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Deciphering Soil-Plant and Soil-Insect Interactions: A Mathematical Modeling Approach

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Abstract: *The Pev and Rv functions were introduced as a way of summarizing the mechanical phenomenon of root-soil interactions and the effect of root growth on soil volume. The Mohr-Coulomb criterion was applied to the Rv function to demonstrate the relationship between root morphology, soil mechanical properties, and soil stability. The theory of elasticity and plasticity was applied in the analysis of the Bio-Geo Interface, which showed that the force F generated by the roots increased with time due to the growth of the roots and the increase in their volume. The forces acting on the Bio-Geo Interface were represented mathematically through the use of vector arithmetic, which helped to calculate the direction of the forces acting on the system. The Pev function was used to study the evolution of the plant and how it affected the Rv function. The results showed that the interplay between plant growth and soil mechanics is crucial in understanding the behavior of the Bio-Geo Interface, and can provide valuable insights into the interactions between plants and soil in natural and engineered systems. In the study of soil-insect interaction, a mathematical function, Iev, was proposed to represent the insect's evolution in the soil environment. The function takes into account factors such as food sources and void ratio, which are important indicators of the biological and geotechnical aspects of the soil respectively. Despite the potential impact of insects on the soil environment, it has been concluded that their force resulting from their movement and behavior is negligible according to theories of elasticity and plasticity. This means that the impact of insects on the soil is minimal and does not significantly affect the soil's mechanical properties. However, the biological aspects of the soil, such as food sources, remain important factors in the study of insect evolution in the soil environment.*

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1. Introduction

Geotechnical Biology refers to the study of biological processes and organisms in geotechnical systems, such as soil and rock mechanics [1]. Continuum Mechanics is a branch of physics that deals with the continuous behavior of matter, typically at a macroscopic scale. The combination of Geotechnical Biology and Continuum Mechanics can be useful in understanding the interactions between living organisms and geotechnical systems. For example, plant roots can affect soil structure, while soil mechanics can influence the growth and stability of plants. These parameters interact with each other and with other factors to determine the elasto-plasticity of soil-plant interactions, and understanding these interactions is essential for predicting and managing changes in soil-plant systems.

The evolution of plants is influenced by both geotechnical and biological factors. Geotechnical indices, such as soil strength, soil moisture content, and soil structure, affect root growth by influencing the plant's ability to penetrate the soil and obtain water and nutrients. On the other hand, biological indices, such as plant species, age, and nutrient status, also play a role in root evolution by affecting root growth and development. For example, some plant species have evolved specialized root structures, such as root hairs or lateral roots, that allow them to better adapt to specific geotechnical conditions. Additionally, the nutrient status of a plant can influences the development of its root system, as the plant will allocate more resources to root growth if it is deficient in certain nutrients [2]. Overall, the interplay between geotechnical and biological indices can lead to a complex and dynamic system, where the evolution of roots is shaped by the combined effects of these factors.

2. Soil Structure Properties

Soil structure refers to the arrangement of soil particles, such as sand, silt, and clay, and the pore spaces between them [3]. Soil structure is important because it affects the physical and hydraulic properties of the soil and influences its ability to support vegetation and structures [4]. Different types of soil have unique physical properties that are related to their composition and structure. Some of the common physical properties of soil include:

- Texture: Refers to the proportion of sand, silt, and clay particles in the soil. Soils can be classified as sandy, silty, or clayey based on their texture.
- Structure: Refers to the arrangement of soil particles and the pore spaces between them. Soils can have different structures, such as granular, blocky, platy, prismatic, and columnar.

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- Density: Refers to the mass of soil per unit volume and is often related to soil compaction. Denser soils have higher dry bulk densities and lower porosity.
- Porosity: Refers to the amount of void space in the soil, which includes both macro-pores (such as cracks and fissures) and micro-pores (such as the spaces between soil particles). Porosity affects the soil's ability to store water and exchange gases.
- Permeability: Refers to the ability of water to flow through the soil and is related to soil structure and porosity. Soils with high permeability allow water to flow through easily, while soils with low permeability have limited water flow.
- Compressibility: Refers to the soil's ability to be compressed or deformed under stress and is related to soil density and structure. Soils with high compressibility are easily compacted, while soils with low compressibility are more resistant to compaction.
- Shear strength: Refers to the soil's ability to resist shearing or sliding and is related to soil structure, density, and water content. Soils with high shear strength are more resistant to erosion and can support greater loads.

The physical properties of soil can play a significant role in shaping the evolution of roots, and vice versa [5]. The physical properties of soil, such as void ratio, density, soil particle diameter, solid particle density, coefficient of uniformity, and coefficient of curvature [6], can have a significant impact on root evolution.

- Void ratio: The void ratio, or the ratio of empty space to solid material in a soil, affects root growth and development by determining the amount of available space for root growth. Soils with higher void ratios are typically more conducive to root growth and development.
- Density: Soil density affects root growth and development by affecting the soil's mechanical resistance to root penetration. Soils with lower densities are typically more conducive to root growth and development.
- Soil particle diameter: The size of soil particles affects root growth and development by determining the size of soil pores, which can affect water and nutrient uptake by roots. Soils with larger particle diameters tend to have larger pores, making it easier for roots to access water and nutrients.
- Solid particle density: The density of solid particles in soil affects root growth and development by determining the soil's mechanical resistance to root penetration. Soils with lower solid particle densities tend to be easier for roots to penetrate.
- Coefficient of uniformity: The coefficient of uniformity, which measures the range of particle sizes in a soil, affects root growth and development by determining the size and distribution of soil pores.

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Soils with a higher coefficient of uniformity tend to have a more uniform distribution of soil pores, which can be more conducive to root growth and development.

- Coefficient of curvature: The coefficient of curvature, which measures the shape of soil particles, affects root growth and development by determining the size and distribution of soil pores. Soils with a higher coefficient of curvature tend to have more irregularly shaped soil particles, which can result in a more complex and varied distribution of soil pores, affecting root growth and development.

These physical properties of soil interact with each other and with other factors, such as soil water and nutrient availability, to determine the suitability of a soil for root growth and development. Understanding these interactions is essential for predicting and managing changes in root growth and development in soil-plant systems [7]. The properties of soil, such as texture, structure, porosity, and water-holding capacity, can affect the growth and development of roots, as well as their ability to access water and nutrients. For example, dense, compacted soils can limit root growth, while well-structured soils with high porosity can allow roots to penetrate deeper into the soil profile. In turn, the presence of roots can influence the physical properties of soil by altering its structure, porosity, and water-holding capacity. For instance, the growth and expansion of roots can create channels that improve soil aeration and water permeability, while the release of exudates and other organic matter by roots can increase soil fertility and microbial activity. This interplay between the physical properties of soil and the evolution of roots is part of the dynamic and complex interactions that occur at the Bio-Geo Interface and help to regulate the functioning of terrestrial ecosystems.

The mechanical properties of soil, such as strength, stiffness, and hardness, can also play a role in shaping the evolution of roots and their ability to grow and penetrate the soil. Soil strength and stiffness, which describe the resistance of soil to deformation and breaking, can affect the ability of roots to penetrate and anchor in the soil. For instance, roots may have difficulty growing in soils that are too hard or too stiff, while softer, more malleable soils may allow for greater root growth and expansion. The mechanical properties of soil, such as cohesion, shear strength, stiffness, deformation, and constraints, can have a significant impact on root evolution:

- Cohesion: Cohesion refers to the internal bonding strength of soil particles and affects root growth and development by determining the soil's resistance to root penetration. Soils with higher cohesion tend to be more difficult for roots to penetrate, while soils with lower cohesion are typically more conducive to root growth and development.
- Shear strength: Shear strength refers to the ability of soil to resist failure when subjected to shear stress and affects root growth and development by determining the soil's resistance to root penetration. Soils with higher shear strengths tend to be more difficult for roots to penetrate, while soils with lower shear strengths are typically more conducive to root growth and development.

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- Stiffness: Stiffness refers to the resistance of soil to deformation when subjected to stress and affects root growth and development by determining the soil's resistance to root penetration. Soils with higher stiffness tend to be more difficult for roots to penetrate, while soils with lower stiffness are typically more conducive to root growth and development.
- Deformation: Deformation refers to the amount of change in soil volume when subjected to stress and affects root growth and development by determining the soil's ability to adjust to changes in root growth and development. Soils with lower deformation are typically more conducive to root growth and development, while soils with higher deformation may be more resistant to root growth and development.
- Constraints: Constraints refer to physical or mechanical impediments to root growth and development, such as rocks or hardpan layers, and affect root growth and development by limiting the available space for root growth. Soils with more constraints tend to be less conducive to root growth and development, while soils with fewer constraints are typically more conducive to root growth and development.

These mechanical properties of soil interact with each other and with other factors, such as soil water and nutrient availability, to determine the suitability of a soil for root growth and development [8]. Understanding these interactions is essential for predicting and managing changes in root growth and development in soil-plant systems. On the other hand, the growth and expansion of roots can also alter the mechanical properties of soil by changing its strength, stiffness, and hardness. For example, roots can create voids and channels in the soil, reducing soil strength and stiffness, while the accumulation of roots and organic matter can increase soil hardness and stability. This interaction between the mechanical properties of soil and the evolution of roots is another example of the dynamic interplay that occurs at the Bio-Geo Interface and helps regulate the functioning of terrestrial ecosystems. In mechanical perspective, plant roots also have several important functions;

- Anchoring: Plant roots help anchor the plant in the soil and prevent it from being knocked over by wind or other external forces.
- Soil penetration: The growth and elongation of roots can help plants penetrate compacted soil and access deeper water and nutrient resources.
- Water uptake: The ability of roots to absorb water is essential for plant survival, especially in conditions of drought. Roots are capable of absorbing water due to the presence of specialized structures such as root hairs and Casparian strips.
- Tension and compression: Roots must withstand both tension and compression forces in order to anchor the plant and absorb water and nutrients from the soil. The mechanical strength of roots is

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determined by the arrangement of their cells and the presence of specialized structures such as lignified secondary walls.

- Deformation and bending: Plant roots are capable of deforming and bending in response to changes in soil moisture and other environmental conditions, allowing them to optimize their position for water and nutrient uptake.

Overall, the mechanical properties of plant roots play a critical role in their ability to perform essential functions such as anchoring, soil penetration, water uptake, and nutrient acquisition.

3. Mathematical Modeling of Soil-Plant Interaction (SPI)

In terms of creating an algorithm to define the rheology and mechanism of this phenomenon, it would be necessary to consider additional factors and processes involved in root-soil interactions, such as soil strength, water content, root morphology, and nutrient availability. To build an accurate model, it would likely require a comprehensive understanding of these factors and the development of mathematical models and software tools that can simulate root-soil interactions. It may be possible to develop a numerical model to simulate root-soil interactions, which could then be used to predict the impact of different soil properties and conditions on root growth. This could involve using software tools such as finite element analysis, computational fluid dynamics, or discrete element analysis, and incorporating empirical data and experimental results to validate the model.

The relationship between plant evolution, biological index, and geotechnical index can be complex and multifaceted. However, based on the extensive research we have done, a general equation summarizing the phenomenon of root-soil interactions and the effect of soil properties on root growth could be represented as follows:

$$\mathrm{Pev} = F(\mathrm{Bi}, \mathrm{Gi}) = F(\text{suction, void ratio}) \qquad (1)$$

where Pev represents plant evolution, Bi represents the biological index, and Gi represents the geotechnical index.

The function F describes the relationship between these variables and suction and void ratio are key factors affecting the process. Suction refers to the pressure difference between the soil solution and the air in the soil pores. It can be expressed as a matric potential, which is the energy required to remove water from the soil to the air, and is typically measured in units of kilopascals (kPa). The suction experienced by plant roots can influence their growth and water uptake, and can be affected by factors such as soil water content, soil structure, and soil type. The void ratio is a measure of the volume of voids (empty spaces) in a soil sample

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relative to the volume of solid material. It can be expressed as the ratio of the volume of voids to the volume of solids, and is typically represented by the symbol "e". The void ratio can affect the mechanical behavior of soil, including soil compaction, strength, and deformation, and can influence root growth and water uptake. With these definitions in mind, the equation Pev = F(Bi, Gi) = F(suction, void ratio) can be written as:

$$\text{Pev} = F(\text{Bi}, \text{Gi}) = F(\text{matric potential, void ratio}) \quad \text{(1-a)}$$

where matric potential is the suction experienced by the roots and void ratio is a measure of the volume of voids in the soil.

The function F describes the relationship between these variables and plant evolution, and takes into account the influence of soil water content, soil structure, and soil type on root growth and water uptake. Matric potential, also known as soil water potential, is a measure of the energy required to extract water from the soil. It is an important factor that affects the water uptake of plant roots, and is influenced by factors such as soil water content, soil structure, and soil type. Matric potential is typically expressed in units of pressure, such as kilopascals (kPa), and is defined as the pressure difference between the soil solution and the air in the soil pores. It can be calculated as the difference between the total soil water potential and the gravitational water potential, which is proportional to the height of water in the soil. The matric potential is a negative value, as it takes energy to remove water from the soil, and is a measure of the suction experienced by the roots. The relationship between matric potential and suction can be represented mathematically as:

$$\text{Matric potential } (\Psi m) = -\text{ suction} \quad (2)$$

where matric potential is expressed in units of pressure, and suction is a measure of the energy required to remove water from the soil. The negative sign indicates that matric potential is a negative value, as it takes energy to remove water from the soil.

Matric potential, suction, and interstitial pressure are all related concepts in soil-root interactions [9]. Matric potential is a measure of the energy required to extract water from the soil and is an important factor that affects the water uptake of plant roots. Suction refers to the pressure difference between the soil solution and the air in the soil pores, and interstitial pressure is the pressure in the spaces between soil particles. The relationship between matric potential, suction, and interstitial pressure can be defined mathematically as follows:

$$\text{Suction (s)} = \rho g \Delta h + \Psi m \quad \text{(2-a)}$$

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where ρ is the density of water, g is the acceleration due to gravity, Δh is the height difference between the soil solution and the air in the soil pores, and Ψm is the matric potential.

The interstitial pressure can influence root growth and development, as well as water and nutrient uptake by roots. Positive interstitial pressure can create a tension that can limit root expansion and water uptake, while negative interstitial pressure can increase water and nutrient availability and facilitate root growth. The interstitial pressure can be affected by several factors, including soil texture and structure, water content, and plant transpiration. In turn, root growth and development can also affect interstitial pressure, by creating channels and voids in the soil and altering soil structure and porosity. In summary, the interstitial pressure is a key aspect of the complex interactions between soil mechanics and root evolution. Further research is needed to better understand the dynamics of interstitial pressure and how it can be used to improve soil and root health, as well as to support sustainable land use and management practices. The interstitial pressure (Ψi) is defined as the pressure in the spaces between soil particles [10] and can be expressed as:

$$\Psi i = \rho g \Delta z \qquad (3)$$

where Δz is the height difference between the soil solution and the reference point.

The relationship between matric potential, suction, and interstitial pressure can be represented graphically using a soil water potential diagram, where the matric potential is plotted along the vertical axis and the height difference along the horizontal axis. The relationship between the three concepts can be described by the soil water retention curve, which describes the relationship between soil water content and soil water potential.

To represent the relationship between soil parameters and plant root parameters, it is possible to use mathematical models that incorporate factors such as root diameter, root length, root density, soil structure, soil type, soil water content, and soil strength. These models can help to understand the complex interactions between roots and soil, and can be used to predict plant growth and water uptake under different soil and climatic conditions. Root-soil interactions and their impact on plant evolution is a complex and interdisciplinary field that involves many variables, and it is challenging to describe the relationship between all these variables with a single equation. However, here's a general overview of some of the factors that can be included in a mathematical model to describe root-soil interactions:

- Root diameter: The diameter of roots can affect the soil water uptake, as wider roots can extract more water from the soil than smaller roots.

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- Root length: The length of roots can influence the amount of water and nutrients that the plant can absorb from the soil.
- Root density: The density of roots in the soil can affect the rate of water uptake and the distribution of roots in the soil.
- Soil structure: The structure of soil can impact root growth, as soil structure can affect root penetration and root water uptake. Soil structure can also affect the distribution of water and nutrients in the soil.
- Soil type: Different soil types have different physical and chemical properties that can affect root growth, water uptake, and nutrient availability.
- Soil water content: The amount of water in the soil can impact root growth and water uptake, as soil water content can influence soil water potential and soil suction.
- Soil strength: Soil strength can affect root penetration, root growth, and root water uptake, as stronger soils can provide more resistance to root growth.

We could summarize the mathematical relationships between interstitial pressure, root suction, void ratio, and soil water content as following equations. The root suction (h) is defined as the pressure difference between the root and the soil, which drives water uptake by the roots. The mathematical relationship between interstitial pressure and root suction is given by:

$$h = -\Psi \qquad (4)$$

where h is in units of pressure (e.g., kPa) and Ψ is in units of pressure (e.g., kPa). A negative value for h indicates that the root suction is pulling water out of the soil, while a positive value indicates that water is being pushed into the soil.

Void ratio and interstitial pressure: The void ratio (e) is defined as the ratio of the volume of voids (empty spaces) in the soil to the volume of solid material. The mathematical relationship between void ratio and interstitial pressure is given by:

$$\Psi = \gamma\,(1 - e) \qquad (5)$$

where γ is the unit weight of water and e is the void ratio, both in units of length (e.g., m).

Suction and void ratio: The relationship between suction and void ratio can be derived from the relationships between suction, interstitial pressure, and void ratio. Substituting the expression for interstitial pressure (Ψ) from the second equation into the first equation gives:

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$h = -\gamma (1 - e)$ (6)

where h is in units of pressure (e.g., kPa) and γ is the unit weight of water, both in units.

The time can be included in the equations to represent changes in the soil water content, interstitial pressure, root suction, and void ratio over time. Here are the equations with time included:

Interstitial pressure and root suction with time:

$h(t) = -\Psi(t)$ (4-a)

where h(t) is the root suction at time t and Ψ(t) is the interstitial pressure at time t, both in units of pressure (e.g., kPa).

Void ratio and interstitial pressure with time:

$\Psi(t) = \gamma (1 - e(t))$ (5-a)

where Ψ(t) is the interstitial pressure at time t, γ is the unit weight of water, and e(t) is the void ratio at time t, all in units of length (e.g., m).

Suction and void ratio with time:

$h(t) = -\gamma (1 - e(t))$ (6-a)

where h(t) is the root suction at time t, γ is the unit weight of water, and e(t) is the void ratio at time t, all in units of length (e.g., m).

Note that in each of these equations, the variables with time (h(t), Ψ(t), and e(t)) can change with time, depending on factors such as water uptake by roots, soil water content, and soil structure.

So, here is the equation for the plants' evolution function Pev with time included:

$Pev(t) = F(Bi(t), Gi(t))$ (1-b)

where Pev(t) is the plants' evolution at time t, Bi(t) is the biological index at time t, and Gi(t) is the geotechnical index at time t.

Bi(t) can be described as a function of root suction at time t, and Gi(t) can be described as a function of void ratio at time t, as follows:

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$$Bi(t) = f(h(t)) \quad (7)$$

$$Gi(t) = g(e(t)) \quad (8)$$

where f and g are functions describing the relationship between root suction and the biological index, and void ratio and the geotechnical index, respectively.

Therefore, the final equation for the plants' evolution function Pev with time included is:

$$Pev(t) = F(f(h(t)), g(e(t))) = F(-\Psi(t), 1 - e(t)) \quad (1\text{-}c)$$

A model that incorporates all of these factors to describe root-soil interactions would be complex, and the exact form of the model would depend on the specific system being studied. However, a mathematical model of root-soil interactions could be represented as a set of equations that describe the relationship between root diameter, root length, root density, soil structure, soil type, soil water content, and soil strength, as well as the relationship between these variables and plant evolution (Pev). However, it is important to note that this is a complex and interdisciplinary field, and many factors that are not included in this simple overview may also be important in describing root-soil interactions and plant evolution. In addition, the relationships between these factors may be non-linear, and the model would need to account for this complexity in order to accurately describe root-soil interactions and plant evolution.

To include the effects of agricultural machinery and human loads on the BIO-GEO Interface, we can modify the function Pev to take into account these external loads. One way to do this is to include terms that represent the magnitude and direction of these loads, as well as the soil's resistance to these loads. This can be done using soil strength parameters, such as shear strength and compression strength, and incorporating these into the equation for Pev. Additionally, we can include factors that describe the soil's response to these loads, such as soil deformation and soil displacement, which can be modeled using constitutive equations and numerical methods. To ensure that the model accurately reflects the complex and dynamic nature of root-soil interactions, it may also be necessary to incorporate time-dependent processes, such as soil moisture dynamics and root growth, into the equation for Pev. Incorporating external loads into the function Pev that describes the evolution of plants would likely require adding additional factors to the equation that account for the effects of these loads. Some possible factors to consider include the magnitude and direction of the loads, the soil strength and stiffness, the soil water content, and the root system geometry and distribution. The exact form of the equation would depend on the specifics of the system being modeled and the specific loads being

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considered. It would be necessary to consult the relevant literature and conduct additional research to determine the best approach for incorporating these factors into the equation for Pev.

Incorporating the height (H), width (D), and depth (Z) of the Bio-Geo Interface (BGi) into the Plants' evolution (Pev) function, as well as taking into account the machinery loads and strength and strain factors, would require considering these parameters as additional inputs to the function. One way to do this could be by using mechanics of materials principles, where the BGi is modeled as a beam subject to bending and shear stresses due to the machinery loads. The plants' roots would then be modeled as additional loads acting on the BGi. The bending stress in the BGi can be calculated using the equation:

$$\sigma = M * y / I \tag{9}$$

where M is the bending moment, y is the distance from the neutral axis to the point where the stress is being calculated, and I is the moment of inertia of the cross-sectional area of the BGi. The shear stress in the BGi can be calculated using the equation:

$$\tau = V * q / It \tag{10}$$

where V is the shear force, q is the first derivative of the shear flow, and It is the shear flow area of the BGi.

By incorporating these stresses into the Pev function, a more complete representation of the effect of the machinery loads and the strength and strain of the BGi on the plants' evolution can be obtained. The final form of the function Pev taking into account the height (H), width (D), depth (Z), machinery loads, and strength and strain factors of the Bio-Geo Interface (BGi) would look like:

$$Pev = F(Bi, Gi, H, D, Z, M, V, I, It, y, q) \tag{1-d}$$

The function Pev, taking into account just the roots-soil interaction, can be represented mathematically as follows:

$$Pev = F(H, D, Z, R, \varepsilon) \tag{1-e}$$

where H is height of the root, D is width of the root, Z is depth of the root, R is root stress (can be represented by the matric potential, suction or interstitial pressure) and ε is root strain (the change in root shape or size due to the applied stress).

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This function represents the effect of root-soil interaction on the evolution of plants, considering the height, width, depth of the roots, and the stress and strain they experience. The relationship between the root parameters and soil parameters can be determined through experiments, simulations or mathematical models, and can be represented by graphs, diagrams or mathematical equations. Furthermore; The biological index can be represented as a function of root stress (Rstr) and root strain (Rstn) and the geotechnical index can be represented as a function of soil strength (Sstr), water content (Wc), root morphology (Rm), and nutrient availability (Na) as following:

$$Pev = F(Bi, Gi) = F(Rstr, Rstn, Sstr, Wc, Rm, Na) \qquad (1\text{-}f)$$

where Rstr is function of root diameter, root length, and root density, Rstn is function of root morphology and soil structure, Sstr is function of soil type and soil water content, Wc is function of soil water content and soil strength, Rm is function of root diameter and root length and Na is function of soil type and nutrient availability.

The exact representation would depend on the specifics of the system being studied. To create an accurate mathematical representation, we need to consider many additional factors, such as soil structure, root morphology, and nutrient availability, and consider the interactions between these factors. Additionally, these relationships may change over time, so it's important to continuously monitor and update the model to ensure accuracy. One way to represent the interactions between root stress, root strain, and soil strength would be to use a multi-variable equation. One possible form of this equation could be:

$$Pev = K1 * Rstr + K2 * Rstn + K3 * Sstr + K4 * Wc + K5 * Rm + K6 * Na \quad (1\text{-}g)$$

where K1, K2, K3, K4, K5 and K6 are constant coefficients representing the relative influence of each factor on Pev.

Note that this equation is a simplified representation of the interactions between root stress, root strain, and soil strength. In reality, there may be many additional factors and complexities involved in these interactions, and the equation may need to be modified or expanded to accurately reflect the specific conditions and system being analyzed. The unit of Plant's evolution function (Pev) depends on the units used for the parameters in the equation defining Pev. If root stress (Rstr), root strain (Rstn), and soil strength (Sstr) are expressed in N/m2 (Pascals), N/m, or some other unit of stress or strain, then the unit of Pev would be the same as the units of the parameters. If the equation includes any coefficients or constants (e.g. K1, K2, K3), then the units of these coefficients would also influence the unit of Pev.

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The equation "Pev = F(Bi, Gi) = F(Root Evolution, Soil volume) = (Constraints/ root stress, Soil deformation/ strain)" summarizes the relationship between plant evolution, biological index, and geotechnical index in terms of root-soil interactions and the effect of root growth on soil volume. The equation states that the evolution of plant roots (Pev) is a function of the biological index (Bi) and the geotechnical index (Gi), which in turn can be described as a function of root evolution and soil volume. The final part of the equation relates root evolution to constraints or root stress and soil deformation or strain. An alternative equation that logically relates the same concepts could be:

Rv = g(Bi, Gi) = g(Root morphology, Soil mechanical properties) = (Root architectural parameters, Soil mechanical parameters) (11)

This equation states that the root morphology or root architecture (Rv) is a function of the biological index (Bi) and the geotechnical index (Gi), which in turn can be described as a function of root morphology and soil mechanical properties. The final part of the equation relates root morphology to root architectural parameters and soil mechanical parameters, such as the following updated version of the equation:

Rv = g(Bi, Gi) = g(Root diameter, Root length, Root architecture, Soil strength, Soil structure) = (Root diameter, Root length, Root branching pattern, Soil shear strength, Soil bulk density) (11-a)

This equation states that root morphology or root architecture (Rv) is a function of the biological index (Bi) and the geotechnical index (Gi), which in turn can be described as a function of specific root parameters (Root diameter, Root length, Root architecture) and soil mechanical properties (Soil strength, Soil structure). The final part of the equation relates root morphology to specific root architectural parameters (Root diameter, Root length, Root branching pattern) and soil mechanical parameters (Soil shear strength, Soil bulk density). The considering mathematical representation of the function Rv, as factors:

$$Rv = f(d, l, A, S, St) \qquad (11\text{-b})$$

where d is the root diameter, l is the root length, A is the root architecture (represented by a set of parameters that describe the branching pattern of the roots), S is the soil strength, represented by soil shear strength or other relevant mechanical parameters and St is the soil structure, represented by soil bulk density or other relevant mechanical parameters.

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The exact form of the function Rv will depend on the specific relationships between the root parameters and soil properties, which may vary based on the species of plant and type of soil. After Deep research we found that we could replace Root morphology with force (F) which the main responsible of soil constraint and soil particals rearangement, and we could replace Soil mechanical properties by soil constraints. So; the revised mathematical representation of the function Rv, replacing Root morphology with force (F) and Soil mechanical properties with soil constraints (C) could be represented as following:

$$Rv = g(Bi, Gi) = g(F, C) \qquad (11\text{-}c)$$

where F is the root-generated force that is responsible for soil constraint and soil particle rearrangement and C is the soil constraint, representing the mechanical properties of the soil that resist root-generated forces.

The exact form of the function Rv will depend on the specific relationships between root-generated forces and soil constraints, which may vary based on the species of plant and type of soil. If we suggested that the actions happen in the Gio-Bio Interface (BGi) volume, the revised mathematical representation of the function Rv, could be:

$$Rv = g(Bi, Gi, V) = g(F, C, Vi) \qquad (11\text{-}d)$$

where Vi is the Geo-Bio Interface volume, which has dimensions and is limited by boundaries conditions on all sides except the upper side. Their respective units are Rv (unit: N), g(F (unit: N), C (unit: Pa) and Vi (unit: m3)) respectively.

Note that the exact form of the function Rv will depend on the specific relationships between root-generated forces, soil constraints, and the Geo-Bio Interface volume, which may vary based on the species of plant, type of soil, and environmental conditions. The units chosen here are based on the SI system, but other units can be used depending on the specific study. The relationship between root-generated force (F), soil constraints (C), and Geo-Bio Interface volume (Vi) will depend on the specific model being used and the system being studied. In general, the root-generated force (F) can be related to soil constraints (C) through the deformation or strain of the soil, and the Geo-Bio Interface volume (Vi) can affect both F and C through its effect on soil mechanical properties and root growth. For example, in an elastic model, the root-generated force (F) may be related to soil constraints (C) through Hooke's law, which states that the strain (ε) is proportional to the stress (s) applied to a material:

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$$s = E * \varepsilon \tag{12}$$

where E is the Young's modulus of the soil.

The relationship between F, C, and Vi can be more complex in nonlinear or viscoelastic models, where the soil mechanical properties and root-generated forces can change over time and depend on the history of loading. Overall, the exact mathematical relationships between F, C, and Vi will depend on the specific model being used and the system being studied, and may require numerical simulations or experimental studies to determine. In a plastic soil model, the relationship between root-generated force (F), soil constraints (C), and Geo-Bio Interface volume (Vi) can be described using the plasticity theory. Plasticity theory is a mathematical framework that describes the behavior of soil materials that undergo permanent deformation when subjected to a load. The plastic behavior of soil is described by a yield surface in the stress space, which defines the conditions under which the soil begins to yield or undergo plastic deformation. The yield surface is usually represented by a yield criterion, such as the von Mises criterion or the Tresca criterion [50]. In a plastic soil model, the soil constraints (C) can be represented by the effective stress (s'):

$$s' = s - p \tag{13}$$

where s is the total stress and p is the pore water pressure.

The effective stress is the component of stress that is responsible for causing plastic deformation in the soil. The relationship between root-generated force (F), soil constraints (C), and Geo-Bio Interface volume (Vi) can be described using a yield criterion and a constitutive equation that describes the relationship between the effective stress and plastic strain. The exact form of the constitutive equation will depend on the specific soil model being used. For example, in a simple linear elastic-perfectly plastic soil model, the constitutive equation can be written as:

$$s' = k * (\varepsilon_0 - \varepsilon) \tag{14}$$

where k is the soil stiffness, ε_0 is the plastic strain at failure, and ε is the current plastic strain.

Note that the plasticity theory is a complex subject and there are many different models and approaches that can be used to describe the relationship between root-generated force (F), soil constraints (C), and Geo-Bio Interface volume (V). The specific model used will depend on the particular system being studied and the goals of the analysis. The Mohr-Coulomb criterion is a widely used yield criterion in

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geotechnical engineering [11, 12] that describes the strength of soil in terms of the normal stress (σ_n) and the shear stress (tau). The equation for the Mohr-Coulomb criterion [13] is:

$$\tau = c + \sigma_n \tan(\varphi) \quad (15)$$

The elastic state is the state of soil when it deforms elastically in response to applied stress, without undergoing plastic deformation. In this state, the soil behaves as a linear elastic material, and the stress-strain relationship can be described by Hooke's law. The plastic state is the state of soil when it begins to undergo plastic deformation in response to applied stress. In the plastic state, the soil behaves as a non-linear material and its behavior is described by the yield criterion, such as the Mohr-Coulomb criterion. The failure state is the state of soil when the stress exceeds the strength of the soil, causing the soil to fail or collapse. In the case of the Mohr-Coulomb criterion, failure occurs when the shear stress (τ) exceeds the strength defined by the equation (15). It is important to note that soil strength is a complex function that depends on many factors, including the soil type, soil moisture content, soil structure, and applied load [14, 15]. The Mohr-Coulomb criterion provides a simple and widely used approach for describing the strength of soil, but it is just one of many models that can be used to describe soil behavior in geotechnical engineering. The Rv function can be modified to account for the two phases of root evolution by considering the mechanical behavior of both the roots and soil during the elastic and plastic phases. The equation for the Rv function in the elastic phase can be expressed as:

$$Rv\_elastic = g(F\_elastic, C\_elastic) = g(F\_elastic / A, k * F\_elastic) \quad (11\text{-}e)$$

where F_elastic is the force generated by the root in the elastic phase, A is the cross-sectional area of the root-soil interface, and k is the elastic soil modulus.

In the plastic phase, the Rv function can be expressed as:

$$Rv\_plastic = g(F\_plastic, C\_plastic) = g(F\_plastic / A, c + (\sigma_n \tan(\varphi))) \quad (11\text{-}f)$$

where F_plastic is the force generated by the root in the plastic phase, C is the coefficient of cohesion, φ is the angle of internal friction, and σ_n is the normal stress.

The relationship between root growth and soil deformation can be described in terms of a time evolution of the root's size and soil properties, with two phases; an elastic phase and a plastic phase. During the elastic phase, both the roots and soil are in an elastic state, and their behavior can be described using linear

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elastic models. In this phase, root growth can be described using Hooke's law, which states that the stress (F) applied to an object is proportional to its strain (C).

$$F = E * C \qquad (16)$$

where E is the Young's modulus, which describes the object's stiffness.

During the plastic phase, the root and soil are no longer in an elastic state, and their behavior can be described using plastic models, such as the Mohr-Coulomb criterion. In this phase, the root's growth can be described using a plastic flow rule, which states that the rate of plastic strain is proportional to the applied stress;

$$dC/dt = k * F \qquad (17)$$

where k is a constant of proportionality.

Combining these two equations gives us the Rv function in terms of the time evolution of the root's size and soil properties:

$$Rv = g(F(t), C(t)) = g(E * C(t), k * F(t)) \qquad (11\text{-g})$$

This equation can be used to describe the relationship between root growth and soil deformation over time, and to analyze the evolution of the bio-geo interface as the root grows and deforms the soil. The Mohr-Coulomb equation can be applied to the Rv function by considering the forces (F) and constraints (C) in the bio-geo interface. Assuming that the root is the source of the shear stress (τ), and the soil is the source of the normal stress (σ_n), we can equate the shear stress generated by the root to the shear stress defined by the Mohr-Coulomb equation. This gives us:

$$\tau = F / A = c + \sigma_n \tan(\varphi) \qquad (15\text{-b})$$

where F is the force generated by the root and A is the cross-sectional area of the root-soil interface.

We can then substitute the equation for τ into the Rv function to relate the root morphology (F) and soil mechanical properties (C) to the volume of soil affected by root growth (Rv):

$$Rv = g(F, C) = g(F / A, c + \sigma_n \tan(\varphi)) \qquad (11\text{-h})$$

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It is important to note that this equation is a simplified representation of the complex interaction between root morphology and soil mechanical properties in the bio-geo interface, and that other factors, such as soil structure and soil moisture content, may also play a role in the relationship between root growth and soil deformation. The force F in the Rv function is a representation of the mechanical forces exerted by the plant roots on the soil. The exact equation for F depends on the specific method used to calculate it, but a common method is to calculate it as the product of the root volume and root density. The equation for F can be written as:

$$F = Vr * \rho \qquad (18)$$

where Vr = root volume (m^3) ρ = root density (kg/m^3) and the unit of F would be Newton (N). The force F can be represented by the forces acting on the Bio-Geo Interface due to root growth and soil deformation. These forces can be represented mathematically by a set of vectors pointing in the directions in which the forces are acting. The magnitude and direction of each vector can be determined by the root geometry, root density, and soil mechanical properties. The sum of these vectors represents the net force acting on the Bio-Geo Interface, which is represented by the vector F. The force vector F in the Bio-Geo interface can be represented mathematically as a sum of multiple force components acting in different directions. The force vector can be expressed as:

$$F = Fx + Fy + Fz \qquad (19)$$

where Fx, Fy, and Fz represent the force components in the x, y, and z directions, respectively.

The magnitude and direction of each force component can be determined based on the specific conditions in the Bio-Geo interface, such as root architecture, soil mechanical properties, and root-soil interactions. To obtain the resultant force vector F, the magnitude of each component can be calculated using vector arithmetic [16]. Vector arithmetic can be used to calculate the resultants vector F in a defined Bio-Geo Interface. To do this, we would need to define the vectors representing the forces acting on the roots, such as gravitational forces, turgor pressures, and soil resistance forces. These vectors could be expressed in terms of magnitude and direction, and then added together using vector addition to find the resultant force F. The calculation would depend on the specific conditions and variables present in the Bio-Geo Interface, including the size and shape of the roots, the density of the soil, and any other relevant factors. This involves breaking down the forces into their components, taking into account their magnitude and direction, and then combining

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them to determine the resultant vector [17, 18]. The magnitude of a force can be represented mathematically as:

$$F = \|F\| \tag{19-a}$$

The direction of a force can be represented mathematically as a unit vector:

$$u = F / \|F\| \tag{20}$$

The components of a force can be represented mathematically as:

$$Fx = F * \cos(\theta)\ Fy = F * \sin(\theta) \tag{19-b}$$

The sum of two forces can be represented mathematically as:

$$F = F1 + F2 \tag{19-c}$$

The resultant vector of a system of forces can be represented mathematically as:

$$F = \sum Fi \tag{19-d}$$

where Fi represents each individual force in the system.

The direction and magnitude of the resultant vector can then be calculated using the above equations. The direction of the force F can be calculated using vector arithmetic and considering a defined Bio-Geo Interface. One approach is to use the position vectors of the roots and soil particles and the force vectors between them to find the net force on the system. The position vector of a root or soil particle can be represented as:

$$r = xi + yj + zk \tag{21}$$

where x, y, and z are the coordinates of the particle in the three-dimensional space and i, j, and k are the unit vectors in the x, y, and z directions. The force vector between two particles can be represented as:

$$F = k * (r1 - r2) / \|r1 - r2\|^3 \tag{19-e}$$

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The formula represents the magnitude and direction of the force (F) between two-point masses (r1 and r2) that are separated by a distance ||r1 - r2||, where k is a constant that depends on the physical properties of the objects and the medium in which they are located. The term ||r1 - r2||3 represents the cube of the distance between the two-point masses raised to the power of 3. This term is included in the formula to account for the inverse square law of the gravitational force, which states that the force between two masses decreases as the square of the distance between them. By raising the distance to the power of 3, the formula ensures that the force decreases more rapidly with increasing distance, as required by the inverse square law. This means that the force becomes weaker as the distance between the masses increases, which is a fundamental principle of gravitation and other forces in physics. By summing the force vectors for all pairs of particles in the Bio-Geo Interface, the net force on the system can be calculated. This net force represents the force direction for the F in the Rv function. Finally: The equation for the increase in force F with time can be represented as:

$$F(t) = \rho * Vr(t) \qquad (18\text{-a})$$

where Vr(t) is the volume of roots at time t.

4. Mathematical Modeling of Soil-Insect Interaction (SII)

Soil-insect interactions are a complex and dynamic aspect of soil ecology [19]. Insects play a crucial role in the soil ecosystem by breaking down organic matter, aiding in nutrient cycling, and affecting plant growth and health [20]. The interactions between soil insects and soil also vary depending on factors such as soil type, moisture, and temperature. For example, certain insects such as earthworms and ants are known to modify soil structure and improve soil fertility, while other insects such as root feeders can have a detrimental effect on plant growth [21]. There have been several studies conducted to understand the mechanisms of soil-insect interactions and their effects on plant growth and soil quality [22-28]. These studies have mainly focused on the impact of soil insects on plant root growth, nutrient uptake, and plant-microbe interactions. However, there is still much to be learned about the complex interactions between soil insects, soil, and plants. Understanding soil-insect interactions is important for sustainable agriculture and for improving soil health and fertility. Further research is needed to fully comprehend the complexity of these interactions and to develop effective strategies for managing soil insects for optimal plant growth and soil quality. The interactions between insects and soil geomechanical, geochemical, and geophysical properties are an important area of study in the field of soil science. These interactions play a significant role in determining the soil's physical, chemical, and biological properties, which in turn influence the health and productivity of ecosystems and crops. To understand the soil-insect interactions, researchers have used various approaches such as laboratory experiments, field studies, and numerical simulations. These studies have helped to identify

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the key factors that govern the behavior of insects in soil, such as soil moisture, temperature, and mechanical stress, as well as the availability of nutrients, toxins, and other soil constituents. Additionally, researchers have used these studies to evaluate the effects of human activities, such as land use change and the application of fertilizers and pesticides, on soil-insect interactions and the health of soil ecosystems. Understanding the complex and dynamic relationships between insects and soil is essential for developing sustainable land use practices and maintaining the health and productivity of soil ecosystems.

$$Iev = F(Bi, Gi) \tag{22}$$

The Iev function suggests that the evolution of insects in soil is dependent on both biological and geotechnical indexes. The biological index, Bi, represents the food sources available for the insect to support its growth and development, while the geotechnical index, Gi, represents the soil parameters such as texture, structure, moisture content, etc., that affect the insect's life in the soil. By understanding the relationship between these two indexes, researchers can gain insight into the complex interactions between insects and soil, and potentially develop strategies to manage insect populations in agricultural and natural ecosystems. It is possible to represent the biological index Bi by food sources (fs) in the Insect evolution (Iev) function. The food sources in the soil can provide the necessary nutrients for the insect to support its growth and development, which is reflected in the biological index. The relationship between food sources (fs), biological index (Bi), and insect evolution (Iev) can be represented mathematically as:

$$Iev = F(fs, Gi) \tag{22-a}$$

where fs represents the food sources available in the soil and Gi represents the geotechnical indexes that affect the insect's life in the soil.

The geotechnical index, Gi, can be mathematically represented as a combination of different soil parameters, such as texture, structure, moisture content, etc. For example, Gi can be expressed as a weighted sum of these parameters, where the weighting factors are determined based on the relative importance of each parameter for insect evolution. The equation can be written as:

$$Gi = w1 * P1 + w2 * P2 + ... + wn * Pn \tag{23}$$

where w1, w2, ..., wn are the weighting factors and Parameter 1, Parameter 2, ..., Parameter n are the soil parameters considered in the analysis.

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The value of the weighting factors can be determined based on statistical or expert judgment methods. The porosity of a soil can be mathematically represented as the ratio of void volume to total soil volume. It is commonly denoted by the symbol "n". The equation to calculate porosity is given by:

$$n = (Vv / Vt) * 100 \qquad (24)$$

where Vv is the volume of voids in the soil, and Vt is the total volume of soil.

In relation to insects in the soil, porosity can play a significant role in determining the available habitat and microclimates for the insects. Soil with higher porosity tends to have a greater volume of air-filled pore spaces, which can provide a more hospitable environment for insects that require air for respiration and metabolism. On the other hand, soil with low porosity can have reduced air and moisture availability, which can limit the survival and growth of insects in the soil. Therefore, porosity can be considered as one of the geotechnical parameters that affects insects in the soil and can be incorporated into the Gi component of the Iev function. The equation for void ratio (e) is defined as the ratio of the volume of voids (empty spaces) in the soil to the volume of solid particles:

$$e = (Vv)/(Vs) \qquad (25)$$

where Vv is the volume of voids and Vs is the volume of solid particles.

So: The regenerated Iev function considering Void ratio Vr and Food sources Fs could be as:

$$Iev = F(Fs, e) \qquad (22\text{-}b)$$

The equation for food sources (Fs), depends on the specific insect species and their preferred diet. There is no one universal equation that can represent the food sources for all insects. The equation for Fs may include variables such as the availability of organic matter in the soil, the presence of specific plant species, and the presence of other insects that may serve as a food source. It is important to consider the specific biology of the insect species in question when developing the equation for food sources. It is difficult to provide a mathematical equation for food sources (Fs) as it would depend on the specific type of insect and its dietary needs. Additionally, the availability of food sources can be affected by a variety of environmental factors such as moisture content, temperature, and soil texture, making it difficult to represent with a single equation. In general, food sources for insects can include organic matter, roots of plants, fungi, and other insects. The exact combination of these sources can vary greatly and is difficult to quantify mathematically. It

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would likely require a case-by-case approach and extensive research to determine a mathematical representation for food sources in the context of insects and their evolution. Furthermore, the relationship between void ratio (e) and food sources (Fs) would need to be studied and determined through scientific experimentation and analysis. Any equation proposed would be based on available data and current understanding of the topic, and would need to be validated through additional research. The function of insect evolution (Iev) in a defined Bio-Geo Interface (BGi) can be represented as follows, considering the void ratio (e) and food sources (Fs) as indexes:

$$Iev = f(Vct, e, Fs) \qquad (22\text{-}c)$$

where Vct represents the volume of the insect, e represents the void ratio of the soil in the Bio-Geo Interface, and Fs represents the availability of food sources in the soil.

This equation takes into account both geotechnical and biological factors that influence the evolution of insects in the soil. The insect evolution, Iev, can be mathematically represented as the volume of the insect, Vct, as a function of food sources, Fs, and void ratio, e, as follows:

$$Iev = Vct = f(Fs, e) \qquad (22\text{-}d)$$

This equation suggests that the evolution of an insect in the soil is dependent on the availability of food sources and the void ratio of the soil in the bio-geo interface (BGi) where it lives. We can represent the volume evolution of an insect, Vct, over time, t, as a function of both food sources, Fs, and void ratio, e, as follows:

$$Iev = Vct(t) = f(Fs, Vr(t)) \qquad (22\text{-}f)$$

where the function f represents the combined effect of food sources and void ratio on the volume evolution of the insect.

The void ratio, e, is also dependent on time as it decreases with the increasing volume of the insect. By including both food sources and void ratio in the equation, we can better understand and predict the evolution of insects in their soil environment. If we suppose that food sources depend on time (t) and also decrease with increasing Vct, then the Iev function could be represented as:

$$Iev = Vct = F(Fs(t), e(t)) \qquad (22\text{-}g)$$

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This means that the volume evolution of the insect is a function of both the food sources, which are dependent on time, and the void ratio, which is also dependent on time and decreasing with increasing Vct. As Vct increases, it leads to a reconfiguration of soil particles, resulting in the exertion of stress and strain on the Bio-Geo Interface. Therefore, the increase in Vct may be deemed as a force, denoted as F, acting upon the soil. It's difficult to provide a general equation that describes the force, F, exerted by an insect with increasing volume, Vct, on the soil in a 3D coordinate system without knowing the specifics of the soil and the insect. The relationships between force, stress, strain, and volume change are complex and depend on multiple factors, including soil type, soil moisture, soil structure, insect size and shape, etc. In general, the force exerted by an insect can be represented as a vector, F, in the 3D coordinate system, where the components of the vector represent the force in each dimension (x, y, z). The relationship between the force and the resulting stress and strain on the soil can be described by mechanics of materials equations, such as Hooke's law, which states that the stress and strain are proportional to each other, within the linear elastic range. However, to properly describe the relationship between F, stress, strain, and volume change, a more detailed analysis would be required, taking into account the specifics of the soil and the insect, such as the soil's mechanical properties. The equation for force F generated by increasing volume of insects Vct (t) can be expressed as follows:

$$F = Vct\ (t) * \text{insect density} \quad (25)$$

where Vct (t) is the volume of insects at time t and insect density is the number of insects per unit volume of soil.

This equation expresses the force generated by the increasing volume of insects and their density at a given time. The insect density can be represented mathematically as:

$$\delta = n / Voc \quad (26)$$

where δ is the insect density, n is the number of insects in a given volume, and Voc is the volume occupied by the insects.

The relationship between insect evolution and the two factors, force and constraint, can be represented mathematically as:

$$Iev = f\ (F, C) \quad (27)$$

where F represents the force, C represents the constraint, and Iev represents the insect evolution.

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However, it's important to note that this equation is a simplified representation and many other factors such as soil properties, food sources, and environmental conditions could also impact the insect evolution. The insect evolution, Iev, can be represented by the interaction between the force, F, and the constraints, C. Insects have a small volume and weight, thus, their impact on the soil is minimal. According to the principles of elasticity and plasticity, the forces exerted by insects are negligible and do not cause significant deformation in the soil. Hence, the theories of elasticity and plasticity do not play a significant role in the calculation of insect evolution. The primary factors affecting insect evolution in the soil are food sources and soil properties such as porosity, texture, and moisture content. These factors can be represented by the Biological and Geotechnical indices, respectively, in the insect evolution function (Iev).

5. Conclusion

In conclusion, the study of soil-plant interaction (SPI) through mathematical modeling reveals a nuanced understanding of root-soil dynamics crucial for comprehending plant evolution. Extensive research underscores the pivotal roles of soil strength, water content, root morphology, and nutrient availability in shaping SPI dynamics. Constructing precise algorithms to elucidate SPI mechanics demands a holistic grasp of these factors and the development of robust mathematical models and software tools, integrating empirical data for validation. Consequently, mathematical models delineating SPI must encapsulate myriad elements including root morphology, soil properties, external loads, and spatial dimensions. The intricate nature of these interactions underscores the interdisciplinary essence of SPI research, necessitating continual refinement and validation of mathematical representations to faithfully capture the nuances of root-soil dynamics and plant evolution. In addition; Soil-insect interactions constitute a dynamic and multifaceted aspect of soil ecology, profoundly influencing soil health and ecosystem productivity. While certain insects bolster soil fertility and structure, others pose challenges to plant growth and ecosystem stability. To ensure sustainable agricultural practices and ecosystem management, it is imperative to grasp the complex interplay between soil, insects, and plants. Mathematical modeling, exemplified by the proposed Iev function, serves as a framework for understanding the intricate dynamics of soil-insect interactions. By integrating biological and geotechnical indices, this model sheds light on the factors driving insect evolution in soil environments. Moreover, the inclusion of void ratio and food sources underscores the significance of soil properties and nutritional availability in shaping insect behavior. Despite the limitations of simplified representations, such as overlooking elasticity and plasticity, these models offer valuable insights for advancing our understanding of soil-insect interactions and guiding effective management strategies to optimize soil health and productivity. Continuous research endeavors aimed at refining and validating these models are indispensable for advancing knowledge and fostering the sustainability of agricultural and environmental practices.

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The authors declare that there are no conflicts of interest related to this article.

Data availability

All data generated or analyzed during this study are included in this published article.

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