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New results on fixed point theorems in n-Banach spaces

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Abstract In this article we prove a common fixed point result for interpolative Kannan type contraction mappings in a nonempty, closed and bounded subset with respect to n linearly independent vectors in an n-Banach space. On the other hand, we introduce interpolative Dass and Gupta rational type contraction mappings on a nonempty, closed and bounded subset with respect to n linearly independent vectors in an n-Banach space. In particular, we discuss the existence and uniqueness of a fixed point of such a mapping on a nonempty, closed and bounded subset with respect to n linearly independent vectors in an n-Banach space.

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1 Introduction

In an n-Banach space framework, Gähler [4] initiated the study of n-normed spaces. The 2-Banach spaces and n-Banach spaces were studied by [3–6, 12].

Harikrishnan and Ravindran [8] demonstrated that a contraction mapping has a unique fixed point in bounded and closed subsets of a 2-Banach space. However, Kir and Kiziltunc [10] demonstrated several results on fixed points in 2-Banach spaces.

In [1], we introduced the notions of Meir-Keeler contraction mappings and Cirić contraction mappings on a 2-Banach space. In particular, we proved that Meir-Keeler contraction mappings and Cirić contraction mappings on a 2-Banach space have a unique fixed point. On the other hand, we defined the concept of Hardy-Rogers contraction mappings on a 2-Banach space. In particular, we established that a Hardy-Rogers contraction mapping on a 2-Banach space has a unique fixed point. However, several results are demonstrated on fixed point theorems of some mappings in 2-Banach spaces. Recently, in [2], we introduced and studied interpolative Boyd-Wong and Matkowski type contractions in 2-Banach spaces. For more details on fixed point theorems in the setting of metric spaces, see [9, 11].

Fixed point theory plays a crucial role in functional analysis. Moreover, it is used in many branches of science such as biology, chemistry, economics, engineering and computer science.

By applying the techniques introduced in [7] within the framework of n-Banach spaces, we establish several new fixed point theorems for certain classes of mappings in n-Banach spaces.

2 Preliminaries

Definition 2.1 ([4–6]). *Let $\mathcal{X}$ be a real vector space with $\dim \mathcal{X} \geq n$. A function $\|\cdot, \cdots, \cdot\| : \mathcal{X}^n \to [0, \infty)$ is called an n-norm if*

(i) $\|x_1, \cdots, x_n\| = 0$ if and only if $x_1, \cdots, x_n$ are linearly dependent;

(ii) $\|x_1, \cdots, x_n\|$ is invariant under permutation;

(iii) for each $x_1, \cdots, x_n \in \mathcal{X}$ and for all $\lambda \in \mathbb{R}$, $\|\lambda x_1, \cdots, x_n\| = |\lambda| \|x_1, \cdots, x_n\|$;

(iv) for all $x, y, x_2 \cdots, x_n \in \mathcal{X}$, $\|x + y, x_2, \cdots, x_n\| \leq \|x, x_2, \cdots, x_n\| + \|y, x_2, \cdots, x_n\|$.

Then $(\mathcal{X}, \|\cdot, \cdots, \cdot\|)$ is called an n-normed space.

Hereafter, let $(\mathcal{X}, \|\cdot, \cdots, \cdot\|)$ be an n-normed space and $\mathcal{A} = \{a_1, a_2, \cdots, a_n\}$ be a collection of n-linearly independent vectors in $\mathcal{X}$. We recall some nedeed results:

Definition 2.2 ([7]). *A sequence $\{x_j\} \subset \mathcal{X}$ is said to be a Cauchy sequence with respect to $\mathcal{A}$ if $\lim\limits_{m,k\to\infty} \|x_k - x_m, a_{i_2}, \cdots, a_{i_n}\| = 0$ for any $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$. If every Cauchy sequence in $\mathcal{X}$ converges to some $x \in \mathcal{X}$, then $(\mathcal{X}, \|\cdot, \cdots, \cdot\|)$ is said to be complete with respect to $\mathcal{A}$.*

Definition 2.3 ([7]). *Let $B \subset \mathcal{X}$ be a nonempty set. Then we say that B is closed if for every sequence $\{x_j\}$ in B which converges in $\mathcal{X}$, its limit is in B.*

Definition 2.4 ([7]). *A sequence $\{x_j\} \subset \mathcal{X}$ converges to an element $x \in \mathcal{X}$ with respect to $\mathcal{A}$, denoted by $x_j \xrightarrow{\mathcal{A}} x$ as $j \to \infty$, if $\lim\limits_{j\to\infty} \|x_j - x, a_{i_2}, \cdots, a_{i_n}\| = 0$ for any $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$.*

Definition 2.5 ([7]). *Let B be a nonempty subset of $\mathcal{X}$. Then B is called bounded with respect to $\mathcal{A}$ if there exists $M > 0$ such that*

$$\|x, a_{i_2}, \cdots, a_{i_n}\| \leq M \tag{2.1}$$

for any $x \in \mathcal{X}$ and $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$.

Lemma 2.1 ([7]). *Let $x \in \mathcal{X}$. If $\|x, a_{i_2}, \cdots, a_{i_n}\| = 0$ for any $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$, then $x = 0$.*

Theorem 2.2 ([7]). *Let $\mathcal{X}$ be an n-Banach space and $\mathcal{F} \subset \mathcal{X}$ be nonempty, closed, and bounded with respect to $\mathcal{A}$. If $S : \mathcal{F} \longrightarrow \mathcal{F}$ is a contraction mapping with respect to $\mathcal{A}$, then S has a unique fixed point in $\mathcal{F}$.*

Theorem 2.3 ([7]). *Let $\mathcal{X}$ be an n-Banach space and $\mathcal{F} \subset \mathcal{X}$ be nonempty, closed, and bounded with respect to $\mathcal{A}$. If $S : \mathcal{F} \longrightarrow \mathcal{F}$ is a φ-contraction mapping with respect to $\mathcal{A}$, then S has a unique fixed point in $\mathcal{F}$.*

The set of all fixed points of a mapping S on the space $\mathcal{X}$ is defined by $\mathrm{Fix(S)} = \{u \in \mathcal{X} : Su = u\}$.

3 Main results

Before stating our main results, we set $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

Theorem 3.1. *Let $\mathcal{X}$ be an n-Banach space and $\mathcal{F} \subset \mathcal{X}$ be nonempty, closed, and bounded with respect to $\mathcal{A}$. Let S, T be self mappings on $\mathcal{F}$ satisfying for each $x \in \mathcal{F} \setminus Fix(S), y \in \mathcal{F} \setminus Fix(T)$ and $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$,*

$$\|Sx - Ty, a_{i_2}, \cdots, a_{i_n}\| \leq \lambda \|x - Sx, a_{i_2}, \cdots, a_{i_n}\|^{\alpha} \cdot \|y - Ty, a_{i_2}, \cdots, a_{i_n}\|^{1-\alpha} \tag{3.1}$$

where $\lambda \in [0, 1)$ and $\alpha \in (0, 1)$. Then S and T have a common fixed point.

Proof. Let $x_0 \in \mathcal{F}$ and let $\{x_j\}$ be a sequence defined by $x_{2j+1} = Sx_{2j}$ and $x_{2j+2} = Tx_{2j+1}$ for each $j \in \mathbb{N}_0$. If there exists $j \in \mathbb{N}_0$ such that $x_j = x_{j+1} = x_{j+2}$, then x_j is a common fixed point of S and T, so assume that there does not exist three consecutive identical terms in the sequence $\{x_j\}$ and that $x_0 \neq x_1$. Using (3.1), we obtain

$$\begin{aligned}\|x_{2j+1} - x_{2j+2}, a_{i_2}, \cdots, a_{i_n}\| =& \|Sx_{2j} - Tx_{2j+1}, a_{i_2}, \cdots, a_{i_n}\| \\ \leq& \lambda \|x_{2j} - x_{2j+1}, a_{i_2}, \cdots, a_{i_n}\|^{\alpha} \|x_{2j+1} - x_{2j+2}, a_{i_2}, \cdots, a_{i_n}\|^{1-\alpha}.\end{aligned}$$

Then

$$\|x_{2j+1} - x_{2j+2}, a_{i_2}, \cdots, a_{i_n}\|^{\alpha} \leq \lambda \|x_{2j} - x_{2j+1}, a_{i_2}, \cdots, a_{i_n}\|^{\alpha}. \tag{3.2}$$

Hence

$$\begin{aligned}\|x_{2j+1} - x_{2j+2}, a_{i_2}, \cdots, a_{i_n}\| &\leq \lambda^{\frac{1}{\alpha}} \|x_{2j} - x_{2j+1}, a_{i_2}, \cdots, a_{i_n}\| \\ &\leq \lambda \|x_{2j} - x_{2j+1}, a_{i_2}, \cdots, a_{i_n}\|.\end{aligned} \tag{3.3}$$

From (3.3), we obtain

$$\begin{aligned}\|x_{2j+1} - x_{2j+2}, a_{i_2}, \cdots, a_{i_n}\| &\leq \lambda \|x_{2j} - x_{2j+1}, a_{i_2}, \cdots, a_{i_n}\| \\ &\leq \lambda^2 \|x_{2j-1} - x_{2j}, a_{i_2}, \cdots, a_{i_n}\| \\ &\leq \cdots \leq \lambda^{2j+1} \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\|.\end{aligned} \tag{3.4}$$

Then

$$\|x_{2j+1} - x_{2j+2}, a_{i_2}, \cdots, a_{i_n}\| \leq \lambda^{2j+1} \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\|. \tag{3.5}$$

Similarly,

$$\begin{aligned}\|x_{2j+1}-x_{2j},a_{i_2},\cdots,a_{i_n}\| &= \|Sx_{2j}-Tx_{2j-1},a_{i_2},\cdots,a_{i_n}\|\\ &\leq \lambda\|x_{2j}-x_{2j+1},a_{i_2},\cdots,a_{i_n}\|^{\alpha}\|x_{2j-1}-x_{2j},a_{i_2},\cdots,a_{i_n}\|^{1-\alpha}.\end{aligned}$$

Then

$$\|x_{2j+1}-x_{2j},a_{i_2},\cdots,a_{i_n}\|^{1-\alpha} \leq \lambda\|x_{2j-1}-x_{2j},a_{i_2},\cdots,a_{i_n}\|^{1-\alpha}. \tag{3.6}$$

Hence

$$\begin{aligned}\|x_{2j+1}-x_{2j},a_{i_2},\cdots,a_{i_n}\| &\leq \lambda^{\frac{1}{1-\alpha}}\|x_{2j-1}-x_{2j},a_{i_2},\cdots,a_{i_n}\|\\ &\leq \lambda\|x_{2j-1}-x_{2j},a_{i_2},\cdots,a_{i_n}\|.\end{aligned} \tag{3.7}$$

Thus

$$\begin{aligned}\|x_{2j+1}-x_{2j},a_{i_2},\cdots,a_{i_n}\| &\leq \lambda\|x_{2j-1}-x_{2j},a_{i_2},\cdots,a_{i_n}\|\\ &\leq \lambda^2\|x_{2j-2}-x_{2j-1},a_{i_2},\cdots,a_{i_n}\|\\ &\leq \cdots \leq \lambda^{2j}\|x_0-x_1,a_{i_2},\cdots,a_{i_n}\|.\end{aligned}$$

Then

$$\|x_{2j+1}-x_{2j},a_{i_2},\cdots,a_{i_n}\| \leq \lambda^{2j}\|x_0-x_1,a_{i_2},\cdots,a_{i_n}\|. \tag{3.8}$$

Using (3.5) and (3.8), we obtain

$$\|x_j-x_{j+1},a_{i_2},\cdots,a_{i_n}\| \leq \lambda^{j}\|x_0-x_1,a_{i_2},\cdots,a_{i_n}\|. \tag{3.9}$$

Now, by (3.9) we demonstrate that $\{x_j\}$ is Cauchy. Let $m,j>0$ with $m>j$. Put $m=j+l$, hence

$$\begin{aligned}\|x_j-x_m,a_{i_2},\cdots,a_{i_n}\| =&\|x_j-x_{j+l},a_{i_2},\cdots,a_{i_n}\|\\ =&\|(x_j-x_{j+1})+(x_{j+1}-x_{j+2})\\ &+\cdots+(x_{j+l-1}-x_{j+l}),a_{i_2},\cdots,a_{i_n}\|\\ \leq&\|x_j-x_{j+1},a_{i_2},\cdots,a_{i_n}\|+\|x_{j+1}-x_{j+2},a_{i_2},\cdots,a_{i_n}\|\\ &+\cdots+\|x_{j+l-1}-x_{j+l},a_{i_2},\cdots,a_{i_n}\|\\ \leq&\lambda^j\|x_0-x_1,a_{i_2},\cdots,a_{i_n}\|+\lambda^{j+1}\|x_0-x_1,a_{i_2},\cdots,a_{i_n}\|\\ &+\cdots+\lambda^{j+l-1}\|x_0-x_1,a_{i_2},\cdots,a_{i_n}\|\\ =&\lambda^j(1+\lambda+\cdots+\lambda^{l-1})\|x_0-x_1,a_{i_2},\cdots,a_{i_n}\|\\ \leq&\frac{\lambda^j}{1-\lambda}\|x_0-x_1,a_{i_2},\cdots,a_{i_n}\|.\end{aligned}$$

Letting $j,m\longrightarrow\infty$, we get

$$\lim_{m,j\to\infty}\|x_j-x_m,a_{i_2},\cdots,a_{i_n}\|=0.$$

Hence $\{x_j\}$ is Cauchy. Then $\{x_j\}$ converges to some $x\in\mathcal{F}$. Now, we demonstrate that $x\in\mathcal{F}$ is a fixed point of S. Hence

$$\begin{aligned}\|Sx-x_{2j+2},a_{i_2},\cdots,a_{i_n}\| &= \|Sx-Tx_{2j+1},a_{i_2},\cdots,a_{i_n}\|\\ &\leq \lambda\|x-Sx,a_{i_2},\cdots,a_{i_n}\|^{\alpha}\cdot\|x_{2j+1}-x_{2j+2},a_{i_2},\cdots,a_{i_n}\|^{1-\alpha}.\end{aligned}$$

Letting $j \longrightarrow \infty$, we get
$$\|Sx - x, a_{i_2}, \cdots, a_{i_n}\| = 0,$$
so $Sx = x$. Similarly
$$\begin{aligned}\|x_{2j+1} - Tx, a_{i_2}, \cdots, a_{i_n}\| &= \|Sx_{2j} - Tx, a_{i_2}, \cdots, a_{i_n}\| \\ &\leq \lambda \|x_{2j} - x_{2j+1}, a_{i_2}, \cdots, a_{i_n}\|^{\alpha} \cdot \|x - Tx, a_{i_2}, \cdots, a_{i_n}\|^{1-\alpha}.\end{aligned}$$
Letting $j \longrightarrow \infty$, we get $x = Tx$. Then $x = Sx = Tx$. □

Theorem 3.2. *Let $\mathcal{X}$ be an n-Banach space and $\mathcal{F} \subset \mathcal{X}$ be nonempty, closed, and bounded with respect to $\mathcal{A}$ and let S be a self mapping on $\mathcal{F}$ satisfying for each $x, y \in \mathcal{F}$ and for any $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$,*
$$\begin{aligned}\|Sx - Sy, a_{i_2}, \cdots, a_{i_n}\| \leq & k_1 \|x - y, a_{i_2}, \cdots, a_{i_n}\| \\ &+ k_2 \frac{\|x - Sx, a_{i_2}, \cdots, a_{i_n}\| \|y - Sy, a_{i_2}, \cdots, a_{i_n}\|}{1 + \|x - y, a_{i_2}, \cdots, a_{i_n}\|},\end{aligned} \tag{3.10}$$
where $k_1, k_2 \geq 0$ such that $k_1 + k_2 < 1$. Then S has a unique fixed point in $\mathcal{F}$.

Proof. Let $x_0 \in \mathcal{F}$ and let $\{x_j\}$ be a sequence defined by $x_{j+1} = Sx_j$ for each $j \in \mathbb{N}_0$. Putting $x = x_{j-1}$ and $y = x_j$ in (3.10), we obtain
$$\begin{aligned}\|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\| =& \|Sx_{j-1} - Sx_j, a_{i_2}, \cdots, a_{i_n}\| \\ \leq & k_1 \|x_{j-1} - x_j, a_{i_2}, \cdots, a_{i_n}\| \\ &+ k_2 \frac{\|x_{j-1} - Sx_{j-1}, a_{i_2}, \cdots, a_{i_n}\| \|x_j - Sx_j, a_{i_2}, \cdots, a_{i_n}\|}{1 + \|x_{j-1} - x_n, a_{i_2}, \cdots, a_{i_n}\|} \\ =& k_1 \|x_{j-1} - x_j, a_{i_2}, \cdots, a_{i_n}\| \\ &+ k_2 \frac{\|x_{j-1} - x_j, a_{i_2}, \cdots, a_{i_n}\| \|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\|}{1 + \|x_{j-1} - x_j, a_{i_2}, \cdots, a_{i_n}\|} \\ =& k_1 \|x_{j-1} - x_j, a_{i_2}, \cdots, a_{i_n}\| + k_2 \|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\|.\end{aligned}$$
Then
$$\|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\| \leq \lambda \|x_{j-1} - x_j, a_{i_2}, \cdots, a_{i_n}\|, \tag{3.11}$$
where $\lambda = \frac{k_1}{1-k_2}$. Since $\lambda < 1$, we deduce that the sequence $\{\|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\|\}$ is decreasing. From (3.11), we obtain
$$\begin{aligned}\|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\| &\leq \lambda \|x_{j-1} - x_j, a_{i_2}, \cdots, a_{i_n}\| \\ &\leq \lambda^j \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\|.\end{aligned} \tag{3.12}$$
Letting $j \longrightarrow \infty$ in (3.12), we obtain $\lim\limits_{j \to \infty} \|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\| = 0$.

Now, we demonstrate that $\{x_j\}$ is Cauchy. Let $m, j > 0$ with $m > j$. Put $m = j + l$, hence

$$\begin{aligned}\|x_j - x_m, a_{i_2}, \cdots, a_{i_n}\| =& \|x_j - x_{j+l}, a_{i_2}, \cdots, a_{i_n}\| \\ =& \|(x_j - x_{j+1}) + (x_{j+1} - x_{j+2}) \\ & + \cdots + (x_{j+l-1} - x_{j+l}), a_{i_2}, \cdots, a_{i_n}\| \\ \leq& \|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\| + \|x_{j+1} - x_{j+2}, a_{i_2}, \cdots, a_{i_n}\| \\ & + \cdots + \|x_{j+l-1} - x_{j+l}, a_{i_2}, \cdots, a_{i_n}\| \\ \leq& \lambda^j \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\| + \lambda^{j+1} \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\| \\ & + \cdots + \lambda^{j+l-1} \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\| \\ =& \lambda^j (1 + \lambda + \cdots + \lambda^{l-1}) \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\| \\ \leq& \frac{\lambda^j}{1-\lambda} \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\|.\end{aligned}$$

Letting $j, m \longrightarrow \infty$, we get

$$\lim_{m,j\to\infty} \|x_j - x_m, a_{i_2}, \cdots, a_{i_n}\| = 0.$$

Hence $\{x_j\}$ is Cauchy. Then $\{x_j\}$ converges to some $x \in \mathcal{F}$. Now, we demonstrate that $x \in \mathcal{F}$ is a fixed point of S. Hence

$$\begin{aligned}\|x_{j+1} - Sx, a_{i_2}, \cdots, a_{i_n}\| =& \|Sx_j - Sx, a_{i_2}, \cdots, a_{i_n}\| \\ \leq& k_1 \|x_j - x, a_{i_2}, \cdots, a_{i_n}\| \\ & + k_2 \frac{\|x_j - Sx_j, a_{i_2}, \cdots, a_{i_n}\| \|x - Sx, a_{i_2}, \cdots, a_{i_n}\|}{1 + \|x_j - x, a_{i_2}, \cdots, a_{i_n}\|}.\end{aligned}$$

Letting $j \longrightarrow \infty$, we get

$$\|Sx - x, a_{i_2}, \cdots, a_{i_n}\| = 0.$$

Thus $Sx = x$ then x is a fixed point of S. Suppose that x_1 and x_2 are two distinct fixed points of S. From

$$\begin{aligned}\|x_1 - x_2, a_{i_2}, \cdots, a_{i_n}\| =& \|Sx_1 - Sx_2, a_{i_2}, \cdots, a_{i_n}\| \\ \leq& k_1 \|x_1 - x_2, a_{i_2}, \cdots, a_{i_n}\| \\ & + k_2 \frac{\|x_1 - Sx_1, a_{i_2}, \cdots, a_{i_n}\| \|x_2 - Sx_2, a_{i_2}, \cdots, a_{i_n}\|}{1 + \|x_1 - x_2, a_{i_2}, \cdots, a_{i_n}\|} \\ =& k_1 \|x_1 - x_2, a_{i_2}, \cdots, a_{i_n}\| \\ & + k_2 \frac{\|x_1 - x_1, a_{i_2}, \cdots, a_{i_n}\| \|x_2 - x_2, a_{i_2}, \cdots, a_{i_n}\|}{1 + \|x_1 - x_2, a_{i_2}, \cdots, a_{i_n}\|} \\ =& k_1 \|x_1 - x_2, a_{i_2}, \cdots, a_{i_n}\|,\end{aligned}$$

which contradicts that $k_1 < 1$. Therefore $\|x_1 - x_2, a_{i_2}, \cdots, a_{i_n}\| = 0$. Then $x_1 = x_2$. □

By the same arguments as in the proof of Theorem 3.2, we obtain:

Theorem 3.3. *Let $\mathcal{X}$ be an n-Banach space, $\mathcal{F} \subset \mathcal{X}$ be nonempty, closed, and bounded with respect to $\mathcal{A}$, and let S be a self mapping on $\mathcal{F}$ satisfying for each $x, y \in \mathcal{F}$,*

$$\begin{aligned}\|Sx - Sy, a_{i_2}, \cdots, a_{i_n}\| \leq& k_1 \|x - y, a_{i_2}, \cdots, a_{i_n}\| \\ & + k_2 \frac{\|x - Sy, a_{i_2}, \cdots, a_{i_n}\| \|y - Sx, a_{i_2}, \cdots, a_{i_n}\|}{1 + \|x - y, a_{i_2}, \cdots, a_{i_n}\|},\end{aligned} \tag{3.13}$$

where $k_1, k_2 \geq 0$ and $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$ such that $k_1 + k_2 < 1$. Then S has a unique fixed point in $\mathcal{F}$.

Now, we introduce an interpolative Dass and Gupta rational type contraction in an n-normed space as follows.

Definition 3.1. *A continuous self-mapping S on an n-normed space $\mathcal{X}$ is called an interpolative Dass and Gupta rational type contraction with respect to $\mathcal{A}$ if there exist $\lambda \in (0,1)$ and $\alpha \in \left(0, \frac{1}{2}\right]$ such that*

$$\begin{aligned} \|Sx - Sy, a_{i_2}, \cdots, a_{i_n}\| \leq & \lambda \left[\frac{\|x - Sx, a_{i_2}, \cdots, a_{i_n}\| \|y - Sy, a_{i_2}, \cdots, a_{i_n}\|}{\|x - y, a_{i_2}, \cdots, a_{i_n}\|}\right]^{\alpha} \\ & \cdot \left[\|x - y, a_{i_2}, \cdots, a_{i_n}\|\right]^{1-\alpha}, \end{aligned} \tag{3.14}$$

for each $x, y \in \mathcal{X} \setminus Fix(S)$ with $x \neq y$ and $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$ where $Fix(S) = \{u \in \mathcal{X} : Su = u\}$.

Theorem 3.4. *Let $\mathcal{X}$ be an n-Banach space and $\mathcal{F} \subset \mathcal{X}$ be nonempty, closed, and bounded with respect to $\mathcal{A}$. Let $S : \mathcal{F} \longrightarrow \mathcal{F}$ be an interpolative Dass and Gupta rational type contraction with respect to $\mathcal{A}$. Then S has a fixed point in $\mathcal{F}$.*

Proof. Let $x_0 \in \mathcal{F}$. We will set a constructive sequence $\{x_j\}$ by $x_{j+1} = Sx_j$ for each $j \in \mathbb{N}_0$. Putting $x = x_j$ and $y = x_{j-1}$ in (3.14), we obtain

$$\begin{aligned} \|x_{j+1} - x_j, a_{i_2}, \cdots, a_{i_n}\| = & \|Sx_j - Sx_{j-1}, a_{i_2}, \cdots, a_{i_n}\| \\ \leq & \lambda \left[\frac{\|x_j - Sx_j, a_{i_2}, \cdots, a_{i_n}\| \|x_{j-1} - Sx_{j-1}, a_{i_2}, \cdots, a_{i_n}\|}{\|x_j - x_{j-1}, a_{i_2}, \cdots, a_{i_n}\|}\right]^{\alpha} \\ & \cdot \left[\|x_j - x_{j-1}, a_{i_2}, \cdots, a_{i_n}\|\right]^{1-\alpha} \\ \leq & \lambda \left[\|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\|\right]^{\alpha} \cdot \left[\|x_j - x_{j-1}, a_{i_2}, \cdots, a_{i_n}\|\right]^{1-\alpha}. \end{aligned}$$

Then

$$\|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\|^{1-\alpha} \leq \lambda \|x_{j-1} - x_j, a_{i_2}, \cdots, a_{i_n}\|^{1-\alpha}. \tag{3.15}$$

Hence

$$\|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\| \leq \lambda \|x_{j-1} - x_j, a_{i_2}, \cdots, a_{i_n}\|. \tag{3.16}$$

Thus, we deduce that the sequence $\{\|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\|\}$ is decreasing. As a result, there exists $M \in [0, \infty)$ such that $\lim\limits_{j \to \infty} \|x_j - x_{j-1}, a_{i_2}, \cdots, a_{i_n}\| = M$. We will indicate that $M = 0$. Indeed, by (3.16), we derive that

$$\|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\| \leq \lambda \|x_{j-1} - x_j, a_{i_2}, \cdots, a_{i_n}\| \leq \lambda^j \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\|. \tag{3.17}$$

Letting $j \longrightarrow \infty$ in (3.17), we obtain $M = 0$.

Now, we demonstrate that $\{x_j\}$ is Cauchy. Let $m, j > 0$ with $m > j$. Put $m = j + l$, hence

$$\begin{aligned}\|x_j - x_m, a_{i_2}, \cdots, a_{i_n}\| =& \|x_j - x_{j+l}, a_{i_2}, \cdots, a_{i_n}\| \\ =& \|(x_j - x_{j+1}) + (x_{j+1} - x_{j+2}) \\ & + \cdots + (x_{j+l-1} - x_{j+l}), a_{i_2}, \cdots, a_{i_n}\| \\ \le& \|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\| + \|x_{j+1} - x_{n+2}, a_{i_2}, \cdots, a_{i_n}\| \\ & + \cdots + \|x_{j+l-1} - x_{j+l}, a_{i_2}, \cdots, a_{i_n}\| \\ \le& \lambda^j \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\| + \lambda^{j+1}\|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\| \\ & + \cdots + \lambda^{j+l-1}\|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\| \\ =& \lambda^j(1 + \lambda + \cdots + \lambda^{l-1})\|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\| \\ \le& \frac{\lambda^j}{1-\lambda}\|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\|.\end{aligned}$$

Letting $j, m \longrightarrow \infty$, we get

$$\lim_{m,j\to\infty} \|x_j - x_m, a_{i_2}, \cdots, a_{i_n}\| = 0.$$

Hence $\{x_j\}$ is Cauchy. Then $\{x_j\}$ converges to some $x \in \mathcal{F}$.

Now, we demonstrate that $x \in \mathcal{F}$ is a fixed point of S. Hence

$$\begin{aligned}\|x_{j+1} - Sx, a_{i_2}, \cdots, a_{i_n}\| =& |Sx_j - Sx, a_{i_2}, \cdots, a_{i_n}\| \\ \le& \lambda \left[\frac{\|x_j - Sx_j, a_{i_2}, \cdots, a_{i_n}\|\|x - Sx, a_{i_2}, \cdots, a_{i_n}\|}{\|x_j - x, a_{i_2}, \cdots, a_{i_n}\|}\right]^\alpha \\ & \cdot [\|x_j - x, a_{i_2}, \cdots, a_{i_n}\|]^{1-\alpha}.\end{aligned}$$

Letting $j \longrightarrow \infty$, we get

$$\|Sx - x, a_{i_2}, \cdots, a_{i_n}\| = 0.$$

Then $Sx = x$, thus x is a fixed point of S. □

We obtain the following theorem.

Theorem 3.5. *Let $\mathcal{X}$ be an n-Banach space and $\mathcal{F} \subset \mathcal{X}$ be nonempty, closed, and bounded with respect to $\mathcal{A}$. Let S be a self mapping on $\mathcal{F}$ satisfying for each $x, y \in \mathcal{F} \setminus Fix(S)$ and $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$,*

$$\begin{aligned}\|Sx - Sy, a_{i_2}, \cdots, a_{i_n}\| \le& \lambda \left[\frac{\|x - Sx, a_{i_2}, \cdots, a_{i_n}\|\|y - Sy, a_{i_2}, \cdots, a_{i_n}\|}{1 + \|x - y, a_{i_2}, \cdots, a_{i_n}\|}\right]^\alpha \\ & \cdot [\|x - y, a_{i_2}, \cdots, a_{i_n}\|]^{1-\alpha},\end{aligned} \tag{3.18}$$

where $\lambda \in [0, 1), \alpha \in (0, 1)$, and $Fix(S) = \{u \in \mathcal{F} : Su = u\}$. Then S has a fixed point in $\mathcal{F}$.

Proof. Let $x_0 \in \mathcal{F}$. We will set a constructive sequence $\{x_j\}$ by $x_{j+1} = Sx_j$ for each $j \in \mathbb{N}_0$. Putting $x = x_j$ and $y = x_{j-1}$ in (3.18), we obtain

$$\begin{aligned}\|x_{j+1} - x_j, a_{i_2}, \cdots, a_{i_n}\| =& \|Sx_j - Sx_{j-1}, a_{i_2}, \cdots, a_{i_n}\| \\ \le& \lambda \left[\frac{\|x_j - Sx_j, a_{i_2}, \cdots, a_{i_n}\|\|x_{j-1} - Sx_{j-1}, a_{i_2}, \cdots, a_{i_n}\|}{1 + \|x_j - x_{j-1}, a_{i_2}, \cdots, a_{i_n}\|}\right]^\alpha \\ & \cdot [\|x_j - x_{j-1}, a_{i_2}, \cdots, a_{i_n}\|]^{1-\alpha} \\ \le& \lambda [\|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\|]^\alpha \cdot [\|x_j - x_{j-1}, a_{i_2}, \cdots, a_{i_n}\|]^{1-\alpha}.\end{aligned}$$

Then

$$\|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\|^{1-\alpha} \leq \lambda \|x_{j-1} - x_j, a_{i_2}, \cdots, a_{i_n}\|^{1-\alpha}. \tag{3.19}$$

Hence

$$\|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\| \leq \lambda \|x_{j-1} - x_j, a_{i_2}, \cdots, a_{i_n}\|. \tag{3.20}$$

Thus, we deduce that the sequence $\{\|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\|\}$ is decreasing. As a result, there exists $M \in [0, \infty)$ such that $\lim_{j\to\infty} \|x_j - x_{j-1}, a_{i_2}, \cdots, a_{i_n}\| = M$. We will indicate that $M = 0$. Indeed, by (3.20), we derive that

$$\|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\| \leq \lambda \|x_{j-1} - x_j, a_{i_2}, \cdots, a_{i_n}\| \leq \lambda^j \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\|. \tag{3.21}$$

Letting $j \longrightarrow \infty$ in (3.21), we obtain $M = 0$.

Now, we demonstrate that $\{x_j\}$ is Cauchy. Let $m, j > 0$ with $m > j$. Put $m = j + l$, hence

$$\begin{aligned}
\|x_j - x_m, a_{i_2}, \cdots, a_{i_n}\| =& \|x_j - x_{j+l}, a_{i_2}, \cdots, a_{i_n}\| \\
=& \|(x_j - x_{j+1}) + (x_{j+1} - x_{j+2}) \\
& + \cdots + (x_{j+l-1} - x_{j+l}), a_{i_2}, \cdots, a_{i_n}\| \\
\leq& \|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\| + \|x_{j+1} - x_{n+2}, a_{i_2}, \cdots, a_{i_n}\| \\
& + \cdots + \|x_{j+l-1} - x_{j+l}, a_{i_2}, \cdots, a_{i_n}\| \\
\leq& \lambda^j \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\| + \lambda^{j+1} \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\| \\
& + \cdots + \lambda^{j+l-1} \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\| \\
=& \lambda^j (1 + \lambda + \cdots + \lambda^{l-1}) \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\| \\
\leq& \frac{\lambda^j}{1 - \lambda} \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\|.
\end{aligned}$$

Letting $j, m \longrightarrow \infty$, we get

$$\lim_{m,j\to\infty} \|x_j - x_m, a_{i_2}, \cdots, a_{i_n}\| = 0.$$

Hence $\{x_j\}$ is Cauchy. Then $\{x_j\}$ converges to some $x \in \mathcal{F}$.

Now, we demonstrate that $x \in \mathcal{F}$ is a fixed point of S. Hence

$$\begin{aligned}
\|x_{j+1} - Sx, a_{i_2}, \cdots, a_{i_n}\| =& \|Sx_j - Sx, a_{i_2}, \cdots, a_{i_n}\| \\
\leq& \lambda \left[\frac{\|x_j - Sx_j, a_{i_2}, \cdots, a_{i_n}\| \|x - Sx, a_{i_2}, \cdots, a_{i_n}\|}{1 + \|x_j - x, a_{i_2}, \cdots, a_{i_n}\|} \right]^{\alpha} \\
& \cdot [\|x_j - x, a_{i_2}, \cdots, a_{i_n}\|]^{1-\alpha}.
\end{aligned}$$

Letting $j \longrightarrow \infty$, we get

$$\|Sx - x, a_{i_2}, \cdots, a_{i_n}\| = 0.$$

Then $Sx = x$, thus x is a fixed point of S. □

We finish this section with the following theorem.

Theorem 3.6. *Let $\mathcal{X}$ be an n-Banach space and $\mathcal{F} \subset \mathcal{X}$ be nonempty, closed, and bounded with respect to $\mathcal{A}$. Let S, T be continuous self mappings on $\mathcal{F}$ satisfying for each $x \in \mathcal{F} \setminus Fix(S), y \in \mathcal{F} \setminus Fix(T)$ with $x \neq y$ and $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$,*

$$\begin{aligned} \|Sx - Ty, a_{i_2}, \cdots, a_{i_n}\| \leq &\lambda \left[\frac{\|x - Sx, a_{i_2}, \cdots, a_{i_n}\| \|y - Ty, a_{i_2}, \cdots, a_{i_n}\|}{\|x - y, a_{i_2}, \cdots, a_{i_n}\|} \right]^{\alpha} \\ &\cdot [\|x - y, a_{i_2}, \cdots, a_{i_n}\|]^{1-\alpha}, \end{aligned} \tag{3.22}$$

where $\lambda \in (0, 1)$ and $\alpha \in \left(0, \frac{1}{2}\right]$. Then S and T have a common fixed point.

Proof. Let $x_0 \in \mathcal{F}$ and let $\{x_j\}$ be a sequence defined by $x_{2j+1} = Sx_{2j}$ and $x_{2j+2} = Tx_{2j+1}$ for each $j \in \mathbb{N}_0$. If there exists $j \in \mathbb{N}_0$ such that $x_{2j} = x_{2j+1} = x_{2j+2}$, then x_{2j} is a common fixed point of S and T, so assume that there does not exist three consecutive identical terms in the sequence $\{x_j\}$ and that $x_0 \neq x_1$. Using (3.22), we obtain

$$\begin{aligned} \|x_{2j+1} - x_{2j+2}, a_{i_2}, \cdots, a_{i_n}\| =& \|Sx_{2j} - Tx_{2j+1}, a_{i_2}, \cdots, a_{i_n}\| \\ \leq& \lambda \left[\frac{\|x_{2j} - Sx_{2j}, a_{i_2}, \cdots, a_{i_n}\| \|x_{2j+1} - Tx_{2j+1}, a_{i_2}, \cdots, a_{i_n}\|}{\|x_{2j} - x_{2j+1}, a_{i_2}, \cdots, a_{i_n}\|} \right]^{\alpha} \\ &\cdot \|x_{2j} - x_{2j+1}, a_{i_2}, \cdots, a_{i_n}\|^{1-\alpha} \\ \leq& \lambda \|x_{2j+1} - x_{2j+2}, a_{i_2}, \cdots, a_{i_n}\|^{\alpha} \|x_{2j} - x_{2j+1}, a_{i_2}, \cdots, a_{i_n}\|^{1-\alpha}. \end{aligned}$$

Then

$$\|x_{2j+1} - x_{2j+2}, a_{i_2}, \cdots, a_{i_n}\|^{1-\alpha} \leq \lambda \|x_{2j} - x_{2j+1}, a_{i_2}, \cdots, a_{i_n}\|^{1-\alpha}. \tag{3.23}$$

Hence

$$\begin{aligned} \|x_{2j+1} - x_{2j+2}, a_{i_2}, \cdots, a_{i_n}\| &\leq \lambda^{\frac{1}{1-\alpha}} \|x_{2j} - x_{2j+1}, a_{i_2}, \cdots, a_{i_n}\| \\ &\leq \lambda \|x_{2j} - x_{2j+1}, a_{i_2}, \cdots, a_{i_n}\|. \end{aligned} \tag{3.24}$$

From (3.24), we obtain for all $z \in \mathcal{F}$,

$$\begin{aligned} \|x_{2j+1} - x_{2j+2}, a_{i_2}, \cdots, a_{i_n}\| &\leq \lambda \|x_{2j} - x_{2j+1}, a_{i_2}, \cdots, a_{i_n}\| \\ &\leq \lambda^2 \|x_{2j-1} - x_{2j}, a_{i_2}, \cdots, a_{i_n}\| \\ &\leq \cdots \leq \lambda^{2j+1} \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\|. \end{aligned} \tag{3.25}$$

Then

$$\|x_{2j+1} - x_{2j+2}, a_{i_2}, \cdots, a_{i_n}\| \leq \lambda^{2j+1} \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\|. \tag{3.26}$$

Similarly,

$$\begin{aligned} \|x_{2j+1} - x_{2j}, a_{i_2}, \cdots, a_{i_n}\| =& \|Sx_{2j} - Tx_{2j-1}, a_{i_2}, \cdots, a_{i_n}\| \\ \leq& \lambda \left[\frac{\|x_{2j} - Sx_{2j}, a_{i_2}, \cdots, a_{i_n}\| \|x_{2j-1} - Tx_{2j-1}, a_{i_2}, \cdots, a_{i_n}\|}{1 + \|x_{2j} - x_{2j-1}, a_{i_2}, \cdots, a_{i_n}\|} \right]^{\alpha} \\ &\cdot [\|x_{2j-1} - x_{2j}, a_{i_2}, \cdots, a_{i_n}\|]^{1-\alpha} \\ =& \lambda \left[\frac{\|x_{2j} - x_{2j+1}, a_{i_2}, \cdots, a_{i_n}\| \|x_{2j-1} - x_{2j}, a_{i_2}, \cdots, a_{i_n}\|}{1 + \|x_{2j} - x_{2j-1}, a_{i_2}, \cdots, a_{i_n}\|} \right]^{\alpha} \\ &\cdot [\|x_{2j-1} - x_{2j}, a_{i_2}, \cdots, a_{i_n}\|]^{1-\alpha}. \end{aligned}$$

Then

$$\|x_{2j+1} - x_{2j}, a_{i_2}, \cdots, a_{i_n}\|^{1-\alpha} \leq \lambda \|x_{2j-1} - x_{2j}, a_{i_2}, \cdots, a_{i_n}\|^{1-\alpha}. \tag{3.27}$$

Hence

$$\begin{aligned} \|x_{2j+1} - x_{2j}, a_{i_2}, \cdots, a_{i_n}\| &\leq \lambda^{\frac{1}{1-\alpha}} \|x_{2j-1} - x_{2j}, a_{i_2}, \cdots, a_{i_n}\| &(3.28)\\ &\leq \lambda \|x_{2j-1} - x_{2j}, a_{i_2}, \cdots, a_{i_n}\|. &(3.29) \end{aligned}$$

Thus

$$\begin{aligned} \|x_{2j+1} - x_{2j}, a_{i_2}, \cdots, a_{i_n}\| &\leq \lambda \|x_{2j-1} - x_{2j}, a_{i_2}, \cdots, a_{i_n}\| \\ &\leq \lambda^2 \|x_{2j-2} - x_{2j-1}, a_{i_2}, \cdots, a_{i_n}\| \\ &\leq \cdots \leq \lambda^{2j} \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\|. \end{aligned} \tag{3.30}$$

Then

$$\|x_{2j+1} - x_{2j}, a_{i_2}, \cdots, a_{i_n}\| \leq \lambda^{2j} \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\|. \tag{3.31}$$

Using (3.26) and (3.31), we obtain

$$\|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\| \leq \lambda^j \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\|. \tag{3.32}$$

Now, by (3.32) we demonstrate that $\{x_j\}$ is Cauchy. Let $m, j > 0$ with $m > j$. Put $m = j + l$, hence

$$\begin{aligned} \|x_j - x_m, a_{i_2}, \cdots, a_{i_n}\| =& \|x_j - x_{j+l}, a_{i_2}, \cdots, a_{i_n}\| \\ =& \|(x_j - x_{j+1}) + (x_{j+1} - x_{j+2}) \\ &+ \cdots + (x_{j+l-1} - x_{j+l}), a_{i_2}, \cdots, a_{i_n}\| \\ \leq& \|x_j - x_{j+1}, a_{i_2}, \cdots, a_{i_n}\| + \|x_{j+1} - x_{j+2}, a_{i_2}, \cdots, a_{i_n}\| \\ &+ \cdots + \|x_{j+l-1} - x_{j+l}, a_{i_2}, \cdots, a_{i_n}\| \\ \leq& \lambda^j \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\| + \lambda^{j+1} \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\| \\ &+ \cdots + \lambda^{j+l-1} \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\| \\ =& \lambda^j (1 + \lambda + \cdots + \lambda^{l-1}) \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\| \\ \leq& \frac{\lambda^j}{1-\lambda} \|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\|. \end{aligned}$$

Letting $j, m \longrightarrow \infty$, we get

$$\lim_{m,j\to\infty} \|x_j - x_m, a_{i_2}, \cdots, a_{i_n}\| = 0.$$

Hence $\{x_j\}$ is Cauchy. Then $\{x_j\}$ converges to some $x \in \mathcal{F}$.

Now, we demonstrate that $x \in \mathcal{F}$ is a fixed point of S. Hence

$$\begin{aligned} \|Sx - x_{2j+2}, a_{i_2}, \cdots, a_{i_n}\| =& \|Sx - Tx_{2j+1}, a_{i_2}, \cdots, a_{i_n}\| \\ \leq& \lambda \left[\frac{\|x - Sx, a_{i_2}, \cdots, a_{i_n}\| \|x_{2j+1} - Tx_{2j+1}, a_{i_2}, \cdots, a_{i_n}\|}{\|x - x_{2j+1}, a_{i_2}, \cdots, a_{i_n}\|} \right]^\alpha \\ &\cdot [\|x - x_{2j+1}, a_{i_2}, \cdots, a_{i_n}\|]^{1-\alpha}. \end{aligned}$$

Letting $j \longrightarrow \infty$, we get

$$\|Sx - x, a_{i_2}, \cdots, a_{i_n}\| = 0.$$

So $Sx = x$. Similarly

$$\begin{aligned}\|x_{2j+1} - Tx, a_{i_2}, \cdots, a_{i_n}\| =& \|Sx_{2j} - Tx, a_{i_2}, \cdots, a_{i_n}\| \\ \leq& \lambda \left[\frac{\|x_{2j} - Sx_{2j}, a_{i_2}, \cdots, a_{i_n}\| \|x - Tx, a_{i_2}, \cdots, a_{i_n}\|}{\|x - x_{2j}, a_{i_2}, \cdots, a_{i_n}\|} \right]^{\alpha} \\ & \cdot [\|x - x_{2j}, a_{i_2}, \cdots, a_{i_n}\|]^{1-\alpha}.\end{aligned}$$

Letting $j \longrightarrow \infty$, we get $x = Tx$. Then $x = Sx = Tx$. □

4 Generalizations of results

Similarly to the proof of Theorem 3.1, we obtain:

Theorem 4.1. *Let $\mathfrak{X}$ be an n-Banach space. Let S, T be self mappings on $\mathfrak{X}$ satisfying for each $x \in \mathfrak{X} \setminus Fix(S), y \in \mathfrak{X} \setminus Fix(T)$ and $y_2, \cdots, y_n \in \mathfrak{X}$,*

$$\|Sx - Ty, y_2, \cdots, y_n\| \leq \lambda \|x - Sx, y_2, \cdots, y_n\|^{\alpha} \cdot \|y - Ty, y_2, \cdots, y_n\|^{1-\alpha}, \tag{4.1}$$

where $\lambda \in [0, 1)$ and $\alpha \in (0, 1)$. Then S and T have a common fixed point.

With a similar proof to that of Theorem 3.2, we obtain:

Theorem 4.2. *Let $\mathfrak{X}$ be an n-Banach space. Let S be a self mapping on $\mathfrak{X}$ satisfying for each $x \in \mathfrak{X} \setminus Fix(S)$ and $y_2, \cdots, y_n \in \mathfrak{X}$,*

$$\|Sx - Sy, y_2, \cdots, y_n\| \leq k_1 \|x - y, y_2, \cdots, y_n\| + k_2 \frac{\|x - Sx, y_2, \cdots, y_n\| \|y - Sy, y_2, \cdots, y_n\|}{1 + \|x - y, y_2, \cdots, y_n\|}, \tag{4.2}$$

where $k_1, k_2 \geq 0$ such that $k_1 + k_2 < 1$. Then S has a unique fixed point in $\mathfrak{X}$.

By the same arguments as in the proof of Theorem 3.3, we obtain:

Theorem 4.3. *Let $\mathfrak{X}$ be an n-Banach space. Let S be a self mapping on $\mathfrak{X}$ satisfying for each $x, y \in \mathfrak{X} \setminus Fix(S)$ and $y_2, \cdots, y_n \in \mathfrak{X}$,*

$$\|Sx - Sy, y_2, \cdots, y_n\| \leq k_1 \|x - y, y_2, \cdots, y_n\| + k_2 \frac{\|x - Sy, y_2, \cdots, y_n\| \|y - Sx, y_2, \cdots, y_n\|}{1 + \|x - y, y_2, \cdots, y_n\|} \tag{4.3}$$

where $k_1, k_2 \geq 0$ such that $k_1 + k_2 < 1$, then S has a unique fixed point in $\mathfrak{X}$.

Now, we introduce an interpolative Dass and Gupta rational type contraction in an n-normed space as follows.

Definition 4.1. *A continuous self-mapping S on an n-normed space $\mathfrak{X}$ is called an interpolative Dass and Gupta rational type contraction if there exist $\lambda \in (0, 1)$ and $\alpha \in \left(0, \frac{1}{2}\right]$ such that*

$$\begin{aligned}\|Sx - Sy, y_2, \cdots, y_n\| \leq& \lambda \left[\frac{\|x - Sx, y_2, \cdots, y_n\| \|y - Sy, y_2, \cdots, y_n\|}{\|x - y, y_2, \cdots, y_n\|} \right]^{\alpha} \\ & \cdot [\|x - y, y_2, \cdots, y_n\|]^{1-\alpha},\end{aligned} \tag{4.4}$$

for each $x, y \in \mathfrak{X} \setminus Fix(S)$ with $x \neq y$ and $y_2, \cdots, y_n \in \mathfrak{X}$, where $Fix(S) = \{u \in \mathfrak{X} : Su = u\}$.

By the same arguments as in the proof of Theorem 3.4, we obtain:

Theorem 4.4. *Let $\mathfrak{X}$ be an n-Banach space and let $S : \mathfrak{X} \longrightarrow \mathfrak{X}$ be an interpolative Dass and Gupta rational type contraction. Then S has a fixed point in $\mathfrak{X}$.*

By the same arguments as in the proof of Theorem 3.5, we obtain:

Theorem 4.5. *Let $\mathfrak{X}$ be an n-Banach space and let $S : \mathfrak{X} \longrightarrow \mathfrak{X}$ be a self mapping on $\mathfrak{X}$ satisfying for each $x, y \in \mathfrak{X} \setminus Fix(S)$ and $y_2, \cdots, y_n \in \mathfrak{X}$,*

$$\begin{aligned} \|Sx - Sy, y_2, \cdots, y_n\| \leq & \lambda \left[\frac{\|x - Sx, y_2, \cdots, y_n\| \|y - Sy, y_2, \cdots, y_n\|}{1 + \|x - y, y_2, \cdots, y_n\|} \right]^{\alpha} \\ & \cdot [\|x - y, y_2, \cdots, y_n\|]^{1-\alpha}, \end{aligned} \tag{4.5}$$

where $\lambda \in [0, 1)$ and $\alpha \in (0, 1)$. Then S has a unique fixed point.

By the same arguments as in the proof of Theorem 3.6, we obtain:

Theorem 4.6. *Let $\mathfrak{X}$ be an n-Banach space and let S, T be continuous self mappings on $\mathfrak{X}$ satisfying for each $x \in \mathfrak{X} \setminus Fix(S)$, $y \in \mathfrak{X} \setminus Fix(T)$ with $x \neq y$ and $y_2, \cdots, y_n \in \mathfrak{X}$,*

$$\begin{aligned} \|Sx - Ty, y_2, \cdots, y_n\| \leq & \lambda \left[\frac{\|x - Sx, y_2, \cdots, y_n\| \|y - Ty, y_2, \cdots, y_n\|}{\|x - y, y_2, \cdots, y_n\|} \right]^{\alpha} \\ & \cdot [\|x - y, y_2, \cdots, y_n\|]^{1-\alpha}, \end{aligned} \tag{4.6}$$

where $\lambda \in (0, 1)$ and $\alpha \in \left(0, \frac{1}{2}\right]$. Then S and T have a common fixed point.

References

[1] J. Ettayb, *Fixed point theorems for some mappings in 2-Banach spaces*, Math. Anal. Contemp. Appl. **7** (3) (2025), 67-75.

[2] J. Ettayb, *On interpolative Boyd-Wong and Matkowski type contractions in 2-Banach spaces*, Eur. J. Math. Appl. **5** (2025), Article ID 15, 10 pp.

[3] R. W. Freese, Y. J. Cho, *Geometry of linear 2-normed spaces*, Nova Publishers, Inc., New York, 2001.

[4] S. Gähler, *Untersuchungen uber verallgemeinerte m-metrische räume I*, Math. Nachr. **40** (1969), 165-189.

[5] S. Gähler, *Untersuchungen uber verallgemeinerte m-metrische räume II*, Math. Nachr. **40** (1969), 229-264.

[6] S. Gähler, *Untersuchungen uber verallgemeinerte m-metrische räume III*, Math. Nachr. **41** (1970), 23-26.

[7] H. Gunawan, O. Neswan, E. Sukaesih, *Fixed point theorems on bounded sets in an n-normed space*, J. Math. Anal. **6** (3) (2015), 51-58.

[8] P. K. Harikrishnan, K. T. Ravindran, *Some properties of accretive operators in linear 2-normed spaces*, Int. Math. Forum **6** (59) (2011), 2941-2947.

[9] R. Kannan, *Some results on fixed points*, Bull. Calcutta Math. Soc. **60** (1968), 71-76.

[10] M. Kir, H. Kiziltunc, *Some new fixed point theorems in 2-normed spaces*, Int. J. Math. Anal. **58** (7) (2013), 2885-2890.

[11] M. Noorwali, *Common fixed point for Kannan type contractions via interpolation*, J. Math. Anal. **9** (6) (2018), 92-94.

[12] A. White, 2-*Banach spaces*, Math. Nachr. **42** (1969), 43-60.

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