

An Immense Approach of High Order Fuzzy Time Series Forecasting of Household Consumption Expenditures with High Precision

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Abstract – Fuzzy Time Series (Fts) models are experiencing an increase in popularity due to their effectiveness in forecasting and modelling diverse and intricate time series data sets. Essentially these models use membership functions and fuzzy logic relation functions to produce predicted outputs through a defuzzification process. In this study, we suggested using a Second Order Type-1 fts (S-O T-1 F-T-S) forecasting model for the analysis of time series data sets. The suggested method was compared to the state-of-the-art First Order Type 1 Fts method. The suggested approach demonstrated superior performance compared to the First Order Type 1 Fts method when applied to household consumption data from the Magene Regency in Indonesia, as measured by absolute percentage error rate (APER).

Keywords – APER, fuzzy numbers, fuzzy relationship, fuzzy sets, fuzzy time series, second-order fuzzy sets, soft computing.

I. INTRODUCTION

The significance of modelling and forecasting cannot be overstated when it comes to making timely decisions. Forecasting time series data are very important in every field of life such as weather, stock exchange prices, sales of any product, household consumption, and in medical science to predict future values. Over the past few decades, a multitude of studies have emerged employing various methodologies to address time series forecasting [1]. Parametric and non-parametric time series models are two distinct categories used to classify models for time series forecasting. Parametric models, including ARMA, ARIMA, and SARIMA models, require adherence to fundamental assumptions such as linearity and stationary processes. These models have been commonly employed for both modelling and forecasting purposes [2], [3] with some new techniques of wavelets. In contrast, non-parametric models do not rely on fundamental assumptions like stationary or others [1]. Fts models have gained significant traction in recent decades among non-parametric models, primarily due to their reduced computational requirements and

high accuracy in modelling and forecasting outcomes. In contrast to parametric models, Fts can also be applied over short-time series data sets. These models have proven to be a powerful tool for foretelling in various fields because of their capability to handle uncertainty and vagueness. In [4], a Fts new model is presented for forecasting exchange prices. The model uses rough computing with binning-based subdividing and entropy-based discretization methods. Experimental results using different stock indices show improved prediction accuracy compared to existing Fts, SVM, and economic models. The suggested model demonstrates optimized linguistic intervals and overall accuracy, validated through paired two-tailed t-tests.

In [5], authors offer a tutorial on FTS methods, covering a literature review, addressing challenges, and presenting the authors' research timeline. The study also introduces to the pyFts library, an open-source Python library developed by the MINDS Lab, which provides pre-processing functions, benchmarking metrics, databases, and implementations of various FTS models from the literature. The study [6] presents a hybrid model that combines Fts forecasting with data reprocessing techniques to forecast major air pollutants. The model includes an uncertainty analysis that examines uncertainties in future air quality predictions. The evaluation module verifies the performance of the developed model, which outperforms benchmark models and baselines in terms of accuracy and stability. The study highlights fuzzy logic as an excellent option for air quality prediction and concludes that the developed system is a valuable tool for pollution analysis and monitoring.

It is suggested to apply a higher-order Fts model using household consumption data from the Magene Regency in Indonesia [7]. The high-order Fts model combines the principles of fuzzy logic and higher-order modelling to provide a more accurate and flexible approach to time series prediction.

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It has been useful in various fields such as finance, economics, and weather forecasting, where capturing nonlinear and higher-order relationships is essential for accurate predictions. The flow diagram of the suggested model is shown in Fig. 3. The suggested model outperforms and finds an AFER of 0.035 % compared to the baseline method [8].

This paper is structured as follows: Section II deals with the literature review on Fts foretelling. Section III presents the highly accurate Fts model procedure. In Section IV, we use the suggested S-O T-1 F-T-S model and present an assessment of the highly accurate Fts method with an earlier model [8] for forecasting using precision measures through AFER. The findings are presented in Section V.

II. LITERATURE REVIEW

The time series data (TSD) are represented using linguistic terms and fuzzy sets, allowing for the handling of uncertainty and imprecision in the data. The model uses a fuzzy inference system to portray the relationships between past and future values of a time series.

This research [9] first suggested the concept of Fts using the fuzzy set theory through fuzzy logic [10]. They developed an Fts model by fuzzifying data enrolled at a University in Alabama. Time variant first-order Fts models were developed and applied for enrolment data [11]. Neural Network provides higher accuracy among the three different defuzzification methods. In [12], the researcher used university data from [9] and applied abridged arithmetic procedures instead of the difficult max-min addition actions. This approach proves to be more effective than the one prescribed in [9]. In this study [13], researchers applied a new multivariate Fts model used for predicting the causal relationships in traffic accidents in Belgium. Compared to other studies, the work shows improved results. In [14], the effective interval lengths were designed using a mean-based length, taking half the mean of the first difference of the TSD. The analysis showed better-predicting results compared to previous Fts methods suggested by various researchers. In [15], the researchers first used an Adaptive Neuro-Fuzzy Inference System (ANFIS) and then an Artificial Neural Network (ANN) methodology to model the price of gold. A Conventional ARIMA time series model was also applied. Compared to these models, the ANFIS model outperformed the other models. In [16], the analysis of the fuzzy process of fuzzy expert systems used in various fields and for the creation of fuzzy expert systems recommended certain rules because the success of the fuzzy system depended on the selection of appropriate membership functions.

In [17], researchers presented an operative method of non-stationary time foretelling method using the intuitionistic Fts clustering technique. The four data sets used for testing and comparison showed that non-stationary time series outperformed other algorithms effectively. In [18], the authors applied a predictive Fts model (FTSOAX) imitative from the new seagull optimization algorithm (ISOA) and XGBoost. The ISOA algorithm was used to create more efficient intervals in the discourse domain. XGBoost was used to achieve accuracy. These changes found greater accuracy in the estimation results

of daily definite cases of Covid-19. In [19], to forecast dependence on oil imports, a hybrid model was applied, which was based on a Fts and a multi-objective optimization algorithm. Two main issues were discussed: stability and accuracy of prediction. In [20], TAIEX and PM10 data sets were used for forecasting using a new hybrid model. Integration of Fts into C-Means and Markov chain clustering techniques with an optimum number of groups increased the accuracy of outcome predictions. In [21], the researchers presented an Integer GARCH-based Fuzzy C-medoids process for bunching the count time series created on Mahalanobis distance by grouping soccer teams based on the number of goals scored. The study [22] presented three DPFTS (Dynamic Panel Fts) model approaches (DPFTS1, DPFTS2, and DPFTS3) that were used to forecast these two data sets. First, the models were applied to artificially generated data sets and, second, six real-TSD sets were used for comparison with traditional equivalent models (TFTS1, TFTS2, and TFTS3). In [23], the authors used a general procedure of fuzzy and intuitionistic fuzzy sets to design an image Fts model with an application to a set of meteorological data. In the study [24], researchers suggested an Fts model using a Particle Swarm Optimization (PSO) algorithm to forecast COVID-19. Compared with different time series models, the model provided better accuracy results. Time series is a powerful forecasting tool that helps in the forecasting of complex data. In this connection, this paper proposes using the higher-order time fuzzy time series method for a household consumption expenditure data set.

III. TOOLS AND SUGGESTED METHODOLOGY

In this study, a high-order Fts model was developed and used to analyse and forecast a data set of Indonesia [7]. The high-order Fts model is a sophisticated approach that uses the power of fuzzy logic to handle qualms and fuzziness in TSD. It extends the traditional Fts model by incorporating higher-order relationships and dependencies between variables, leading to more accurate and robust predictions.

A. Data Set

The data set used in this study is related to the percentage contribution of household consumption expenditure to the GRDP of the Majene district, Indonesia from 2010 to 2019 [7].

B. High-order Fts

A time series can be characterised as a group of data organised into equal intervals over a certain time period [2]. Let $Y(t)$ be a subset of real numbers ($t = \dots, 0, 1, 2, \dots$). The set can be thought of as the realm of discourse that allows for the definition of fuzzy sets $h_i(t)$. This sequence of values is known as Fts with a crisp output Y , which is represented in Fig. 1, if $h(t)$ is a group of $h_1(t), h_2(t), \dots$. Figure 1 depicts a system, based on fuzzy inference, on which Fts have been created [25].

The Fuzzy Logical Relationship (FLR) plays a significant part in the Fts. A first order fuzzy logical connection, such as $H(t-1) = B_i$ and $H(t) = B_j$, can be used to describe an FLR. B_i and B_j , where B_i is considered the left-hand side (LHS) and B_j is considered the right-hand side (RHS) of the FLR, can be

presented as the relationship between two consecutive observations, $H(t)$ and $H(t-1)$, which is known as FLR [9]. The second order FLR can be described as three consecutive observations, $H(t-2) = B_j$, $H(t-1) = B_k$, and $H(t) = B_l$ and can be indicated as $\tilde{B}j, \tilde{B}k \rightarrow \tilde{B}l$ [26].

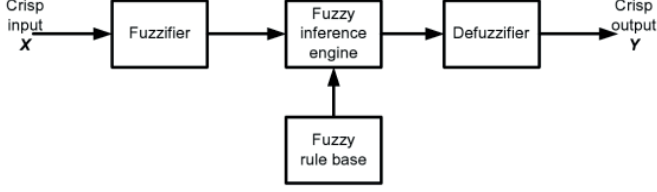


Fig. 1. Fuzzy inference system.

If $H(t)$ is an FTS, then the high order N th fuzzy logical connection can be described as such. If $H(t)$ is represented in $H(t-1)$, $H(t-2)$, ..., $H(t-n-1)$ and $H(t-n)$, with each observation marked by $H(t-n) = \tilde{B}n$, $H(t-n-1) = \tilde{B}n-1$, ..., $H(t-2) = \tilde{B}2$, $H(t-1) = \tilde{B}1 \rightarrow H(t) = \tilde{B}l$ then this FLR can be presented in $\tilde{B}n, \tilde{B}n-1, \dots, \tilde{B}2, \tilde{B}1 \rightarrow \tilde{B}l$.

In this FLR, the LHS sets are $\tilde{B}n, \tilde{B}n-1, \dots, \tilde{B}2, \tilde{B}1$ and the RHS set is denoted by $\tilde{B}l$. This idea of FLR is known as the “ N th order” FLR. Trapezoidal membership function (TMF) is applied in this suggested Second-Order Type-1 Fts (S-O T-1 F-T-S) model [27]. It presents forecasted output values in fuzzy numbers (FN). We utilised [26] methods as a basis to develop it. The basic steps are listed below:

1. Determine the number of equally long intervals using the improved concept which is suggested by [14].
2. Fuzzy sets are based over Trapezoidal Fuzzy Numbers (TFN) in Fts.
3. Create the Second Order FLR.

The method to find the average based interval length, which is defined in [26] is substituted with the [14] concept. Then fuzzy numbers are utilised in place of discrete fuzzy sets. The third big difference is the use of high-order FLR. The following stages describe the model:

Stage 1: Collect TSD ($Y(t)$).

Stage 2: Define the universe of discourse as U .

$$U = [D_{\min} - D_1, D_{\max} + D_2], \quad (1)$$

where

$D_{\min}$ – minimum observation of $Y(t)$;

$D_{\max}$ – maximum observation of $Y(t)$;

D_1 and D_2 – proper observations to define the “ U ”.

Stage 3: The interval length “ l ” determined using “average based length” discussed in [14].

- (i) Find the first absolute difference (FAD) of time series data and compute the average.
- (ii) Use length as the half of the average.
- (iii) Match the length range using Table I by [14].
- (iv) Use round off value for the length of interval “ l ” as match in step (iii).

TABLE I
AVERAGE BASED LENGTH METHOD

Range	Base
0.1 - 1.0	0.1
1.1 - 10	1
11 - 100	10
101 - 1000	100
1001 - 10000	1000

Stage 4: The count of intervals “ m ” can be determined using the suitable “ l ” from (2).

$$m = (D_{\max} + D_2 - D_{\min} + D_1) / l \quad (2)$$

Notations $u_1, u_2, \dots, u_m$ are denoted as “ m ” intervals. These are defined first as $u_1 = [d_1, d_2]$, second as $u_2 = [d_2, d_3]$, so on as $u_{m-2} = [d_{m-2}, d_{m-1}]$, $u_{m-1} = [d_{m-1}, d_m]$, and $u_m = [d_m, d_{m+1}]$. Hence, the discrete fuzzy sets $\tilde{B}_1, \tilde{B}_2, \dots, \tilde{B}_m$ are presented in (3).

$$\begin{aligned} B_1 &= \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \dots + \frac{0}{u_{m-1}} + \frac{0}{u_m} \\ B_2 &= \frac{0.5}{u_1} + \frac{1}{u_2} + \frac{0.5}{u_3} + \dots + \frac{0}{u_{m-1}} + \frac{0}{u_m} \\ &\dots \end{aligned} \quad (3)$$

$$B_m = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \dots + \frac{0.5}{u_{m-1}} + \frac{1}{u_m}$$

Note: u_i shows the intervals midpoints.

Stage 5: Establish the fuzzy numbers.

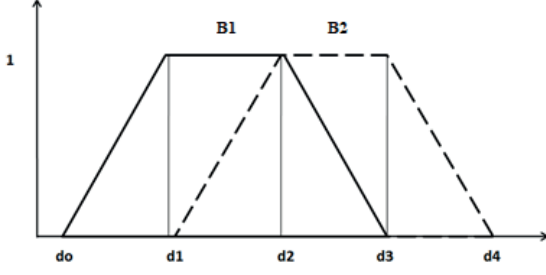
Trapezoidal Fuzzy Numbers (TFN) are substituted by the discrete fuzzy sets, which are determined through the Matlab software (R12a) [27]. A fuzzy membership function with parameters (a, b, c, d) is presented by (4).

$$\mu B(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{d-x}{d-c}, & c \leq x \leq d \\ 0, & x > d \end{cases} \quad (4)$$

Parameters of the Trapezoidal Membership function are “ a ”, “ b ”, “ c ”, and “ d ”. The fuzzy numbers $B_1, B_2, \dots, B_m$ are defined in (5).

$$\begin{aligned} B_1 &= (d_0, d_1, d_2, d_3) \\ B_2 &= (d_1, d_2, d_3, d_4) \\ &\dots \\ B_m &= (d_{m-2}, d_{m-1}, d_m, d_{m+1}) \end{aligned} \quad (5)$$

Fuzzy numbers B_1 and B_2 use the TMF as shown in Fig. 2.

Fig. 2. Fuzzy numbers B_1 and B_2 using the TMF.**Stage 6: Fuzzify the TSD data $Y(t)$.**

Historical Time Series Data (TSD) can be fuzzified by supposing that the value of time series $Y(t)$ exists in the range of interval u_j , then it has its place in the FN $\tilde{B}_j$. Use this condition for placing all the observations of TSD.

Stage 7: FLR of Type-1 in Fts.

A second-order FLR was generated using the following relationship as discussed in Section III, Part b.

$$\tilde{B}_i, \tilde{B}_j \rightarrow \tilde{B}_k.$$

Establish the FLR for all the fuzzified data.

Stage 8: Groups of FLR of Type-1 Fts.

Groups of FLR can be formed when the FN on the LHS in the FLR are the same as shown in Table II.

TABLE II
FUZZY LOGICAL RELATIONSHIP GROUPS

Second Order
$\tilde{B}_i, \tilde{B}_j \rightarrow \tilde{B}_{k1}$
$\tilde{B}_i, \tilde{B}_j \rightarrow \tilde{B}_{k2}$
...
$\tilde{B}_i, \tilde{B}_j \rightarrow \tilde{B}_{kp}$

Stage 9: Process of defuzzification and predicted output.

The result of forecasting can be determined using Table III.

TABLE III
FORECASTED OUTPUT

Second Order	
Condition 1:	$\tilde{B}_i, \tilde{B}_j \rightarrow \tilde{B}_k$
	$\tilde{B}_i, \tilde{B}_j \rightarrow \tilde{B}_{k1}$
	$\tilde{B}_i, \tilde{B}_j \rightarrow \tilde{B}_{k2}$
	...
Condition 2:	$\tilde{B}_i, \tilde{B}_j \rightarrow \tilde{B}_{kp}$
	$\tilde{B}_i, \tilde{B}_j = \frac{\tilde{B}_{k1} + \tilde{B}_{k2} + \dots + \tilde{B}_{kp}}{p}$
Condition 3:	$\tilde{B}_i, \tilde{B}_j \rightarrow \emptyset$

In Condition 1, FN on the RHS is the predicted result. When the FN on the left side are the same, it is necessary to take the average from the right side of the FN as Condition 2. In Condition 3, when there is an unoccupied FN on the right side, then the forecast result of the second order output is $\tilde{B}_j$. Centroid formula to defuzzify these numbers is presented in (6).

$$W = \frac{\sum_{i=1}^n \mu B(x_i) x_i}{\mu B(x_i)} \quad (6)$$

The defuzzify value is denoted by "W". MATLAB is used to defuzzify the predicted FN. For accuracy measure of the forecasting output for the suggested second order type 1 fuzzy time series (S-O T-1 F-T-S) approach, we used the average forecasting error rate "AFER" as a performance evaluation.

$$AFER = \frac{\sum_{i=1}^n \left| \frac{(Actual\ Value - Forecasted\ Value)}{Actual\ Value} \right|}{n} \quad (7)$$

Algorithm of the Second Order Type I Fts (S-O T-1 F-T-S) Model is presented in Fig. 3.

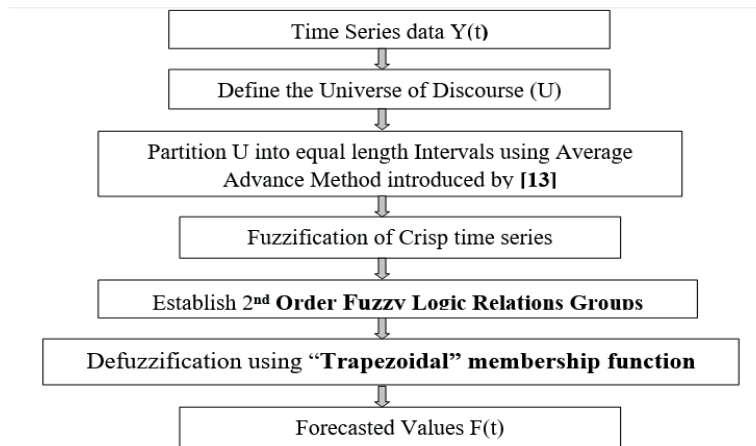


Fig. 3. (S-O T-1 F-T-S) Model Algorithm of Fuzzy Time Series

IV. IMPLEMENTATION AND RESULTS

The application of S-O T-1 F-T-S model is applied to the percentage of household expenditure consumption on the gross domestic product GRDP of Majene Regency Indonesia [7].

Stage 1: The Actual TSD of household consumption is presented in Table IV.

TABLE IV
ACTUAL DATA

Year	Household Consumption
2010	66.58
2011	65.83
2012	64.25
2013	64.66
2014	62.62
2015	64.21
2016	64.61
2017	62.91
2018	61.64
2019	60.34

Stage 2: Compute “ U ” using (1), where

$$D_{\min} = 60.34;$$

$$D_{\max} = 66.58;$$

$$D_1 \text{ \& } D_2 = 0.34 \text{ \& } 0.42. \quad U = [60], [67].$$

Stage 3: Length of interval.

(i) The mean of the FAD between $Y(t-i)$ and $Y(t)$ is 1.2267.

(ii) The half of the average of the FAD is 0.6133.

(iii) The table base value is 0.1 because 0.6133 is in the interval 0.1 to 1.0 [14].

(iv) Using (iii), the interval length “ l ” is 0.1.

Stage 4: The “ m ” segments are computed through (2):

$$m = (D_{\max} + D_2 - D_{\min} + D_1) / l \\ = (67 - 60) / 0.1 = 70.$$

There are 70 intervals constructed based on stage 4, i.e., they are obtained by $u_1, u_2, \dots, u_{70}$. Segments are classified as $u_1 = [60 \ 60.1]$, $u_2 = [60.1 \ 60.2]$, ..., $u_{70} = [66.9 \ 67]$. Fuzzy sets of these number of intervals are presented through B_1, B_2 to B_{70} using (3).

$$B_1 = \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \dots + \frac{0}{u_{69-1}} + \frac{0}{u_{70}}$$

$$B_2 = \frac{0.5}{u_1} + \frac{1}{u_2} + \frac{0.5}{u_3} + \dots + \frac{0}{u_{69-1}} + \frac{0}{u_{70}}$$

...

$$B_{70} = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \dots + \frac{0.5}{u_{69-1}} + \frac{1}{u_{70}}$$

FN is applied using TMF instead of fuzzy sets [14] and [28] strategies.

Stage 5: Establish the fuzzy numbers:

The TMF is used to establish the FN presented as (5).

$$B_1 = (59.9, 60.0, 60.1, 60.2)$$

$$B_2 = (60.0, 60.1, 60.2, 60.3)$$

...

$$B_{70} = (69.8, 69.9, 67.0, 67.1)$$

Stage 6: Fuzzify the TSD $Y(t)$:

Historical TSD can be fuzzified through this concept, when considering the year 2010 the consumption is 66.58 which is in the interval range $u_{66} = [66.5 \ 66.6]$. Hence, for the given year the corresponding fuzzy number is B_{66} . Likewise, we match the range of each data observation and present FN as provided in Table V.

TABLE V
FUZZY NUMBERS

Year	Household Consumption	Fuzzy Numbers
2010	66.58	B66
2011	65.83	B59
2012	64.25	B43
2013	64.66	B47
2014	62.62	B27
2015	64.21	B43
2016	64.61	B47
2017	62.91	B30
2018	61.64	B17
2019	60.34	B04

Figure 4 portrays the FN B_1 and B_2 using TMF.

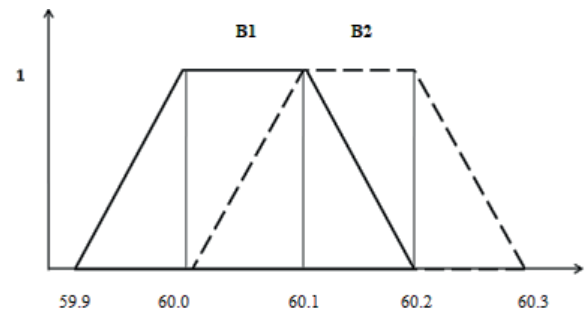


Fig. 4. FN B_1 and B_2 using TMF.

Stage 7: FLR of Type-1 in Fts:

The FLR of the second order is presented in Table VI.

TABLE VI
SECOND ORDER FUZZY LOGICAL RELATIONSHIP

$\tilde{B}_{66}, \tilde{B}_{59} \rightarrow \tilde{B}_{43}$	$\tilde{B}_{47}, \tilde{B}_{27} \rightarrow \tilde{B}_{43}$	$\tilde{B}_{47}, \tilde{B}_{30} \rightarrow \tilde{B}_{17}$
$\tilde{B}_{59}, \tilde{B}_{43} \rightarrow \tilde{B}_{47}$	$\tilde{B}_{27}, \tilde{B}_{43} \rightarrow \tilde{B}_{47}$	$\tilde{B}_{30}, \tilde{B}_{17} \rightarrow \tilde{B}_{24}$
$\tilde{B}_{43}, \tilde{B}_{47} \rightarrow \tilde{B}_{27}$	$\tilde{B}_{43}, \tilde{B}_{47} \rightarrow \tilde{B}_{30}$	

Stage 8: Groups of FLR of Type-1 Fts:

The groups of FLR were created using the alike FN on the left side of the FLR as defined in Stage 7. Second-order FLR groups are shown in Table VII.

TABLE VII

GROUPS

Groups	Second Order
1	$\tilde{B}_{66}, \tilde{B}_{59} \rightarrow \tilde{B}_{43}$
2	$\tilde{B}_{59}, \tilde{B}_{43} \rightarrow \tilde{B}_{47}$
3	$\tilde{B}_{43}, \tilde{B}_{47} \rightarrow \tilde{B}_{27}$
4	$\tilde{B}_{47}, \tilde{B}_{27} \rightarrow \tilde{B}_{43}$
5	$\tilde{B}_{27}, \tilde{B}_{43} \rightarrow \tilde{B}_{47}$
6	$\tilde{B}_{43}, \tilde{B}_{47} \rightarrow \tilde{B}_{30}$
7	$\tilde{B}_{47}, \tilde{B}_{30} \rightarrow \tilde{B}_{17}$
8	$\tilde{B}_{30}, \tilde{B}_{17} \rightarrow \tilde{B}_{24}$

Stage 9: Process of Defuzzification and Forecasted Output:

The outputs of forecasting using the (S-O T-1 F-T-S) model utilizing TMF are presented in Table VIII.

TABLE VIII

DEFUZZIFICATION AND FORECASTED OUTPUT

Year	Household Consumption	Fuzzy Numbers	Forecasted S-O T-1 F-T-S Trapmf	AFER
2010	66.58	B_{66}		
2011	65.83	B_{59}		
2012	64.25	B_{43}	64.25	0
2013	64.66	B_{47}	64.65	0.000155
2014	62.62	B_{27}	62.65	0.000479
2015	64.21	B_{43}	64.25	0.000623
2016	64.61	B_{47}	64.65	0.000619
2017	62.91	B_{30}	62.95	0.000653
2018	61.64	B_{17}	61.65	0.000162
2019	60.34	B_4	60.35	0.000166

It is necessary to defuzzify the fuzzified observations using software MATLAB through the centroid formula as in Stage 9. Performance comparison of our suggested model (S-O T-1 F-T-S) and the baseline approach [8] is evaluated by using the accuracy measurement called Average Forecasted Error Rate (AFER) as shown in Table IX.

TABLE IX

COMPARISON USING ACCURACY MEASUREMENTS

	S-O T-1 F-T-S Model	Abdy M. [7]
AFER	0.035 %	0.59 %

The comparison shows that S-O T-1 F-T-S model using the TMF outperformed the baseline method by getting higher accuracy in forecasted output (see Fig. 5).

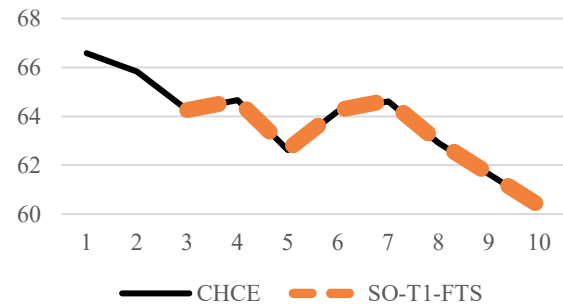


Fig. 5. Actual and forecasted household consumption using S-O T-1 F-T-S model.

V. CONCLUSION

This study identified which Fts model provided higher accurate forecasted results to the suggested Fts Model S-O T-1 F-T-S and baseline Fts model with an application of percentage of household consumption. The main change between the two models is the use of high order FLR. We use a second-order FLR that outperforms the other Fts model. This change increases the accuracy of results and opens future research options such as trying different membership functions. Data sets from econometrics, medical, and engineering fields will be used in future research. High order multivariate Fts modelling will also be a future interest of the researchers. Finally, we can conclude that the suggested Fts model S-O T-1 F-T-S outperforms the other model in foretelling the percentage of household consumption of GRDP of Magene Region in Indonesia [7]. This research cannot develop a generalized model for a defuzzification procedure based on a high-order fuzzy logical relationship. Future studies will find ways to predict a few steps onward values in fuzzy time series, which present studies lack.

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