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SIMULATION OF THE ELECTROMAGNETIC WAVES PROPAGATION IN CONDUCTIVE MEDIA USING ANALYTICAL FORMULAE

BY

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Abstract. The paper aims to investigate the electromagnetic waves behavior in conductive media using analytical formulae. An example of a very long vertical conductive wall of given thickness placed in front of a vertically polarized electromagnetic wave is considered. The variation of the electromagnetic field quantities inside the conductive wall are detailed using analytic formulae. The numerical example considered was aimed to easily understand the important properties of the field behavior inside the conductive wall. The main conclusions end the paper.

Keywords: direct and inverse waves, wave shapes, skin depth, attenuation.

1. Introduction

The continuous increase of the electric and electronic devices usage lead to their miniaturization in the context of the increase of the working

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frequencies. Moreover, many of these devices are nowadays connected to the internet via wireless systems so that electromagnetic waves are present more and more in houses, office buildings, public locations. In this circumstance, the accurate work of these devices in the presence of external signal waves is critically important. To assure this, the protection of the sensible electronic circuits against unintended electromagnetic radiated interferences becomes compulsory. As is well-known, the suppression of unintended electromagnetic radiation is done using shielding devices (Gao, 2015; Orfanidis, 2016; Tirkey and Gupta, 2018). To correctly design and use such a device, the electromagnetic phenomena inside a shielding wall must be well understood. In this light, the paper aims to detail the behaviour of the electromagnetic field quantities inside conductive walls by using analytical formulae. It must be outlined that the usage of analytic formulae gives users flexibility for rapid change of any of the parameters and thus to better understand the sensitivity of the global design with respect to these parameters for a wide range of frequencies (Watanabe, 2017; Weng *et al.*, 2022; Opris and Munteanu, 2023).

2. Problem formulation

It is considered the problem presented in Fig. 1.

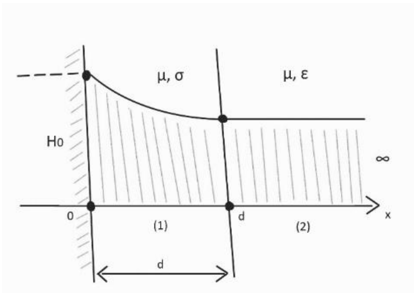


Fig. 1 – The problem formulation.

A vertically polarized sinusoidal electromagnetic wave coming from a long distance through a (μ, ε) dielectric space hits a vertical conductive wall at $x = 0$. The effective value of the magnetic field is known, namely H_0, and the wave frequency f is known as well. Both the electric and magnetic field components are tangent to the conductive wall at $x = 0$, in the plane $y0z$. The (μ, σ) conductive space has a thickness noted with d. At the opposite side of the conductive wall ($x = d$), another (μ, ε) dielectric space exists extended to infinity. The problem is how the magnetic and electric fields components behave inside the conductive wall $x \in (0, d)$ (Simion and Maghiar, 1981; Orfanidis, 2016).

Considering the wave refractions / reflections at $x = 0$ and $x = d$ planes respectively, one can find the expression of the magnetic field distribution inside the conductive media as (Purcar and Munteanu, 2020; Hortopan, 1998):

$$H_1(x) = H_0 \frac{Z_1\, Cosh[\gamma(x-d)] - Z_2\, Sinh[\gamma(x-d)]}{Z_1\, Cosh[\gamma d] + Z_2\, Sinh[\gamma d]} \quad (1)$$

where $\gamma = \sqrt{\pi f \mu \sigma}(1+j)$ is the propagation constant in conductive media and:

$$Z_1 = \frac{\alpha}{\sigma}(1+j)$$
$$Z_2 = \sqrt{\frac{\mu}{\varepsilon}} \quad (2)$$

are the conductive and dielectric media impedances respectively.

The direct and inverse components of the magnetic field inside the conductive media are determined as (Purcar and Munteanu, 2020):

$$H_{1d}(\mathrm{x}) = \frac{0.5\, H_0\, (Z_1+Z_2)}{Z_1\, Cosh[\gamma d] + Z_2\, Sinh[\gamma d]} e^{-\gamma(x-d)} \quad (3)$$

$$H_{1i}(\mathrm{x}) = \frac{0.5\, H_0\, (Z_1-Z_2)}{Z_1\, Cosh[\gamma d] + Z_2\, Sinh[\gamma d]} e^{\gamma(x-d)} \quad (4)$$

The direct and inverse components of the electric field inside the conductive media can be found afterwards as:

$$E_{1d}(\mathrm{x}) = Z_1 H_{1d}(x)$$
$$E_{1\mathrm{i}}(\mathrm{x}) = -Z_1 H_{1\mathrm{i}}(x) \quad (5)$$

The sinusoidal time-varying field components can be found later as:

$$H_{dsin}(x,t) = H_{1dmax}(\mathrm{x}) \sin[\varphi + Arg[H_{1d}(\mathrm{x})]] \quad (6)$$

$$H_{isin}(x,t) = H_{1imax}(\mathrm{x}) \sin[\varphi + Arg[H_{1i}(\mathrm{x})]] \quad (7)$$

where $H_{1dmax}(\mathrm{x}) = Abs[H_{1d}(\mathrm{x})]\sqrt{2}$ and $H_{1imax}(\mathrm{x}) = Abs[H_{1\mathrm{i}}(\mathrm{x})]\sqrt{2}$.

Similar formulae can be deducted for the electric field components:

$$E_{dsin}(x,t) = E_{1dmax}(\mathrm{x}) \sin[\varphi + Arg[E_{1d}(\mathrm{x})]] \quad (8)$$

$$E_{isin}(x,t) = E_{1imax}(\mathrm{x}) \sin[\varphi + Arg[E_{1i}(\mathrm{x})]] \quad (9)$$

where $E_{1dmax}(\mathrm{x}) = Abs[E_{1d}(\mathrm{x})]\sqrt{2}$ and $E_{1imax}(\mathrm{x}) = Abs[E_{1\mathrm{i}}(\mathrm{x})]\sqrt{2}$.

In equations (6) ÷ (9) $\varphi = \omega t$ is the time moment considered in the graphical representations.

3. Numerical example

For the numerical example one consider an electromagnetic wave vertically polarized with $H(t) = H_0\sqrt{2}\sin\omega t$ where $H_0 = 10^5\ ^{\text{A}}/_{\text{m}}\ at\ x = 0$ (it corresponds to $B_0 = 125.6$ mT value of the magnetic induction in vacuum) and the wave frequency is set to $f = 5$ kHz.

It is considered that the conductive media is made of copper ($\mu_0, \sigma = 5.5\ 10^7\ ^1/_{\Omega\text{m}}$) while the dielectric media is vacuum (ε_0, μ_0).

The above-mentioned numerical data lead to a skin depth in the conductive media $\delta = 0.96$ mm while the wavelength is $\lambda = 6.03$ mm and the wave speed in the conductive media is $v = 30.15\ ^{\text{m}}/_{\text{s}}$.

The attenuation of the electromagnetic wave in conductive media can be computed with the formula (Purcar and Munteanu, 2020):

$$Attn = Abs(Cosh[\gamma d] + \frac{Z_2}{Z_1}\ Sinh[\gamma d]) \qquad (10)$$

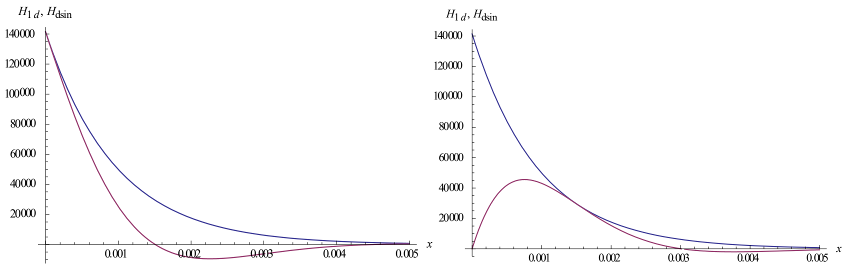
Considering the thickness of the conductive media $d = 5$ mm the attenuation is found to be $Attn = 1.287\ 10^9$.

In order to analyze the variation of the electric and magnetic field components of the electromagnetic wave inside the conductive media using the analytical formulae (1) ÷ (9), the Wolfram Mathematica™ software was used.

One must mention that in all graphic representations presented below, the measurement units considered are: *x* position in meters [m]; magnetic field intensity *H* in Amper/meter [A/m]; electric field intensity *E* in Volt/meter [V/m].

In Fig. 2 there are presented, with blue lines, the magnitude of the magnetic field distribution $H_{1d}(x)$ inside the conductive media using relation (3) and two cases for the direct magnetic field wave $H_{dsin}(x,t)$ (red line) using (6) and corresponding to two different time moments, $\varphi = 0.5\,\pi$ and $\varphi = \pi$ respectively.

As one can notice from Fig. 2, the direct magnetic field wave along the *0x* axis is at any moment tangent to the $H_{1d}(x)$ magnetic field graphic which consists in fact in an asymptotic behavior. This fact can be better visualized if the Animation function is further used in Mathematica™.



$\varphi = 0.5\,\pi$ $\varphi = \pi$

Fig. 2 – The direct magnetic field wave for two different time moments, where H_{1d}, H_{dsin} are in [A/m] and the *0x* coordinate is in [m].

The inverse magnetic field wave representation using (7) for $\varphi = 0.5\,\pi$ can be seen in Fig. 3. As expected, the field values are significantly smaller than in the case of the direct magnetic field wave. The wave is coming from the end of the conductive media at $d = 0.005$ m back to the $x = 0$ starting location.

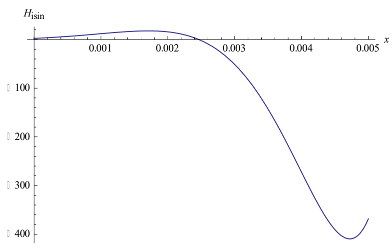
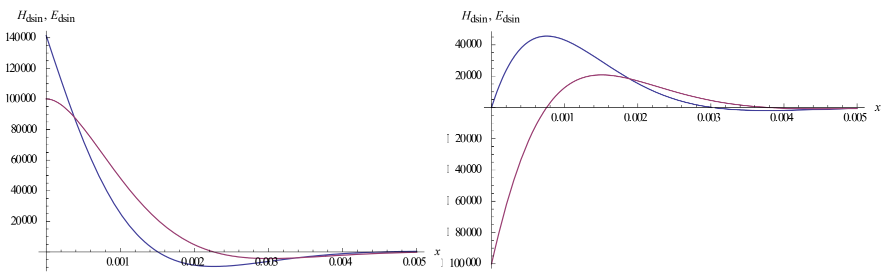


Fig. 3 – The inverse magnetic field wave in the conductive media, where H_{isin} is in [A/m] and the *0x* coordinate is in [m].

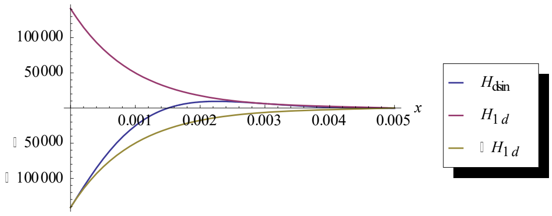
Similar analysis can be done for the electric field distribution using relations (8) – (9). Figure 4 shows a comparison between the direct magnetic field wave (blue) and the direct electric field wave (red) along the 0x axis inside the conductive media, considering two different time moments.



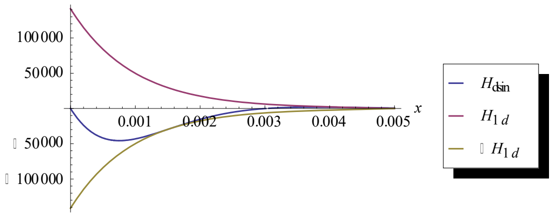
$\varphi = 0.5\,\pi$ $\varphi = \pi$

Fig. 4 – The magnetic and electric direct field waves distribution in conductive media, where H_{dsin} is in [A/m], E_{dsin} is in [V/m] scaled with the numerical value of the conductive impedance and the *0x* coordinate is in [m].

One must outline that in all graphic comparisons between the direct magnetic and electric field wave distributions, the electric field values are scaled with the numerical value of the conductive impedance, in order to have comparable figures in the graphic representations.

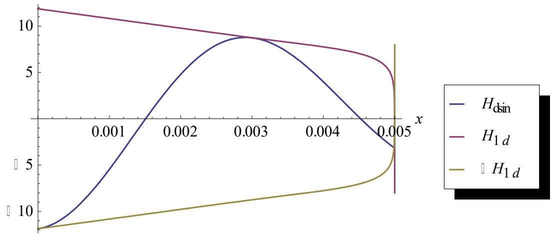


$\varphi = 1.5\,\pi$

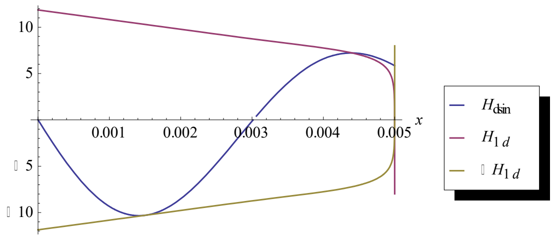


$\varphi = 2\pi$

(a) LogLinear Plot of the direct magnetic field wave



$\varphi = 1.5\,\pi$



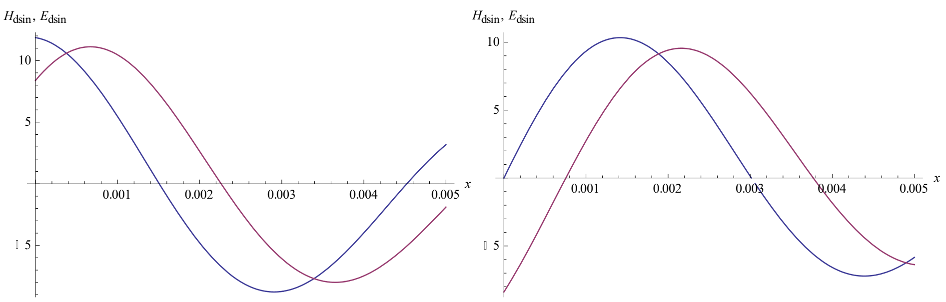
$\varphi = 2\pi$

(b) Plot Log of the direct magnetic field wave

Fig. 5 – Representations of the direct magnetic field wave along *0x* axis for two different time moments using different logarithmic scales, with *H* values in [A/m] and the *0x* coordinate in [m].

As can be noticed from Fig. 2 to Fig. 4 graphics, since the numerical values vary on a very large scale, the usage of logarithmic representation helps for the proper visualization of the wave's behavior. Thus, Fig. 5 proposes two types of representation: first, the usage of the LogLinearPlot function in Mathematica™ that is applied to the direct magnetic field wave along the 0x axis for two different time moments, while in the second case one has the plot of the logarithmic expression of the direct magnetic field wave along the 0x axis, again for two different time moments considered. In both cases, the asymptotic limits are given by the positive and negative magnitude values of the $H_{1d}(x)$ magnetic field distribution in the conductive media.

As it can be seen from Fig. 5, the plot of the logarithmic value of the direct magnetic field wave distribution along the 0x axis seems to be the most appropriate representation to follow the phenomenon inside the conductive media. Therefore, in Fig. 6 there are presented both the variation of the direct magnetic field (blue) and the direct electric field (red) waves on logarithmic scale, again for two different time moments. Moreover, from Fig. 6 one can easily notice the π/4 phase difference between the magnetic and electric field waves.



$\varphi = 0.5\,\pi$ $\qquad$ $\varphi = \pi$

Fig. 6 – Representations of the direct magnetic field (blue) and direct electric field (red) waves along 0x axis for two different time moments using logarithmic scale, where H_{dsin} is in [A/m], E_{dsin} is in [V/m] scaled with the numerical value of the conductive impedance and the *0x* coordinate is in [m].

The further usage of animation in Mathematica™ is recommended for a qualitative visualization of the (H_{dsin}, E_{dsin}) waves propagation inside the conductive media.

4. Conclusions

The paper outlines the electromagnetic wave field quantities distribution inside a conductive wall using analytical formulae. Using analytic computation, one better understands how the electromagnetic field is distributed inside the conductive media, the wave shapes and the related computation values. The example proposed was aimed at efficiently presenting the main ideas and conclusions. Of course, other input data e.g. working frequency, materials used, wall thickness, and so, can be further considered using the proposed analytic formulae and the results will be interpreted accordingly in a similar way.

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SIMULAREA PROPAGĂRII UNDELOR ELECTROMAGNETICE ÎN MEDIUL CONDUCTOR UTILIZÂND FORMULE ANALITICE

(Rezumat)

Lucrarea propune investigarea comportamentului undelor electromagnetice în mediul conductor utilizând formule analitice. Este considerat exemplul unui perete vertical conductor foarte lung de grosime dată amplasat în calea propagării unei unde electromagnetice polarizată vertical. Comportarea mărimilor electromagnetice în interiorul mediului conductor este detaliată prin intermediul formulelor analitice. Exemplul numeric considerat a fost ales astfel încât sa poată fi ușor urmărite proprietățile esențiale ale atenuării câmpului electromagnetic în interiorul peretelui conductor. Concluziile principale încheie lucrarea.