sciendo

ASN, Vol. 12, No 1, Pages 73–80, 2025

Acta Scientifica Naturalis

Former Annual of Konstantin Preslavsky University of Shumen: Chemistry, Physics, Biology, Geography

Journal homepage: https://content.sciendo.com/view/journals/asn/asn-overview.xml

A new look at a second basic geodetic task at short distances

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Abstract: *The article presents new formulas for calculating directional angle* α_{AB}. *Through the formulas presented, the directional angle* α_{AB} *is directly calculated, without using quadrants. It also avoids cases in which, if* $\Delta X_{AB} = 0$, *the problem had no solution and additional requirements had to be applied to solve it.*

Keywords: Second basic geodetic task, coordinates, directional angle, distance, quadrants

Introduction

The most common tasks in geodesy are related to the calculation of coordinates, distances and directional angles. When it comes to no more than two points, the first and second basic geodetic tasks at short distances are most often used. With a larger number of points, different methods are used: polygon moves, geodetic notches, different types of alignments, etc. [1-6].

The topic of the article is related to the second basic geodetic task at short distances, in which the distance and the directional angle are calculated from the coordinates of two points.

Theoretical staging

1. A standard way to solve a second basic geodetic task at short distances

A second basic geodetic task at short distances is used to calculate the distance ($\boldsymbol{S_{AB}}$) and directional angle ($\propto_{\boldsymbol{AB}}$), using the coordinates of two known points – point **A** and point **B** (**Figure 1**).

From Figure 1, it can be seen that solving the problem is reduced to solving a triangle.

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DOI: 10.2478/asn-2025-0008

$$tan \propto'_{AB} = \frac{\Delta Y_{AB}}{\Delta X_{AB}} \quad (1)$$

$$\propto'_{AB} = arc\, tan \frac{\Delta Y_{AB}}{\Delta X_{AB}} \quad (2)$$

knowing that:

$$\Delta Y_{AB} = Y_B - Y_A \quad (3)$$

$$\Delta X_{AB} = X_B - X_A \quad (4)$$

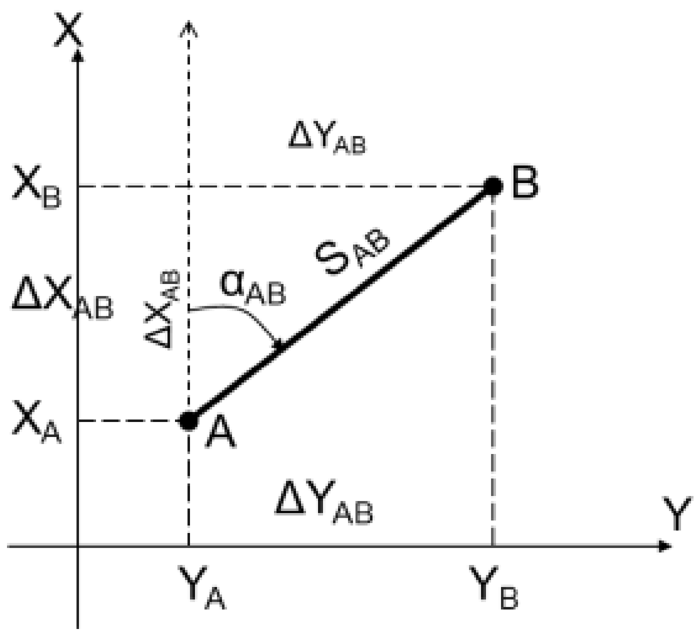

Figure 1. Second basic geodetic task at short distances

It should be borne in mind that the angle thus calculated $\propto'_{AB}$ is not the angle sought $\propto_{AB}$. Since coordinate increments (ΔY_{AB}, ΔX_{AB}) can have positive (+) and negative (-) values, then the directional angle can be positive or negative. In order to avoid negative values and calculate the desired directional angle, we should always bring the angle calculated according to formulas (1) and (2) to the first quadrant, according to the signs of the coordinate increments.

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The alignment is determined using **Table 1**.

Table 1. Table for calculating the Directional angle $\propto_{AB}$

Sign on		*Quadrant*	*Directional angle* $\propto_{AB}$
ΔX_{AB}	ΔY_{AB}		
$+$	$+$	*I*	$\propto_{AB}=\propto'_{AB}$
$-$	$+$	*II*	$\propto_{AB}=(-\propto'_{AB})+200^g$
$-$	$-$	*III*	$\propto_{AB}=(+\propto'_{AB})+200^g$
$+$	$-$	*IV*	$\propto_{AB}=(-\propto'_{AB})+400^g$

After calculating the directional angle, we proceed to calculate the distance.

$$S_{AB} = \frac{\Delta Y_{AB}}{sin \propto_{AB}} = \frac{\Delta X_{AB}}{cos \propto_{AB}} \tag{5}$$

$$S_{AB} = \sqrt{\Delta Y_{AB}^2 + \Delta X_{AB}^2} \tag{6}$$

A problem arises when $\boldsymbol{\Delta X_{AB} = 0}$. Then Formulas (1) and (2) has no solution. In this case, additional conditions apply, namely: if $\boldsymbol{\Delta X_{AB} = 0}$ and $\boldsymbol{\Delta Y_{AB} > 0}$ then the $\boldsymbol{\propto_{AB}= 100^g}$, if $\boldsymbol{\Delta Y_{AB} < 0}$ then the $\boldsymbol{\propto_{AB}= 300^g}$.

2. A new way to solve a second basic geodetic task at short distances

To derive the new formulas for solving the problem, instead of the trigonometric **tan** function, the **sin** function is used. As can be seen in **Figures 2** through **5**, we first calculate the angle $\boldsymbol{\alpha}$ using formula (7) and then we find the directional angle $\boldsymbol{\alpha_{AB}}$.

$$\alpha^g = \left(arcsin\frac{\Delta X_{AB}}{S_{AB}}\right)^g \tag{7}$$

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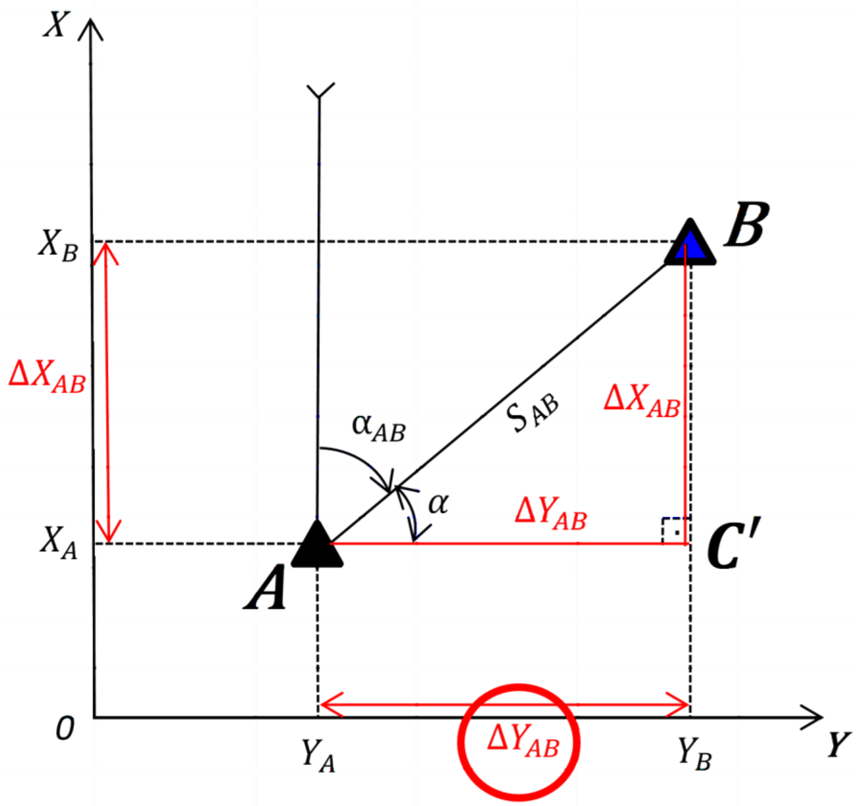

Figure 2. Second basic geodetic task (first quadrant)

$$S_{AB} = \sqrt{(\Delta X_{AB})^2 + (\Delta Y_{AB})^2} \tag{8}$$

$$\alpha_{AB} = 100^g - \left(arcsin\frac{\Delta X_{AB}}{S_{AB}}\right)^g \tag{9}$$

In the first quadrant $\boldsymbol{\Delta X_{AB} > 0}$, angle $\boldsymbol{\alpha}$ has a positive value. As you can see in **Figure 2**, if we subtract the angle $\boldsymbol{\alpha}$ from $\boldsymbol{100^g}$, we will get the desired directional angle (Formula 9).

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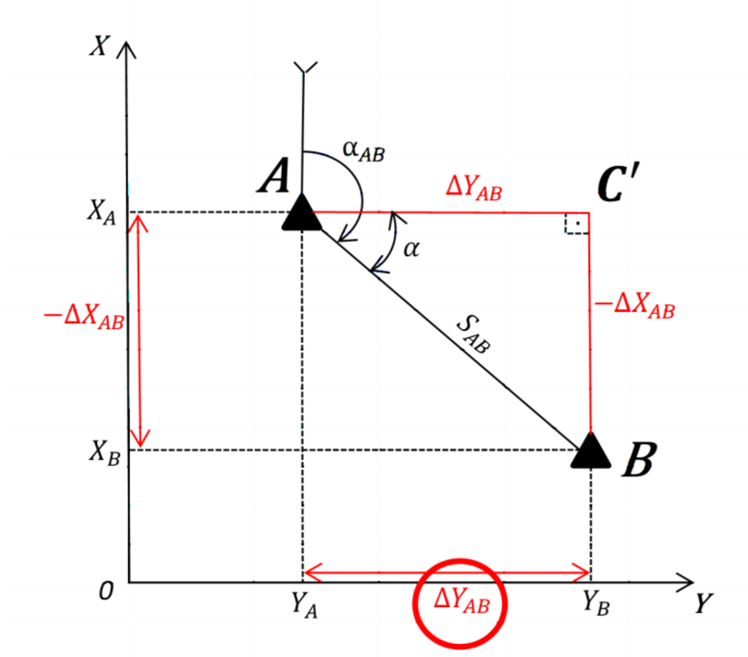

Figure 3. Second basic geodetic task (second quadrant)

In the second quadrant $\Delta \boldsymbol{X_{AB}} < \mathbf{0}$, angle $\boldsymbol{\alpha}$ has a negative value. As you can see in **Figure 3**, if we subtract the angle $-\boldsymbol{\alpha}$ from $\mathbf{100^g}$, we will get the desired directional angle (Formula 9).

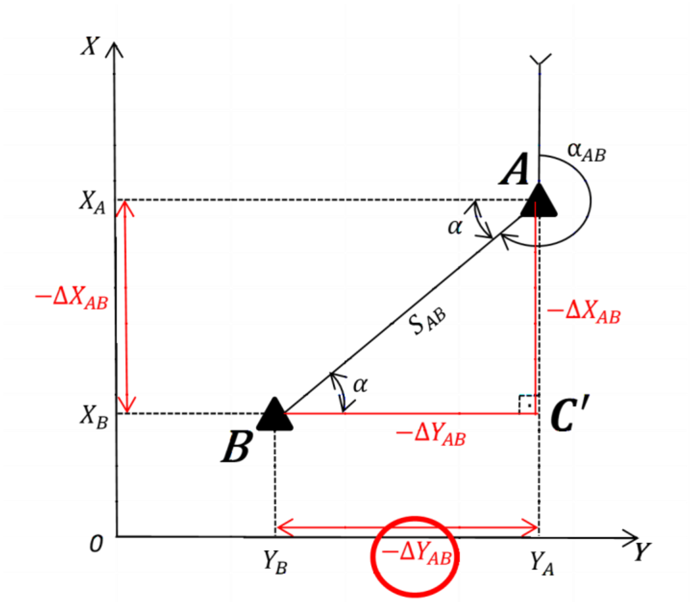

Figure 4. Second basic geodetic task (third quadrant)

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In the third quadrant $\Delta \boldsymbol{X}_{\boldsymbol{AB}} < \mathbf{0}$, angle $\boldsymbol{\alpha}$ has a negative value. As you can see in **Figure 4**, if we add the angle $-\boldsymbol{\alpha}$ to $\mathbf{300^g}$, we will get the desired directional angle (Formula 10).

$$\alpha_{AB} = 300^g + \left(arcsin\frac{\Delta X_{AB}}{S_{AB}}\right)^g \tag{10}$$

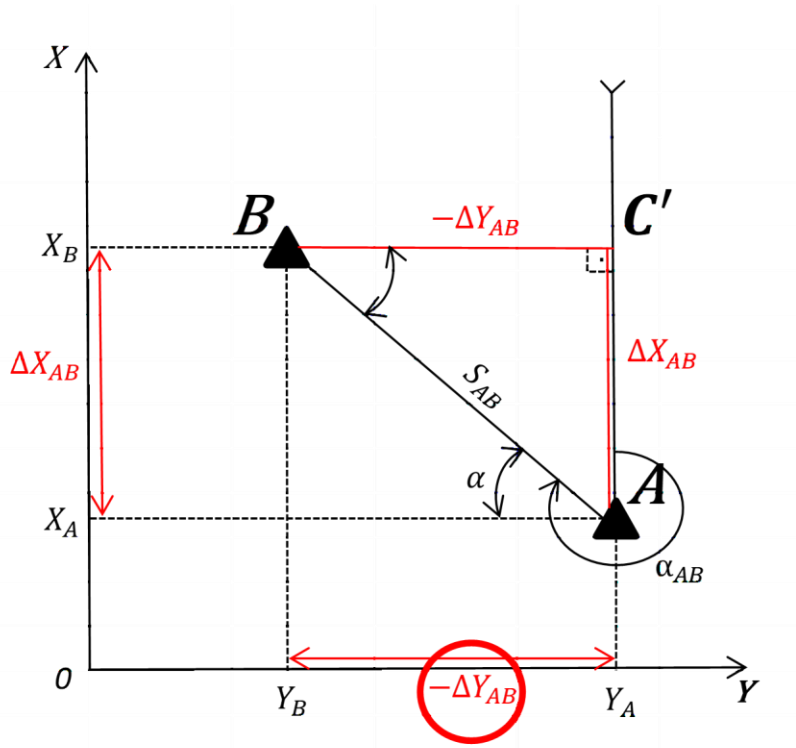

Figure 5. Second basic geodetic task (fourth quadrant)

In the fourth quadrant $\Delta \boldsymbol{X}_{\boldsymbol{AB}} > \mathbf{0}$, angle $\boldsymbol{\alpha}$ has a positive value. As you can see in **Figure 5**, if we add the angle $\boldsymbol{\alpha}$ to $\mathbf{300^g}$, we will get the desired directional angle (Formula 10).

Results and Discussion

Because it becomes clear that in the first and second quadrants one formula can be used, and in the third and fourth another. In short, as a conclusion of the above, it can be said that:

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$$\left.\begin{array}{c} \text{If} \quad \Delta Y_{AB} \geqq 0 \\ \alpha_{AB} = 100^g - \left(arcsin\dfrac{\Delta X_{AB}}{S_{AB}}\right)^g \\ \\ \text{If} \quad \Delta Y_{AB} < 0 \\ \alpha_{AB} = 300^g + \left(arcsin\dfrac{\Delta X_{AB}}{S_{AB}}\right)^g \\ \\ S_{AB} = \sqrt{(\Delta X_{AB})^2 + (\Delta Y_{AB})^2} \end{array}\right] \tag{11}$$

Conclusion

The proposed formulas (8) for solving the Second basic geodetic task at short distances have the following advantages:

1. The directional angle $\boldsymbol{\alpha_{AB}}$ is calculated directly;
2. You don't have to use the quadrants;
3. There are no cases in which the task has no solution;
4. The formulas can also be used for $\Delta \boldsymbol{Y_{AB}} = \mathbf{0}$;
5. The time for solving the task is significantly reduced.

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