is given by the feedback nonlinear algebraic building block, composed by the parallel topology based on three elementary functions. The chosen functions are: a limited increasing function, *sat*(*u*), a periodic one, sin(*u*) and a discontinuous residue function, *res*(*u*).

$$f(\mathrm{u}) = sat(u) + \sin(u) + res(u) \tag{2}$$

In equation (2), the saturation amplifier part, *sat*(*u*), is a unity gain given by relation (3) and leading to the characteristic in Fig. 3:

$$sat(u) = \begin{cases} 1 & u > 1 \\ u & -1 < u < 1 \\ -1 & u < -1 \end{cases} \tag{3}$$

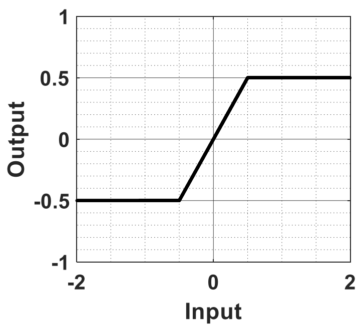

Fig. 3 – Graphical representation of the sat(u) nonlinear function characteristic.

The periodic function, *sin*(*u*), is given by equation (4) and graphically shown in Fig. 4.

$$\sin(u) = Amp \cdot \mathrm{sine}(w \cdot u) \quad Amp = 1 \quad w = 1 \tag{4}$$

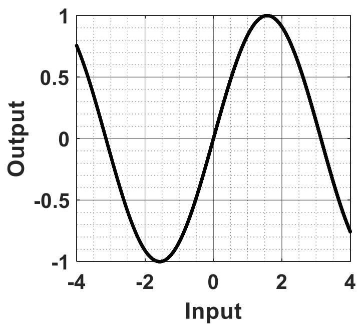

Fig. 4 – The perioic sin(u) nonlinear function characteristic.

The third nonlinear algebraic function, $res(\cdot)$ is given by the 2's complement overflow behavior, as denoted in equation (5) and shown in Fig. 5.

$$res(u) = u - round(u) \tag{5}$$

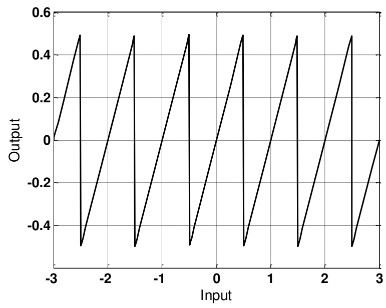

Fig. 5 – Residue nonlinear function characteristic.

The resulting block diagram used in the algebraic nonlinear function implementation is shown in Fig. 6.

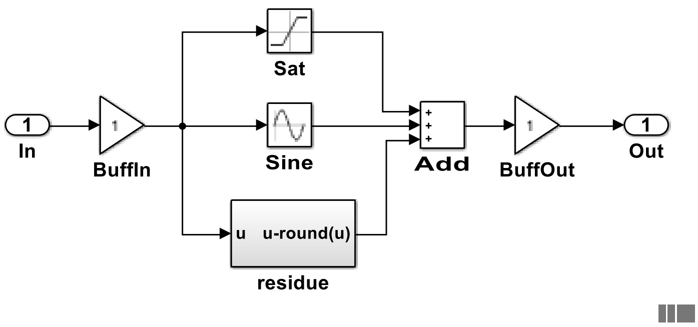

Fig. 6 – Composed algebraic function schematic.

As presented in Fig. 7, the input-output graph of the designed algebraic function is more complex than the elementary building block functions.

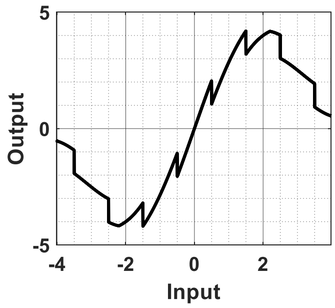

Fig. 7 – Composed algebraic function input-output characteristic.

Combining the dynamic linear and algebraic nonlinear subsystems in the Lure feedback loop connection, gives us the state equations (6):

$$\begin{cases} \dfrac{dx}{dt} = -a \cdot x(t) + \omega_0 \cdot y(t) - z(t) \\ \dfrac{dy}{dt} = -\omega_0 \cdot x(t) + z(t) \\ \dfrac{dz}{dt} = -x(t) - y(t) - \omega_0 \cdot z(t) + f\left(v(t)\right) \\ \dfrac{dv}{dt} = b \cdot x(t) - c \cdot v(t) + d \cdot f\left(v(t)\right) \end{cases} \tag{6}$$

The resulting system is implemented in a complete block diagram presented in Fig. 8.

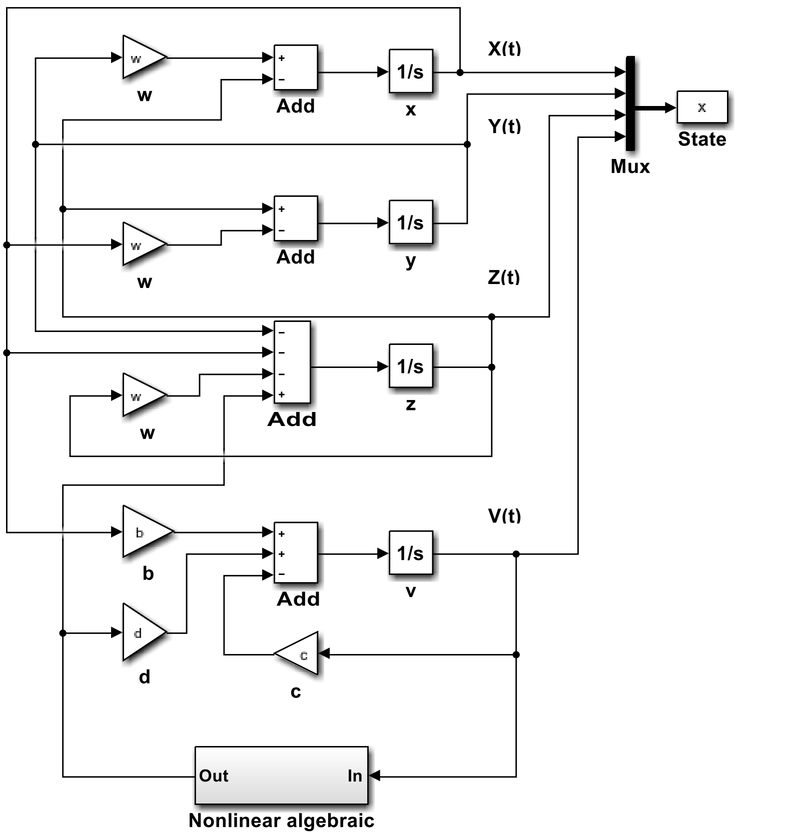

Fig. 8 – Nonlinear system block diagram.

For the resulting analogue non-linear system, the dissipative behaviour can be demonstrated analytically by computing the divergence of the state equations (6). We denote with $F(\mathbf{x})$ the state transition vector function, with the state vector argument, resulting from the state equation of the analysed system (6):

$$F(\mathbf{x}) = \begin{pmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ f_3(\mathbf{x}) \\ f_4(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} -a \cdot x(t) + \omega_0 \cdot y(t) - z(t) \\ -\omega_0 \cdot x(t) + z(t) \\ -x(t) - y(t) - \omega_0 \cdot z(t) + f\left(v(t)\right) \\ b \cdot x(t) - c \cdot v(t) + d \cdot f\left(v(t)\right) \end{pmatrix} \tag{7}$$

The calculated relation of the system divergence highlights that, for the chosen coefficients, used also in the following simulations - $\omega_0 = 2.2$, $a = 1.3$ and b, c, d in equation (9), the resulting value is constant and is strictly negative, -10.4, leading to the dissipative conclusion.

$$\begin{gathered} \nabla F(x) = \frac{df_1}{dx} + \frac{df_2}{dy} + \frac{df_3}{dz} + \frac{df_4}{dv} = \\ -a + 0 - \omega_0 - c + d \cdot \frac{df}{dv} = -a - \omega_0 - c + 3d < 0 \end{gathered} \tag{8}$$

The dynamical time evolution of the proposed non-linear system state vector is analysed by simulation. The obtained results highlight both ergodicity and sensitivity properties, confirming the chaotic behaviour of the proposed structure.

In order to verify the possibility of noise generation use, statistical analysis is also performed on the system state variables.

3. Simulation Results

To justifiably chose the state equations coefficients, bifurcation diagrams were performed. For example, choosing $\omega_0 = 2.2$, and $a = 1.3$, the 'b', 'c' and 'd' parameters were modified with a slow speed, compared to the system dynamical behavior, leading to the bifurcation diagrams in Fig. 9. The parameter variation is 30 to 35 for the c parameter and 8 to 9 for the d one, during the simulation time, T = 10^6 / ω_0. Using the state variable $y(t)$ to obtain the Poincare sampling moments, the x(t), z(t) and v(t) stare variables were sampled to obtain the $x[k]$, $z[k]$ and $v[k]$ state variables of the discrete counterpart system having the order three.

The obtained graphical representation for the three discrete state variables at the three parameters variation highlights chaotic behavior for a large value range. The partial range of values depicted, show the system coefficients that were chosen in the following simulations. For the '*b*' parameter the represented value range of 0.7*10^5 to 1.4*10^5, for the '*c*' parameter in the interval 30 to 35 and 8 to 9 range for the '*d*' one, we chose reference values:

$$b = 1.1 \cdot 10^5; \quad c = 33; \quad \mathrm{d} = 8.7. \tag{9}$$

The highlighted nonlinear dynamical behavior resulting from the parametric analysis performed for all system coefficients is complex enough to allow noise generation as a regular application of the proposed system.

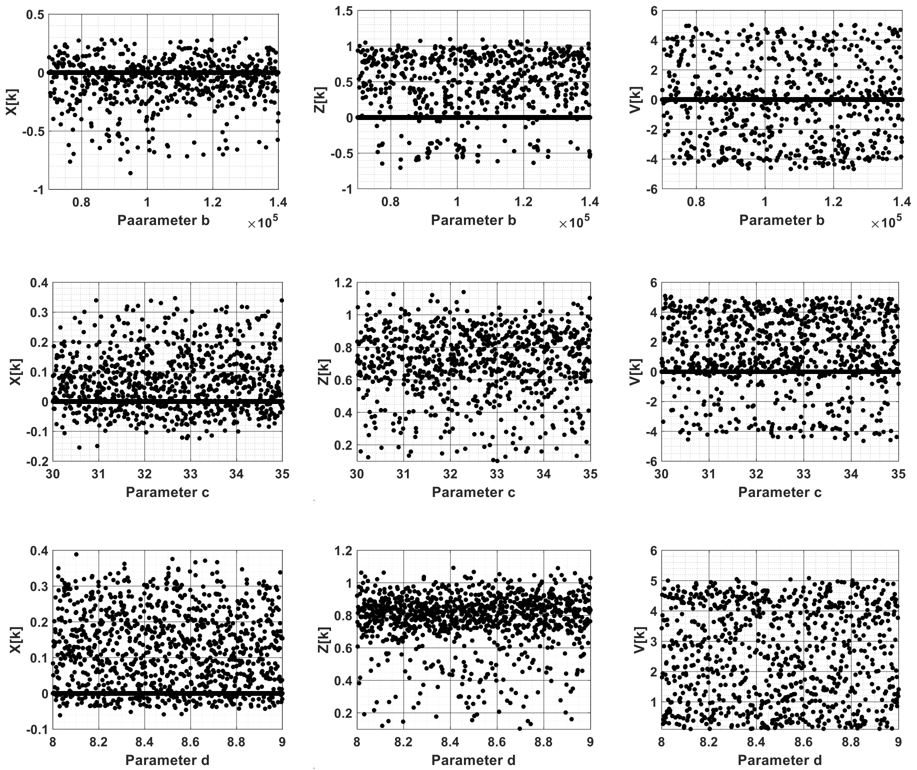

Fig. 9 – Variation of the state variables x[k], z[k] and v[k], for coefficients 'b', 'c' and 'd'.

Simulations performed for the proposed system having the coefficient values chosen (9) from the bifurcation diagrams, gave time evolution of the analogue state variables similar to the ones shown in Fig. 10. The fact that the state variables of the four-order system are non-periodic and unpredictable is evidently shown in the graphical representation, zoomed to 100 seconds for better dynamics view. The same phenomena are visible even for simulations made for time periods 10000 to 100000 times longer. What we could also notice as results of such simulations is the fact that the initial condition for system start do not influence the signal shapes. This aspect is evident also in the statistic simulations shown later on.

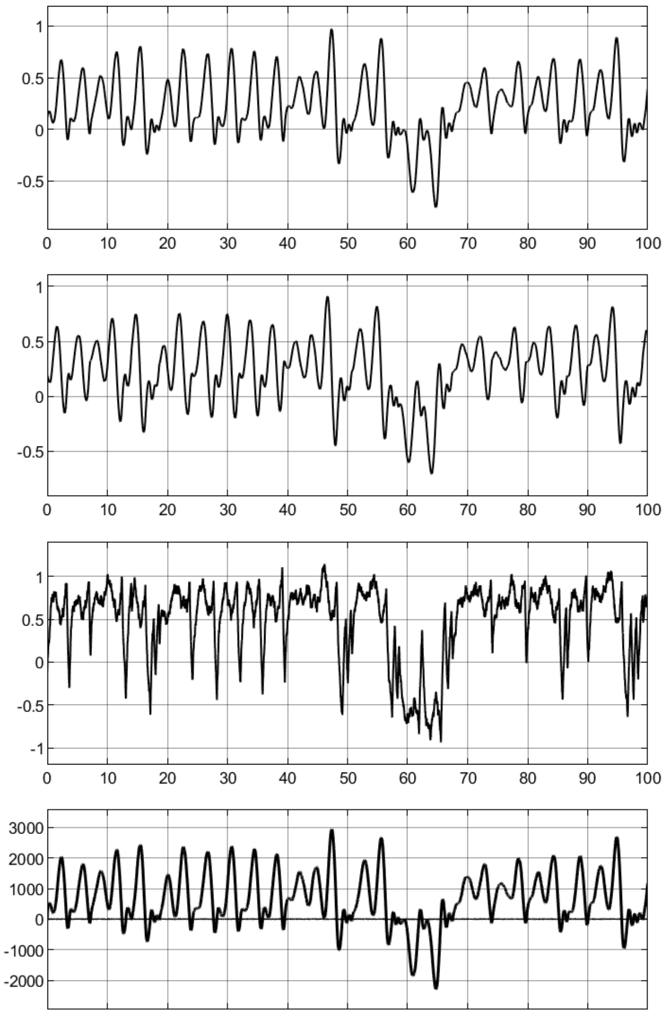

Fig. 10 – Time evolution of the system state variables.

The analysis of the dynamic behaviour of the order four non-linear system, led to a strange attractor with three-dimensional projections depicted in Fig.11. The graphical representations of the simulation results highlight the topological transitivity tendency of the analogue system by the tendency of the state vector to densely fill an area of the state space.

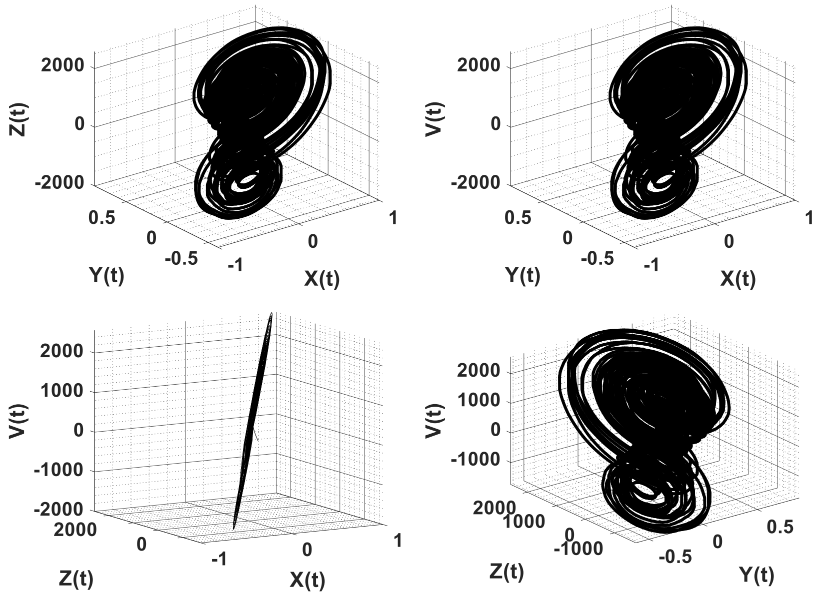

Fig. 11 – Three-dimensional projections of the system attractor for the state variables indicate on axes.

The complexity of the chaotic behaviour of the non-linear system is also visible in Poincare sections, as seen in the example in Fig. 12.

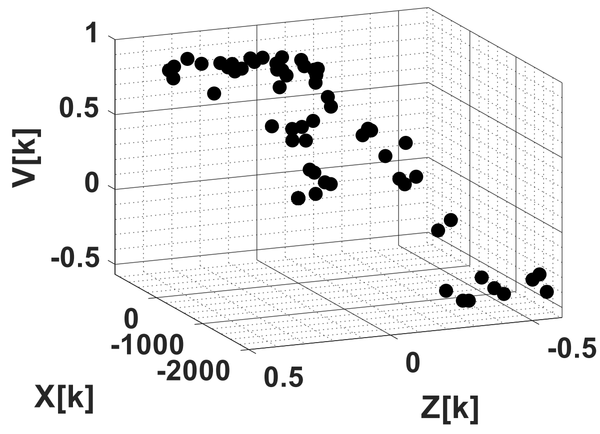

Fig. 12 – Graphical representation of the Poincare section of the nonlinear system attractor.

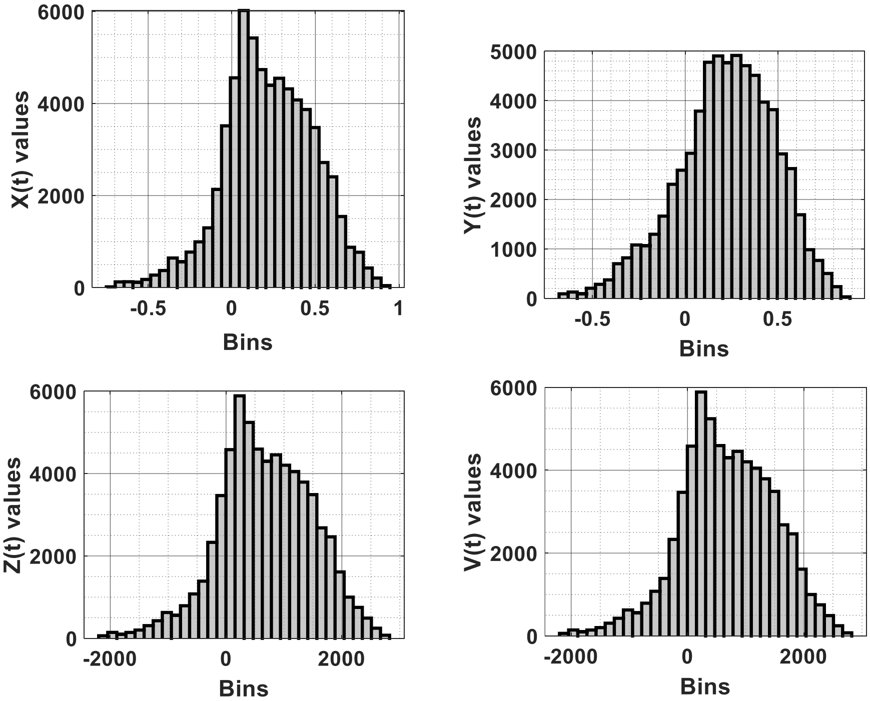

Fig. 13 – State variables histograms in order: Xa, Ya Xd, Yd.

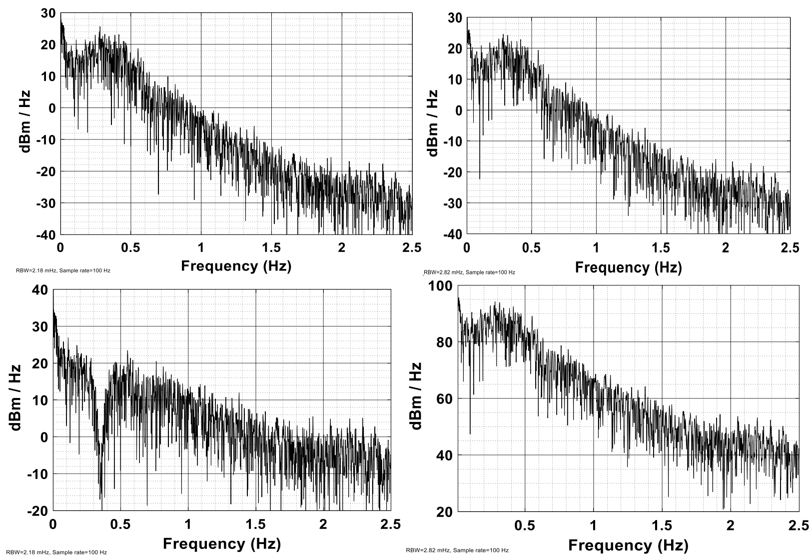

Fig. 14 – State variables power spectra: X(w), Y(w), Z(w) and V(w).

To highlight the possibility of noise generation, statistical analysis of the time evolution of the state variables was also performed. For instance, first-order statistic results, presented in Fig. 13, show that analog state variables, x and especially y are the best choice for approximately Gaussian re-partition of the probability values in the case of noise generation.

Second-order statistic examples are given in the power spectra $X(\omega)$, $Y(\omega)$, $Z(\omega)$ and $V(\omega)$ graphs depicted in Fig. 14 for the analog state variables, $x(t)$, $y(t)$, $z(t)$ and $v(t)$. The approximately uniform low-frequency power re-partition, in the zero to 0.5 Hz frequency band, is useful for noise generation. Due to the relatively constant low frequency power spectra, the frequency bandwidth is better estimated by the ratio of the highest frequency of significant power value to the minimum one, that is approximately zero, leading to a ratio tending to infinity. The fact that the spectrum extends to very low frequencies instead of higher ones suggests a narrow time extension of the state variables auto-correlation functions of the state variables.

4. Conclusions

This paper presents the influence of combining several elementary algebraic functions in the dynamics of an analog nonlinear system. We developed an order four nonlinear system aiming at applications of noise generation, starting from a Lure feedback loop. The proposed design is based on the feedback interconnection of a linear sub-system connected to the developed complex algebraic function. The behavior of the resulting system is unpredictable and generates wide frequency band signals, as presented in the power spectrum analysis (Fig. 14). Simulation performed on the nonlinear system confirmed both dynamically and statistically its complex behavior useful for noise generation.

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SISTEME NELINIARE CU FUNCȚIE ALGEBRICĂ MULTIPLĂ

(Rezumat)

Pentru a obține o comportare dinamică mai complexă a unui sistem neliniar, au fost folosite mai multe metode, inclusiv proiectarea de ordin superior, variația parametrilor sistemului și interconectarea subsistemelor analogice și discrete. Lucrarea de față analizează o abordare de a genera semnale de bandă largă prin conectarea a trei blocuri constructive algebrice pentru a obține o funcție algebrică mai complicată, care să fie introdusă în sistemul dinamic neliniar. Se prezintă atractorul sistemului rezultat, este analizată secțiunea Poincare și sunt arătate rezultatele statistice ale simulărilor.