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Research Article

Reinforced Concrete slabs designed by Finite Element Methods - A Case study

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Abstract

The traditional methodology for designing slab systems typically commences with manual calculations of support and span moments based on linear elastic theory. These moments are subsequently redistributed according to the adopted design framework to achieve an optimal balance between structural performance and cost efficiency. The reinforcement areas are then computed for each region of moment distribution, with supplementary reinforcement introduced to control cracking and mitigate punching shear effects.
Contemporary computer-aided design approaches, employing finite element (FE) analysis, determine internal forces at the elemental level under the same linear elastic assumptions. The resulting reinforcement requirements are provided for each mesh element. Accurate control of internal force magnitudes, particularly bending moments is essential when employing linear elastic FE analyses. The quality of the FE mesh is a critical factor, requiring attention to parameters such as element shape, aspect ratio, and the relative size of adjacent elements. Mesh-related irregularities may also arise in slabs supported by interior, edge, or corner columns, as well as in areas containing openings. In the case study below, the design sequence of reinforced concrete slabs using the SAP2000 program (FEM analysis) is described to meet the requirements of flexural behavior, punching shear capacity and overall deformation performance according to the Eurocodes.

Keywords: slab, finite element, reinforced concrete, Eurocodes.

1. Introduction

The scope of this paper is to present the structural concept of Three Underground Levels Parking Structure built in Square "Çerçiz Topulli", Gjirokastra, in the location indicated into the following Figure 1.

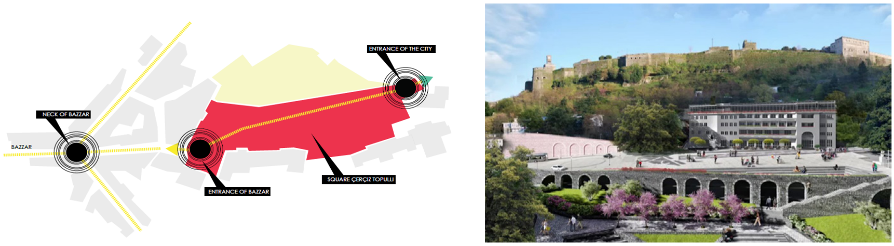


Figure 1. Location of "Three Underground Levels Parking Structure" in Square "Çerçiz Topulli", Gjirokastra, Albania

The structure is fully underground with three levels, that will stand in front of hotel "Çajupi" in the Old Center of the City. Below are shown structural design codes, principles and report the checks performed to define the size of the main structural elements.[1], [2]
The main category of use for the Structure is Parking Areas for Underground levels (Category F). The Square at ±0.00 Groundlevel Areas is susceptible to large crowds (Category C5), access areas for medium vehicles (>30 kN, ≤160 kN gross vehicle weight, on 2 axles) (Category G) up to heavy vehicles (≤250 kN gross vehicle weight, on 3 axles).

1.2 Description of structures

Floors are 25 cm and 30 cm-thick reinforced concrete solid slabs, supported by beams, columns and walls with the following size:

- Level from solid slab raft foundation to ground floor slab: reinforced concrete walls 35cm and 40cm thick for elevator core and retaining wall respectively;
- Levels from solid slab raft foundation to ground floor slab: reinforced concrete columns 40x100cm;
- Levels from solid slab raft foundation to ground floor slab: reinforced concrete beams 70x65cm, 70x50cm, 60x50cm;
- The solid slab raft foundation is 60cm thick.

All the parameters used for design and checks are included in these European norms ("Eurocode") and KTP-N.2-89 Earthquake Resistant Design Regulations, issued by the Seismic Center and Academy of Science of Albania.

2. Materials

2.1 Steel for rebar	500C
Characteristic breaking stress	f_{tk} = 500 MPa
Characteristic yielding stress	f_{yk} = 435 MPa
Characteristic ratio breaking/yielding	$1.3 \leq (f_t / f_y)_k \leq 1.35$
Elastic Modulus	E = 210 GPa

Elongation	≥ 12 %

2.2 Concrete for elevation (slabs, beams)

Strength class of concrete	C30/37
Slump	S4
Maximum dimension of aggregate	Dmax = 25 mm
Concrete cover	30÷40mm
Cubic strength at 28 days	$f_{ck,cube} \geq 37$ MPa
Cylindrical strength at 28 days	$f_{ck} \geq 30$ MPa
Strength at ULS $f_{cd}=\alpha_{cc}f_{ck}/\gamma_c$;(α_{cc}=0.8÷1), α_{cc}=0.85; γ_c=1.5	f_{cd} = 17 MPa
Tensile strength at ULS $f_{ctd} = \alpha_{ct}f_{ctk,0.05}/\gamma_c$; α_{ct}=1.00; γ_c=1.5	f_{ctd} =1.33MPa
Elastic Modulus	E_{cm}= 32837 MPa

3. Load cases

Design and verification of structural elements has been carried out using Semi-Probabilistic limit states method. Every single load case is combined with other load cases using combination that maximize the member forces and the displacements in every structural element. The verifications are made using the most unfavorable combination. Verifications are developed for Ultimate Limit State (ULS) and Service Limit State (SLS). Main load cases are self-weight, dead loads, live loads, snow load, transversal and longitudinal wind load, increment and decrease of temperature, horizontal earthquake in transversal and longitudinal direction. In the following paragraphs the entities and the applications of the loads are explained. Actions on structure are calculated according to Eurocode 0 or Eurocode 1.

3.1 Self-weight

Structural self-weight depends on the size of the structural elements and the specific weight of the material: γ_{steel}= 78.5 kN/m3; $\gamma_{concrete}$ = 25.0 kN/m3
Self- weights are automatically calculated by the FEM software.

3.2 Dead loads

The weight of non-structural elements is calculated separately from the structural self-weight and is applied to the structure as an external load.
In case of seismic analysis, dead loads are considered as masses applied to the structural elements and participating to the global mass of the building.

3.3 Live (imposed) loads

The value for Live Loads is correlated to the suitable use of each part of the building. The Eurocode1 – Basis of structural design, [5] point 6.3, tables 6.1, 6.2, 6.4 prescribes:

Category C5 (Areas susceptible to large crowds) ....qk,1 = **5.0** ÷ 7.5 kN/m2
Category F (Parking Areas for Underground levels) ....qk,2 = 1.5 ÷ **2.5** kN/m2

Category G (Access areas for medium vehicles) .... qk,3 = **5.0** kN/m2
Based on 6.3.1.2 (11) the reduction factor for loads considering a multi-storey building is:

$$\alpha_n = (2 + (n-2)\ \psi_0)/n = (2+(3-2)\cdot 0.7)/3 = 0.9$$

n- is the number of storeys (> 2) above the loaded structural elements from the same category.
ψ_0- is in accordance with EN 1990, Annex A1, Table A1.1. EN 1990:2002 (E)

3.4 Snow load

Snow load has been determined according to climatic records as 0.6 kN/m2.

3.5 Wind load

Records show that average wind velocity in Gjirokastra, at ground level, is 31m/s, considered as average reference wind velocity $v_{b,0}$. Wind pressure is calculated according to Eurocode 1.

3.6 Temperature load

According to Eurocode 1 - Actions on structures - Part 1-5: General actions – Thermal actions, we consider the structure with a temperature change of ±10 °C.

3.7 Seismic action

Seismic forces are calculated by means of a modal analysis with response spectrum. Linear vibration analyses have been carried out to investigate natural frequencies of the structure and calculate the horizontal acceleration given by the design spectra. According to the Albanian Seismic Report (Sh.Aliaj, S.Allkja, Tirane 2020) the ground condition at construction site is classified of type B according to Eurocode 8 . The response spectra are calculated according to Eurocode 8, considering:

a_{gR}-reference peak ground acceleration on type A ground; a_{gR}=0.242g;
γ_I-Importance factor; EC-8, Table 4.3, Importance Class III, γ_I=1.2;
a_g-design ground acceleration on type A ground; $a_g=\gamma_I \cdot a_{gR}$=0.2904g;
EC-8; Table 3.1, νs,30=(180÷360)m/sec◎Ground Type B; NSPT≥ 50; Table 3.2:
S=1.2, T_B(s)=0.15, T_C(s)=0.5, T_D(s)=2;

a_{gv}- design ground acceleration in the vertical direction
$a_{vg}=0.9a_g$; S=1.00, T_B(s)=0.05, T_C(s)=0.15, T_D(s)=1;

The following Figures show the horizontal and vertical design response spectra.

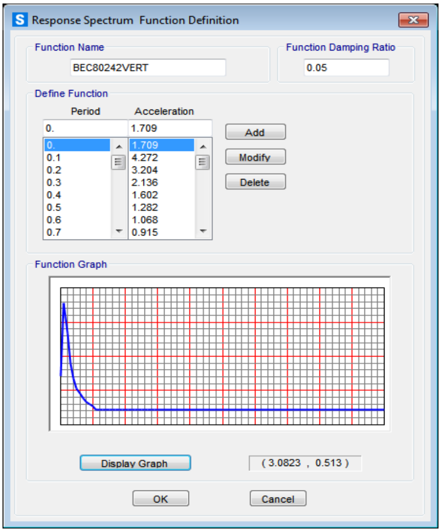


Figure 2. Horizontal Design Response Spectra

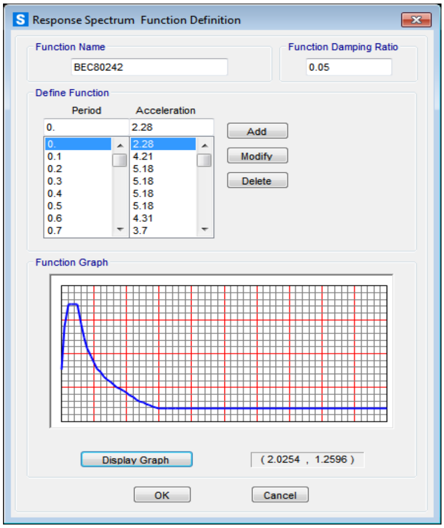


Figure 3. Vertical Design Response Spectra

4. Load combinations

4.1 Ultimate limit states (ULS)

$F_d = \gamma_G G + \gamma_{Q1} Q_1 + \Sigma\, \gamma_{Qi}\, \psi_{0i}\, Q_{ki}$ static actions

$F_d = G + E + \Sigma \psi_{2i} Q_{ki}$ seismic actions

4.2 Serviceability limit state (SLS)

$F_d = G + \Sigma \psi_{0i}\, Q_{ki}$ characteristic

$F_d = G + \Sigma \psi_{2i} Q_{ki}$ quasi permanent

The creep effect for long term actions it is considered according to 3.1.4; prEN 1992-1-1:2003 (E)
obtaining creep coefficient $\varphi(\infty, t_0)=2.3$ for concrete under normal environmental conditions.

5. FEM Calculation model

Calculations have been accomplished with the aid of the following software:
SAP2000, Version 22.0.0 Build 1587; Linear and Nonlinear Static and Dynamic Analysis and Design of Three-Dimensional Structures; Berkeley, California 94704 USA.
From FEM Analyses (Fig.4) results of the dual system applied it is performed design and capacity check for horizontal and vertical reinforced concrete elements according to Eurocode 2, Eurocode 8. In the following Figure 5 is presented an elevation view of frames and wall elements along axis 3.

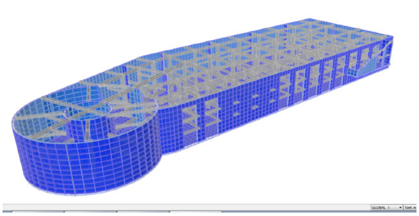


Figure 4. FEM Calculation Model

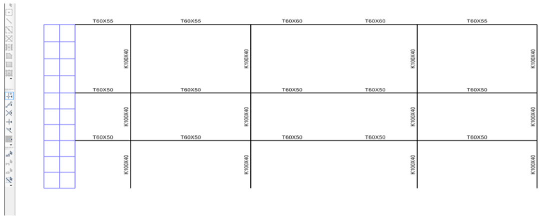


Figure 5. Elevation view along Axis 3

Through the refinement of the numerical model, it is possible to analyze the behavior of any structural component, considering the actual participation of every single element in the global stiffness. Modeling parameters reflect the replication of any structure as they are designed and prescribed to be realized. There are no plans for symmetry or particularly significant sections; it is impossible to carry out a planar analysis. A three-dimensional model has been made to evaluate the effects of forces on the basement and elevations. Using Frame elements are realized columns and beams, using Shell elements (3 nodes and 4 nodes) a simulated solid raft foundation plate, slabs and walls. Loads are applied as non-structural mass, pressure load, distributed load according to the nature of the load itself. Linear spring supports are applied under the solid slab raft foundation which represent the subgrade reaction k_{wink} of soil Winkler model.
Lateral restraints are applied on the nodes of the raft foundation in order to prevent lability.

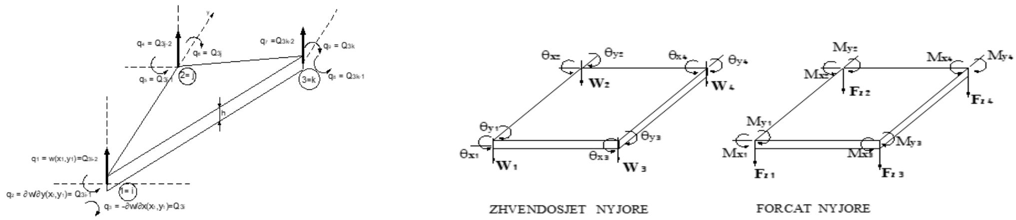
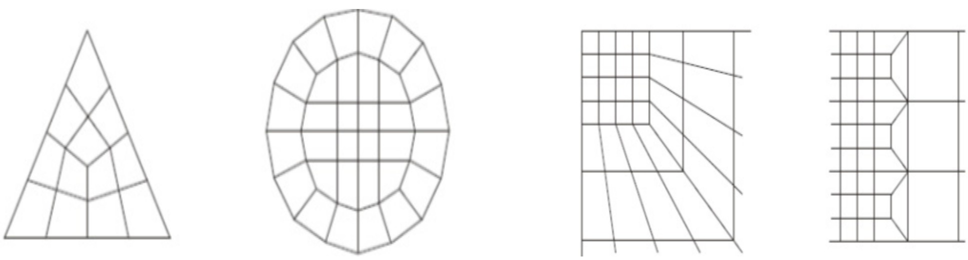


Figure 6 a) Triangular element with 3 nodes b) c)
Quadrilateral element with 4 nodes b) displacements c) nodal forces



Triangular area Circular area Infinite area Transit area

Figure 7 Mesh refinement

6. Results

6.1 Modal analysis results

Mode	Period Sec	Frequency Cyc/sec	CircFreq rad/sec	Eigenvalue rad2/sec2
1	0.1098	9.1088	57.2322	3275.5290
2	0.1066	9.3838	58.9604	3476.3279
3	0.0967	10.3411	64.9748	4221.7297
4	0.0953	10.4928	65.9281	4346.5171

Table 2 Modal Periods and Frequencies

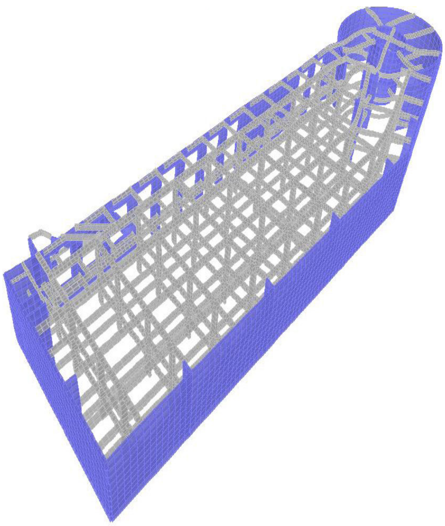
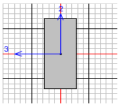


Figure 8. 4th natural vibration mode

6.2. Concrete frame elements

Groundfloor column (100x40) cm ; Groundfloor beam (60x55)cm



Units

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Eurocode 2-2004 COLUMN SECTION DESIGN  Type: DC MEDIUM MRF  Units: Tonf, m, C  (Summary)

L=4.6
Element      : 422              B=0.4          D=1.           dc=0.067
Section ID   : K100X40          E=3200000.     fck,cyl=3000.  Lt.Wt. Fac.=1.
Combo ID     : ENVES            fyk=50000.     fywk=40000.
Station Loc  : 4.6              RLLF=1.        SOM:  Nominal Curvature
Combo Eq.    :  Eq. 6.10

Gamma(Concrete):  1.5            AlphaCC = 1.    AlphaCT = 1.
Gamma(Steel)   :  1.15           AlphaLCC = 0.8  AlphaLCT = 0.8
Gamma(cE)      :  1.2            Alpha_e = 15.   phi_ef = 1.21

SLENDERNESS CHECK (governing permutation)
                     Slenderness Slenderness     Column   Governing
                           Ratio Limit Ratio  Condition Permutation
   Major Bending(M3)      15.935      26.525        N/A         N/A
   Minor Bending(M2)      39.837      26.525        N/A         N/A

AXIAL FORCE & BIAXIAL MOMENT DESIGN FOR NEd, MEd2, MEd3 (governing permutation)
                 Rebar    Capacity      Design      Design      Design     Minimum     Minimum
                  Area       Ratio         NEd        MEd3        MEd2          M3          M2
                 0.004       0.258     144.476      30.032        2.89       4.816        2.89

FIRST-ORDER MOMENTS (governing permutation)
                           End I       End J     Section        Mimp         M02         M01
                          Moment      Moment      Moment      Moment      Moment      Moment
   Major Bending(M3)      -8.294      28.482      28.482       1.549      30.032      -6.745
   Minor Bending(M2)       0.102       0.094       0.094       1.549       0.102       0.094

DESIGN DETAILS FOR ALL PERMUTATIONS CONSIDERED
          Imperfection Direction        None     Pos. M3     Neg. M3     Pos. M2     Neg. M2
   MAJOR BENDING(M3)
      (Analysis)            Mana      28.482      28.482      28.482      28.482      28.482
      (Imperfection)        Mimp           0       1.549      -1.549           0           0
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          Imperfection Direction        None     Pos. M3     Neg. M3     Pos. M2     Neg. M2Units  Tonf, m, C
   MAJOR BENDING(M3)
      (Analysis)            Mana      28.482      28.482      28.482      28.482      28.482
      (Imperfection)        Mimp           0       1.549      -1.549           0           0
            Mai = Mana + Mimp          N/A      30.032      26.933      28.482      28.482
      (Minimum)             Mmin         N/A       4.816       4.816       4.816       4.816
         M3des1 = Mai or Mmin          N/A      30.032      26.933      28.482      28.482
      (Neg. Moment Capacity) MRd-   -116.422    -116.422    -116.422    -116.422    -116.422
      (Pos. Moment Capacity) MRd+    116.422     116.422     116.422     116.422     116.422
   MINOR BENDING(M2)
                            Mana       0.094       0.094       0.094       0.094       0.094
                            Mimp           0           0           0       1.549      -1.549
            Mai = Mana + Mimp          N/A       0.094       0.094       1.643      -1.455
                            Mmin         N/A        2.89        2.89        2.89        2.89
         M2des1 = Mai or Mmin          N/A        2.89        2.89        2.89       -2.89
                            MRd-     -44.927     -44.927     -44.927     -44.927     -44.927
                            MRd+      44.927      44.927      44.927      44.927      44.927
   DEMAND/CAPACITY PMM RATIO
      (M3des1, M2des1) PMM Ratio     0.242       0.258       0.245       0.252       0.252
    (M3des1, M2des1) PMM Governs        No         Yes          No          No          No

SHEAR DESIGN FOR V2,V3
                           Rebar       Shear       Shear       Shear  Tan(Theta)
                           Asw/s         VEd        VRdc        VRds       Ratio
   Major Shear(V2)            0.      10.011      36.971          0.          0.
   Minor Shear(V3)            0.      18.229      36.878          0.          0.

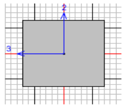
AXIAL COMPRESSION RATIO
                       Conc.Capa   CompRatio   CompRatio     Seismic   CompCheck       Ratio
                           A*fcd Ned/(A*fcd)       Limit       Load?     Needed?         OK?
                            800.       0.181        0.65         Yes         Yes         Yes

JOINT SHEAR DESIGN
                     Joint Shear       Shear       Shear       Shear       Shear       Joint
                             Ash     VEd,Top        Vjhd    Vrd,Conc       Ratio        Area
   Major Shear(V2)           N/N         N/N         N/N         N/N         N/N         N/N
   Minor Shear(V3)           N/A         N/A         N/A         N/A         N/A         N/A

(1.3) BEAM/COLUMN CAPACITY RATIOS
                           Major       Minor
                           Ratio       Ratio
                             N/A         N/A


Notes:
   N/A: Not Applicable
   N/C: Not Calculated
   N/N: Not Needed
```

Column element check.

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Eurocode 2-2004 BEAM SECTION DESIGN  Type: DC MEDIUM MRF  Units: Tonf, m, C  (Summary)

L=6.5
Element      : 487          D=0.55          B=0.6           bf=0.6
Section ID   : T60X55       ds=0.           dct=0.05        dcb=0.05
Combo ID     : ENVES        E=3100000.      fck=2500.       Lt.Wt. Fac.=1.
Station Loc  : 6.           fyk=50000.      fywk=40000.

Gamma(Concrete):  1.5
Gamma(Steel)   :  1.15


Design Moments, M3
                       Positive    Negative
                         Moment      Moment
                         18.392     -36.784

Flexural Reinforcement for Moment, M3
                       Required     +Moment     -Moment     Minimum
                          Rebar       Rebar       Rebar       Rebar
   Top    (+2 Axis)       0.002          0.       0.002   7.745E-04
   Bottom (-2 Axis)   8.797E-04   8.797E-04          0.   7.745E-04

Shear Reinforcement for Shear, V2
                          Rebar       Shear       Shear       Shear  Tan(Theta)
                          Asw/s         VEd        VRdc        VRds       Ratio
                          0.002      34.699      14.789      34.699          1.

Torsion Reinforcement for Torsion, T
              Rebar       Rebar     Torsion    Critical        Area   Perimeter
               At/s         Asl         TEd           T          Ak          uk
                 0.   4.006E-06       0.037       0.758       0.186       1.726
```

Beam element check

6.3 ULS – Punching shear check

The punching shear resistance of a slab should be assessed for the basic control section according to 6.4.2. prEN 1992-1-1:2003 (E)
No shear reinforcement is required

6.4 Concrete slabs

Groundfloor level ±0.00 (t=25cm; t=30cm)

6.4.1 ULS – Flexural checks
Design moments of the slab: Base reinforcement: 8Ø16/ml bottom 6Ø16/ml top
M_{Rd1} = 15 Tm (bottom); M_{Rd1} = 12Tm (top)
Additional reinforcement (top): 6Ø16/ml top
M_{Rd2} = 22 Tm (top)

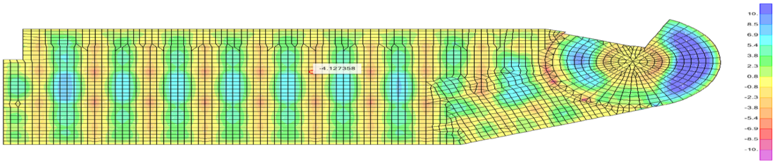

Figure 9. Bending moments in direction XX with base reinforcement

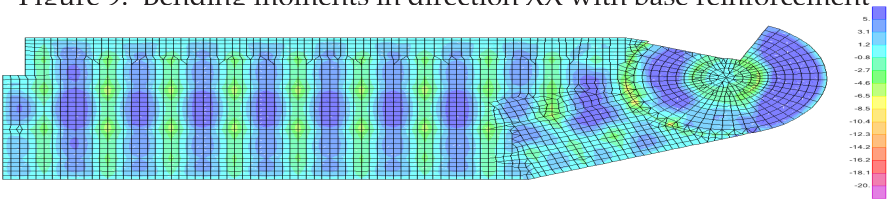

Figure 10. Bending moments in direction XX with additional reinforcement

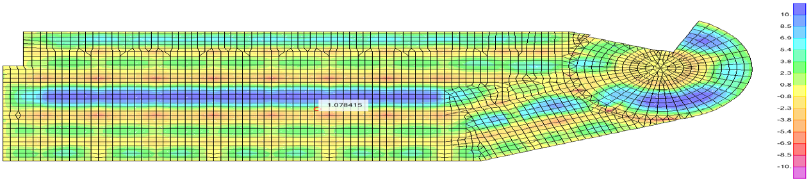

Figure 11. Bending moments in direction YY with base reinforcement

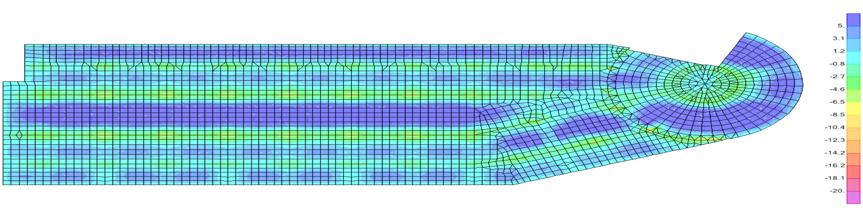

Figure 12. Bending moments in direction YY with additional reinforcement

6.4.2 SLS – Deflection

The maximum elastic deflection is u_{zel}=18.91mm for span length L=800cm that corresponds L/423; verified.

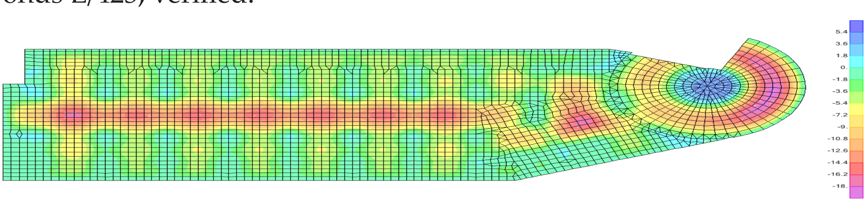

Figure 13. Elastic deflections u_{zel} (mm)

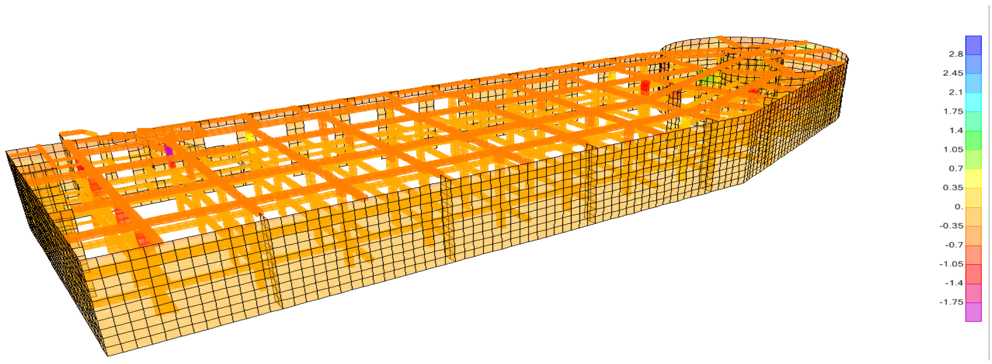

Figure 14. Horizontal displacements XX under seismic action (mm)

Height h = 11.5 m
Displacements/height ratio: ΔXmax /h =0.38/11500 =1/30000 height. The check is satisfied.
The maximum horizontal displacement under seismic action in YY direction is ΔYmax=8.6mm:

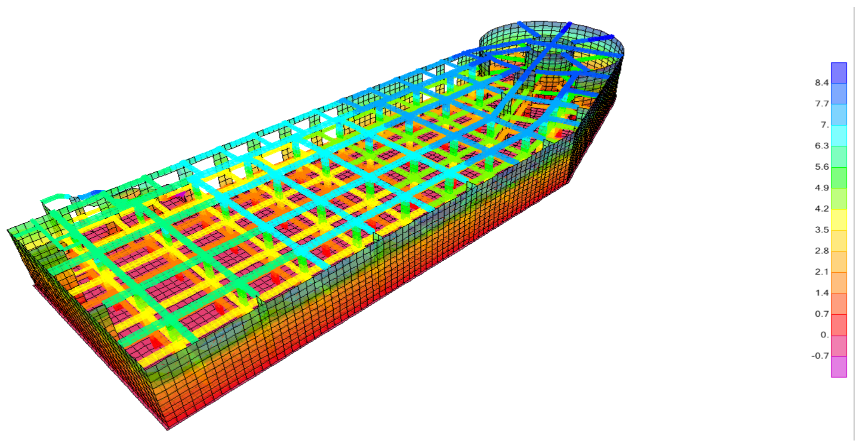

Figure 15. Horizontal displacements YY under seismic action (mm)
Displacements/height ratio: ΔYmax /h =8.6/11500 =1/1330 height. The check is satisfied.

6.4.3 Interstorey drifts

Interstorey Drifts													
Level	Storey height; **h**	Elastic displacements; **d_e(cm)**		Importance factor; γ_I	Behaviour factor; **q**	Absolute displacements; **d_s(cm)**		Relative displacements; **d_r(cm)**		υ-Reduction factor	Interstorey Drifts **d_rυ** (% **h**)	
(m)	(m)	X	Y			X	Y	X	Y		X	Y
-11.55		0.00	0.00	1.2	1.65	0.00	0.00					
-8.4	3.15	0.01	0.26	1.2	1.65	0.03	0.51	0.03	0.51	0.5	0.004	0.082
-5.25	3.15	0.03	0.49	1.2	1.65	0.05	0.97	0.03	0.46	0.5	0.004	0.072
±0.00	5.25	0.04	0.86	1.2	1.65	0.08	1.70	0.02	0.73	0.5	0.002	0.070

Table 2 Interstorey Drifts

The relative displacement is satisfied floor by floor.

7. Conclusion

This article is a guide for the design of a reinforced concrete structure for parking purposes, starting from load calculation, modeling, seismic analysis, and checks according to European standards. Manually solving the slab is an accurate method, but it takes a lot of time. Verification of the results obtained from computer solutions based on the finite element method must be done manually. The finite element method on which the computational programs are based gives very satisfactory results, facilitates calculations, is applicable to any type of slab shape, takes into account the effect of lateral forces on the model, but there is always a need for verification of the results. The latest versions of computational programs based on the Finite Element Method have not only increased the accuracy of the results, but also provide the possibility of modeling the structure together with the slabs. This method allows the modeling to be more complete, to obtain more results in the areas of joining vertical and horizontal elements, and the seismic analysis is more comprehensive. These software programs require higher computer capacities to have benefits during the analysis of the program. In order to obtain accurate results sudden changes in the shell element size must be avoided. The refinement mesh should be applied to structural parts where rapid change of internal forces and material properties is expected. Mesh refinement is a very important task of the pre-processing. It can be a very time-consuming task but an experienced engineer will produce a more credible mesh for a complex problem. The structural model has to be meshed properly into elements of specific shapes such as quadrilaterals, triangles etc., using as many advantages of automated mesh generators as possible.

Since there is no a priori method of an efficient finite element model that provides a certain degree of accuracy, numerical tests are necessary.

References

Akademia e Shkencave, Qendra Sizmologjike. (1989). *Kusht teknik projektimi për ndërtimet antisizmike KTP-N.2-89.* Tiranë, Shqipëri

Bowles, J. E. (1996). *Foundation analysis and design* (5th ed.). McGraw-Hill.

Das, B. M., & Sobhan, K. (2018). *Principles of foundation engineering* (9th ed.)

European Committee for Standardization. (2002). *EN 1991-1-1: Eurocode 1 — Actions on structures — Part 1-1: General actions — Densities, self-weight, imposed loads for buildings.* CEN. https://eurocodes.jrc.ec.europa.eu/

European Committee for Standardization. (2002). *EN 1991-1-2: Eurocode 1 — Actions on structures — Part 1-2: General actions — Actions on structures exposed to fire.* CEN. https://eurocodes.jrc.ec.europa.eu/

European Committee for Standardization. (2003). *EN 1991-1-3: Eurocode 1 — Actions on structures — Part 1-3: General actions — Snow loads.* CEN. https://eurocodes.jrc.ec.europa.eu/

European Committee for Standardization. (2005). *EN 1991-1-4: Eurocode 1 — Actions on structures — Part 1-4: General actions — Wind actions.* CEN. https://eurocodes.jrc.ec.europa.eu/

European Committee for Standardization. (2004). *EN 1992-1-1: Eurocode 2 — Design of concrete structures — Part 1-1: General rules and rules for buildings.* CEN. https://eurocodes.jrc.ec.europa.eu/

European Committee for Standardization. (2004). EN 1998-1: Eurocode 8 — Design of structures for earthquake resistance — Part 1: General rules, seismic actions and rules for buildings. CEN. https://eurocodes.jrc.ec.europa.eu/

European Committee for Standardization. (2004). *EN 1997-1: Eurocode 7 — Geotechnical design — Part 1: General rules.* CEN. https://eurocodes.jrc.ec.europa.eu/

Lee, C. K. and Hobbs, R. E., *"Automatic Adaptive Refinement for Plate Bending Problems Using Reissner-Mindlin Plate Bending Elements,"* International Journal for in Numerical Methods in Engineering, Vol. 41, p. 1-63, 1998

MacGregor, J. G. (2002). *Reinforced concrete: Mechanics and design* (3rd ed.). Prentice Hall.

Park, R., & Paulay, T. (1975). *Reinforced concrete structures*. John Wiley & Sons.

Penelis, G. G., & Kappos, A. J. (2000). *Earthquake-resistant concrete structures.* E & FN Spon.