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Application of functional analysis in dendrometry using five-year growth of selected dendrometric traits of scots pine (*Pinus sylvestris* L.)

Bogna Zawieja[1], Katarzyna Kaźmierczak[2], Laura Slebioda[1]

[1] Poznań University of Life Sciences, Faculty of Agronomy and Bioengineering, Department of Mathematical and Statistical Methods, Wojska Polskiego 28, 60-637 Poznań, Poland,
e-mail: bogna13@up.poznan.pl
[2] Poznań University of Life Sciences, Faculty of Forestry, Department of Forest Management, Wojska Polskiego 28, 60-637 Poznań, Poland

SUMMARY

The differentiation between age classes of Scots pine (*Pinus sylvestris* L.) was analyzed with regard to the five-year increment of seven traits: height growth (zh_5), diameter growth at breast height (zd_5), cross-sectional area growth at breast height (zg_5), volume growth (zv_5), volume growth intensity coefficient (i_5), and slenderness (s). Measurements were made in five periods for 24-year-old trees and six periods for 33-year-old trees, all growing in fresh mixed coniferous forest sites. Repeated measures data analysis was conducted separately for all traits. Multivariable functional data analysis (FDA) was proposed to compare age classes of trees. The functional variables which resulted from this analysis can be used, as data, in many analyses (designate functions representing each of trees, FPCA – functional principal component analysis, FLDC – discriminant analysis, permutation analysis of variance). The results of the above analyses revealed significant differences between age groups. Furthermore the functions and FPCA were used to detect outliers. This procedure had not previously been used for such a purpose. FPCA explained 55% of the total variance, with the first two components clearly separating the groups. The study showed that 33-year-old trees exhibit stable growth, while 24-year-old trees show greater variability, highlighting the impact of age on growth dynamics. Permutation analysis of variance confirmed significant growth differences between the groups. The findings highlight the importance of age as a factor influencing tree growth and demonstrate the effectiveness of the multivariable FDA approach for analyzing such data.

Key words: age group, forest productivity, functional analysis, multivariable data, functional data, FPCA, permutation ANOVA.

1. Introduction

Each year, trees expand in size. They grow taller and thicker at various points along their trunks, which ultimately leads to an increase in their overall volume. This growth occurs during the vegetative season through annual increments and is influenced by numerous factors (Lian et al., 2021). These include species characteristics, tree age, site potential, weather conditions in the current and preceding years, as well as silvicultural practices applied to the stand. The growth of specific traits varies among trees within the same stand, and the rate of growth determines the development of these traits (Bongers et al., 2021). Key increments in tree growth include the current volume increment and its components: height increment, diameter increment at breast height, and cross-sectional area increment at breast height. Additional important factors related to volume growth include the volume growth intensity coefficient and slenderness ratio. The volume increment of a stand, which is essentially the sum of the volume increments of individual trees within it, is considered in forest management as a critical indicator of the stand's productive potential. It also serves as a reliable measure of the stand's response to silvicultural treatments. Consequently, determining volume increment is one of the fundamental tasks in forest management. Research into the dynamics of this process contributes to improving the accuracy of these calculations. However, the variability in volume increments among trees within a stand complicates this task. A significant breakthrough in refining methods for estimating current volume increments was the introduction by Borowski (1954) of the concept of volume growth intensity. This is defined as the ratio of the current periodic volume increment of the tree trunk excluding bark to the cross-sectional area in the bark at the end of the growth period.

Extensive research on this concept formed the basis for the labor-intensive method of determining current stand volume increment proposed by Borowski (1954, 1958, 1961a, b, 1964). Studies on volume growth intensity have predominantly focused on pine stands (Borowski 1954, 1961a, b, 1971; Dudek 1969, 1994, 2000; Lemke 1971, 1984), with limited research in a single spruce stand in the Węgierska Górka Forest District (Dudzińska, Wirowski 1995), and oak stands studied by Remi (1981). Borowski's research (1961a, 1974) revealed significant variability in volume growth intensity, with an average coefficient of variation of 30.5% for 10-year growth periods and 33.0% for 5-year periods across the studied stands. In an oak stand, Remi (1981) observed coefficients of variation of 33.0% and 35.4% for 10-year and 5-year growth periods, respectively. A less labor-intensive method with comparable accuracy was developed by Bruchwald (1971). In his studies, Borowski (1971) examined the variability of and relationships between volume growth intensity and tree dimensions, creating volume growth tables. After adjustments (Borowski, Rekosz 1974; Bruchwald, Michalak 1984), these tables can be applied regardless of stand age. Another approach for determining current volume increments in pine stands is Dudek's (1994) "Volume Growth Tables for Pine," implemented as a computer program. Given the importance of volume growth for forest planning and updating stand volume data in the SILP system, there is a pressing need to develop new procedures for estimating stand volume increments. According to Bruchwald (1999), the trial-tree method proposed by Borowski (1954) remains one of the most accurate techniques for assessing volume growth. However, its application in forestry practice has been limited by labor intensity, the need for extensive data collection and storage for each stand, and concerns about measurement accuracy (Nieuwenhuis and Barrett, 2002). Direct measurement of volume increment and other growth traits in individual trees can enhance understanding of the relationships between these traits and their influence on overall tree volume increment, improving the accuracy of stand-level estimates. Volume growth intensity also serves as an indicator of tree growth dynamics and provides an objective measure of trees' growth responses to external factors. The strong dependence of volume growth intensity on a tree's social position within the stand highlights the importance of comparing growth energy among trees within the same Kraft class (Borowski 1974; Lemke 1984). Dominant trees exhibit the highest growth intensity, with average values decreasing in successive growth periods in proportion to the tree's biosocial position.

Another significant characteristic is tree slenderness, defined as the ratio of tree height (in meters) to its diameter at breast height (in centimeters). This metric is closely associated with volume growth, as an increase in slenderness typically results in a decline in growth traits (Kaźmierczak, 2013). Slenderness is a synthetic measure commonly used for evaluating stand stability, particularly against windthrow (Bruchwald, Dmyterko, 2010, 2012; Burschel, Huss, 1997; Zajączkowski, 1991). According to Zajączkowski (1991), Scots pine is relatively resistant to damage risk. Research by Burschel and Huss (1997) indicates that a slenderness value below 1 m/cm renders a stand unstable. Orzeł (2007) conducted studies on the slenderness of various tree species, including Scots pine, in the Niepołomice Forest. Detailed analyses were carried out by Rymer-Dudzińska (1992a, 1992b), revealing relationships between slenderness and factors such as tree age, average diameter at breast height, stand height, site quality, and density. Furthermore, studies in pure pine stands in the Zielonka Forest demonstrated a correlation between slenderness and biosocial class (Kaźmierczak, 2012).

Studies on the slenderness of Scots pine have shown that this metric slenderness decreased with increasing tree age. This reduction occurred alongside increases in average diameter and stand height, while slenderness tended to rise with improvements in site quality and greater stand density (Rymer-Dudzińska, 1992b). Orzeł (2007) demonstrated that Scots pine exhibited lower slenderness compared to deciduous species but similar slenderness to European larch. Research by Kaźmierczak et al. (2011) indicated that larch slenderness depends on age and the tree's biosocial position, while site fertility had no detectable effect. Similar patterns were observed for other tree species. For example, the slenderness of ash trees in fresh forest stands was most strongly influenced by age and height class (Turczański, Kaźmierczak, 2018), while for beech, slenderness was determined by average diameter, age, height, and bark thickness (Rymer-Dudzińska, Tomusiak, 2000). Recent studies by Turczański and

Kaźmierczak (2020) confirmed a general decline in Scots pine slenderness with age, regardless of site conditions, in the examined habitats LMśw and BMśw.

From the above, it follows that periodic increments in tree height, diameter at breast height (DBH), cross-sectional area at breast height, tree volume, DBH form factor, volume growth intensity coefficient, and slenderness are critical characteristics (referred to as variables) that define the condition and vitality of forest stands.

General trends in growth, determined from a sample of trees over a specific period, can facilitate predictions about future trends in height, volume, and other traits, as well as the potential uses of harvested wood.

Understanding growth and increments in both individual trees and entire stands is one of the primary research goals in forestry. This requires tree measurements and statistical analysis of the collected data. Typically, one-dimensional analyses are conducted separately for each characteristic during each growth period, or regression and variance analyses are performed with repeated measures. However, these methods do not fully capture the comprehensive characteristics of the studied stand. A more holistic approach is provided by Functional Data Analysis (FDA) (Ramsay and Dalzell, 1991) and Multivariate Functional Data Analysis (MFDA) (Waszak, 2015; Krzyśko et al., 2022).

MFDA transforms multidimensional data (with respect to time and traits) into new variables that integrate these two dimensions into a single one. This results in a unidimensional dataset, often with a reduced dimensionality compared to the number of time points. A key advantage of MFDA is that measurements do not need to be taken at equal intervals, and the number of time points can vary across variables (traits) and objects (trees). Coefficients calculated with this method are determined separately for each studied object, preserving object independence. These transformed data can then be subjected to well-known multivariate analyses. If the objects are grouped by levels of a studied factor (e.g., two age classes), permutation-based analysis of variance can be conducted to test for significant differences between groups. Additionally, other multivariate methods can be applied, such as Functional Principal Component Analysis (FPCA), and –

– when more than two groups are compared – discriminant coordinates analyses (linear – FLDC or quadratic – FQDC).

The aim of this study was to apply a novel methodological approach – functional data analysis (a multivariate analysis technique for dependent data) – to explore and quantify the differentiation between two age groups of trees. This analysis was conducted over five-year periods (dependent traits) and focused on six key traits, providing a comprehensive tool for understanding temporal and multivariate patterns in forest dynamics.

2. Materials

The research material was collected from two Scots pine stands located in the Zielonka Experimental Forest District (temperate climate, E 17° 6' 48.996", N 52° 33' 13.752"). These stands were divided into two age groups: 24 years and 33 years. From each stand, 25 trees were randomly selected using the Draudt method, and measurements were taken on felled trunks. The following parameters were measured over five-year increments: height growth (zh_5), diameter growth at breast height (zd_5), cross-sectional area growth at breast height (zg_5), volume growth (zv_5). Additionally, coefficients of volume growth intensity (i_5) and slenderness (s) were calculated (Table 1).

3. Statistical methods

Traits measured in five-year periods on the same trees can be treated as functional data, as suggested by Ramsay and Dalzell (1991). In functional data analysis (FDA), discrete time intervals are transformed into continuous functions, effectively converting the data from vectors to smooth functions. When the analysis extends beyond a single trait to include multiple features simultaneously, a multivariate version of FDA, known as multivariate functional data analysis (MFDA), can be applied (Waszak, 2015; Krzyśko et al., 2022).

Table 1. Descriptive statistics: means, standard deviations of means, minimum and maximum of analyzed traits

Age group	Trait	Years	Mean±SE	Min	Max
24	zh_5	5	2.83±0.10	1.70	3.75
		10	2.85±0.05	2.00	3.55
		15	3.76±0.07	3.22	4.38
		20	2.89±0.08	2.21	3.72
		25	1.05±0.05	0.52	1.43
	zd_5	5	1.54±0.18	0.52	4.25
		10	1.46±0.13	0.55	3.30
		15	3.37±0.16	2.40	6.20
		20	4.39±0.20	2.20	6.33
		25	0.43±0.12	0.00	1.80
	zg_5	5	0.0028±0.0005	0.0006	0.0105
		10	0.0022±0.0003	0.0005	0.0062
		15	0.0035±0.0003	0.0015	0.0071
		20	0.0019±0.0002	0.0004	0.0042
		25	0.00004±0.00001	0.0000	0.0003
	zv_5	5	0.0333±0.0048	0.0080	0.1017
		10	0.0191±0.0020	0.0063	0.0464
		15	0.0184±0.0017	0.0047	0.0340
		20	0.0046±0.0005	0.0010	0.0109
		25	0.0004±0.0001	0.0000	0.0011
	i_5	5	2.91±0.15	1.40	4.23
		10	2.45±0.09	1.70	3.35
		15	3.29±0.10	2.40	4.20
		20	2.32±0.08	1.80	3.29
		25	1.25±0.52	0.00	7.00
	s	5	1.25±0.04	0.81	1.75
		10	1.13±0.04	0.79	1.62
		15	0.97±0.03	0.73	1.32
		20	0.86±0.03	0.61	1.34
		25	0.66±0.17	0.00	2.13
Age group	Trait	Years	Mean±SE	Min	Max
33	zh_5	5	2.18±0.12	0.75	3.28
		10	1.95±0.06	1.39	2.52
		15	2.91±0.07	2.14	3.54
		20	2.84±0.05	2.47	3.42
		25	3.19±0.05	2.70	3.60
		30	1.53±0.07	0.82	2.11
	zd_5	5	1.15±0.14	0.25	2.50
		10	0.98±0.11	0.31	2.38
		15	1.64±0.13	0.81	3.50
		20	1.78±0.13	0.95	3.13
		25	3.33±0.23	1.95	7.30
		30	2.11±0.22	0.00	3.98

zg_5	5	0.0022±0.0004	0.0002	0.0073
	10	0.0016±0.0002	0.0004	0.0052
	15	0.0022±0.0003	0.0008	0.0075
	20	0.0018±0.0002	0.0007	0.0049
	25	0.0020±0.0002	0.0010	0.0061
	30	0.0004±0.0001	0.0000	0.0012
zv_5	5	0.0274±0.0046	0.0029	0.0938
	10	0.0175±0.0022	0.0030	0.0477
	15	0.0198±0.0024	0.0066	0.0608
	20	0.0117±0.0011	0.0040	0.0278
	25	0.0062±0.0005	0.0029	0.0112
	30	0.0013±0.0046	0.0002	0.0029
i_5	5	2.37±0.18	0.97	5.06
	10	2.06±0.12	1.07	3.68
	15	2.98±0.16	1.81	6.02
	20	2.76±0.12	1.55	4.56
	25	2.68±0.18	1.61	6.62
	30	3.67±0.88	0.00	21.00
s	5	1.41±0.06	0.85	1.93
	10	1.33±0.05	0.79	1.86
	15	1.23±0.05	0.77	1.74
	20	1.08±0.04	0.71	1.65
	25	0.90±0.03	0.60	1.26
	30	0.93±0.14	0.00	2.58

Let x_j be the observed value of variable X at the j-th time point t_j, where $j = 1,2,\ldots,T$ (T being the number of time points). Then the data are represented by T pairs (t_j, x_j). These discrete data can be smoothed using a continuous function $x(t)$, represented by a linear combination of a finite number, say B, of orthonormal base functions, as shown in the equation:

$$x(t) = \sum_{b=0}^{B} c_b \, \varphi_b(t),$$

where $\{\varphi_b\}$ is a system of orthonormal base functions, and $c_0, \ldots, c_B$ are the coefficients subject to estimation. In this study the orthonormal Fourier base is used.

Let $\boldsymbol{x} = (x_1, x_2, \ldots, x_T)'$ be the observations vector of one trait, let $\boldsymbol{c} = (c_0, \ldots, c_B)'$ be the vector of unknown coefficients, and $\boldsymbol{\Phi}(t)$ the $T \; x \; (B+1)$ matrix of functions represented by the values $\varphi_b(t_j)$ at the successive time points

t_j. The vector $\boldsymbol{c} = (c_0, \dots, c_B)'$ is estimated by the least squares method, so as to minimize the function as given in the equation:

$$S(c) = (\boldsymbol{x} - \boldsymbol{\Phi}(t)\boldsymbol{c})'(\boldsymbol{x} - \boldsymbol{\Phi}(t)\boldsymbol{c}).$$

By solving the above minimization problem we obtain the least square estimator:

$$\hat{\boldsymbol{c}} = \left(\boldsymbol{\Phi}'(t)\boldsymbol{\Phi}(t)\right)^{-1}\boldsymbol{\Phi}'(t)\boldsymbol{x}.$$

The optimal number of basic elements *B* is selected using the Bayesian Information Criterion (BIC, Shmueli, 2010) separately for each function $x(t)$ (where: $x(t) = \sum_{b=0}^{B} c_b\, \varphi_b(t)$) and then taking the minimal value over all functions calculated for different *B*. The above calculation (fixing *B*) is repeated for each trait separately. Finally, the coefficients matrix $\widehat{\boldsymbol{C}}$**,** for all traits, is composed of the matrices $\hat{\boldsymbol{c}}$ determined for each trait separately. The functions $x(t)$ can be presented on a graph separately for all traits, where each function represents a single tree.

The x-axis represents time, while the y-axis shows the values of $x(t)$. Such graphs can be helpful for detecting groups of objects (trees) with similar patterns of $x(t)$ as well as identifying outliers or unusual observations. Using the matrix $\widehat{\boldsymbol{C}}$ as new data (called functional variables) PCA analysis can be conducted, known as FPCA – functional principal component analysis (Derengowski, Krzyśko, 2011) or if more than two groups are compared, FLDC – functional linear discrimination coordinates (Górecki et al., 2014). These analyses can be conducted either for all dimensions simultaneously or separately for each trait. PCA is performed on the new variables $\hat{\boldsymbol{c}}$. The percentage of variability explained by individual principal components is determined using the eigenvalues of the covariance Σ of the variables $\hat{\boldsymbol{c}}$. The contribution of variables (traits) to individual components is obtained by calculating the area under the absolute value of weight functions, which are the product of the matrix Φ and the eigenvectors of the covariance matrix Σ. In FPCA, each object is represented by a single point that encapsulates information about all analyzed traits and time points, allowing us to determine whether the analyzed objects group based on traits evolving over time.

However, these plots do not show temporal changes, which are visible in the plots of the calculated functions. It should be noted that FPCA can also be performed separately for each trait. Moreover, permutation analysis of variance (Górecki, Smaga, 2015) for $\widehat{\boldsymbol{C}}$ was conducted to check the significance of differences between age groups in all analyzed periods. Calculations were performed using a custom program written in the Scilab package.

4. Results

The 3, 5, 3, 5, 3, 5 elements of the Fourier base were determined as the optimal number B of unknown coefficients using the Bayesian Information Criterion (BIC) for each of the traits (zh_5, zd_5, zg_5, zv_5, i_5, s). Thus, the matrix of functional variables $\widehat{\boldsymbol{C}}$ consisted of 24 columns. This matrix was used to calculate the functions $x(t)$. Figure 1 presents the functions $x(t)$, where each function corresponds to an individual tree. Unlike previous studies that focused on single traits, this analysis integrates multiple traits simultaneously, providing a more comprehensive view of growth patterns.

Detailed analysis for each trait:

Height growth (zh_5) – the curves for 24-year-old and 33-year-old trees show distinctly different patterns. This may indicate that height growth occurred at different rates in the two stands.

Diameter Increment (zd_5) – the curves of both age groups overlap at ages 10 and 25, suggesting that diameter increments were similar between the groups during these periods.

Cross-sectional area growth at breast height (zg_5), volume growth (zv_5), volume growth intensity coefficient (i_5) and slenderness (s) – the curves for the two groups are poorly distinguishable, showing only minor differences. However, the shapes of the curves in both groups are somewhat different. In Figure 1, some curves representing individual trees deviate significantly from the others. Notably, these are the curves for trees 25, 50, and 49, which exhibited high average increments in the respective traits, marking them as outliers.

A comprehensive analysis of the data was performed using FPCA. This analysis resulted in a projection of points representing transformed observations in the space of the first two principal components, where each point represents a single tree. Each point contains combined information about all analyzed time periods and traits. The results of the analysis are shown in Figure 2. The percentage of total variance explained by the first three principal components was 55% (33% FPCA1, 22% FPCA2, 15% FPCA3). A plot of the projection of the transformed observations onto the first two principal components is presented in Figure 2a, and the projection onto the first and third components in Figure 2b. The projection onto the first two components reveals two distinct groups, with most trees in each group belonging to one of the studied plots. The projection onto FPC1 and FPC3 (Figure 2) indicates that trees 50, 25, and 49 stand out.

Table 2 summarizes the contributions of the variables to the first three principal components, calculated using the area under the weighting functions (Figure 3). Variables (zd_5), (i_5) and (zh_5) had the highest contributions to FPC1, while (zd_5), (zh_5) and (s) had the highest contributions to FPC2. These traits had the greatest impact in differentiating the studied plots. Variables (zg_5) and (zv_5) had minimal contributions to all three components, indicating that the two plots did not differ in terms of the increments of these traits.

Table 2. The contributions of variables to individual principal components

Variable	Contribution in FPC1	Contribution in FPC2	Contribution in FPC3
zh5	0.041674	0.035174	0.039099
zd5	0.091475	0.038761	0.098766
zg5	0.000045	0.000009	0.000190
zv5	0.000504	0.000259	0.001811
i5	0.067164	0.004606	0.065459
S	0.002492	0.030476	0.013376

Both the graph depicting functional curves and the PCA graph were not clear enough to explain the differences between age groups, as there was no distinct separation of trees belonging to each age group, and the results did not show statistical significance of the differences between groups. Thus, for the matrix of coefficients $\hat{\boldsymbol{c}}$, a permutation analysis of variance was conducted. The number of permutations was 50,000, and the permutation *p*-value was 0.00002, which is smaller than 0.05, indicating that the age groups of trees differed significantly. It should be noted that the permutation analysis of variance demonstrated significant multidimensional differences (time, traits) in a bidirectional manner, enabling a comprehensive understanding of the studied phenomenon.

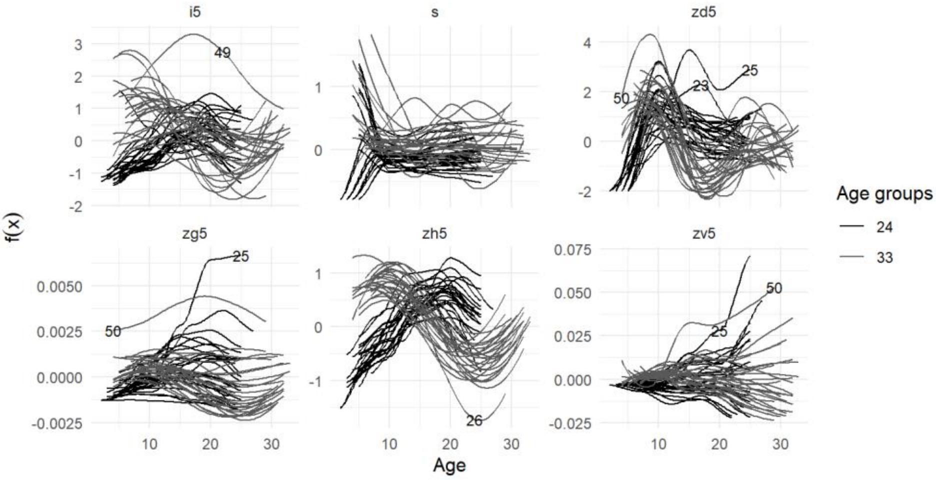

Figure 1. Functions representing all trees separately for all traits given from functional data analysis (zh_5, zd_5, zg_5, zv_5, i_5, s)

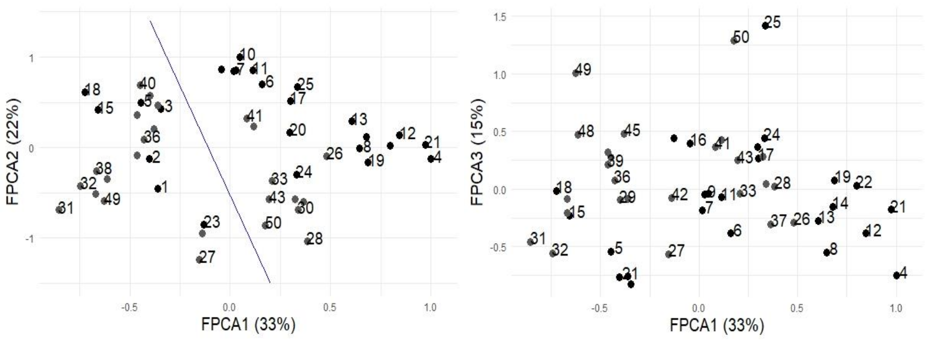

Figure 2. Functional principal component analysis representing each tree as a single point through all traits and time points. Left: FPCA1, FPCA2; right: FPCA1, FPCA3

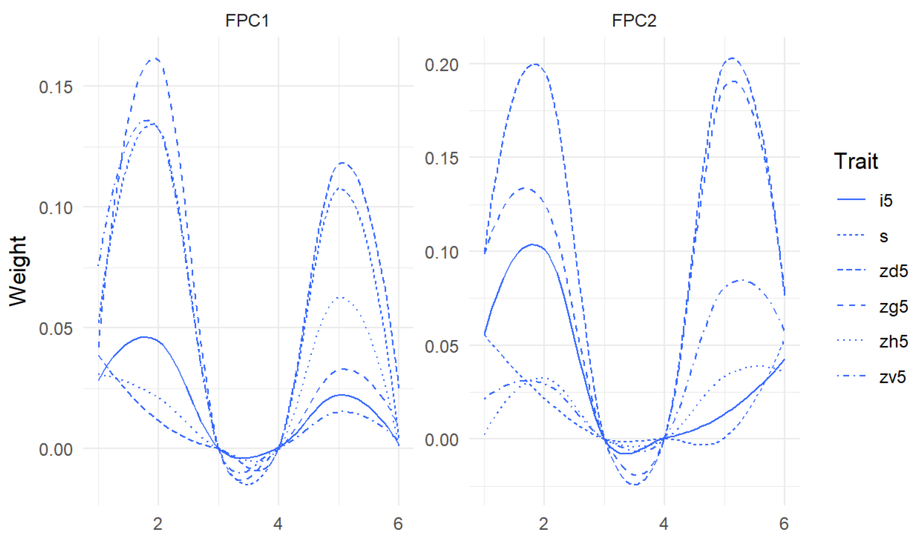

Figure 3. Weight functions defining the contribution (area under the curve) of traits to the first two principal components. Left: PC1; right: PC2

5. Discussion and conclusion

In the paper by Crecente-Campoa et al. (2010), an individual-tree growth model was developed. Stankova and Diéguez-Aranda (2013) presented modeling of height increments based on other characteristics of trees. Gerelbaatar and Baatarbileg (2011) studied height increment changes in close correlation with diameter increment for different vegetation zones. Profile analyses do not give a model of growth, but on the basis of this analysis it can be seen whether the increments in the different groups of trees are the same or different in successive years. If the increments are not the same, it can be concluded which groups differ from each other. As mentioned above, the results of profile analysis provide information such as that obtained in the analysis of growth, namely, one can conclude at which age the increments of trees do not change from year to year. In the analysis of growth, autocorrelation is often observed. In profile analysis the effects of autocorrelation and weather conditions are excluded, in view of the fact that the analysis is tailored to data concerning repeated measurement on the same experimental units. The applied method of analysis for repeated measures seems to be a very good method for evaluating the increments of tree heights, and may also be used for tree species other than Scots pine.

In the paper by Zawieja and Kaźmierczak (2014), profile analysis was applied to longitudinal data, using the repeated ANOVA method. The analysis showed that tree 33 and 43, next 43 and 55, and next 55, 62, 72, 84 33, 43, 55, 62, 72, and 84 did not differ significantly in terms of growth increments, while trees 24 and 92 differed from all other age groups. These trees belong to the second phase of growth, while the remaining trees are in the third phase, divided into two groups: trees with still relatively high annual increments (33, 43, 55) and trees with more stable, smaller increments (62, 72, 84, and 92). In this paper, when functional analysis was applied, a similar result was obtained. Specifically, during the study period (5–30 years), the growth increments of 24-year-old trees were significantly different from those of 33-year-old trees. However, it should be noted that this result was based on the overall analysis (whereas the previous result was based on a single trait). This finding may be linked to the different environmental conditions in which the studied trees were growing.

Observations of European forests conducted in recent decades have shown increased growth in tree volume (Nabuurs et al., 2001; Hunter, Schuck, 2002). The primary cause of this phenomenon is believed to be climate change (Bernadzki, 1995; Rykowski, 2007). Increased site productivity may be influenced by rising temperatures (Kellomäki, Kolström, 1994) and greater nitrogen accumulation (Solberg et al., 2004). Studies conducted on permanent forest monitoring plots in Europe indicate a clear impact of nitrogen deposition and higher summer temperatures on the increased volume growth of spruce and pine stands (Solberg et al., 2009). At the same time, further increases in forest resources are forecast for European forests (Pussinen et al., 2009). However, according to Socha and Durło (2012), a significant increase in carbon dioxide levels in the atmosphere, along with reduced precipitation, may lead to a decrease in forest resources, especially in mountain stands. One of the reasons for the currently observed positive growth responses in trees in the Sudetes is the reduction of industrial emissions (Korzybski et al., 2013). The current volume increment of pine stands is significantly higher than the values listed in the volume tables compiled by Szymkiewicz. In practice, the intensity of pre-thinning usage is much lower than the indicators based on volume growth in the resource tables, and it is around 30−50% of the actual volume increment. Given the importance of volume growth for planning purposes and updating of tree volumes in forest management plans (SILP), there is an urgent need to develop new procedures for estimating volume growth in forest stands (Jabłoński, 2013).

In this study, distinct growth patterns were observed between 24-year-old and 33-year-old trees, indicating differing growth rates across the two stands. Overlapping curves at certain ages, such as 10 and 25 years, suggested similar diameter increment dynamics between age groups during those periods. For other traits, only minor differences were noted between the age groups; however, individual variations in curve shapes were evident, particularly for outlier trees, such as trees 25, 50, and 49. The FPCA projections identified two distinct groups corresponding to the studied plots, with outlier trees standing out in the multidimensional space. Among the traits, zd_5, zh_5, and i_5 contributed most to FPC1, while zg_5 and zv_5 showed negligible contributions, suggesting limited differentiation between plots for these traits. Although the graphical representations of functional curves and PCA projections did not distinctly separate the age groups, the permutation analysis of variance, based on 50,000 permutations, revealed statistically significant multidimensional differences (*p*-value = 0.00002). These findings highlight the significant age-related differences in tree growth dynamics across traits and time. The combination of functional data analysis, PCA, and permutation tests proved to be a robust approach, emphasizing the multidimensional and temporal complexities of the studied traits.

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